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YÖS

MATEMATİK
MATHEMATICS

Konu Açıklamalı - Örnek Çözümlü

Detailed Subject Explanations
Detailed Solutions of Sample Questions

1080 çözümlü soru
questions with solutions

457 alıştırma sorusu
exercise questions

1040 cevaplı test sorusu
practice questions
with answers

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YAYINLARI

asb

Türkiye'de ve Dünyada!
In Turkey and Around the World!

YÖS

أفضل ما أنتجت متروبول
كتاب الرياضيات المدمج
الإصدار العاشر 2016
كتاب شامل يحوي الأبحاث
العشرة في الرياضيات مع شرح
و أمثلة محلولة و اختبارات

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PREFACE

Dear YÖS candidates;

METROPOL, *is a well-established educational institution which has provided students with a comprehensive training assistance on courses and publications for over* ***20*** *years. In parallel,* ***METROPOL*** *has been in the service of guidance for you.*

The Examination for Foreign Students, referred to as YÖS, has been designed for foreign students wishing to study in Higher Education Institutions in Turkey. Depending on the results of this examination, they can apply to Turkish Institutions of Higher Education. YÖS consists of two different parts as follows: ***Basic Learning Skills Test*** *and* ***Turkish Language Proficiency Test.*** *In order to study in the best universities in Turkey, first of all, you need to have received at least a* ***40*** *standard score and then, aim at receiving more than this score in the Basic Learning Skills Test. Basic Learning Skills Test the in total exam it self is composed of 80 questions considering* ***45 questions of the IQ (Intelligence Quotient)*** *and* ***35 questions related to Mathematics*** *and* ***Geometry*** *issues.*

This book has been prepared by the expert educator staff of ***Metropol Educational Institutions.*** *This book includes* ***1022*** *examples with solutions,* ***430*** *exercises, and* ***1038*** *test questions. Moreover, there are the solution of* ***YÖS*** *questions between the years* ***2008*** *and* ***2016****.*

We wish you success in exams to all students. ***Welcome to TURKEY.***

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ÜNİTE 1

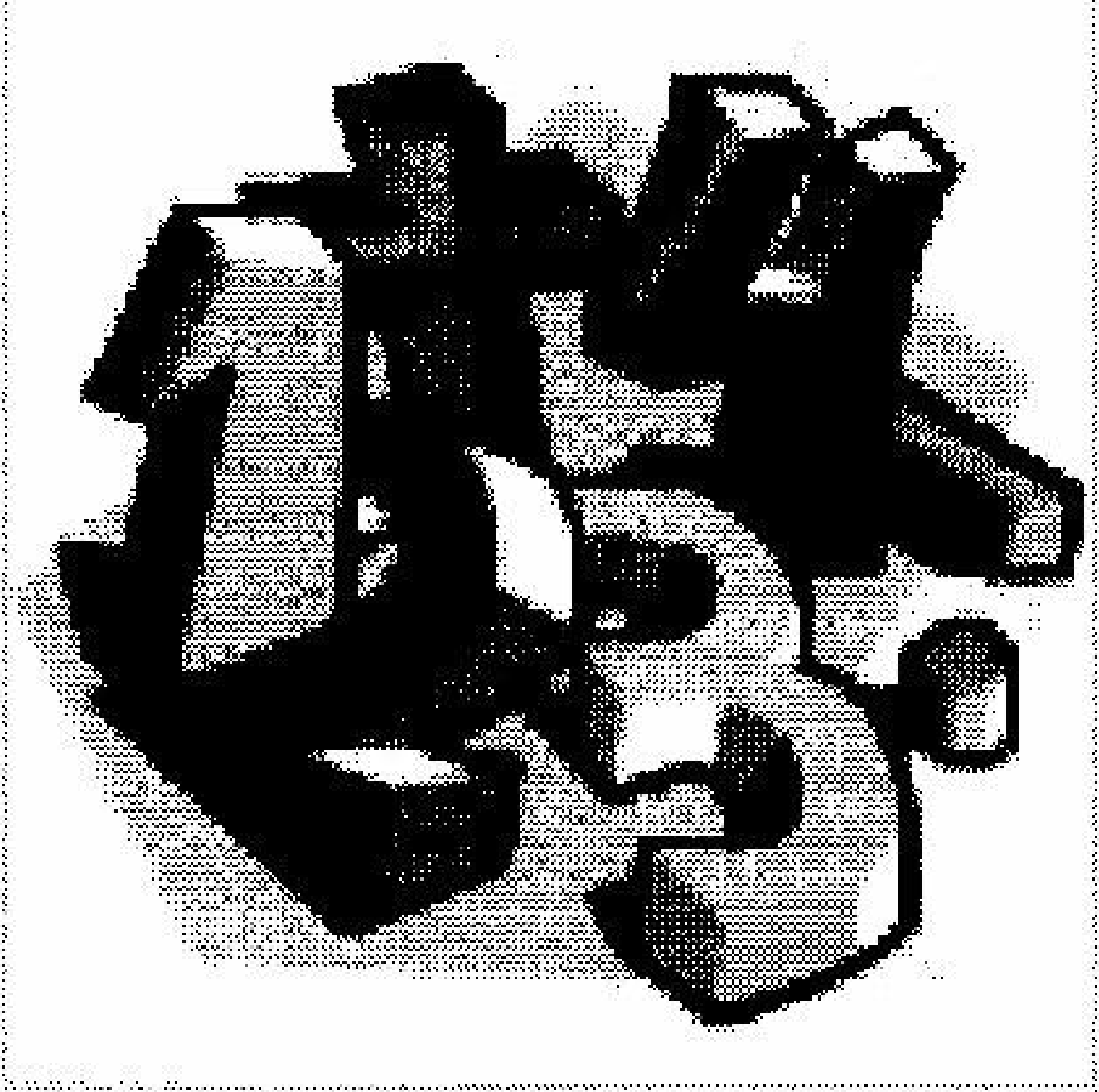
UNIT

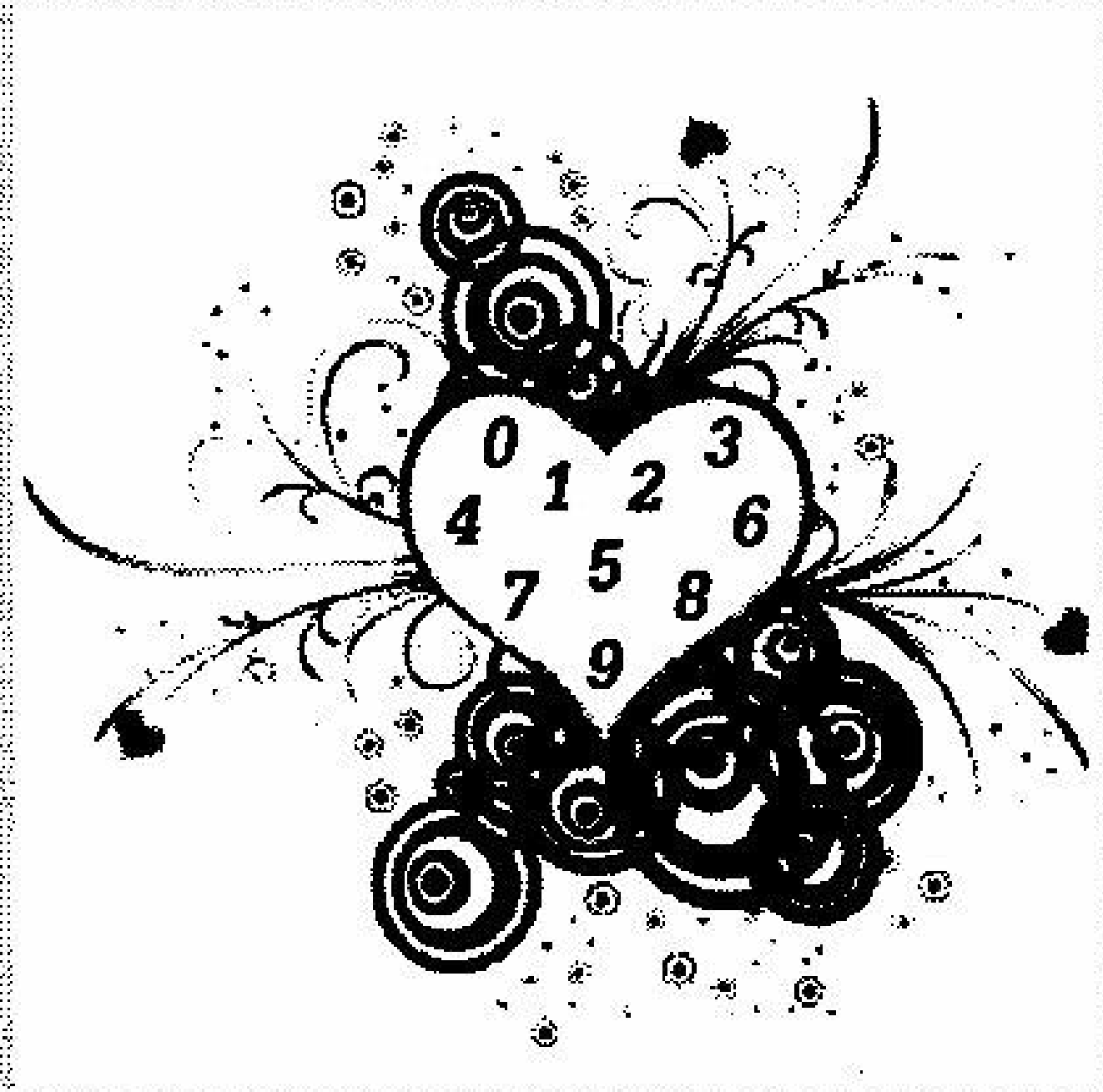
Sayılar, günlük yaşamımızda çok sık kullandığımız kavramlardandır. Matematiğin temel taşlarından olan bu kavram, insanların sayma gereksinimlerini karşılamak amacıyla geliştirilmiştir. Bu ünitede; sayılar, rasyonel ve ondalık sayılar kavramları ve bu kavramların özellikleri ayrıntılı bir şekilde anlatılmıştır. Öğrencilerin bu konuyu iyi öğrenmiş olmaları, bundan sonraki konulara temel oluşturacağından önemlidir.

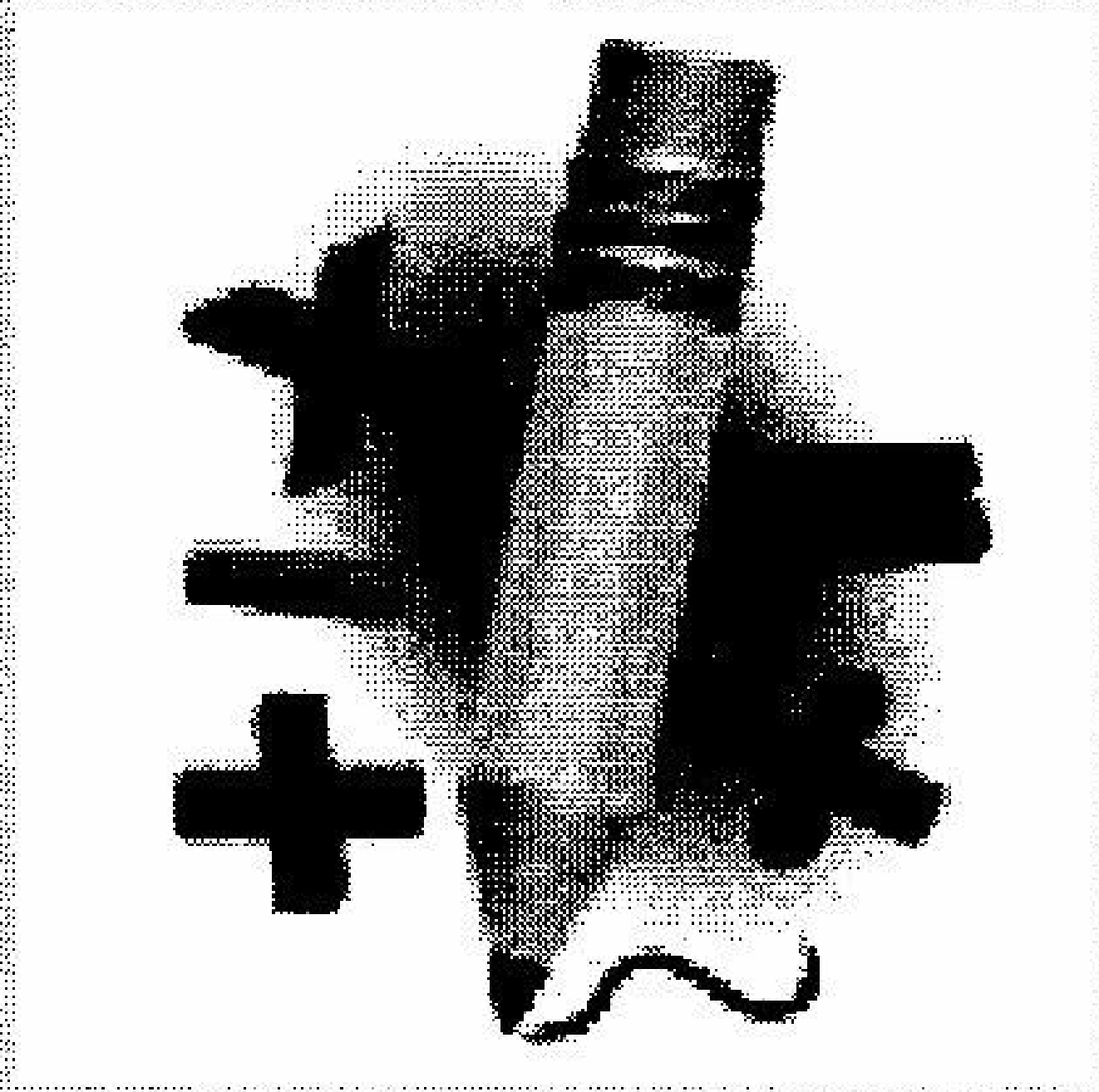
Bu ünite, 64 çözümlü örnek, 36 alıştırma, 120 test sorusu içermektedir.

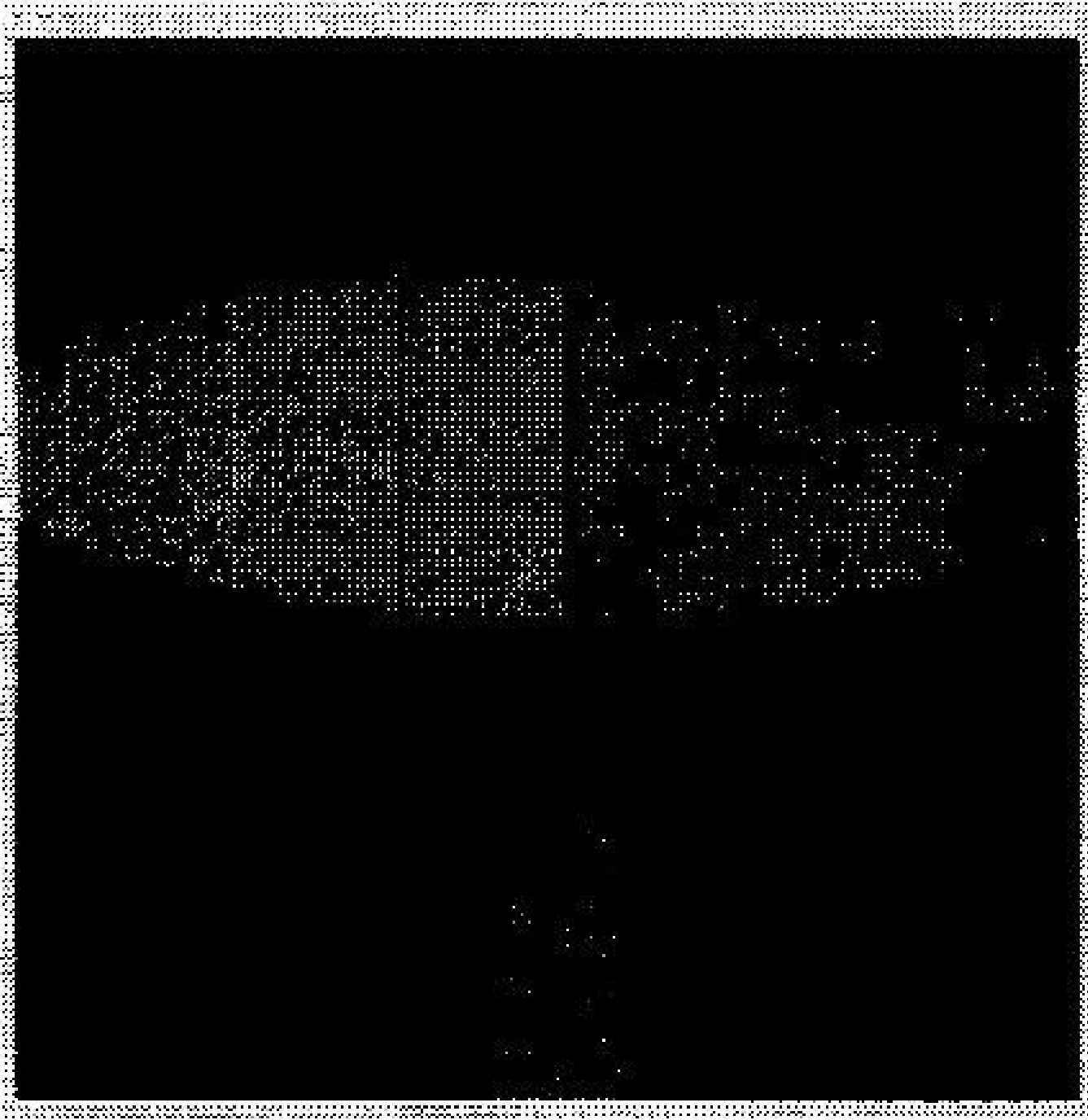
We are using the number concept in real-life. This concept is product for the counting necessarily that fundamental stone of mathematics. In this unit, numbers, rational and decimal numbers are explained in detailed. Since these concepts are essential of mathematics, students must be very good learning.

This unit includes; 64 examples, 36 exercises, and 120 test questions.



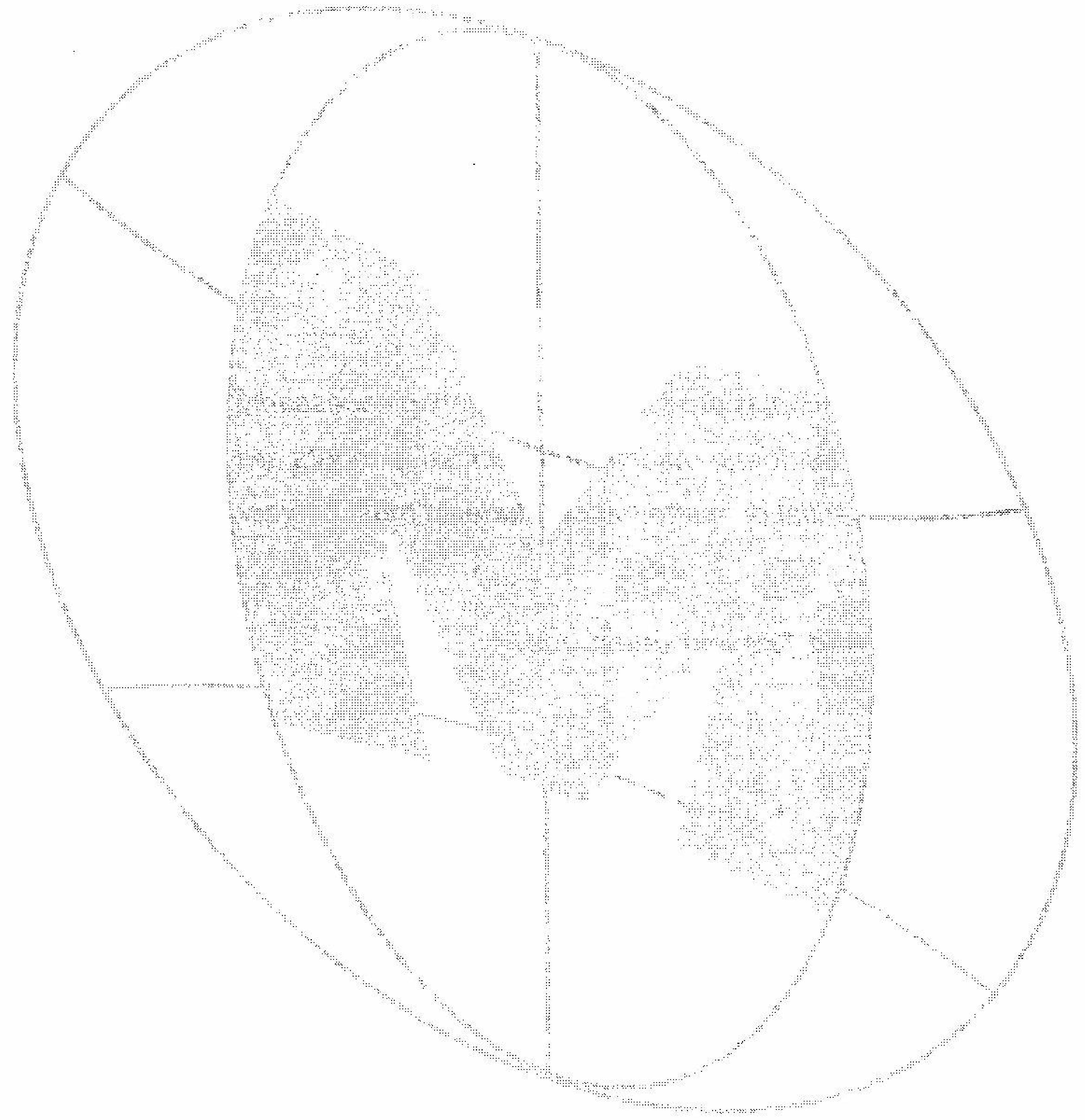






◆ SAYILAR / *NUMBERS* .... 3 – 38

◆ RASYONEL ve ONDALIK SAYILAR
RATIONAL and DECIMAL NUMBERS .... 39 – 68



SAYILAR
NUMBERS

BÖLÜM
CHAPTER
1

TEMEL KAVRAMLAR

Rakam: Sayıları yazmak için kullanılan sembollerden her birine *rakam* denir.

Rakamlar kümesi; { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Örneğin; 716 sayısının rakamları 7, 1 ve 6 dır.

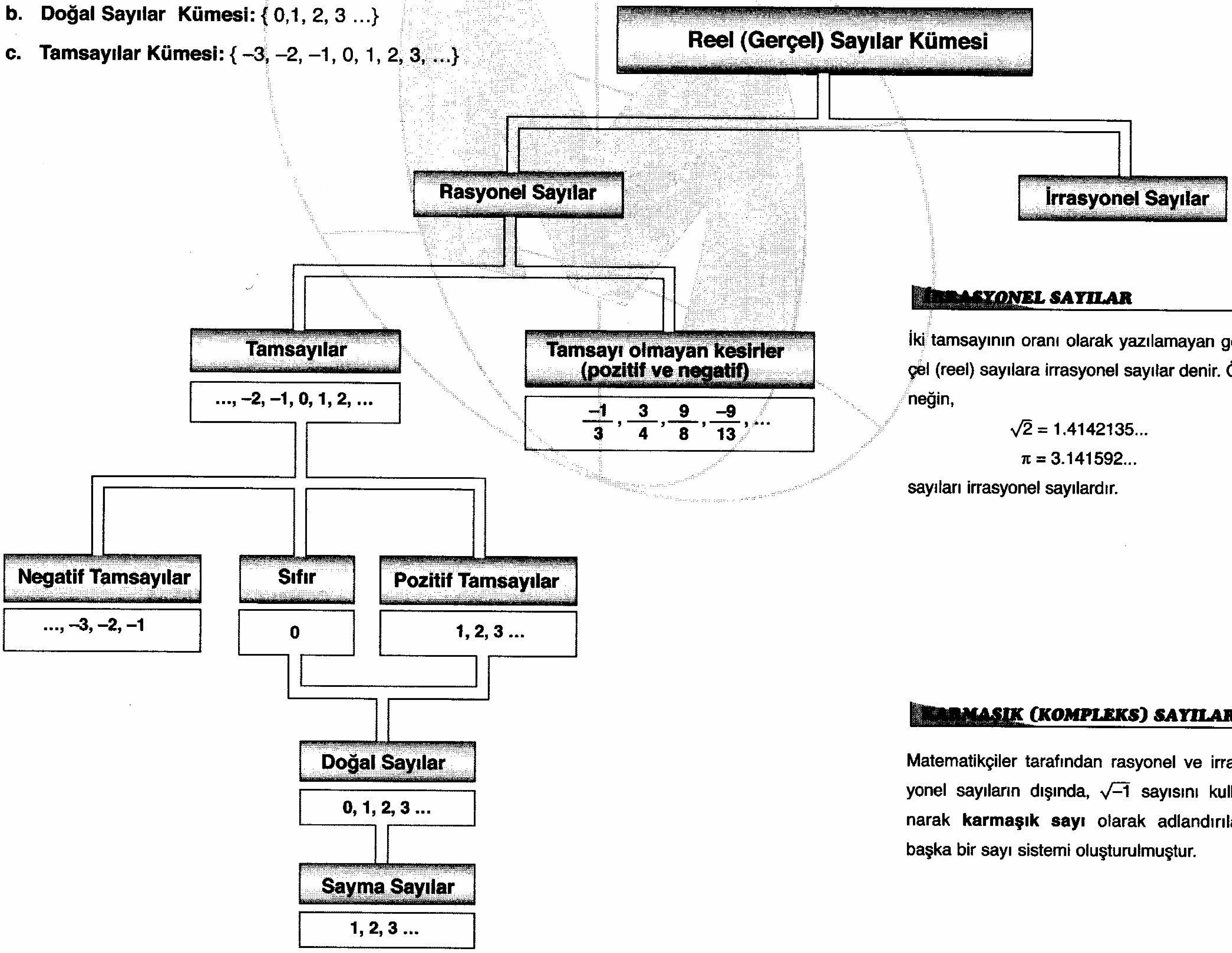
SAYI KÜMELERİ

Sayı: Rakamların tek başlarına belirttiği veya birden fazla rakamın bir araya getirilmesi ile oluşturulan çokluğa **sayı** denir.

a. **Sayma Sayılar Kümesi:** {1, 2, 3 ...}

b. **Doğal Sayılar Kümesi:** { 0,1, 2, 3 ...}

c. **Tamsayılar Kümesi:** { –3, –2, –1, 0, 1, 2, 3, ...}


Reel (Gerçel) Sayılar Kümesi
Rasyonel Sayılar
İrrasyonel Sayılar
Tamsayılar
..., –2, –1, 0, 1, 2, ...
Tamsayı olmayan kesirler (pozitif ve negatif)
$\frac{-1}{3}, \frac{3}{4}, \frac{9}{8}, \frac{-9}{13}, \ldots$
Negatif Tamsayılar
..., –3, –2, –1
Sıfır
0
Pozitif Tamsayılar
1, 2, 3 ...
Doğal Sayılar
0, 1, 2, 3 ...
Sayma Sayılar
1, 2, 3 ...

AÇIKLAMA

Aritmetikte kullanılan çeşitli işlemleri iyi anlayabilmek için, ortak sayı sistemine gereksinim vardır.

Sayı sisteminde, rakam olarak adlandırılan 10 sembol kullanılır. Bunlar; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 dur. Bu on sembolü kullanarak sayıları yazabiliriz.

Bu sayıların her biri, nesnelerin miktarlarını belirtmek için kullanılır. Semboller ilk olarak, sayma amacıyla kullanılmıştır.

İRRASYONEL SAYILAR

İki tamsayının oranı olarak yazılamayan gerçel (reel) sayılara irrasyonel sayılar denir. Örneğin,

$$\sqrt{2} = 1.4142135...$$
$$\pi = 3.141592...$$

sayıları irrasyonel sayılardır.

KARMAŞIK (KOMPLEKS) SAYILAR

Matematikçiler tarafından rasyonel ve irrasyonel sayıların dışında, $\sqrt{-1}$ sayısını kullanarak **karmaşık sayı** olarak adlandırılan başka bir sayı sistemi oluşturulmuştur.

BASIC CONCEPTS

Digit: *Each symbol used to expression of the numbers is called digit.*

Set of digits: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

For example; digits of the number 716 are 7, 1, and 6.

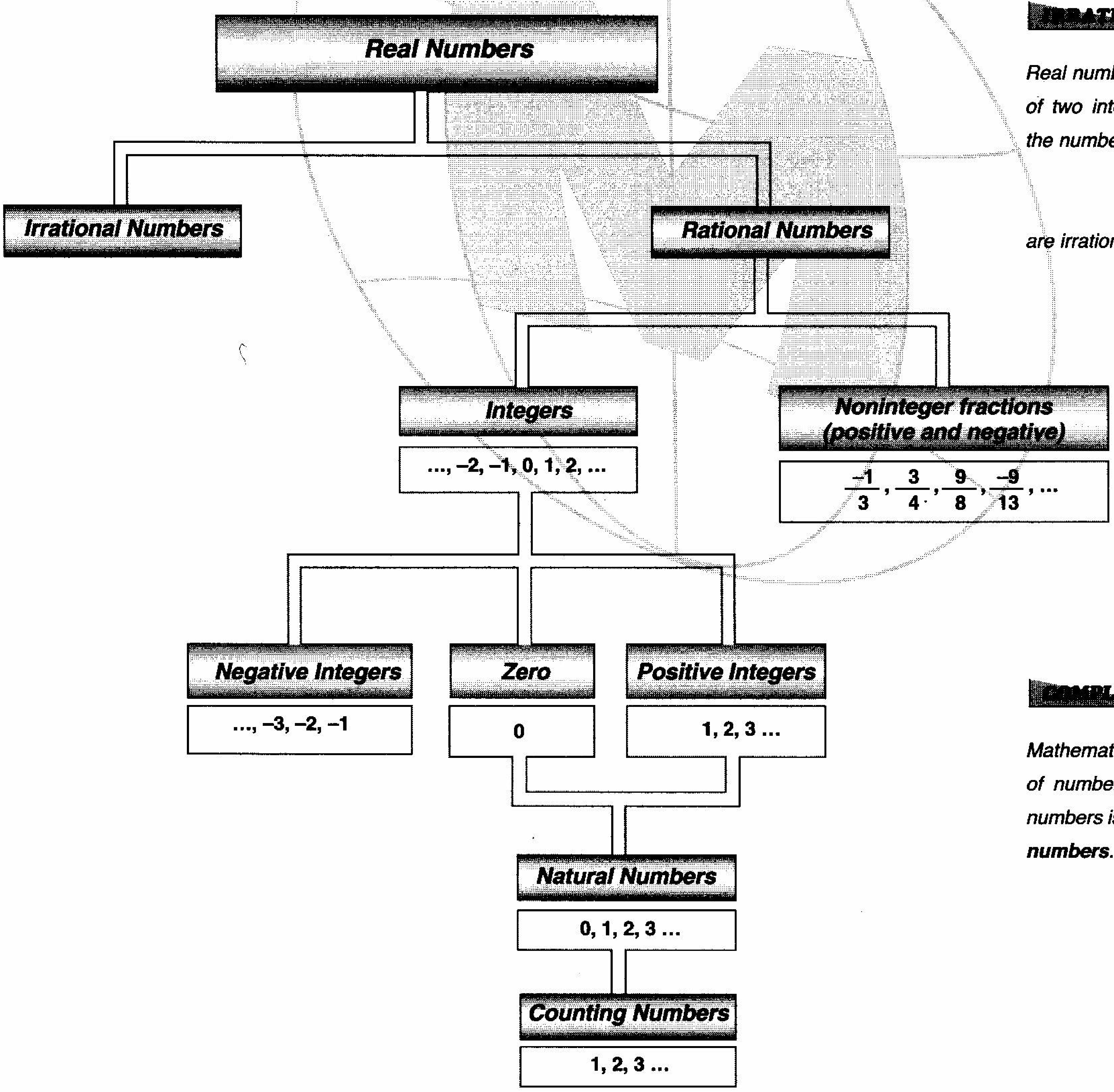
NUMBER SETS

Number: *The muchness determined by a digit itself or constituded by gathering more than one digit named as number.*

a. Counting Numbers: *{1, 2, 3 ...}*

b. Natural Numbers: *{ 0,1, 2, 3 ...}*

c. Integers: *{ –3, –2, –1, 0, 1, 2, 3, ...}*


Real Numbers
Irrational Numbers
Rational Numbers
Integers
..., –2, –1, 0, 1, 2, ...
Noninteger fractions (positive and negative)
$\frac{-1}{3}, \frac{3}{4}, \frac{9}{8}, \frac{-9}{13}, \ldots$
Negative Integers
..., –3, –2, –1
Zero
0
Positive Integers
1, 2, 3 ...
Natural Numbers
0, 1, 2, 3 ...
Counting Numbers
1, 2, 3 ...

EXPLANATION

In order to get a good understanding of the various operations used in arithmetic, one should have a clear idea of the formation of our common number system.

Our system of numbers makes use of only ten symbols, called digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. By means of these ten symbols we can write a number of any size.

Each of these symbols was invented by early man to stand for a certain number of objects. The symbols were first used for counting.

IRRATIONAL NUMBERS

Real numbers that cannot be written as ratio of two integers are irrational. For instance, the numbers

$$\sqrt{2} = 1.4142135...$$

$$\pi = 3.141592...$$

are irrational.

COMPLEX NUMBERS

Mathematicians created an expanded system of numbers by using $\sqrt{-1}$. This system of numbers is called as ***the system of complex numbers.***

İRDELEME / STUDY TIP

ab ve ba iki basamaklı birer doğal sayı olmak üzere aşağıdaki eşitlikler vardır:

If ab and ba are two place natural numbers, there are following equalities:

i. $ab + ba = (10 \cdot a + b) + (10 \cdot b + a)$
$= 11 \cdot a + 11 \cdot b = 11 \cdot (a + b),$

ii. $ab - ba = (10 \cdot a + b) - (10 \cdot b + a)$
$= 9 \cdot a - 9 \cdot b$
$= 9 \cdot (a - b).$

PEKİŞTİRME / REINFORCE

$xy + yx = 55$

$\Rightarrow \max(x \cdot y) = ?$

YANIT / ANSWER : 6

DOĞAL SAYILARDA ÇÖZÜMLEME
ANALYSIS of the NATURAL NUMBERS

Rakamın Basamak Değeri: Bir veya birden çok basamaklı bir sayıdaki herhangi bir rakamın bulunduğu basamağa göre aldığı değere, bu rakamın basamak değeri denir.

The place value of a digit: *In a decimal number system, the value of a digit depends on its place or position in the number. This value is called place value of this digit.*

Rakamın Sayı Değeri: Bir veya birden çok basamaklı bir sayıdaki, herhangi bir rakamın bulunduğu basamağa bağlı olmadan tek başına gösterdiği sayıya, bu rakamın sayı değeri denir. Rakamın sayı değeri hiç değişmez.

The number value of a digit: *In a decimal number, the number value of a digit is the value that digit has. This value does not depend on the place of the digit and it never changes.*

a b c d e

	Sayı değeri *The value of the number*	Basamak değeri *The value of the place*
Birler basamağı / *ones place* ⟶	e	$1 \cdot e$
Onlar basamağı / *tens place* ⟶	d	$10 \cdot d$
Yüzler basamağı / *hundreds place* ⟶	c	$100 \cdot c$
Binler basamağı / *thousands place* ⟶	b	$1000 \cdot b$
Onbinler basamağı / *ten thousands place* ⟶	a	$10000 \cdot a$

ÖRNEK / EXAMPLE 1

$ab + bb - aa = 15$

$\Rightarrow$ **a + b = ?**

A) 8 B) 10 C) 11 D) 14 E) 18

ÇÖZÜM / SOLUTION

Yanıt / Answer C

$ab + bb - aa = 15$ [Verilen eşitlik / *Given equality*]

$10a + b + 10b + b - 10a - a = 15$ [Çözümleme / *Analyze*]

$12b - a = 15$ [Sadeleştir. / *Simplify.*]

$\Rightarrow b = 2, \ a = 9$

$a + b = 9 + 2 = 11$

ÖRNEK / EXAMPLE 2

$abc - cba = 396$

$\Rightarrow$ **max(abc) + min(abc) = ?**

A) 1320 B) 1345 C) 1390 D) 1450 E) 1496

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$abc - cba = 396$ [Verilen eşitlik / *Given equality*]

$100a + 10b + c - 100c - 10b - a = 396$ [Çözümleme / *Analyze*]

$99a - 99c = 396$ [Sadeleştir. / *Simplify.*]

$a - c = 4$ [Sonuç / *Conclusion*]

max(abc) = 995

min(abc) = 501 $\Rightarrow$ max(abc) + min(abc) = 995 + 501 = 1496

ÖRNEK / EXAMPLE 3

$ABA + BAB = 1332$

$\Rightarrow$ **max(A) + min(A) = ?**

A) 11 B) 12 C) 13 D) 14 E) 15

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$ABA + BAB = 101A + 10B + 101B + 10A = 1332$

$111(A + B) = 1332$

$A + B = 12$

max(A) = 9 , min(A) = 3

$\Rightarrow$ max(A) + min(A) = 9 + 3 = 12

TAMSAYILAR / INTEGERS

$Z = \{\ldots\ldots -3, -2, -1, 0, 1, 2, 3, 4 \ldots\}$ kümesinin herbir elemanına *tamsayı* denir.

Each element of the set $\{\ldots\ldots -3, -2, -1, 0, 1, 2, 3, 4 \ldots\}$ *is called integer.*

Pozitif tamsayılar / *Positive integers* : $Z^+ = \{1, 2, 3, 4 \ldots\}$

Negatif tamsayılar / *Negative integers* : $Z^- = \{\ldots\ldots -3, -2, -1\}$

$$Z = Z^+ \cup \{0\} \cup Z^-$$

TEK ve ÇİFT TAMSAYILAR / ODD and EVEN INTEGERS

Tanım: $\{\ldots, -5, -3, -1, 1, 3, 5\ldots\}$ kümesinin her bir elemanına tek tamsayı denir.
n bir tamsayı iken, tek sayılar $2n - 1$ ile gösterilir.
Definition: *Each element of the set* $\{\ldots, -5, -3, -1, 1, 3, 5\ldots\}$ *is an odd number. While n is integer, the odd numbers are represented by* $2n - 1$.

Tanım: $\{\ldots, -4, -2, 0, 2, 4,\ldots\}$ kümesinin her bir elemanına çift tamsayı denir.
n bir tamsayı iken, çift sayılar $2n$ ile gösterilir.
Definition: *Each element of the set* $\{\ldots, -4, -2, 0, 2, 4,\ldots\}$ *is an even number. While n is integer, the even numbers are represented by* $2n$.

Tek ve Çift Tamsayılarda Yapılan İşlemler
Operations on Odd and Even Integers

Tek tamsayılar / *Odd integers*: T,

Çift tamsayılar / *Even integers*: Ç

a. T ± T = Ç
b. T · T = T
c. T ± Ç = T
d. Ç ± Ç = Ç
e. T · Ç = Ç
f. Ç · Ç = Ç

ÖRNEK / EXAMPLE 4

$$\begin{array}{r|l} x & b \\ \cline{2-2} - & c \\ \hline b-1 & \end{array} \qquad \begin{array}{r|l} y & b \\ \cline{2-2} - & c+1 \\ \hline 0 & \end{array} \qquad \Rightarrow x + y = ?$$

A) 2x + 2 B) 2x + 1 C) 2x

D) 2x – 1 E) 2x – 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left.\begin{array}{l} \begin{array}{r|l} x & b \\ \cline{2-2} - & c \\ \hline b-1 & \end{array} \Rightarrow x = bc + b - 1 \\ \\ \begin{array}{r|l} y & b \\ \cline{2-2} - & c+1 \\ \hline 0 & \end{array} \Rightarrow y = bc + b \end{array}\right\} \Rightarrow y = x + 1$$

$x + y = x + (x + 1) = 2x + 1$

ÖRNEK / EXAMPLE 5

$$\begin{array}{r|l} 32323 & 32 \\ \cline{2-2} - & x \\ \hline y & \end{array} \qquad \Rightarrow x + y = ?$$

A) 113 B) 1013 C) 1030 D) 1130 E) 1133

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{array}{r|l} 32323 & 32 \\ \cline{2-2} - & \mathbf{1010} \\ \hline \mathbf{3} & \end{array}$$

x = 1010 ve (and) y = 3 $\Rightarrow$ x + y = 1010 + 3 = 1013

İŞLEM ÖNCELİĞİ / THE ORDER of OPERATIONS

Herhangi bir sayı kümesinde toplama, çıkarma, çarpma ve bölme işlemleri (yani, dört işlem) yapılırken aşağıdaki sıra uygulanır:

In any set of numbers addition, subtraction, multiplication and division must be done as the following order:

1. Parantez ( ), köşeli parantez [ ], mutlak değer I I ve kesir çizgisi gibi grup sembolleri içeren işlemler yapılır.
   Perform operations inside grouping symbols. Grouping symbols include parantheses ( ), brackets [ ], braces { }, absolute value symbols I I, and the fraction bar.

2. Üslü ifadeler sadeleştirilir.
   Simplify exponential expressions.

3. Soldan sağa doğru çarpma ve bölme yapılır.
   Do multiplication and division as they occur from left to right.

4. Soldan sağa doğru toplama ve çıkarma yapılır.
   Do addition and subtraction as they occur from left to right.

ÖRNEK / EXAMPLE 6

$4 - [(15 : 5) + 3 \cdot 5 - 9 : (-3)] = ?$

A) –17 B) –15 C) 0 D) 15 E) 17

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned} 4 - [(15 : 5) + 3 \cdot 5 - 9 : (-3)] &= 4 - [3 + 15 + 3] \\ &= 4 - 21 \\ &= -17 \end{aligned}$$

PEKİŞTİRME / REINFORCE

$$\begin{array}{r|l} x & \underline{5} \\ -\underline{} & 6 \\ y & \end{array} \qquad \begin{array}{r|l} K & \underline{y} \\ -\underline{} & \\ 3 & \end{array}$$

$\Rightarrow x = ?$

YANIT / ANSWER : 34

ARDIŞIK SAYILAR / CONSECUTIVE NUMBERS

Tanım: Belirli bir kurala göre sıralanmış sayılara ardışık sayılar denir.
Definition: *It is group of terms going forward with a certain rule.*

Örneğin / *For example:*

1, 2, 3, 4 ......, n, ... ardışık sayılar / *consecutive numbers*

2, 4, 6, 8 ......, 2n, ... ardışık çift sayılar / *consecutive even numbers*

1, 3, 5, 7 ......, 2n – 1, ... ardışık tek sayılar / *consecutive odd numbers*

3, 8, 13, 18 ...., 78 ...... ardışık sayı dizisi / *consecutive number sequence*

ARDIŞIK SAYILARIN TOPLANMASI / SUM of CONSECUTIVE NUMBERS

Genel Toplam Formülü / General Sum Formula

Dizide kaç tane eleman olduğunu ve bu elemanların toplamını bulmak için aşağıda verilen formüller kullanılır.

To find the number of elements and the sum of these elements, in a sequence, these formulas use.

a. Terim sayısı / *Number of terms* : $T = \dfrac{\text{Son terim} - \text{İlk terim} \ [\textit{Last term} - \textit{First term}]}{\text{Ortak fark} \ [\textit{Common difference}]} + 1$

b. Terimlerin toplamı / *Sum of terms* : $S = \dfrac{\text{Son terim} + \text{İlk terim} \ [\textit{Last term} + \textit{First term}]}{2} \cdot T$

I. $1+2+3+\ldots+n=\dfrac{n\cdot(n+1)}{2}$ [Ardışık tamsayıların toplamı / *Sum of consecutive integers*]

II. $2+4+6+\ldots+2n=n\cdot(n+1)$ [Ardışık çift tamsayıların toplamı / *Sum of consecutive even integers*]

III. $1+3+5+\ldots+(2n-1)=n^2$ [Ardışık tek tamsayıların toplamı / *Sum of consecutive odd integers*]

Etkinlikler / Activities

1. $1+2+3+\ldots\ldots\ldots+12=\dfrac{12\cdot(12+1)}{2}=78$

2. $2+4+6+\ldots\ldots\ldots+40=20\cdot(20+1)=420$

3. $1+3+5+\ldots\ldots\ldots+27=14^2=196$

PEKİŞTİRME / REINFORCE

3 + [12 – 4 : 2] + 5 : 1 = ?

YANIT / ANSWER : 18

ÖRNEK / EXAMPLE 7

$2 + 5 + 8 + \ldots + 50 = ?$

A) 430 B) 436 C) 442 D) 450 E) 456

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

Terim sayısı : T = $\dfrac{\text{Son terim} - \text{İlk terim [\textit{Last term} – \textit{First term}]}}{\text{Ortak fark [\textit{Common difference}]}} + 1$

Number of terms

$$= \frac{50-2}{3} + 1 = 17$$

Terimlerin toplamı : S = $\dfrac{\text{Son terim + İlk terim [\textit{Last term} + \textit{First term}]}}{2} \cdot T$

Sum of terms

$$= \frac{50+2}{2} \cdot 17 = 442$$

ÖRNEK / EXAMPLE 8

$3 + 8 + 13 + \ldots + x = 429 \Rightarrow$ **x = ?**

A) 53 B) 58 C) 63 D) 68 E) 73

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$3+8+13+\ldots+x=429$ [Verilen eşitlik / *Given equality*]

$\left(\dfrac{x-3}{5}+1\right)\cdot\left(\dfrac{x+3}{2}\right)=429$ [Genel toplam formülü / *General sum formula*]

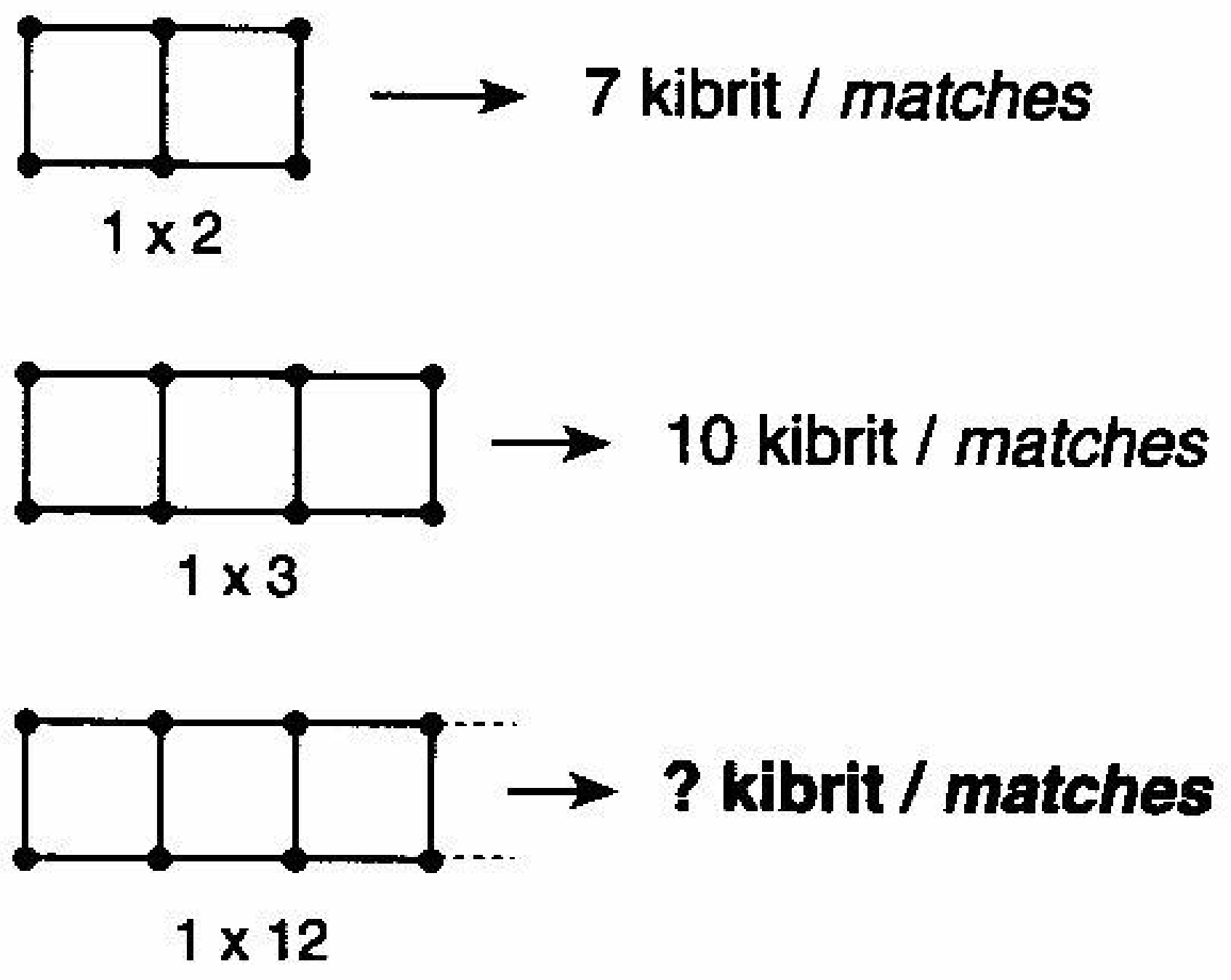
$\dfrac{x+2}{5}\cdot\dfrac{x+3}{2}=429$ [Sadeleştir. / Simplify.]

$(x+2)\cdot(x+3)=4290$ [Her iki tarafı 10 ile çarp. / *Multiply both sides by 10.*]

$(x+2)\cdot(x+3)=65\cdot 66$ [Sadeleştir. / *Simplify.*]

$x=63$ [Sonuç / *Conclusion*]

Düşünün / Think about it



Yanıt / Answer

37

PEKİŞTİRME / REINFORCE

$3 + 7 + 11 + 15 + \ldots + 43 = ?$

YANIT / ANSWER : 253

ÖRNEK / EXAMPLE 9

$1 + 2 + 3 + \ldots + n = A$

$10 + 11 + 12 + \ldots + n = B$

$A + B = 885 \Rightarrow$ **$n = ?$**

A) 25 B) 30 C) 32 D) 35 E) 40

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$1 + 2 + 3 + \ldots + n = A$

$10 + 11 + 12 + \ldots + n = B$

$\Rightarrow A - B = 1 + 2 + 3 + \ldots + 9 = 45$

$$\left.\begin{aligned} A+B &= 885 \\ A-B &= 45 \end{aligned}\right\} \Rightarrow A = 465$$

$$\frac{n \cdot (n+1)}{2} = 465 \Rightarrow n = 30$$

ÖRNEK / EXAMPLE 10

$2011 - 2010 + 2009 - 2008 + \ldots + 3 - 2 + 1 = ?$

A) 1004 B) 1008 C) 1000 D) 1006 E) 1002

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$2011 - 2010 + 2009 - 2008 + \ldots + 3 - 2 + 1$

$= (2011 + 2009 + 2007 + \ldots + 3 + 1) - (2010 + 2008 + 2006 + \ldots + 2)$

$= (1006)^2 - (1005) \cdot 1006 = 1006$

FAKTÖRİYEL / FACTORIAL

Tanım: 1' den n' ye kadar olan ardışık doğal sayıların çarpımına *n faktöriyel (n!)* denir.
Definition: *For any positive integer n, n factorial (n!) represents the product of the first n positive integer.*

$1! = 1$
$2! = 1 \cdot 2 = 2$
$3! = 1 \cdot 2 \cdot 3 = 6$
$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$
$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$
$6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$

Etkinlikler / Activities

$n! = 1 \cdot 2 \cdot 3 \cdot 4 \ldots. (n-2) \cdot (n-1) \cdot n$
$n! = n \cdot (n-1)!$
$n! = n \cdot (n-1) \cdot (n-2)!$

ÖRNEK / EXAMPLE 11

$$\frac{10!+9!}{11} = x \cdot 8! \quad \Rightarrow \quad x = ?$$

A) 6 B) 7 C) 8 D) 9 E) 10

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{10!+9!}{11} = x \cdot 8!$$

$$9!(10+1) = 8! \cdot x \cdot 11$$

$$9 = x$$

PEKİŞTİRME / REINFORCE

$191 - 190 + 189 - 188 + \ldots + 171 - 170 = ?$

YANIT / ANSWER : 11

NOT / NOTE

$0! = 1$

ÖRNEK / EXAMPLE 12

$$\frac{(2n-1)!}{(2n-3)!}+\frac{(2n)!}{(2n-1)!}=50 \quad \Rightarrow \quad n=?$$

A) 4 B) 5 C) 6 D) 7 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{(2n-1)!}{(2n-3)!}+\frac{(2n)!}{(2n-1)!}=50$$ [Verilen eşitlik / *Given equality*]

$$(2n-1)\cdot(2n-2)+(2n)=50$$ [İndirge / *Reduce*]

$$4n^2-6n+2+2n=50$$ [Dağılma özelliği / Distributive Property]

$$4n^2-4n-48=0$$ [Sadeleştir. / Simplify.]

$$n^2-n-12=0$$ [Her iki tarafı 4'e böl. / *Divide both sides by 4.*]

$$n=4$$ [Sonuç / *Conclusion*]

PEKİŞTİRME / REINFORCE

$$\frac{4!-3!}{5!}=\frac{x}{100} \quad \Rightarrow \quad x=?$$

YANIT / ANSWER : 15

ÖRNEK / EXAMPLE 13

$x, A \in \mathbb{N}$,

$25! = 9^x \cdot A \Rightarrow$ **max(x) = ?**

A) 10 B) 9 C) 8 D) 7 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$25! = 9^x \cdot A \Rightarrow 25! = 3^{2x} \cdot A$

$$\begin{array}{r|r|l}
25 & 3 & \\
\underline{-24} & \mathbf{8} & 3 \\
1 & \underline{-6} & \mathbf{2} \Rightarrow \max(2x) = 10 \\
 & 2 & \quad \max(x) = 5
\end{array}$$

TABAN ARİTMETİĞİ / BASE ARITHMETICS

Tanım: Sayının tanımlandığı sayma sistemi o sayının tabanını oluşturur. Bu tabanda yapılan işlemlere de *taban aritmetiği* adı verilir.

Definition: *The countion system on which the number is defined forms the base. Operations that are being done in the base are called base arithmetics.*

✔ **Farklı Tabanda Verilen Bir Sayının Onluk Tabana Çevrilmesi:** Herhangi bir tabanda verilen sayıyı onluk tabana çevirmek için, sayının en sağından başlayarak sayının basamak değerleri yazılır. Soldan başlanarak sayının basamak kuvvetleri ile sayı değerleri çarpılır. Sonuçlar toplanır.

Changing a number from any other base to base ten: *To change a number to base 10 from any other base, write down the digit at the extreme right as the number of units. Then, moving toward the left, multiply each digit by successive powers of the base. Finally, add the results.*

a, b, c, d birer rakam, $t \in N^+$, $t > 1$ ve $a, b, c, d < t$ olmak üzere aşağıdakiler yazılır.

Provided a, b, c, d are digits, $t \in N^+$, $t > 1$ and $a, b, c, d < t$

$$(abcd)_t = a\cdot t^3 + b\cdot t^2 + c\cdot t + d \qquad (ab,cd)_t = a\cdot t^1 + b\cdot t^0 + c\cdot t^{-1} + d\cdot t^{-2}$$

$$d \to d\cdot t^0,\quad c \to c\cdot t^1,\quad b \to b\cdot t^2,\quad a \to a\cdot t^3 \qquad d \to d\cdot t^{-2},\quad c \to c\cdot t^{-1},\quad b \to b\cdot t^0,\quad a \to a\cdot t^1$$

✔ **10'luk tabanda verilen bir sayıyı başka bir tabandaki bir sayıya çevirme:** 10 tabanında verilen sayı, hangi tabanda yazılacaksa o tabana bölüm sıfır olana kadar bölünür. Sıfır dahil olmak üzere tüm kalanlar yazılır. Son kalandan başlanarak yazılan rakamlar yeni sayıyı oluşturur.

Changing a number from base ten to any other base: *Divide the number(base 10) by the new base as many times as possible and write down all remainders, including zeros. The remainders are the digits of the number written in the desired base, starting with the last remainder.*

ÖRNEK / EXAMPLE 14

$(130)_x = (7x)_9 \Rightarrow x = ?$

A) 8 B) 7 C) 6 D) 5 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$(130)_x = (7x)_9$	[Verilen eşitlik / *Given equality*]
$0\cdot x^0 + 3\cdot x^1 + 1\cdot x^2 = x\cdot 9^0 + 7\cdot 9^1$	[10 tabanına çevir. / *Change to base 10.*]
$x^2 + 3x = x + 63$	[Sadeleştir. / *Simplify.*]
$x^2 + 2x - 63 = 0$	[Benzer terimleri topla. / *Combine like terms.*]
$x = 7$	[Sonuç / *Conclusion*]

PEKİŞTİRME / REINFORCE

$$\frac{n!}{(n-1)!} + \frac{(n+1)!}{n!} = 7 \Rightarrow n = ?$$

YANIT / ANSWER : 3

PEKİŞTİRME / REINFORCE

$A, x \in N,$

$35! = 8^x \cdot A \Rightarrow \max(x) = ?$

YANIT / ANSWER : 10

ÖRNEK / EXAMPLE 15

$x \in Z^+$,

$(21)_5 < x \leq (212)_3 \Rightarrow \sum x = ?$

A) 210 B) 220 C) 230 D) 240 E) 250

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$(21)_5 < x \leq (212)_3 \Rightarrow$

$1 \cdot 5^0 + 2 \cdot 5^1 < x \leq 2 \cdot 3^0 + 1 \cdot 3^1 + 2 \cdot 3^2$

$11 < x \leq 23$

$x \in Z^+ \Rightarrow \sum x = 12 + 13 + \ldots + 23 = 210$

ÖRNEK / EXAMPLE 16

$(213)_4 \cdot (32)_4 = (x)_4 \Rightarrow x = ?$

A) 20202 B) 20213 C) 21103 D) 21020 E) 23102

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{array}{lr} & (213)_4 \\ \times & (32)_4 \\ \hline & (1032)_4 \\ + & (1311)_4 \\ \hline & (20202)_4 \end{array}$$

ÖRNEK / EXAMPLE 17

$(231,4)_5 = (x)_{10} \Rightarrow$ **x = ?**

A) $\frac{312}{5}$

B) $\frac{318}{5}$

C) $\frac{324}{5}$

D) $\frac{328}{5}$

E) $\frac{334}{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$(231,4)_5 = 1\cdot5^0 + 3\cdot5^1 + 2\cdot5^2 + 4\cdot5^{-1}$$

$$= 66 + \frac{4}{5}$$

$$= \frac{334}{5}$$

PEKİŞTİRME / REINFORCE

$(11)_5 \cdot (13)_5 = (x)_6 \Rightarrow x = ?$

YANIT / ANSWER : 120

ÖRNEK / EXAMPLE 18

$(24)_5 + 1 = (x)_5 \Rightarrow$ **x = ?**

A) 30 B) 31 C) 32 D) 33 E) 34

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$(1)_{10} = (1)_5 \Rightarrow (24)_5 + (1)_5 = (30)_5$

$\Rightarrow x = 30$

PEKİŞTİRME / REINFORCE

$(121,04)_5 = (x)_{10} \Rightarrow x = ?$

YANIT / ANSWER : 36,16

ÖRNEK / EXAMPLE 19

ON DOKUZ MAYIS ÜNİVERSİTESİ YÖS 2013

$(xy)_7 = (yx)_9$

$x + y = ?$

A) 4 B) 5 C) 7 D) 11 E) 12

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$y \cdot 7^0 + x \cdot 7^1 = x \cdot 9^0 + y \cdot 9$$

$$y + 7x = x + 9y$$

$$6x = 8y$$

$$3x = 4y \Rightarrow x = 4,\ y = 3$$

$$x + y = 4 + 3 = 7$$

ÖRNEK / EXAMPLE 20

YILDIZ TEKNİK ÜNİVERSİTESİ YÖS 2013

$x \in R^+$

$$\frac{12!x^3}{4! \cdot 9!} = \frac{1}{24} \cdot \frac{6! \cdot x^8}{3!} \Rightarrow x = ?$$

A) $\sqrt[5]{11}$ B) $\sqrt[10]{12}$ C) $3\sqrt{5}$ D) $\sqrt[12]{17}$ E) $\sqrt[7]{7}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{12!x^3}{4! \cdot 9!} = \frac{1}{24} \cdot \frac{6! \cdot x^8}{3!}$$

$$\frac{12 \cdot 11 \cdot 10 \cdot x^3}{24} = \frac{1}{24} \cdot \frac{6 \cdot 5 \cdot 4 \cdot x^8}{1}$$

$$11 = x^5$$

$$\sqrt[5]{11} = x$$

ÖRNEK / EXAMPLE 21

İSTANBUL ÜNİVERSİTESİ YÖS 2014

$$\frac{6! - 4!}{3!} = ?$$

A) 720 B) 696 C) 116 D) 20 E) 6

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{6! - 4!}{3!} = \frac{6 \cdot 5 \cdot 4! - 4!}{3!}$$

$$\frac{4! \cdot (30-1)}{3!} = \frac{4 \cdot 3! \cdot 29}{3!} = 116$$

ÖRNEK / EXAMPLE 22

EGE ÜNİVERSİTESİ YÖS 2016

Aşağıdaki bölme işleminde A sayısı en çok kaçtır?

What is the greatest A number in division below?

$$\begin{array}{r|l} A & 20 \\ \cline{2-2} \vdots & x \\ - \quad & \\ \hline x^2 & \end{array}$$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$A = 20 \cdot x + x^2$

$x^2 < 20$

$\max(A) \Rightarrow x = 4$

$\max(A) = 20 \cdot 4 + 4^2$

$= 96$

PEKİŞTİRME / REINFORCE

$$\frac{n! - (n-1)!}{(n-1)!} = ?$$

YANIT / ANSWER : $n - 1$

ÖRNEK / EXAMPLE 23

EGE ÜNİVERSİTESİ YÖS 2016

$$\frac{16!+17!}{18!}=?$$

A) $\frac{1}{9}$ B) $\frac{1}{14}$ C) $\frac{1}{15}$ D) $\frac{1}{16}$ E) $\frac{1}{17}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{\cancel{16!}(1+17)}{18\ 17\ \cancel{16!}}$$

$$\frac{\cancel{18}}{\cancel{18}\cdot 17}=\frac{1}{17}$$

ÖRNEK / EXAMPLE 24

KARADENİZ TEKNİK ÜNİVERSİTESİ YÖS 2016

$$\begin{array}{r|l} A & \underline{\ 5\ \ } \\ - & 8 \\ \hline 2 & \end{array} \qquad \begin{array}{r|l} A+9 & \underline{B+2} \\ - & 5 \\ \hline x & \end{array} \qquad \Rightarrow x=?$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$A=5\cdot 8+2$

$A=42$

$$\begin{array}{r|l} 51 & \underline{\ 5\ \ } \\ - & 10 \\ \hline 1 & \end{array} \quad \Rightarrow x=1$$

1. $2A - B = 10$

   $A + 4B = 8$

   $\Rightarrow$ **$A + B = ?$**

   A) 4

   B) 6

   C) 8

   D) 10

   E) 12

2. **$-4 - (-7)\cdot(-2) + [9 : 3 + 6] = ?$**

   A) −13

   B) −12

   C) −11

   D) −10

   E) −9

3. $AB + BA - BB = 22 \Rightarrow$ **$A = ?$**

   A) 1

   B) 2

   C) 3

   D) 4

   E) 5

4. $AB - BA = 63 \Rightarrow$ **$\max(B) + \min(B) = ?$**

   A) 2

   B) 3

   C) 4

   D) 5

   E) 6

5. $A, B, C \in Z^+$

   $A \cdot B = 42$

   $B \cdot C = 63$

   $\Rightarrow$ **$\min(A + B + C) = ?$**

   A) 22

   B) 26

   C) 34

   D) 40

   E) 46

6. **$\frac{7! + 8! + 9!}{7!} = ?$**

   A) 65

   B) 68

   C) 72

   D) 78

   E) 81

7. $3! \cdot x + 5! \cdot x = 8! \Rightarrow \mathbf{x = ?}$

A) 240

B) 260

C) 300

D) 320

E) 360

8. $\dfrac{5+7+9+......+103}{27} = ?$

A) 100

B) 110

C) 120

D) 130

E) 140

9. $A = 3 + 5 + 7 + \ldots + 19$

$B = 1 + 2 + \ldots + 20$

$\Rightarrow \mathbf{A + B = ?}$

A) 301

B) 303

C) 306

D) 309

E) 311

10. $(214)_5 + 12 = (x)_5 \Rightarrow \mathbf{x = ?}$

A) 221

B) 230

C) 234

D) 241

E)243

11. $(30a2)_5 = (576)_8 \Rightarrow \mathbf{a = ?}$

A) 0

B) 1

C) 2

D) 3

E) 4

12. $(x)_6 = 2 \cdot 6^3 + 4 \cdot 6^2 + 5 \Rightarrow \mathbf{x = ?}$

A) 5402

B) 5142

C) 4025

D) 2415

E) 2405

YANITLAR / ANSWERS

1. B 2. E 3. B 4. B 5. A 6. E 7. D 8. A

9. D 10. D 11. B 12. E

1.

$$\begin{array}{rl} & 2A - B = 10 \\ + & A + 4B = 8 \\ \hline & 3A + 3B = 18 \\ & 3(A + B) = 18 \\ & A + B = 6 \end{array}$$

Yanıt / The Answer is B

2. $-4 - (-7)\cdot(-2) + [9 : 3 + 6] = -4 + 7\cdot(-2) + [3 + 6]$

$= -18 + 9$

$= -9$

Yanıt / The Answer is E

3. $AB + BA - BB = 11(A + B) - 11B$

$11A = 22$

$= 2$

Yanıt / The Answer is B

4. $AB - BA = 9(A - B) = 63$

$A - B = 7$

$\max(B) = 2,\ \min(B) = 1$

$\Rightarrow \max(B) + \min(B) = 2 + 1 = 3$

Yanıt / The Answer is B

5. $A\cdot B = 42,$

$A, B \in Z^{+} \Rightarrow \min(A) = 6,\ \min(B) = 7$

$B\cdot C = 63,$

$B, C \in Z^{+}\ \min(B) = 7 \Rightarrow \min(C) = 9$

$\Rightarrow \min(A + B + C) = 6 + 7 + 9 = 22$

Yanıt / The Answer is A

6. $\dfrac{7! + 8! + 9!}{7!} = \dfrac{7!(1 + 8 + 8\cdot 9)}{7!}$

$= 81$

Yanıt / The Answer is E

7. $3! \cdot x + 5! \cdot x = 8!$

$3! \cdot x\,(1 + 4 \cdot 5) = 8!$

$x \cdot (21) = 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$

$x = 320$

Yanıt / The Answer is D

8. $$\frac{5+7+9+\ldots+103}{27} = \frac{\left(\frac{103-5}{2}+1\right)\cdot\left(\frac{103+5}{2}\right)}{27}$$

$$= \frac{50 \cdot 54}{27}$$

$$= 100$$

Yanıt / The Answer is A

9. $A = 3+5+\ldots+19 = 10^2 - 1 = 99$

$B = 1+2+\ldots+20 = \dfrac{20 \cdot 21}{2} = 210$

$\Rightarrow A+B = 99+210 = 309$

Yanıt / The Answer is D

10. $(12)_{10} = (a)_5 \;\Rightarrow\; a = 22$

$(214)_5 + (22)_5 = (x)_5$

$x = 241$

Yanıt / The Answer is D

11. $(30a2)_5 = (576)_8$

$\Rightarrow 2 \cdot 5^0 + a \cdot 5^1 + 0 \cdot 5^2 + 3 \cdot 5^3 = 6 \cdot 8^0 + 7 \cdot 8^1 + 5 \cdot 8^2$

$2 + 5a + 0 + 375 = 6 + 56 + 320$

$377 + 5a = 382$

$5a = 382 - 377$

$5a = 5$

$a = 1$

Yanıt / The Answer is B

12. $(x)_6 = 2 \cdot 6^3 + 4 \cdot 6^2 + 5$

$(x)_6 = 2 \cdot 6^3 + 4 \cdot 6^2 + 0 \cdot 6^1 + 5 \cdot 6^0$

$\Rightarrow x = 2405$

Yanıt / The Answer is E

1. $2A + B = 9$

$B - C = 5$

$A - D = 4 \Rightarrow \mathbf{A + C + D = ?}$

Yanıt / The Answer is: **0**

2. $A, B \in Z^+$

$2A + 3B = 45 \Rightarrow \mathbf{(A + B)_{min} = ?}$

Yanıt / The Answer is: **16**

3. $$\frac{6! + 7!}{5! + 6!} = ?$$

Yanıt / The Answer is: $\mathbf{\frac{48}{7}}$

4. $$\frac{(n+1)!}{(n-1)!} = 42 \Rightarrow \mathbf{n = ?}$$

Yanıt / The Answer is: **6**

5. $$\frac{(n+1)! + n! + (n-1)!}{(n-1)!} = 49 \Rightarrow \mathbf{n = ?}$$

Yanıt / The Answer is: **6**

6. $$\frac{(n-2)!}{(n-4)!} = 20 \Rightarrow \mathbf{n = ?}$$

Yanıt / The Answer is: **7**

7. $\frac{(n+1)!}{2(n-1)!}+\frac{(n+1)!}{6(n-2)!}=\frac{8(n+1)!}{n!}$ $\Rightarrow$ **$n=?$**

Yanıt / The Answer is: **6**

8. $\frac{25!}{8^n}=A$ $\quad n, A \in N \Rightarrow$ **max(n) = ?**

Yanıt / The Answer is: **7**

9. $\frac{10! \cdot 25!}{12!}=x \cdot 10^n$ $\quad n, x \in N \Rightarrow$ **max(n) = ?**

Yanıt / The Answer is: **6**

10. $32 + 33 + \ldots + 58 = ?$

Yanıt / The Answer is: **1215**

11. $10 + 11 + 12 + \ldots + 89 + 90 + 89 + 88 + \ldots + 9 + 8 = ?$

Yanıt / The Answer is: **8027**

12. $12 + 15 + \ldots + 6n = 390$ $\Rightarrow$ **$n = ?$**

Yanıt / The Answer is: **8**

13. $\mathbf{13 - 50 + 15 - 48 + 17 - 46 + \ldots 27 - 36 + 29 = ?}$

Yanıt / The Answer is: **−155**

14. $A = 4{\cdot}6 + 5{\cdot}7 + 6{\cdot}8 + \ldots + 24{\cdot}26$

$B = 4{\cdot}4 + 5{\cdot}5 + 6{\cdot}6 + \ldots + 24{\cdot}24$

$\Rightarrow \mathbf{A - B = ?}$

Yanıt / The Answer is: **588**

15. $(213)_5 = (20x1)_3 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **1**

16. $(a3)_{a+2} = 38 \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **5**

17. $(a15)_{3a} = 83 \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **2**

18. $(123)_5 \cdot 6 = (x)_5 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **1403**

19. $(13, 12)_4 = (x)_{10} \Rightarrow$ **x = ?**

Yanıt / The Answer is: $\frac{59}{8}$

20. $\frac{a+4b-c}{b+c} = 4, \; a,b,c \in Z^+ \Rightarrow \mathbf{min(a+b+c) = ?}$

Yanıt / The Answer is: **7**

21. $a + b = 9, \quad \frac{a}{b} + 1 = c,$

$a, b, c \in Z^+ \Rightarrow \sum \mathbf{b = ?}$

Yanıt / The Answer is: **4**

22. $abc - cba = 594 \Rightarrow \sum \mathbf{c = ?}$

Yanıt / The Answer is: **6**

23.

$$\begin{array}{r|l} A & 75 \\ \cline{2-2} \underline{-\quad} & n+1 \\ n^3 & \end{array} \Rightarrow \mathbf{max(A) = ?}$$

Yanıt / The Answer is: **439**

24.

$$\begin{array}{r|l} A & 8 \\ \cline{2-2} \underline{-\quad} & C \\ 2 & \end{array} \qquad \begin{array}{r|l} C & 6 \\ \cline{2-2} \underline{-\quad} & B \\ 4 & \end{array} \qquad \begin{array}{r|l} A & 24 \\ \cline{2-2} \underline{-\quad} & \\ x & \end{array}$$

$\Rightarrow$ **x = ?**

Yanıt / The Answer is: **10**

1. $7 - \{[(-1) - 7 - (-9)] - (-5)\} = ?$
   A) 0
   B) 1
   C) 2
   D) 3
   E) 4

2. $8 : 2 - \{[(-2) - (-3) - (-4)] - 3\} = ?$
   A) 2
   B) 3
   C) 4
   D) 5
   E) 6

3. $[3\cdot(3^{-1} + 2^{-1}) + 1] + 2 = ?$
   A) $\frac{7}{2}$
   B) $\frac{9}{2}$
   C) $\frac{11}{2}$
   D) $\frac{13}{2}$
   E) $\frac{15}{2}$

4. $[2\cdot(3^{-1} - 4^{-1}) + 1]^{-1} = ?$
   A) $\frac{2}{7}$
   B) $\frac{3}{7}$
   C) $\frac{4}{7}$
   D) $\frac{5}{7}$
   E) $\frac{6}{7}$

5. $\frac{AB + BA}{AA} = 3 \Rightarrow \max(B - A) = ?$
   A) 2
   B) 3
   C) 4
   D) 5
   E) 6

6. $AB + BA = 88$
   $AB - BA = 54 \Rightarrow 2A + B = ?$
   A) 14
   B) 15
   C) 16
   D) 17
   E) 18

7. $a, b \in Z^+$,

$(17 - a)\cdot(a - 6) = b \Rightarrow \boldsymbol{\sum a = ?}$

A) 110

B) 115

C) 120

D) 130

E) 140

8. $a \neq b \neq c$,

$a, b, c \in Z^+$,

$2a + b + \frac{c}{3} = 12 \Rightarrow \mathbf{max(c) = ?}$

A) 3

B) 6

C) 9

D) 24

E) 27

9.

$$\begin{array}{r|l} A & C+2 \\ \cline{2-2} - \quad & 5 \\ \hline C & \end{array} \qquad \begin{array}{r|l} B & C \\ \cline{2-2} - \quad & 6 \\ \hline 2 & \end{array} \Rightarrow \mathbf{A - B = ?}$$

A) 6

B) 7

C) 8

D) 9

E) 10

10.

$$\begin{array}{r|l} A & 5 \\ \cline{2-2} - \quad & B \\ \hline 2 & \end{array} \qquad \begin{array}{r|l} A+1 & B-1 \\ \cline{2-2} - \quad & 5 \\ \hline C & \end{array} \quad \Rightarrow \mathbf{C = ?}$$

A) 4

B) 5

C) 6

D) 7

E) 8

11.

$$\begin{array}{r|l} ABAB3 & AB \\ \cline{2-2} - \quad & x \\ \hline y & \end{array} \quad \Rightarrow \mathbf{x + y = ?}$$

A) 103

B) 104

C) 114

D) 1003

E) 1013

12.

$$\begin{array}{r|l} A & 18 \\ \cline{2-2} - \quad & B \\ \hline B^2 & \end{array} \quad \Rightarrow \mathbf{max(A) = ?}$$

A) 88

B) 90

C) 92

D) 94

E) 96

YANITLAR / ANSWERS

1. B 2. A 3. C 4. E 5. C. 6. B 7. B 8. D

9. C 10. E 11. E 12. A

1. **$3 - 4 : 2 + 5\cdot[3 - 4 - (-2)] = ?$**

A) 5
B) 6
C) 7
D) 8
E) 9

2. **$[-2^2 + (-3)^3 : (-3)] : [(-2)^2 + 1] = ?$**

A) 1
B) 2
C) 3
D) 4
E) 5

3. **$(-3)^2 - 2^2 - (-2)^3\cdot[-3 - (-2)]^{-1} = ?$**

A) –6
B) –5
C) –4
D) –3
E) –2

4. **$2x - [-(x + z) - 2(y - z) + 3x] = ?$**

A) $2x + y$
B) $x - z$
C) $y + z$
D) $2y - z$
E) $2x + z$

5. $AB + BA = 7(A + B) + 32 \Rightarrow$ **$\max(A) = ?$**

A) 4
B) 5
C) 6
D) 7
E) 8

6. $A, B \in \mathbb{N}$,
$A + B = 20 \Rightarrow$ **$\min(A\cdot B) = ?$**

A) 0
B) 25
C) 50
D) 75
E) 100

7. $A, B \in Z^+$

$A + B = 50 \Rightarrow$ **min(A·B) + max(A·B) = ?**

A) 625

B) 632

C) 645

D) 668

E) 674

8.
$$\begin{array}{r|l} AA & 9 \\ \hline - \quad & A \\ 8 & \end{array} \quad \Rightarrow \mathbf{A = ?}$$

A) 1

B) 2

C) 3

D) 4

E) 5

9.
$$\begin{array}{r|l} AB & 8 \\ \hline - \quad & x-4 \\ x+1 & \end{array} \quad \Rightarrow \mathbf{max\ (AB) = ?}$$

A) 18

B) 21

C) 23

D) 29

E) 35

10. $A, B, C \in Z^+$

$3 \cdot A = 7 \cdot B, \; C = 2 \cdot A$

$\Rightarrow$ **min(C) = ?**

A) 10

B) 12

C) 14

D) 16

E) 18

11. $A, B, C \in Z^+$,

$A \neq B \neq C$

$A + 2B + 3C = 45 \Rightarrow$ **max(A) = ?**

A) 34

B) 35

C) 36

D) 37

E) 38

12. $a + \frac{12}{b} = 10$,

$a, b \in Z^+ \;\; \Rightarrow \;\; \mathbf{\sum a = ?}$

A)23

B) 25

C) 26

D) 30

E) 34

YANITLAR / ANSWERS

1. B	**2. A**	**3. D**	**4. D**	**5. D**	**6. A**	**7. E**	**8. D**
9. C	**10. C**	**11. E**	**12. E**				

1. $3 + 6 + 9 + \ldots + 72 = ?$

A) 895
B) 896
C) 897
D) 900
E) 903

2. $7 + 11 + 15 + \ldots + 91 = ?$

A) 1078
B) 1082
C) 1096
D) 1104
E) 1108

3. $\frac{2}{3} + \frac{3}{2} + \frac{4}{3} + \frac{5}{2} + \ldots\ldots + \frac{49}{2} = ?$

A) 508
B) 512
C) 516
D) 520
E) 524

4. $9 + 11 + 13 + \ldots + (2n - 1) = 425 \Rightarrow n = ?$

A) 17
B) 19
C) 21
D) 23
E) 25

5. $8 + 10 + 12 + \ldots + 2n = 408 \Rightarrow n = ?$

A) 16
B) 18
C) 20
D) 22
E) 24

6. $\frac{4 + 8 + 12 + \ldots\ldots + 92}{16 + 24 + 32 + \ldots\ldots + 192} = ?$

A) $\frac{3}{13}$
B) $\frac{4}{13}$
C) $\frac{5}{13}$
D) $\frac{6}{13}$
E) $\frac{7}{13}$

7. $3 - 4 + 5 - 6 + \ldots - 30 = ?$

A) -15

B) -14

C) -13

D) -12

E) -11

8. $-9 - 5 - 1 + 3 + \ldots + 43 = ?$

A) 224

B) 230

C) 238

D) 242

E) 246

9. $1 + 2 + 3 + \ldots + 16 = A$

$2 + 4 + 6 + \ldots + 16 = B$

$\Rightarrow$ **A + B = ?**

A) 206

B) 208

C) 209

D) 210

E) 212

10. $7 + 9 + \ldots + n = A$

$10 + 12 + \ldots + (n + 1) = B$

$A + B = 181 \Rightarrow$ **n = ?**

A) 27

B) 25

C) 23

D) 21

E) 19

11. $A = 6 + 7 + 8 + \ldots + n$

$B = 2 + 3 + 4 + \ldots + n + 1$

$A + B = n + 225 \Rightarrow$ **n = ?**

A) 11

B) 12

C) 13

D) 14

E) 15

12. $A = 5 + 6 + 7 + \ldots + n$

$B = 2 + 3 + 4 + \ldots + n - 2$

$A - B = 80 \Rightarrow$ **n = ?**

A) 41

B) 42

C) 43

D) 44

E) 45

YANITLAR / ANSWERS

1. D 2. A 3. B 4. C 5. C 6. D 7. B 8. C

9. B 10. E 11. E 12. E

1. $\dfrac{7!-2\cdot 8!+9!}{8!+6!}=?$

A) 5
B) 6
C) 7
D) 8
E) 9

2. $\dfrac{6!-5\cdot 7!+8!}{5!+2\cdot 3!}=?$

A) 80
B) 90
C) 100
D) 110
E) 120

3. $\dfrac{(n-3)!+(3-n)!}{(n-2)!}=?$

A) 0
B) 1
C) 2
D) 3
E) 4

4. $\dfrac{(n+1)!}{8!}=90 \Rightarrow n=?$

A) 9
B) 10
C) 11
D) 12
E) 13

5. $\dfrac{(n-2)!+(n-3)!}{2(n-4)!+(n-3)!}=8 \Rightarrow n=?$

A) 7
B) 8
C) 9
D) 10
E) 11

6. $(ab)! = (9a+4b)! \Rightarrow \sum ab = ?$

A) 174
B) 186
C) 192
D) 202
E) 214

7. $50! = A \cdot 2^n \cdot 15^m$,

$n, m, A \in \mathbb{N}^+$

$\Rightarrow$ **max(n + m) = ?**

A) 56

B) 57

C) 58

D) 59

E) 60

8. $n, A \in \mathbb{N}^+$,

$\dfrac{48!}{8^n} = A \;\Rightarrow$ **max(n) = ?**

A) 12

B) 13

C) 14

D) 15

E) 16

9. $n, A \in \mathbb{N}^+$,

$\dfrac{38! + 39!}{10^n} = A \;\Rightarrow$ **max(n) = ?**

A) 7

B) 8

C) 9

D) 10

E) 11

10. $n, A \in \mathbb{N}^+$,

$\dfrac{90! - 86!}{10^n} = A \;\Rightarrow$ **max(n) = ?**

A) 18

B) 20

C) 24

D) 26

E) 32

11. $1! + 4! + 7! + \ldots + 2027! = \ldots\ldots x \;\Rightarrow$ **x = ?**

A) 2

B) 3

C) 4

D) 5

E) 6

12.

$$\begin{array}{r|l} 2! + 4! + 6! + \ldots + 2006! & 15 \\ \cline{2-2} - \underline{\qquad\qquad\qquad\qquad\qquad} & \\ k & \end{array} \quad \Rightarrow \textbf{k = ?}$$

A) 7

B) 8

C) 9

D) 10

E) 11

YANITLAR / ANSWERS

1. C	**2. E**	**3. C**	**4. A**	**5. E**	**6. B**	**7. D**	**8. D**
9. C	**10. B**	**11. D**	**12. E**				

1. $(102)_a + (a2)_4 = (x)_{10} \Rightarrow$ **$x = ?$**

A) 15

B) 20

C) 25

D) 30

E) 35

2. $(302)_4 = (x)_5 \Rightarrow$ **$x = ?$**

A) 103

B) 104

C) 200

D) 210

E) 213

3. $(23)_x = (102)_3 \Rightarrow$ **$x = ?$**

A) 4

B) 5

C) 6

D) 7

E) 8

4. $(xyx)_6 = (yyx)_5 \Rightarrow$ **$x + y = ?$**

A) 9

B) 8

C) 7

D) 6

E) 5

5. $(231)_5 + (142)_5 - (204)_5 = (x)_5 \Rightarrow$ **$x = ?$**

A) 210

B) 211

C) 212

D) 213

E) 214

6. $(314)_6 \cdot (52)_6 = (x)_6 \Rightarrow$ **$x = ?$**

A) 24203

B) 25252

C) 25312

D) 25402

E) 31023

7. $(3a)_{a+1} = (4b)_a \Rightarrow \mathbf{b = ?}$

A) 1

B) 2

C) 3

D) 4

E) 5

8. $(xx2)_5 = (yy)_7 \Rightarrow \mathbf{x + y = ?}$

A) 3

B) 4

C) 5

D) 6

E) 7

9. $(34x)_5 < (xxx)_6 \Rightarrow \mathbf{min(x) = ?}$

A) 1

B) 2

C) 3

D) 4

E) 5

10. $3a^8 + 2a^6 + a^2 + 1 = (x)_{a^2}$

$\Rightarrow \mathbf{x = ?}$

A) 3211

B) 30211

C) 32101

D) 32011

E) 32010

11. $(x)_8 = 3\cdot8^3 + 7\cdot8^2 + 43 \Rightarrow \mathbf{x = ?}$

A) 3643

B) 3673

C) 3726

D) 3753

E) 3761

12. $(12,31)_4 = (x)_{10} \Rightarrow \mathbf{x = ?}$

A) $\frac{35}{4}$

B) $\frac{47}{4}$

C) $\frac{81}{16}$

D) $\frac{105}{16}$

E) $\frac{109}{16}$

YANITLAR / ANSWERS

1. C	**2. C**	**3. A**	**4. E**	**5. E**	**6. B**	**7. C**	**8. C**
9. C	**10. D**	**11. D**	**12. E**				

RASYONEL ve ONDALIK SAYILAR
RATIONAL and DECIMAL NUMBERS

BÖLÜM
CHAPTER **2**

RASYONEL SAYILAR / RATIONAL NUMBERS

Kesir: a ve b tamsayı ve $b \neq 0$ olmak üzere $\frac{a}{b}$ ifadesi bir kesirdir.

Fraction: *Provided that a and b are integers and* $b \neq 0$*, the expression* $\frac{a}{b}$ *is called a fraction.*

Rasyonel Sayı: Birbirine denk kesirlerin oluşturduğu her kümenin temsilcisine rasyonel sayı denir. Bu rasyonel sayıların oluşturduğu kümeye **rasyonel sayılar kümesi** denir ve Q ile gösterilir.

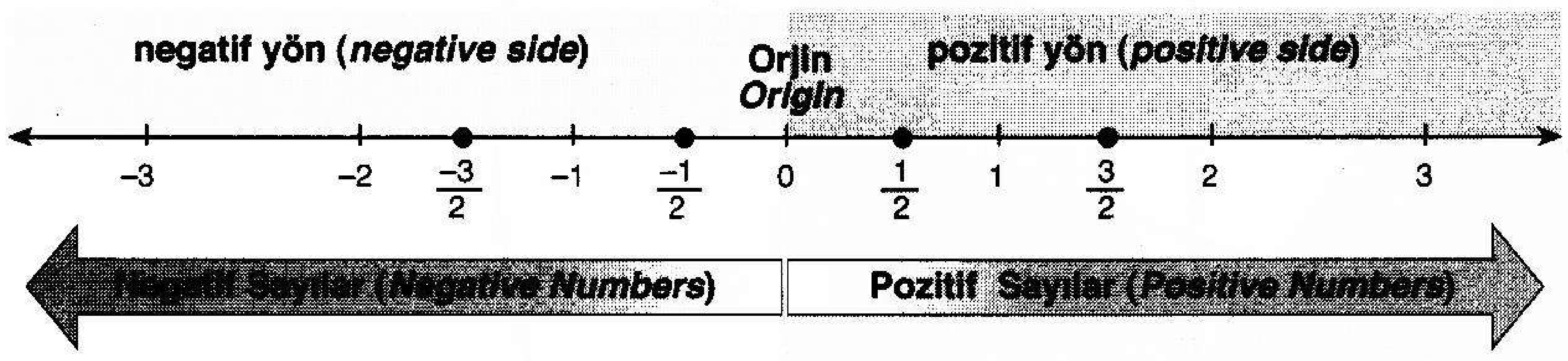
Rational Number: *Any number that can be represented by a fraction* $\frac{a}{b}$*, where* $a, b \in Z$ *and* $b \neq 0$*, is a rational number. The set of rational number is shown by the letter Q.*

NOT / NOTE

*$\frac{a}{b}$ kesrinde, a sayısına **pay**, b sayısına **payda**, aradaki çizgiye **kesir çizgisi** denir*

*In fraction $\frac{a}{b}$ a is **numerator**, b is **denominator** and the center line is called **fraction bar**.*

kesir çizgisi ← $\frac{a}{b}$ $\longrightarrow$ pay / numerator
fraction bar $\quad$ b $\longrightarrow$ payda / denominator


negatif yön (negative side)
Orjin
Origin
pozitif yön (positive side)
-3
-2
-3/2
-1
-1/2
0
1/2
1
3/2
2
3
Negatif Sayılar (Negative Numbers)
Pozitif Sayılar (Positive Numbers)

- ▼ **0 ın sağındaki rasyonel sayılara pozitif rasyonel sayılar denir.**

  The rational numbers at the right side of zero are called positive rational numbers.

$$Q^{+} = \left\{ \ldots\ldots\frac{1}{4},\ \frac{1}{2},\ldots\ldots1\ldots\frac{3}{2}\ldots2,\ldots3,\ldots\ldots \right\}$$

- ▼ **0 ın solundaki rasyonel sayılara negatif rasyonel sayılar denir.**

  The rational numbers at the left side of zero are called negative rational numbers.

$$Q^{-} = \left\{ \ldots\ldots,-4,-3,-2,-\frac{3}{2},-1,-\frac{1}{2}\ldots\ldots \right\}$$

- ▼ **Her tamsayı bir rasyonel sayıdır.**

  Every integer is a rational number.

  $3=\frac{3}{1},\ 4=\frac{4}{1},\ \ldots\ldots\ldots$

  O halde, $N \subset Z \subset Q$ dür.

  Therefore, $N \subset Z \subset Q$*.*

RASYONEL SAYILAR KÜMESİNDE İŞLEMLER
OPERATIONS in the SET of RATIONAL NUMBERS

NOTLAR / NOTES

✓ $a\frac{b}{c} = a + \frac{b}{c} = \frac{a \cdot c + b}{c}$

✓ $-a\frac{b}{c} = -\left(a + \frac{b}{c}\right) = -\frac{a \cdot c + b}{c}$

TOPLAMA ve ÇIKARMA İŞLEMİ / ADDITION and SUBTRACTION

Paydaları eşit olan iki kesir toplanırken veya çıkarılırken payların toplamı paya, ortak payda da paydaya yazılır. Bunun için aşağıdaki genel kural yazılabilir.

To add or subtract rational numbers with the same denominators, add the numerators and write the sum or difference as the numerator. Then we write the common denominator as the denominator. We can give the following general rule:

$$a, b, c, d \in Z \text{ ve(and) } b, d \neq 0 \Rightarrow \frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d \pm b \cdot c}{b \cdot d}$$

ÇARPMA ve BÖLME İŞLEMİ / MULTIPLICATION and DIVISION

Çarpma işleminde iki rasyonel sayının çarpımı bulunurken, payların çarpımı pay,a paydaların çarpımı ise paydaya yazılır.

Bölmede ise birinci kesrin aynısı yazılıp, ikinci kesir ters çevrilir ve işlem çarpmaya dönüştürülür.

To multiply two fractions multiply their numerators and denominators. To divide any number by a fraction, multiply the first fraction by the reciprocal of the second fraction.

☐ $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ ☐ $\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

☐ $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$ ☐ $\frac{1}{\frac{a}{b}} = \frac{b}{a}$

PEKİŞTİRME / REINFORCE

$\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{3}\right) = ?$

YANIT / ANSWER : 3 / 4

ÖRNEK / EXAMPLE 1

$$\frac{1}{5} - \left(\frac{1}{5} - \frac{1}{6}\right) - \left(\frac{1}{4} + \frac{1}{6}\right) - \left(-\frac{5}{4}\right) = ?$$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{1}{5} - \left(\frac{1}{5} - \frac{1}{6}\right) - \left(\frac{1}{4} + \frac{1}{6}\right) - \left(-\frac{5}{4}\right) = \frac{1}{5} - \frac{1}{5} + \frac{1}{6} - \frac{1}{4} - \frac{1}{6} + \frac{5}{4}$$

$$= -\frac{1}{4} + \frac{5}{4}$$

$$= \frac{4}{4} = 1$$

ÖRNEK / EXAMPLE 2

$$\frac{4}{7}-\left(\frac{2}{5}+\frac{4}{7}\right)+\frac{9}{5}:4=?$$

A) $\frac{1}{9}$ B) $\frac{1}{12}$ C) $\frac{1}{18}$ D) $\frac{1}{20}$ E) $\frac{1}{27}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{aligned}\frac{4}{7}-\left(\frac{2}{5}+\frac{4}{7}\right)+\frac{9}{5}:4&=\frac{4}{7}-\frac{2}{5}-\frac{4}{7}+\frac{9}{20}\\&=\frac{9}{20}-\frac{2}{5}\\&=\frac{1}{20}\end{aligned}$$

ÖRNEK / EXAMPLE 3

$$2\frac{1}{3}\cdot\frac{1}{4}+\frac{2}{3}:\frac{3}{4}-\frac{1}{3}\cdot 5=?$$

A) $-\frac{7}{36}$ B) $-\frac{5}{36}$ C) $-\frac{1}{12}$ D) $-\frac{1}{36}$ E) $\frac{1}{36}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\begin{aligned}2\frac{1}{3}\cdot\frac{1}{4}+\frac{2}{3}:\frac{3}{4}-\frac{1}{3}\cdot 5&=\frac{7}{3}\cdot\frac{1}{4}+\frac{8}{9}-\frac{5}{3}\\&=\underset{(3)}{\frac{7}{12}}+\underset{(4)}{\frac{8}{9}}-\underset{(12)}{\frac{5}{3}}\\&=\frac{21+32-60}{36}\\&=-\frac{7}{36}\end{aligned}$$

PEKİŞTİRME / REINFORCE

$$\left(2\frac{1}{3}-1\frac{1}{2}\right)\cdot 12=?$$

YANIT / ANSWER : 10

ZİNCİRLEME KESİRLER / CONTINUED FRACTIONS

a ≠ 0, a bir tamsayı olmak üzere, e ≠ 0

a is an integer and a ≠ 0, e ≠ 0

$$x = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e}}}}$$

biçiminde tanımlanan kesirlere *sonlu zincirleme kesirler* denir.

The fractions defined as are called *finite continued fractions.*

a ≠ 0, k ≠ 0 ve a, k ∈ Z^+ olmak üzere,

a. k ∈ Z^+ and a ≠ 0, k ≠ 0.

$$x = a + \cfrac{k}{a + \cfrac{k}{a + \cfrac{k}{\vdots}}}$$

biçiminde tanımlanan kesirlere *sonsuz zincirleme kesirler* denir.

The fractions defined as are called *infinite continued fractions.*

ÖRNEK / EXAMPLE 4

$$\frac{1}{8} - \cfrac{1}{1 + \cfrac{1}{2 - \cfrac{1}{3}}} = ?$$

A) $-\frac{1}{2}$ B) -1 C) $-\frac{3}{2}$ D) -2 E) $-\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{1}{8} - \cfrac{1}{1 + \cfrac{1}{2 - \cfrac{1}{3}}} = \frac{1}{8} - \cfrac{1}{1 + \cfrac{1}{\frac{5}{3}}}$$

$$= \frac{1}{8} - \cfrac{1}{1 + \frac{3}{5}}$$

$$= \frac{1}{8} - \frac{5}{8}$$

$$= -\frac{4}{8} = -\frac{1}{2}$$

PEKİŞTİRME / REINFORCE

$$1 + \cfrac{1}{3 + \cfrac{1}{2}} = ?$$

YANIT / ANSWER : 9/7

ÖRNEK / EXAMPLE 5

$$6+\cfrac{x}{6+\cfrac{x}{6+\cfrac{x}{\vdots}}}=11 \Rightarrow x=?$$

A) 22 B) 33 C) 44 D) 55 E) 66

ÇÖZÜM / SOLUTION

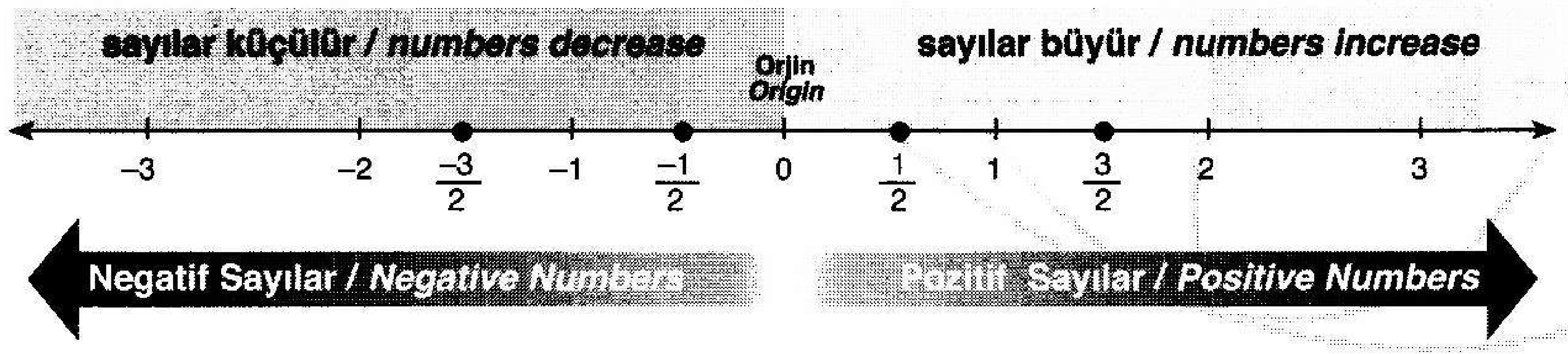
Yanıt / Answer **D**

$$6+\cfrac{x}{\boxed{6+\cfrac{x}{6+\cfrac{x}{\vdots}}}\longrightarrow 11}=11$$

$$\Rightarrow 6+\frac{x}{11}=11$$

$$x=55$$

RASYONEL SAYILARIN SIRALANMASI
THE ORDER of RATIONAL NUMBERS



sayılar küçülür / *numbers decrease*
Orjin
Origin
sayılar büyür / *numbers increase*
–3
–2
$\frac{-3}{2}$
–1
$\frac{-1}{2}$
0
$\frac{1}{2}$
1
$\frac{3}{2}$
2
3
Negatif Sayılar / *Negative Numbers*
Pozitif Sayılar / *Positive Numbers*

Sayı doğrusu üzerinde, 0 dan sağa doğru uzaklaştıkça sayıların değeri büyür. 0 dan sola doğru uzaklaştıkça sayıların değeri küçülür.

On the number line the values of numbers increase by going to the right from zero and decrease by going to the left from zero.

SIRALAMANIN ÖZELLİKLERİ / PROPERTIES of ORDER

1. Negatif bir rasyonel sayı, pozitif bir rasyonel sayıdan daha küçüktür.

A negative rational number is smaller than a positive one.

$-\frac{5}{6}<\frac{5}{6},\ -\frac{3}{4}<\frac{3}{4}\ \ldots$

2. Paydaları eşit olan iki pozitif rasyonel sayıdan, payı küçük olan daha küçüktür.

If the fractions have the same positive denominator, the fraction with the small numerator is smaller.

$\frac{3}{4} < \frac{5}{4}, \frac{2}{5} < \frac{3}{5} \ldots$

3. Payları eşit olan pozitif iki rasyonel sayıdan, paydası küçük olan sayı daha büyüktür.

The fractions have the same numerator and the denominators are positive, the fraction with the smaller denominator is bigger.

$\frac{4}{3} < \frac{4}{2}, \frac{3}{4} < \frac{3}{2} \ldots$

4. Payı ile paydası arasındaki fark aynı olan, pozitif basit kesirlerden payı küçük olan daha küçüktür.

In positive proper fractions whose differences between numerators and denominators are same, the fraction with smaller numerator is the smallest fraction.

$\frac{23}{27} < \frac{25}{29} < \frac{37}{41},$

$\frac{1}{2} < \frac{15}{16} < \frac{51}{52}.$

5. Payı ile paydası arasındaki fark aynı olan, pozitif bileşik kesirlerden payı büyük olan daha küçüktür.

In positive improper fractions whose differences between numerators and denominators are same, the fraction with bigger numerator is the smaller fraction.

$\frac{36}{31} < \frac{24}{19} < \frac{12}{7},$

$\frac{50}{49} < \frac{25}{24} < \frac{3}{2}.$

ÖRNEK / EXAMPLE 6

$$\left.\begin{aligned} a &= \frac{2}{3} \\ b &= \frac{4}{5} \\ c &= \frac{1}{9} \end{aligned}\right\} \Rightarrow \; ? < ? < ?$$

A) c < a < b
B) a < b < c
C) a < c < b
D) c < b < a
E) b < a < c

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\left.\begin{aligned} a &= \frac{2\cdot 15}{3\cdot 15} = \frac{30}{45} \\ b &= \frac{4\cdot 9}{5\cdot 9} = \frac{36}{45} \\ c &= \frac{1\cdot 5}{9\cdot 5} = \frac{5}{45} \end{aligned}\right\} \Rightarrow c < a < b$$

ÖRNEK / EXAMPLE 7

$$\frac{5}{6} < \frac{x}{9} \le \frac{9}{8}, \quad x \in Z^{+} \quad \Rightarrow \quad \sum x = ?$$

A) 24
B) 25
C) 26
D) 27
E) 28

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{5}{6} < \frac{x}{9} \le \frac{9}{8} \Rightarrow \frac{60}{72} < \frac{8x}{72} \le \frac{81}{72}$$

$$\Rightarrow 60 < 8x \le 81$$

$x \in Z^{+} \Rightarrow x_1 = 8,\ x_2 = 9,\ x_3 = 10$

$\sum x = 8 + 9 + 10 = 27$

PEKİŞTİRME / REINFORCE

$$x = \frac{1}{3},\ y = \frac{1}{6},\ z = \frac{1}{4} \quad \Rightarrow \; ? < ? < ?$$

YANIT / ANSWER : $x > z > y$

ONDALIK SAYILAR / DECIMAL NUMBERS

Tanım: Paydası 10, 100, 1000 gibi 10 un kuvveti olan kesirlere *ondalık kesir*, bu kesrin belirttiği sayıya da *ondalıklı sayı* denir.

Definition: *The fractions whose denominators are power of 10 like 10, 100, 1000 are called decimal fractions and the values of these fractions are called decimal numbers.*

BİR RASYONEL SAYIYI ONDALIKLI SAYIYA ÇEVİRME
CONVERTING a RATIONAL NUMBER into a DECIMAL NUMBER

Bir rasyonel sayıyı, ondalıklı sayıya çevirirken, aşağıdaki iki kuraldan biri uygulanır:

To convert a rational number into a decimal number we use either of the two following rules:

1. Paydaki sayı, paydadaki sayıya bölünür.
   Divide the numerator into the denominator.
2. Paydadaki sayı, 10 un kuvveti olarak yazıldıktan sonra ondalık sayıya çevrilir.
   After writing denominator as a power of 10 convert it into decimal number.

ONDALIKLI SAYILARDA İŞLEMLER
OPERATIONS in DECIMAL NUMBERS

TOPLAMA İŞLEMİ / ADDITION

Ondalık sayıları toplamak için / *To add a collection of decimals;*

✓ Ondalık virgülünün sağındaki her bir sütunda yer alan her sayı için ondalık virgülünün sağına sıfır eklenir.
Add zeros to the right of the decimal comma for all of the number having an entry in each column on the right of the decimal comma.

✓ Sayılar toplandıktan sonra ondalık virgülü yerleştirilir.
After adding the numbers a decimal comma is placed.

ÇIKARMA İŞLEMİ / SUBTRACTION

Ondalıklı sayılarda çıkarma işlemi yapılırken / *To subtract one decimal from another;*

- ✓ Eksilen sayının, çıkan ondalıklı kesire göre eksik basamağı varsa, yerine sıfır yazılır.
  If the minuend number has deficient places according to the subtrahend decimal fraction, add zeros instead.
- ✓ Bulunan fark, virgüllerin hizasından yine virgülle ayrılır.
  The difference is seperated with a comma from the comma by.

ÇARPMA İŞLEMİ / MULTIPLICATION

Ondalıklı sayılarda çarpma yapılırken, işlem virgül yokmuş gibi yapılır. İşlemin sonunda, çarpılan sayıların virgülden sonraki basamak sayılarının toplamı kadar, sağdan sola doğru virgül ile ayrılır.

In the multiplication of decimal numbers, do the multiplication like there is no comma. At the end of the operation, the result is seperated with a comma from right to left as much as sum of the numbers of the digits after comma in multiplicand number.

ÖRNEK / EXAMPLE 8

$$\frac{4{,}8}{0{,}016} - \frac{0{,}51}{0{,}017} + \frac{0{,}4}{0{,}04} = ?$$

A) 280 B) 290 C) 300 D) 320 E) 350

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{4{,}8}{0{,}016} - \frac{0{,}51}{0{,}017} + \frac{0{,}4}{0{,}04} = \frac{4800}{16} - \frac{510}{17} + \frac{40}{4}$$

$$= 300 - 30 + 10$$

$$= 280$$

PEKİŞTİRME / REINFORCE

$$\frac{2{,}13}{0{,}01} - \frac{0{,}13}{0{,}01} = ?$$

YANIT / ANSWER : 200

ÖRNEK / EXAMPLE 9

$a, b, c \in N^+$,

$$3{,}12 = \frac{a}{100} + \frac{b}{20} + c \Rightarrow \min(a+b+c) = ?$$

A) 3 B) 5 C) 7 D) 9 E) 11

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$3{,}12 = \frac{a}{100} + \frac{b}{20} + c$$

$$\frac{312}{100} = \frac{a}{100} + \frac{5b}{100} + \frac{100c}{100}$$

$$312 = a + 5b + 100c$$

$$\Rightarrow c = 3\ ,\ b = 2\ ,\ a = 2$$

$$\min(a+b+c) = 2+2+3 = 7$$

PEKİŞTİRME / REINFORCE

$$\frac{0{,}05}{0{,}01} - \frac{0{,}04}{0{,}02} = ?$$

YANIT / ANSWER : 3

ÖRNEK / EXAMPLE 10

$$\frac{0{,}3}{\frac{3}{0{,}6} - \frac{0{,}2}{0{,}04} + \frac{0{,}06}{0{,}012}} = ?$$

A) 0,006 B) 0,06 C) 0,6 D) 6 E) 60

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{0{,}3}{\frac{3}{0{,}6} - \frac{0{,}2}{0{,}04} + \frac{0{,}06}{0{,}012}} = \frac{0{,}3}{5-5+5}$$

$$= \frac{3}{50} = 0{,}06$$

DEVİRLİ ONDALIK SAYILAR / PERIODIC DECIMALS

Tanım: Bir rasyonel sayı ondalıklı sayı biçiminde yazıldığında virgülden sonra gelen kısımdaki sayılar belli bir rakamdan sonra tekrar ediyorsa bu sayılara *devirli ondalık sayı* denir.

Definition: *When a rational number is written in the decimal form, if the numbers coming after the common repeats infinitely afterwards a certain number, then these numbers are called periodic decimals.*

$$ab,cd\overline{efg} = \frac{abcdefg - abcd}{99900}$$

ÖRNEK / EXAMPLE 11

40 + 4 + 0,4 + 0,04 + 0,004 + ... = ?

A) $\frac{40}{9}$ B) $\frac{44}{9}$ C) $\frac{400}{9}$ D) $\frac{404}{9}$ E) $\frac{440}{9}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$40+4+0,4+0,04+0,004+\ldots = 40+4+0,\overline{4}$$

$$= 44+\frac{4}{9} = \frac{400}{9}$$

PEKİŞTİRME / REINFORCE

3 + 0,3 + 0,03 + ... = ?

YANIT / ANSWER : 10 / 3

ÖRNEK / EXAMPLE 12

$2,\overline{3} + 3,\overline{4} + 4,\overline{5} + \ldots + 8,\overline{9} = ?$

A) $\frac{116}{3}$ B) $\frac{119}{3}$ C) $\frac{121}{3}$ D) $\frac{122}{3}$ E) $\frac{124}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$2,\overline{3}+3,\overline{4}+4,\overline{5}+\ldots+8,\overline{9} = \frac{21}{9}+\frac{31}{9}+\frac{41}{9}+\ldots+\frac{81}{9}$$

$$= \frac{357}{9} = \frac{119}{3}$$

ÖRNEK / EXAMPLE 13

$$\frac{0,2}{0,0\overline{2}}+\frac{6}{0,0\overline{3}}=?$$

A) 176 B) 181 C) 189 D) 192 E) 196

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{0,2}{0,0\overline{2}}+\frac{6}{0,0\overline{3}}=\frac{0,2}{\frac{2}{90}}+\frac{6}{\frac{3}{90}}$$

$$=9+180$$

$$=189$$

PEKİŞTİRME / REINFORCE

$$\frac{1,0\overline{3}}{0,0\overline{3}}+\frac{0,0\overline{9}}{0,\overline{03}}=?$$

YANIT / ANSWER : 34,3

ÖRNEK / EXAMPLE 14

$$\left(\frac{16,\overline{27}-15,\overline{17}}{109}\right)^{-1}=?$$

A) 89 B) 90 C) 98 D) 99 E) 100

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\left(\frac{16,\overline{27}-15,\overline{17}}{109}\right)^{-1}=\left(\frac{\frac{1627-16}{99}-\frac{1517-15}{99}}{109}\right)^{-1}$$

$$=\left(\frac{\frac{1611-1502}{99}}{109}\right)^{-1}$$

$$=\left(\frac{\frac{109}{99}}{109}\right)^{-1}$$

$$=\left(\frac{1}{99}\right)^{-1}=99$$

ÖRNEK / EXAMPLE 15

$$\frac{1}{2}-\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{6}\right)=?$$

A) $-\frac{1}{3}$ B) $-\frac{1}{4}$ C) $-\frac{1}{6}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{1}{2}-\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{6}\right)$$

$$=\frac{1}{2}-\left(\frac{1}{6}\right)-\left(\frac{4}{6}\right)$$

$$=\frac{1}{2}-\frac{1}{6}-\frac{4}{6}$$

$$=\frac{-2}{6}$$

$$=\frac{-1}{3}$$

PEKİŞTİRME / REINFORCE

$$2005\frac{1}{2}-2004\frac{1}{3}=?$$

YANIT / ANSWER : 7 / 6

ÖRNEK / EXAMPLE 16

YILDIZ TEKNİK ÜNV. YÖS 2013

$$\frac{99999999}{9999}+\frac{3333}{33}=?$$

A) 1001 B) 10102 C) 10100 D) 11012 E) 10210

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{99999999}{9999}+\frac{3333}{33}=10001+101=10102$$

PEKİŞTİRME / REINFORCE

$$\frac{1}{\frac{1}{2}}+\frac{\frac{1}{2}}{\frac{1}{6}}=?$$

YANIT / ANSWER : 5

ÖRNEK / EXAMPLE 17

KARADENİZ TEKNİK ÜNİVERSİTESİ YÖS 2016

$$\frac{0,7}{0,07}+\frac{3,6}{6}+\frac{3}{20}=?$$

A) 1,05 B) 1,75 C) 10,5 D) 10,75 E) 100,7

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{0,7}{0,07}+\frac{3,6}{6}+\frac{3}{20}=\frac{70}{7}+\frac{36}{60}+\frac{3}{20}$$

$$=10+\frac{3}{5}+\frac{3}{20}$$

$$=\frac{215}{20}=\frac{1075}{100}=10,75$$

ÖRNEK / EXAMPLE 18

EGE ÜNİVERSİTESİ YÖS 2016

$$\frac{\frac{15}{0,3}+\frac{1}{0,1}}{\frac{0,06}{0,003}-\frac{1}{0,2}}=?$$

A) 4 B) $\frac{11}{3}$ C) $\frac{10}{3}$ D) 3 E) $\frac{8}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{\frac{15}{0,3}+\frac{1}{0,1}}{\frac{0,06}{0,003}-\frac{1}{0,2}}=\frac{\frac{150}{3}+\frac{10}{1}}{\frac{60}{3}-\frac{10}{2}}=\frac{60}{15}=4$$

1. $\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{3}+\frac{1}{5}\right)-\left(\frac{1}{2}-\frac{1}{5}\right)=?$

A) $-\frac{1}{2}$ B) $-\frac{1}{3}$ C) $-\frac{1}{4}$

D) $-\frac{1}{5}$ E) $-\frac{1}{6}$

---

2. $\dfrac{\left(3-\frac{1}{2}\right)-\left(5-\frac{3}{2}\right)}{\left(4+\frac{2}{3}\right)-\left(2-\frac{4}{3}\right)}=?$

A) $-\frac{1}{6}$ B) $-\frac{1}{5}$ C) $-\frac{1}{4}$

D) $-\frac{1}{3}$ E) $-\frac{1}{2}$

---

3. $3-\dfrac{6}{1+\dfrac{2}{4-\dfrac{2-x}{6}}}=1 \Rightarrow x=?$

A) –16 B) –15 C) –14

D) –13 E) –12

---

4. $1+\dfrac{2-\dfrac{3}{\frac{1}{2}+1}}{3+\dfrac{4}{2-\frac{1}{3}}}=?$

A) –1 B) 1 C) 2

D) 3 E) 4

---

5. $\left(1-\frac{1}{2}\right)\left(2-\frac{2}{3}\right)\left(3-\frac{3}{4}\right)\ldots\ldots\left(6-\frac{6}{7}\right)=?$

A) $\frac{680}{7}$ B) $\frac{690}{7}$ C) $\frac{710}{7}$

D) $\frac{720}{7}$ E) $\frac{730}{7}$

---

6. $a=\frac{2}{13}$, $b=\frac{4}{15}$, $c=\frac{4}{11}$ $\Rightarrow$? < ? < ?

A) a < c < b B) a < b < c C) b < a < c

D) c < b < a E) b < c < a

7. $0{,}04:\left[(0{,}6:0{,}7):\dfrac{3}{14}\right]=?$

A) 0,01 B) 0,02 C) 0,03

D) 0,04 E) 0,05

8. $$\frac{\dfrac{1}{0{,}1}-\dfrac{3}{0{,}5-\dfrac{1}{0{,}5}}}{3+\dfrac{1}{0{,}5}}=?$$

A) $\frac{8}{5}$ B) 2 C) $\frac{12}{5}$

D) $\frac{14}{5}$ E) $\frac{16}{5}$

9. $\dfrac{a}{0{,}03}=\dfrac{b}{0{,}05}=\dfrac{c}{0{,}07} \Rightarrow ?<?<?$

A) c < a < b B) a < b < c C) c < b < a

D) b < a < c E) b < c < a

10. $3{,}\overline{a}+5{,}\overline{a}+7{,}\overline{a}=16 \Rightarrow a=?$

A) 3 B) 4 C) 5

D) 6 E) 7

11. $\dfrac{0{,}2\cdot10^{-4}+0{,}04\cdot10^{-3}}{1{,}2\cdot10^{-5}}=?$

A) 1 B) 2 C) 3

D) 4 E) 5

12. $\dfrac{1{,}\overline{1}+2{,}\overline{2}+3{,}\overline{3}}{0{,}\overline{2}}=?$

A) 40 B) 30 C) 20

D) 10 E) $\frac{1}{10}$

YANITLAR / ANSWERS

1. C 2. C 3. A 4. B 5. D 6. B 7. A 8. C
9. B 10. A 11.E 12. B

1. $\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{3}+\frac{1}{5}\right)-\left(\frac{1}{2}-\frac{1}{5}\right)$

$=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{3}-\frac{1}{5}-\frac{1}{2}+\frac{1}{5}$

$=-\frac{1}{4}$

Yanıt / The Answer is C

2. $\dfrac{\left(3-\frac{1}{2}\right)-\left(5-\frac{3}{2}\right)}{\left(4+\frac{2}{3}\right)-\left(2-\frac{4}{3}\right)}=\dfrac{\frac{5}{2}-\frac{7}{2}}{\frac{14}{3}-\frac{2}{3}}=-\dfrac{1}{4}$

Yanıt / The Answer is C

3. $3-\dfrac{6}{1+\dfrac{2}{4-\dfrac{2-x}{6}}}=1$

$\Rightarrow \dfrac{6}{1+\dfrac{2}{4-\dfrac{2-x}{6}}}=2 \Rightarrow 1+\dfrac{2}{4-\dfrac{2-x}{6}}=3$

$\dfrac{2}{4-\dfrac{2-x}{6}}=2$

$4-\dfrac{2-x}{6}=1$

$\dfrac{2-x}{6}=3$

$2-x=18 \Rightarrow x=-16$

Yanıt / The Answer is A

4. $1+\dfrac{2-\dfrac{3}{\frac{1}{2}+1}}{3+\dfrac{4}{2-\frac{1}{3}}}=1+\dfrac{2-\frac{6}{3}}{3+\frac{12}{5}}$

$=1+\dfrac{0}{\frac{27}{5}}=1$

Yanıt / The Answer is B

5. $\left(1-\frac{1}{2}\right)\left(2-\frac{2}{3}\right)\left(3-\frac{3}{4}\right)\ldots\left(6-\frac{6}{7}\right)=\frac{1}{2}\cdot\frac{4}{3}\cdot\frac{9}{4}\cdot\ldots\cdot\frac{36}{7}$

$=\frac{720}{7}$

Yanıt / The Answer is D

6.

$$\left.\begin{array}{l} a=\frac{2}{13} \Rightarrow a=\frac{4}{26} \\ b=\frac{4}{15} \Rightarrow b=\frac{4}{15} \\ c=\frac{4}{11} \Rightarrow c=\frac{4}{11} \end{array}\right\} \Rightarrow c>b>a$$

Yanıt / The Answer is B

7. $0{,}04:\left[(0{,}6:0{,}7):\dfrac{3}{14}\right]=0{,}04:\left[\dfrac{6}{7}\cdot\dfrac{14}{3}\right]$

$=0{,}04:4$

$=0{,}01$

Yanıt / The Answer is A

8. $$\frac{\dfrac{1}{0{,}1}-\dfrac{3}{0{,}5-\dfrac{1}{0{,}5}}}{3+\dfrac{1}{0{,}5}}=\frac{10-\dfrac{3}{0{,}5-2}}{3+2}=\frac{10+2}{5}=\frac{12}{5}$$

Yanıt / The Answer is C

9. $\dfrac{a}{0{,}03}=\dfrac{b}{0{,}05}=\dfrac{c}{0{,}07}$

$\dfrac{a}{\frac{3}{100}}=\dfrac{b}{\frac{5}{100}}=\dfrac{c}{\frac{7}{100}}$

$\dfrac{100a}{3}=\dfrac{100b}{5}=\dfrac{100c}{7}$

$\Rightarrow a<b<c$

Yanıt / The Answer is B

10. $3,\overline{a}+5,\overline{a}+7,\overline{a}=16$

$3+5+7+3\cdot(0,\overline{a})=16$

$15+3\cdot\dfrac{a}{9}=16$

$\dfrac{3a}{9}=1$

$a=3$

Yanıt / The Answer is A

11. $\dfrac{0{,}2\cdot10^{-4}+0{,}04\cdot10^{-3}}{1{,}2\cdot10^{-5}}=\dfrac{2\cdot10^{-5}+4\cdot10^{-5}}{1{,}2\cdot10^{-5}}$

$=\dfrac{10^{-5}(2+4)}{10^{-5}(1{,}2)}$

$=\dfrac{6}{1{,}2}=5$

Yanıt / The Answer is E

12. $\dfrac{1,\overline{1}+2,\overline{2}+3,\overline{3}}{0,\overline{2}}=\dfrac{1+2+3+\frac{1}{9}+\frac{2}{9}+\frac{3}{9}}{\frac{2}{9}}$

$=\dfrac{6+\frac{6}{9}}{\frac{2}{9}}$

$=30$

Yanıt / The Answer is B

1. $\dfrac{3}{\frac{2}{3}}-\dfrac{\frac{3}{2}}{3}=?$

Yanıt / The Answer is: **4**

---

2. $5:\dfrac{2+\frac{1}{3}:\frac{2}{3}}{2\frac{1}{4}-1}=?$

Yanıt / The Answer is: $\dfrac{5}{2}$

---

3. $3-\cfrac{1}{1-\cfrac{1}{2-\cfrac{1}{3+\cfrac{1}{2}}}}=?$

Yanıt / The Answer is: $\dfrac{3}{5}$

---

4. $4-\cfrac{x}{4-\cfrac{x}{4-\cfrac{x}{\vdots}}}=5 \Rightarrow x=?$

Yanıt / The Answer is: **–5**

---

5. $A=\dfrac{2}{5}-\dfrac{4}{7}+\dfrac{5}{9}$

$B=-\dfrac{3}{5}+\dfrac{10}{7}-\dfrac{13}{9} \Rightarrow A=?$

Yanıt / The Answer is: **1+ B**

---

6. $\left(2+\frac{1}{2}\right)+\left(3+\frac{1}{2}\right)+\left(4+\frac{1}{2}\right)+\ldots\left(17+\frac{1}{2}\right)=?$

Yanıt / The Answer is: **160**

7. $5+\dfrac{5}{0,05}-\dfrac{0,5}{0,05}=?$

Yanıt / The Answer is: **95**

8. $\dfrac{2,3+0,02-0,305}{0,15-0,01-0,04}=?$

Yanıt / The Answer is: **20,15**

9. $\dfrac{\dfrac{3}{0,01}-\dfrac{2}{0,02}}{\dfrac{3}{0,004}}=?$

Yanıt / The Answer is: $\dfrac{4}{15}$

10. $a \in Z^+$

$\dfrac{1}{3}<\dfrac{a}{8}<\dfrac{7}{6} \quad \Rightarrow \sum a=?$

Yanıt / The Answer is: **42**

11. $\dfrac{x,\overline{x}-y,\overline{y}}{x,\overline{0x}-y,\overline{0y}}=?$

Yanıt / The Answer is: $\dfrac{11}{10}$

12. $a=0,\overline{7}$

$b=0,77$

$c=\dfrac{7}{10} \quad \Rightarrow \; ?<?<?$

Yanıt / The Answer is: $c<b<a$

1. $\frac{2}{3}-\frac{3}{4}:\frac{3}{5}-\frac{1}{6}=?$

A) $-\frac{3}{4}$ B) $-\frac{5}{4}$ C) $-\frac{7}{4}$

D) $-\frac{1}{2}$ E) $-\frac{3}{2}$

2. $\dfrac{\frac{2}{3}-\frac{1}{4}+\frac{3}{7}}{\frac{1}{2}+\frac{1}{7}-\frac{4}{3}}+\dfrac{\frac{5}{4}-\frac{1}{7}-\frac{10}{3}}{\frac{1}{7}-\frac{4}{3}+\frac{1}{2}}=?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1

D) $\frac{3}{2}$ E) 2

3. $\dfrac{\frac{1}{2}-\frac{1}{3}+\frac{1}{6}}{2-\frac{1}{2}-\frac{2}{3}}-\frac{1}{2}=?$

A) $-\frac{1}{12}$ B) $-\frac{1}{10}$ C) $-\frac{1}{9}$

D) $-\frac{1}{6}$ E) $-\frac{1}{3}$

4. $\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)......\left(1+\frac{1}{n}\right)=\frac{29}{2}$

$\Rightarrow$ **n = ?**

A) 27 B) 28 C) 29

D) 30 E) 3

5. $\dfrac{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)......\left(1-\frac{1}{x-1}\right)}{\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)......\left(1+\frac{1}{x-1}\right)}=\frac{1}{15}\quad\Rightarrow x=?$

A) 3 B) 4 C) 5

D) 6 E) 7

6. $5\cdot\left[1-\dfrac{3}{4-\frac{1}{4}}\cdot(-3)\right]=?$

A) 13 B) 14 C) 15

D) 16 E) 17

7. $\left(\dfrac{3}{2-\dfrac{1}{3}}\right)^{-1} : \dfrac{4}{3} + 1\dfrac{1}{2} = ?$

A) $1\frac{1}{12}$ B) $1\frac{5}{12}$ C) $1\frac{7}{12}$ D) $1\frac{3}{4}$ E) $1\frac{11}{12}$

---

8. $\left(\dfrac{1+\dfrac{2}{3}}{2}\right)^{-1} : 2 - \dfrac{4}{\dfrac{2}{3}} = ?$

A) $-\frac{24}{5}$ B) $-\frac{26}{5}$ C) $-\frac{27}{5}$ D) $-\frac{28}{5}$ E) $-\frac{29}{5}$

---

9. $\dfrac{x}{\dfrac{y}{z}} \cdot \dfrac{\dfrac{y}{x}}{z} = ?$

A) 1 B) x C) y D) z E) $\frac{x}{z}$

---

10. $\left.\begin{array}{l} a-b=8 \\ a\cdot b=-2 \end{array}\right\} \Rightarrow \left(\dfrac{1}{a}-\dfrac{1}{b}\right)^{-1} = ?$

A) -4 B) $-\frac{1}{4}$ C) 1 D) $\frac{1}{4}$ E) 4

---

11. $\dfrac{a-3b}{a+3b} = 4 \Rightarrow \dfrac{a}{b} + \dfrac{b}{a} = ?$

A) $-\frac{26}{5}$ B) $-\frac{12}{5}$ C) $\frac{4}{5}$ D) $\frac{16}{5}$ E) $\frac{26}{5}$

---

12. $\left.\begin{array}{l} x = a-\dfrac{1}{b} \\ y = b-\dfrac{1}{a} \end{array}\right\} \Rightarrow \dfrac{x}{y} = ?$

A) -1 B) 1 C) $\frac{a}{b}$ D) $\frac{b}{a}$ E) a·b

YANITLAR / ANSWERS

1. A	2. E	3. B	4. B	5. D	6. E	7. E	8. C
9. A	10. D	11. A	12. C				

1. $2-\cfrac{1}{1+\cfrac{3}{4+\cfrac{1}{2}}}=?$

A) $\frac{4}{5}$ B) $\frac{6}{5}$ C) $\frac{7}{5}$ D) $\frac{8}{5}$ E) $\frac{9}{5}$

2. $2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2}}}}=?$

A) $\frac{4}{5}$ B) $\frac{6}{5}$ C) $\frac{7}{5}$ D) $\frac{8}{5}$ E) $\frac{9}{5}$

3. $2+\cfrac{4}{5-\cfrac{6}{2+\cfrac{8}{x}}}=3 \Rightarrow x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $2+\cfrac{3}{2+\cfrac{3}{2+\cfrac{3}{\vdots}}}=?$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

5. $4+\cfrac{4+\cfrac{4+\cfrac{\vdots}{5}}{5}}{5}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{\vdots}}}=\frac{3}{x-1} \Rightarrow x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

7. $\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\left(1-\frac{1}{25}\right)\cdots\left(1-\frac{1}{144}\right)=?$

A) $\frac{1}{3}$ B) $\frac{3}{5}$ C) $\frac{2}{9}$ D) $\frac{5}{18}$ E) $\frac{13}{18}$

8. $\left(1-\frac{1}{3}\right)+\left(2-\frac{1}{3}\right)+\ldots+\left(15-\frac{1}{3}\right)=?$

A) 105 B) 110 C) 115 D) 120 E) 125

9. $A=\frac{1}{2}-\frac{2}{3}+\frac{3}{5}$

$B=\frac{1}{2}+\frac{5}{3}+\frac{2}{5} \Rightarrow A=?$

A) B + 1
B) B – 3
C) 1 – B
D) 3 – B
E) 4 – B

10. $A=\frac{9}{7}+\frac{4}{9}-\frac{5}{11}$

$B=\frac{12}{7}-\frac{13}{9}+\frac{27}{11} \Rightarrow B=?$

A) 1 – A
B) 2 – A
C) 3 – A
D) 4 – A
E) 5 – A

11. $\dfrac{1905\frac{2}{7}-1907\frac{5}{7}}{3+1\frac{6}{7}}=?$

A) $-\frac{1}{2}$ B) -1 C) $-\frac{3}{2}$ D) -2 E) $-\frac{5}{2}$

12. $3+\dfrac{2}{5-\dfrac{x}{3+\dfrac{1}{2-\frac{1}{2}}}}=4 \Rightarrow x=?$

A) 9 B) 11 C) 15 D) 18 E) 21

YANITLAR / ANSWERS

1. C 2. B 3. B 4. E 5. E 6. D 7. E 8. C
9. D 10. D 11. A 12. B

1. $$3-\cfrac{2+\cfrac{1}{2-\cfrac{4}{3}}}{1+\cfrac{1}{3-\cfrac{1}{2}}}=?$$

A) -1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$

D) 1 E) $\frac{3}{2}$

2. $$4+\cfrac{3}{5-\cfrac{4}{x-\cfrac{2}{1+\cfrac{1}{3}}}}=5 \Rightarrow x=?$$

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$

D) $\frac{7}{2}$ E) $\frac{9}{2}$

3. $$1+\cfrac{x}{2+\cfrac{x}{1+\cfrac{x}{2+\cfrac{x}{\vdots}}}}=4 \quad \Rightarrow \quad x=?$$

A) 12 B) 16 C) 18

D) 20 E) 24

4. $$3+\cfrac{3+\cfrac{3+\cfrac{\iddots}{x}}{x}}{x}=5 \Rightarrow x=?$$

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$

D) $\frac{7}{2}$ E) $\frac{9}{2}$

5. $$A=\frac{2}{3}-\frac{7}{4}+\frac{9}{5}$$

$$B=-\frac{1}{3}+\frac{1}{4}+\frac{4}{5} \Rightarrow A=?$$

A) 2 – B

B) 1 – B

C) B

D) 1 + B

E) 2 + B

6. $$8+\cfrac{x}{5+\cfrac{x}{5+\cfrac{x}{\ddots}}}=9 \Rightarrow x=?$$

A) 4 B) 5 C) 6

D) 7 E) 8

7. $\left.\begin{aligned} a &= \frac{3}{7} \\ b &= \frac{2}{13} \\ c &= \frac{4}{17} \end{aligned}\right\} \Rightarrow \mathbf{? < ? < ?}$

A) $b < a < c$
B) $a < b < c$
C) $c < a < b$
D) $a < c < b$
E) $b < c < a$

---

8. $\left.\begin{aligned} a &= -\frac{2}{5} \\ b &= -\frac{3}{11} \\ c &= -\frac{1}{9} \end{aligned}\right\} \Rightarrow \mathbf{? < ? < ?}$

A) $a < b < c$
B) $b < c < a$
C) $c < a < b$
D) $c < b < a$
E) $a < c < b$

---

9. $\left.\begin{aligned} a &= -\frac{5}{6} \\ b &= -\frac{4}{3} \\ c &= -\frac{7}{8} \end{aligned}\right\} \Rightarrow \mathbf{? < ? < ?}$

A) $a < b < c$
B) $c < a < b$
C) $a < c < b$
D) $b < c < a$
E) $b < a < c$

---

10. $a, b, c \in Z^+$

$$\frac{25}{7} = a + \cfrac{1}{b + \cfrac{3}{c}} \Rightarrow \mathbf{a+b+c=?}$$

A) 8
B) 9
C) 10
D) 11
E) 12

---

11. $$\frac{\mathbf{1}}{\mathbf{2}-\frac{\mathbf{2}}{\mathbf{a}}} + \frac{\mathbf{1}}{\frac{\mathbf{2}}{\mathbf{a}}-\mathbf{2}} = \mathbf{?}$$

A) $-a$
B) -1
C) 0
D) 1
E) a

---

12. $$\mathbf{5} + \cfrac{\mathbf{2}}{\mathbf{2} + \cfrac{\mathbf{3}}{\mathbf{2} + \cfrac{\mathbf{3}}{\mathbf{2} + \cfrac{\mathbf{3}}{\ddots}}}} = \mathbf{?}$$

A) $\frac{11}{3}$
B) $\frac{13}{3}$
C) $\frac{14}{3}$
D) $\frac{16}{3}$
E) $\frac{17}{3}$

YANITLAR / ANSWERS

1. C 2. D 3. E 4. C 5. C 6. C 7. E 8. A
9. D 10. A 11. C 12. E

1. $\frac{0,6}{0,12}-\frac{3,6}{0,9}+\frac{0,72}{0,018}=?$

A) 4
B) 12
C) 20
D) 34
E) 41

2. $2^2+\frac{1}{10}+\frac{3^2}{1000}=?$

A) 4,019
B) 4,109
C) 4,901
D) 9,401
E) 1,904

3. $\frac{0,3}{0,03}+\frac{0,03}{0,003}-\frac{0,003}{0,0003}=?$

A) 1
B) 5
C) 10
D) 15
E) 20

4. $\frac{0,63}{0,7}-\frac{3}{0,1}+\frac{0,48}{1,2}=?$

A) –2,17
B) –2,64
C) –25,4
D) –28,7
E) –32,4

5. $\frac{22,22}{0,22}-\frac{1+0,25}{1-1,25}:\frac{0,2}{0,04}=?$

A) 90
B) 94
C) 95
D) 100
E) 102

6. $\frac{\frac{3}{4}+0,75}{0,125+\frac{23}{8}}=?$

A) $\frac{1}{2}$
B) 1
C) $\frac{3}{2}$
D) 2
E) $\frac{5}{2}$

7. $\dfrac{2,\overline{3}+4,\overline{5}+7,\overline{2}}{4,\overline{5}-3,\overline{4}-1,\overline{2}}=?$

A) –134 B) –127 C) –113 D) –105 E) –92

8. $x = 0,4 + 0,04 + 0,004 + \ldots\ldots\ldots$

$y=1+\dfrac{3}{10}+\dfrac{3}{100}+\dfrac{3}{1000}+\ldots\ldots$

$\Rightarrow$ **x + y = ?**

A) 1 B) $\dfrac{13}{9}$ C) $\dfrac{14}{9}$ D) $\dfrac{16}{9}$ E) 2

9. $0,\overline{6}+\dfrac{1}{1-\dfrac{1+0,\overline{6}}{0,\overline{6}-\dfrac{1}{0,\overline{6}}}}=?$

A) -1 B) $-\dfrac{1}{2}$ C) $\dfrac{1}{2}$ D) 1 E) $\dfrac{3}{2}$

10. $\dfrac{0,\overline{2}}{2}-\dfrac{3}{0,\overline{3}}+\dfrac{0,\overline{4}}{4}-\dfrac{5}{0,\overline{5}}+18=?$

A) $\dfrac{2}{9}$ B) $\dfrac{5}{9}$ C) $\dfrac{16}{9}$ D) $\dfrac{49}{9}$ E) $\dfrac{74}{9}$

11. $\dfrac{1}{3\cdot 4}+\dfrac{1}{4\cdot 5}+\ldots+\dfrac{1}{12\cdot 13}=\dfrac{10}{39}$

$\Rightarrow \dfrac{13}{6\cdot 20}+\dfrac{13}{8\cdot 25}+\ldots+\dfrac{13}{24\cdot 65}=?$

A) $\dfrac{1}{2}$ B) $\dfrac{2}{3}$ C) $\dfrac{1}{3}$ D) $\dfrac{2}{7}$ E) $\dfrac{1}{17}$

12. $\dfrac{x,\overline{x}+y,\overline{y}}{x,y+y,x}=?$

A) $\dfrac{98}{99}$ B) 1 C) $\dfrac{100}{99}$ D) $\dfrac{101}{99}$ E) $\dfrac{110}{99}$

YANITLAR / ANSWERS

1. E 2. B 3. C 4. D 5. E 6. A 7. B 8. D
9. D 10. A 11. C 12. C

1. $\dfrac{xy0,xy}{0,xy}+\dfrac{x0,x}{0,0x}-\dfrac{xyx,y}{0,xy}=?$

A) 1 B) 10 C) 11 D) 101 E) 1001

2. $(xy,z + zx,y + yz,x)\cdot a = b$

$b \in Z \Rightarrow$ **min(a) = ?**

A) $\dfrac{11}{10}$ B) $\dfrac{111}{10}$ C) $\dfrac{10}{11}$ D) $\dfrac{10}{111}$ E) $\dfrac{100}{1111}$

3. $\left.\begin{array}{r}\dfrac{1}{x^{-1}}+\dfrac{1}{y^{-1}}=0,\overline{5}\\ x-y=0,\overline{3}\end{array}\right\} \Rightarrow \dfrac{x}{y}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $\dfrac{a,0\overline{a}-b,0\overline{b}}{a,\overline{a}-b,\overline{b}}=?$

A) $\dfrac{33}{100}$ B) $\dfrac{79}{100}$ C) $\dfrac{91}{100}$ D) 1 E) $\dfrac{11}{10}$

5. $a = 3,02$

$b = 3,0\overline{2}$

$c = 3,\overline{02} \Rightarrow$ **? < ? < ?**

A) $a < c < b$ B) $a < b < c$ C) $b < c < a$ D) $c < a < b$ E) $b < a < c$

6. $\dfrac{0,3\overline{4}+0,4\overline{5}}{0,\overline{3}+0,2}\cdot(1-0,6)=?$

A) $\dfrac{5}{2}$ B) $\dfrac{3}{2}$ C) 1 D) $\dfrac{3}{4}$ E) $\dfrac{3}{5}$

7. $3,\overline{x} + 4,\overline{x} + 5,\overline{x} = 13 \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3

D) 4 E) 5

8. $\dfrac{x,yz}{xy,z} + \dfrac{xy,z}{0,xyz} \cdot \dfrac{x,yz}{xyz} = ?$

A) 0,1 B) 0,101 C) 1

D) 1,1 E) 1,11

9. $1,\overline{1} + 2,\overline{2} + 3,\overline{3} + \ldots + 9,\overline{9} = ?$

A) 45 B) 50 C) 55

D) 60 E) 65

10. $x = 0,7 + 0,07 + 0,007 + \ldots\ldots$

$y = 0,5 + 0,05 + 0,005 + \ldots\ldots$

$\Rightarrow \mathbf{x + y = ?}$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1

D) $\frac{4}{3}$ E) $\frac{5}{3}$

11. $\left.\begin{array}{l} a = 4,34\overline{6} \\ b = 4,3\overline{46} \\ c = 4,\overline{346} \end{array}\right\} \Rightarrow \mathbf{? < ? < ?}$

A) b < c < a B) c < b < a C) c < a < b

D) a < c < b E) a < b < c

12. $\dfrac{a,\overline{a} + a,\overline{aa}}{0,\overline{2}} = 10 \Rightarrow \mathbf{a = ?}$

A) 1 B) 2 C) 3

D) 4 E) 5

YANITLAR / ANSWERS							
1. E	**2. D**	**3. D**	**4. C**	**5. A**	**6. E**	**7. C**	**8. D**
9. B	**10. D**	**11. B**	**12. A**				

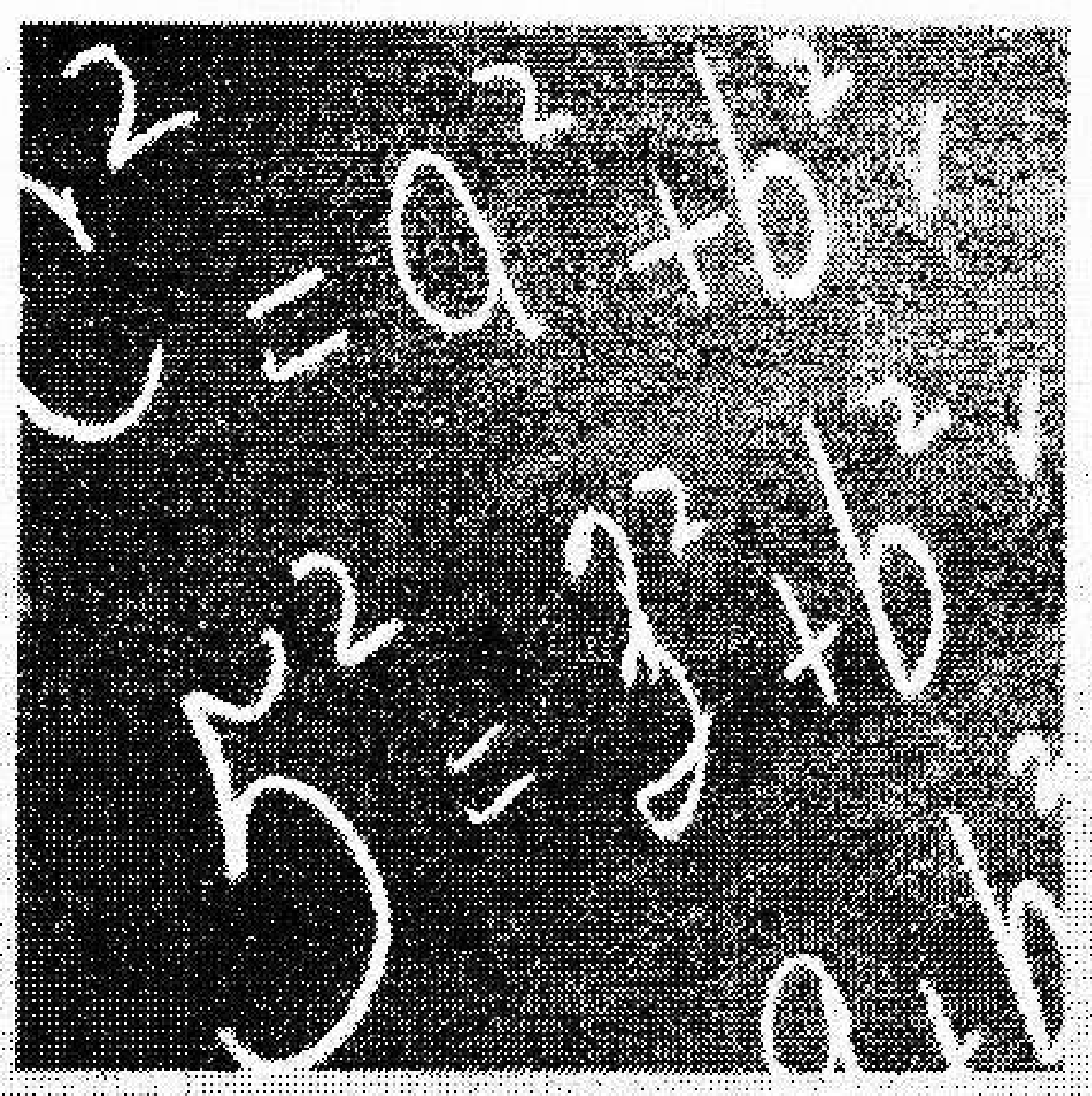
ÜNİTE 2
UNIT

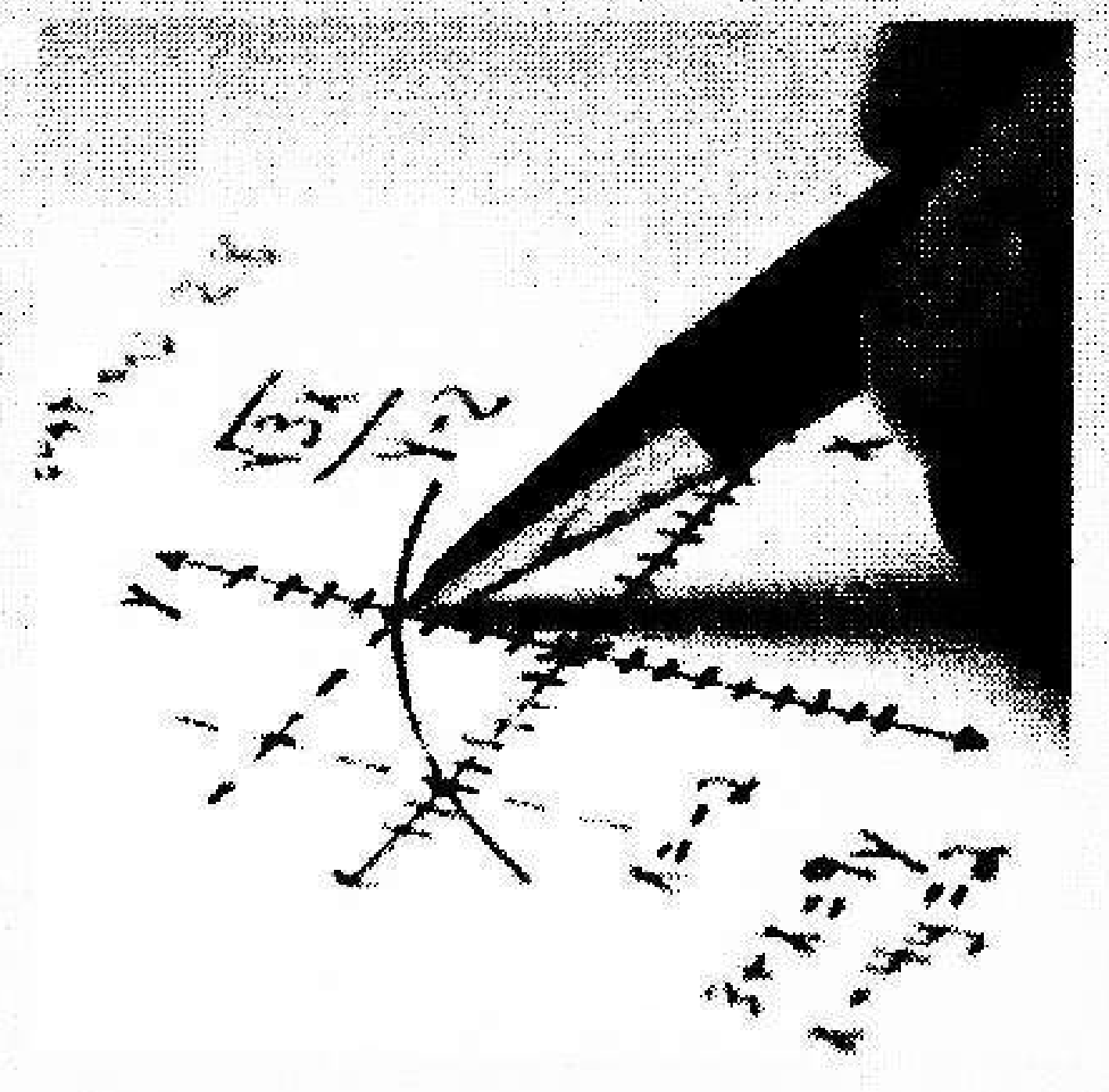
Bu ünitede özdeşlikler ve çarpanlara ayırma ve oran – orantı konusu ayrıntılı anlatılmıştır. Öğrencilerin özdeşlikler bölümünde verilen ifadeleri iyi öğrenmeleri; ikinci dereceden denklem, limit, türev, integral gibi konularda gerekli olacağından önemlidir.

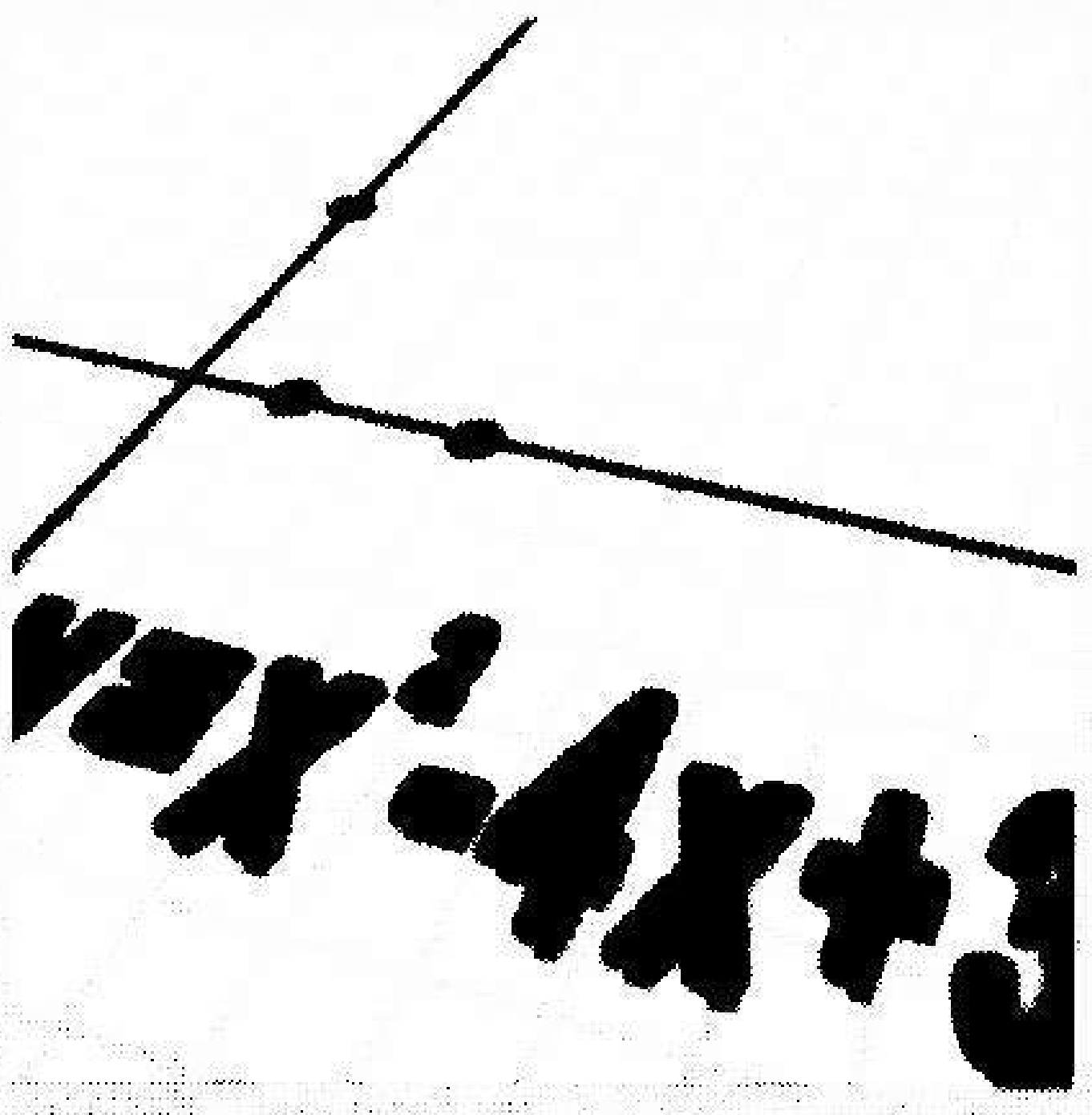
Bu ünite, 67 çözümlü örnek, 24 alıştırma, 96 test sorusu içermektedir.

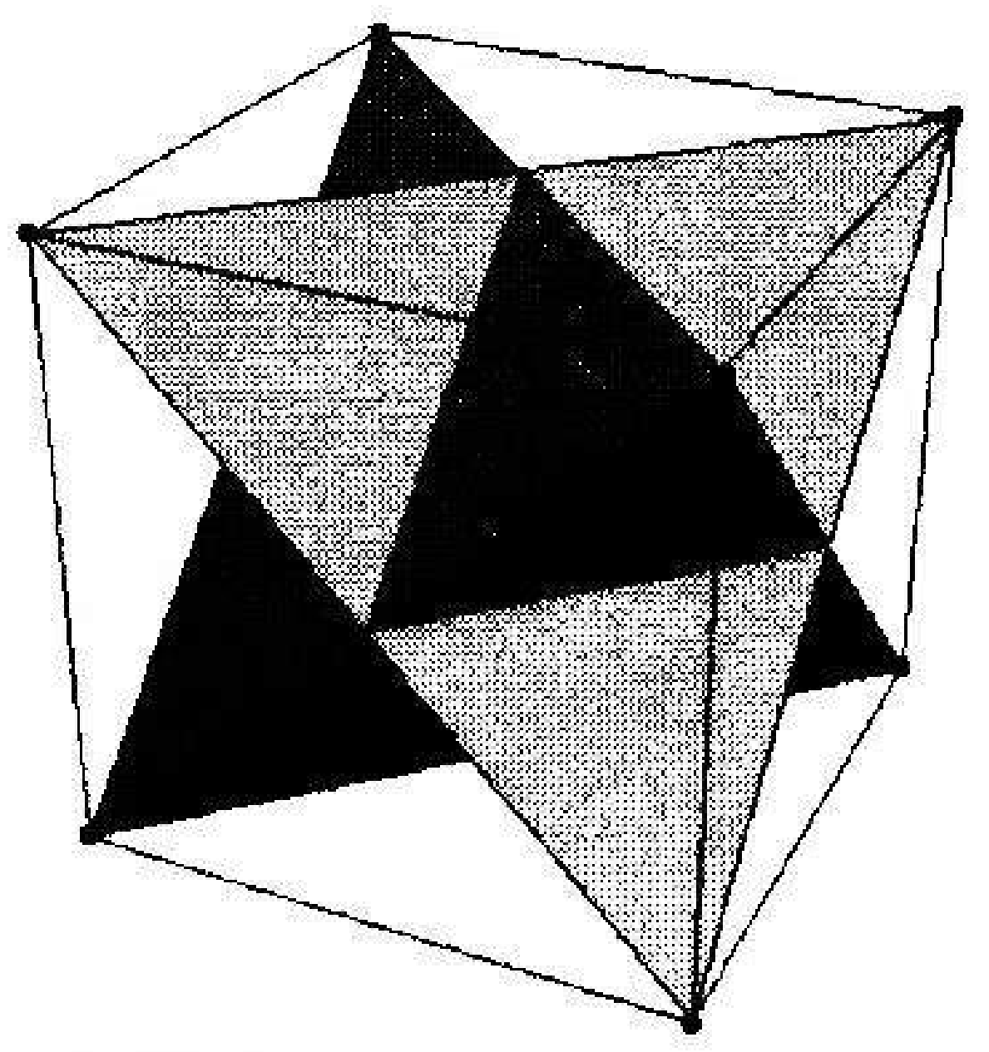
In this unit, identites, factorization and ratio-proportion are explained in detailed. Expressions with identity are very important. Because these expressions are necessary in quadratic equations, limit, derivative, integral.

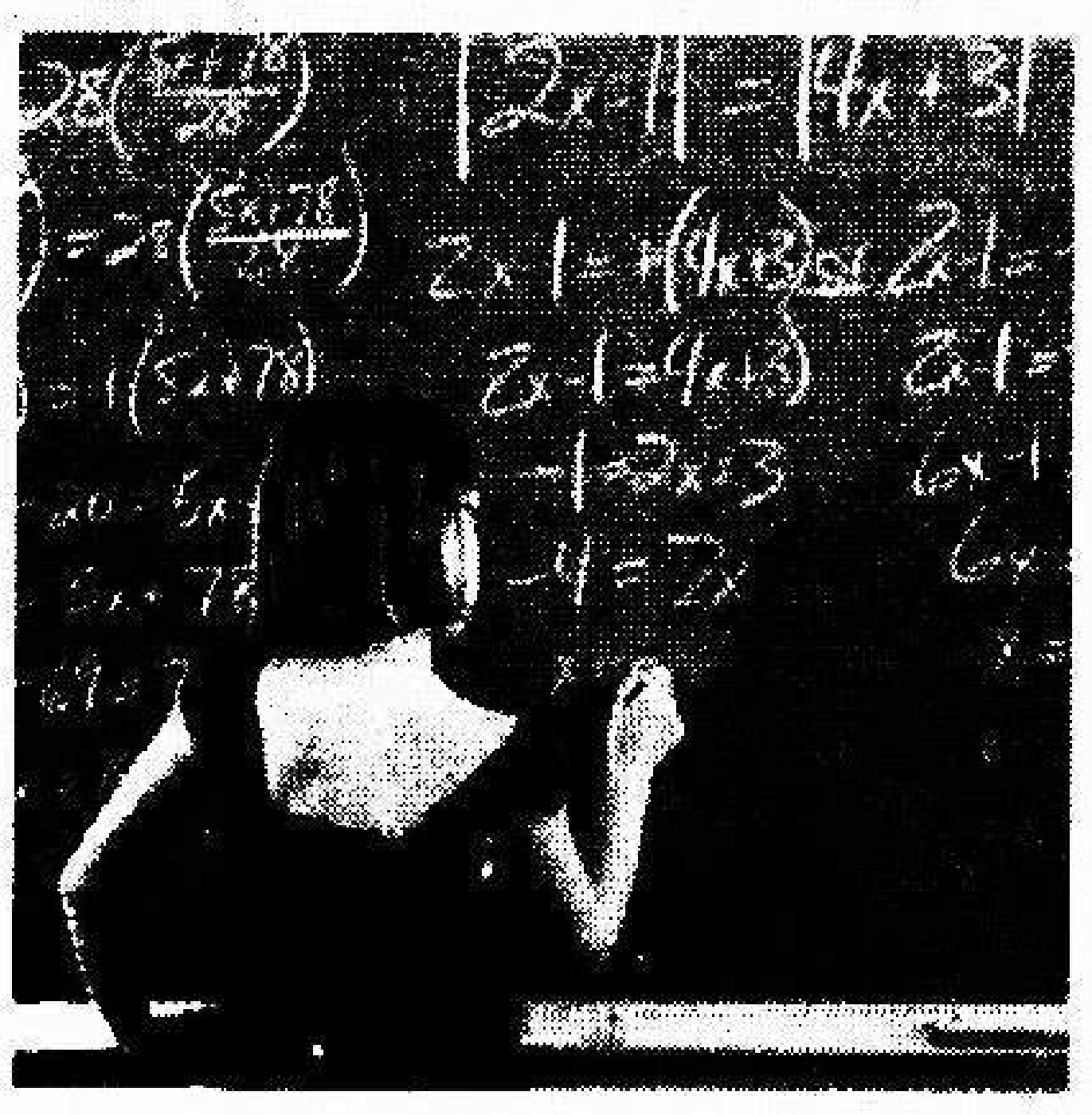
This unit includes; 67 examples, 24 exercises, and 96 test questions.





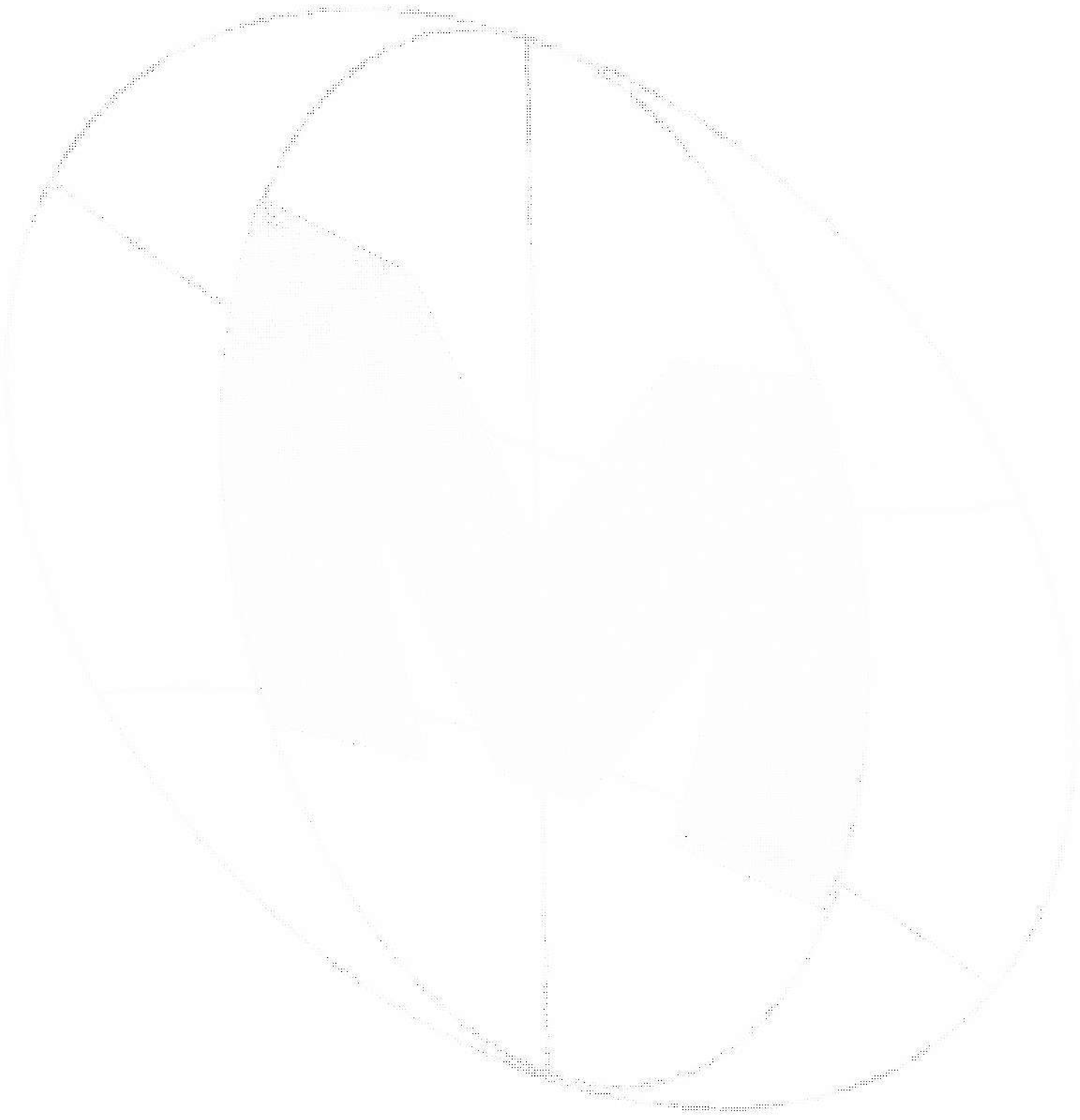






- **ÖZDEŞLİKLER ve ÇARPANLARA AYIRMA**
  IDENTITIES and FACTORIZATION . . . . 71 – 100
- **ORAN ve ORANTI / *RATIOS and PROPORTIONS*** . . . . 101 – 120

BU SAYFADA KONU MATERYALİ YOKTUR.



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ÖZDEŞLİKLER ve ÇARPANLARA AYIRMA
IDENTITIES and FACTORIZATION

BÖLÜM 1
CHAPTER

ÖZDEŞLİKLER / IDENTITIES

Tanım: Bilinmeyenlerinin her gerçek değeri için doğru olan eşitliklere *özdeşlik* denir.

Definition: *The equalities that holds true for all of their unknowns real values are called identities.*

Örnek / *Example:*

In equality, $(x + 3)^2 = x^2 + 6x + 9$ eşitliğinde,

For x = 1 için: $(1 + 3)^2 = 1^2 + 6 \cdot 1 + 9$
$4^2 = 1 + 6 + 9$
$16 = 16$

For x = –2 için: $(-2 + 3)^2 = (-2)^2 + 6\cdot(-2) + 9$
$(1)^2 = 4 - 12 + 9$
$1 = 1$

Burada x'e verilen her değer için eşitlik sağlanır. O halde, bu eşitlik bir özdeşliktir.

For all values assigned to x equality can be obtained. Therefore, this equality is an identity.

A. TAMKARE İFADELER / PERFECT SQUARE EXPRESSIONS

i. $(a + b)^2 = a^2 + 2ab + b^2$
ii. $(a - b)^2 = a^2 - 2ab + b^2$

yukarıdaki ifadelere *tam kare ifadeler* denir.

These are called the *perfect square expressions* or *the squares of the sum and difference.*

Sonuç / Conclusion 1

- $(a + b)^2 = a^2 + 2ab + b^2 \Rightarrow \mathbf{a^2 + b^2 = (a + b)^2 - 2ab}$
- $(a - b)^2 = a^2 - 2ab + b^2 \Rightarrow \mathbf{a^2 + b^2 = (a - b)^2 + 2ab}$

Sonuç / Conclusion 2

- $\left(ax + \frac{b}{x}\right)^2 = a^2x^2 + \frac{b^2}{x^2} + 2ab$
- $\left(ax - \frac{b}{x}\right)^2 = a^2x^2 + \frac{b^2}{x^2} - 2ab$

AÇIKLAMALAR / EXPLANATIONS

✓ **İki sayının toplamının karesi;** birinci sayının karesi, iki sayının çarpımının iki katı, ikinci sayının karesinin toplamına eşittir.

The square of the sum of two *number is equal to the square of the first numbers, plus two times the product of the two numbers, plus the square of the second number.*

✓ **İki sayının farkının karesi;** birinci sayının karesi, iki sayının çarpımının –2 katı, ikinci sayının karesinin toplamına eşittir.

The square of the difference of two *numbers is equal to the square of the first number, minus two times the product of the two numbers, plus the square of the second number.*

ÖRNEK / EXAMPLE 1

a = 2,871,

b = 1,129

$\Rightarrow a^2 + 2ab + b^2 = ?$

A) 2 B) 4 C) 8 D) 16 E) 18

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$= (2{,}871 + 1{,}129)^2$$
$$= 4^2 = 16$$

ÖRNEK / EXAMPLE 2

$$a - \frac{1}{a} = 3 \Rightarrow a^2 + \frac{1}{a^2} = ?$$

A) 7 B) 8 C) 9 D) 10 E) 11

PEKİŞTİRME / REINFORCE

$x - \frac{1}{x} = 4 \Rightarrow x^2 + \frac{1}{x^2} = ?$

YANIT / ANSWER : 18

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$a - \frac{1}{a} = 3 \Rightarrow a^2 - 2 \cdot a\frac{1}{a} + \frac{1}{a^2} = 9$$
$$a^2 - 2 + \frac{1}{a^2} = 9$$
$$a^2 + \frac{1}{a^2} = 11$$

Sonuç / Conclusion 3

- $(a + b)^2 = (a - b)^2 + 4ab$
- $(a - b)^2 = (a + b)^2 - 4ab$

ÖRNEK / EXAMPLE 3

$a > b$, $a + b = 4\sqrt{3}$,

$a \cdot b = 3 \Rightarrow \mathbf{a - b = ?}$

A) 3 B) 6 C) 9 D) 12 E) 15

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned}(a - b)^2 &= (a + b)^2 - 4ab\\ &= (4\sqrt{3})^2 - 4\cdot 3\\ &= 48 - 12\\ &= 36\end{aligned}$$

$(a - b)^2 = 36,\ a > b \Rightarrow a - b = 6$

Sonuç / Conclusion 4

✓ $(a + b)^2 = a^2 + b^2 + 2ab$ özdeşliği, üç terimli genişletilirse aşağıdaki sonuç bulunur.

The square of the trinomials up to this point, we have only dealt with the product of two binomials (two term expressions). $(a + b)^2 = a^2 + b^2 + 2ab$

- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2\cdot(a\cdot b + a\cdot c + b\cdot c)$

ÖRNEK / EXAMPLE 4

$$\left.\begin{aligned}a^2 + b^2 + c^2 = 12\\ ab - bc - ac = 2\end{aligned}\right\} \Rightarrow \mathbf{max(a + b - c) = ?}$$

A) –4 B) –2 C) 2 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{aligned}(a + b - c)^2 &= a^2 + b^2 + c^2 + 2(ab - ac - bc)\\ &= 12 + 2\cdot(2) = 16\end{aligned}$$

$(a + b - c)^2 = 16 \Rightarrow a + b - c = \mp 4$

$\Rightarrow \max(a + b - c) = 4$

Sonuç / Conclusion 5

- $a^2 + b^2 = 0 \Rightarrow a = 0,\ b = 0$

ÖRNEK / EXAMPLE 5

$x^2 + y^2 - 2x + 6y + 10 = 0 \Rightarrow \mathbf{x - y = ?}$

A) –2 B) 0 C) 2 D) 4 E) 6

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$x^2 + y^2 - 2x + 6y + 10 = (x^2 - 2x + 1) + (y^2 + 6y + 9)$

$(x - 1)^2 + (y + 3)^2 = 0$

$\Rightarrow x = 1,\ y = -3$

$x - y = 1 - (-3) = 4$

B. İKİ KARE FARKI İFADELER
DIFFERENCE of two SQUARES EXPRESSIONS

$x^2 - y^2 = (x + y)(x - y)$ ifadesine *iki kare farkı* denir. İki sayının toplamı ile farkının çarpımı, birinci sayının karesi ile ikinci sayının karesinin farkına eşittir.

$x^2 - y^2 = (x + y)(x - y)$ is called difference of two squares. The product of the sum and difference of two numbers is the square of the first number, minus the square of the second number.

Örneğin, aşağıdaki ifade iki kare farkıdır.

For example; the expression below is a difference of two square.

$(x + 3)\ (x - 3) = x^2 - 3x + 3x - 9 = x^2 - 9$

ÖRNEK / EXAMPLE 6

$$\left.\begin{aligned} x - y &= 16 \\ \sqrt{x} - \sqrt{y} &= 2 \end{aligned}\right\} \Rightarrow \mathbf{x = ?}$$

A) 1 B) 4 C) 9 D) 16 E) 25

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x - y = 16$ [Verilen eşitlik / *Given equality*]

$(\sqrt{x} - \sqrt{y}) \cdot (\sqrt{x} + \sqrt{y}) = 16$ [İki kare farkı / *Difference of two squares*]

$(\sqrt{x} + \sqrt{y}) \cdot 2 = 16$ [$\sqrt{x} - \sqrt{y}$ için 2 yaz. / *Substitute 2 for $\sqrt{x} - \sqrt{y}$.*]

$\sqrt{x} + \sqrt{y} = 8$ [Sonuç / *Conclusion*]

$$\left.\begin{aligned} \sqrt{x} - \sqrt{y} &= 2 \\ \sqrt{x} + \sqrt{y} &= 8 \end{aligned}\right\} \Rightarrow \begin{aligned} 2\sqrt{x} &= 10 \\ \sqrt{x} &= 5 \Rightarrow x = 25 \end{aligned}$$

NOT / NOTE

x ve y pozitif tamsayılar, a bir asal sayıdır.

x and y are positive integers, a is a prime number.

$x^2 - y^2 = a \Rightarrow x + y = a,\ x - y = 1$

PEKİŞTİRME / REINFORCE

$x, y \in \mathbb{N}$,

$x^2 - y^2 = 13 \Rightarrow x = ?$

YANIT / ANSWER : 7

C. İKİ KÜP TOPLAMI ve FARKI
SUM and DIFFERENCE of TWO CUBES

$a^3 - b^3 = (a - b) \cdot (a^2 + ab + b^2)$

$a^3 + b^3 = (a + b) \cdot (a^2 - ab + b^2)$

ÖRNEK / EXAMPLE 7

$$\frac{x\sqrt{x} + y\sqrt{y}}{x + y - \sqrt{xy}} = ?$$

A) $x + y$ B) $\sqrt{xy}$ C) $\sqrt{x} + \sqrt{y}$ D) xy E) $x - y$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\sqrt{x} = a \Rightarrow x = a^2$

$\sqrt{y} = b \Rightarrow y = b^2$

$$\frac{x\sqrt{x} + y\sqrt{y}}{x + y - \sqrt{xy}} = \frac{a^2 \cdot a + b^2 \cdot b}{a^2 + b^2 - a \cdot b}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$$

$a + b = \sqrt{x} + \sqrt{y}$

D. TAM KÜP İFADELER / PERFECT CUBE EXPRESSIONS

$(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$

ÖRNEK / EXAMPLE 8

$$\left.\begin{aligned} x^3 + 3xy^2 &= 25 \\ y^3 + 3x^2y &= 39 \end{aligned}\right\} \Rightarrow x + y = ?$$

A) –1 B) 1 C) 2 D) 3 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x^3 + 3xy^2 = 25$

$y^3 + 3x^2y = 39$

$\Rightarrow x^3 + 3x^2y + 3xy^2 + y^3 = 64$

$(x + y)^3 = 64$

$x + y = 4$

NOT / NOTE

✓ $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

✓ $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

PEKİŞTİRME / REINFORCE

$a^3 + 3ab^2 = 13,$

$3a^2b + b^3 = 5 \Rightarrow a - b = ?$

YANIT / ANSWER : 2

ÇARPANLARA AYIRMA / FACTORIZATION

Bir cebirsel ifadeyi, iki veya daha fazla ifadenin çarpımı biçiminde yazmaya *çarpanlara ayırma* denir.

When the algebraic expression can be multiplication of two or more than expressions, this operation is called factorization(or factoring).

A. ORTAK ÇARPAN PARANTEZİNE ALMA BRACKETING of COMMON FACTOR

Cebirsel bir ifadenin tüm terimlerinde ortak ifade varsa, bu ortak terim ifadenin çarpanıdır.
If all the terms of an algebraic expression contain a common factor, then that term can be factored out of the expression.

B. GRUPLANDIRMA YOLUYLA ÇARPANLARA AYIRMA FACTORIZATION GROUPING

Her terimde ortak olan bir ifade yoksa terimler, ikişer ikişer veya üçer üçer gruplandırılır.
If there is not one single common expressions, then expressions are grouped to two two or three three groups.

ÖRNEK / EXAMPLE 9

$$\left.\begin{aligned} a-c&=3 \\ b+a&=5 \end{aligned}\right\} \Rightarrow \mathbf{bc-ab-ac+c^2=?}$$

A) –8 B) –6 C) –4 D) –2 E) –1

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned} bc-ab-ac+c^2 &= b(c-a)-c(a-c) \\ &= (a-c)(-b-c) \end{aligned}$$

$$\left.\begin{aligned} a-c&=3 \\ b+a&=5 \end{aligned}\right\} \Rightarrow (a-c)\cdot(-b-c)=3\cdot(-2)=-6$$

PEKİŞTİRME / REINFORCE

$x-y=2,$

$m+n=3 \Rightarrow mx-my+nx-ny=?$

YANIT / ANSWER : 6

C. $Ax^2 + Bx + C$ ŞEKLİNDEKİ İFADELERİN ÇARPANLARA AYRILMASI
FACTORIZATION of THE EXPRESSIONS in $Ax^2 + Bx + C$ FORMS

1. A = 1 için $x^2 + Bx + C$ üç terimlisinin son terimi C den yararlanılarak çarpanlarına ayrılır. $C = m{\cdot}n$ ve $B = m + n$ olacak biçiminde m ve n sayıları verilmiştir.

   For A = 1, we use last term of $x^2 + Bx + C$ expression C to factorize the expression. n and m numbers are given such that $C = m{\cdot}n$ and $B = m + n$.

   $x^2 + Bx + C = (x + m)(x + n)$

   ↓ ↓ ↓

   x m+n m

   x n

2. $a \neq 1$ ise, $ax^2 + bx + c$ üç terimlisi, ax^2 ve c nin çarpanlarından yararlanılarak çarpanlarına ayrılabilir.

   Using factors of ax^2 and c for $a \neq 1$ to factorization the $ax^2 + bx + c$.

   $ax^2 + bx + c$

   ↓ ↓

   mx ↖↗ p

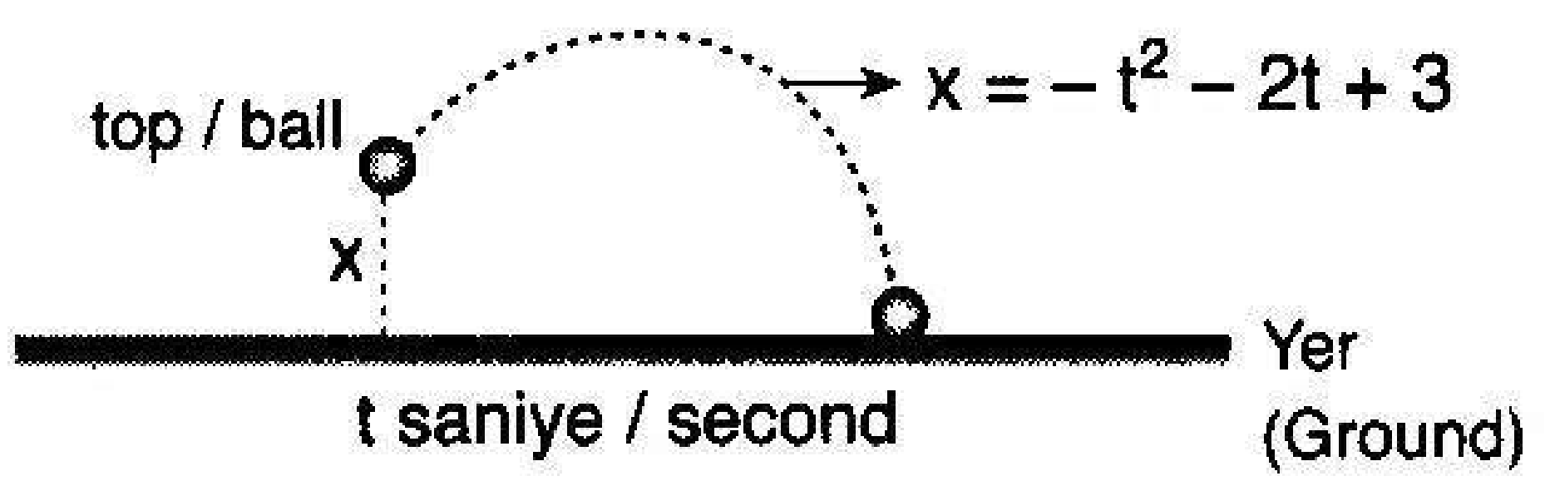
   kx ↙↘ n

   $pkx + mnx = bx$ [Çapraz çarpım ortadaki terime eşit olmalıdır. *The cross – multiply is equal to the middle term.*]

   $(mx)\,(kx) = mkx^2 = ax^2$

   $p{\cdot}n = c \Rightarrow ax^2 + bx + c = (mx + p)\,(kx + n)$

Düşünün / Think about it



$\Rightarrow t = ?$

ÖRNEK / EXAMPLE 10

$$\frac{6x^2 + x - 1}{4x^2 - 1} = ?$$

A) $\frac{2x-1}{2x+1}$ B) $\frac{1}{2x-3}$ C) $\frac{2x-2}{3x-1}$

D) $\frac{3x+1}{2x+1}$ E) $\frac{3x-1}{2x-1}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{6x^2 + x - 1}{4x^2 - 1} = \frac{(3x-1)\cdot(2x+1)}{(2x-1)\cdot(2x+1)}$$

$$= \frac{3x-1}{2x-1}$$

Yanıt / Answer

1

PEKİŞTİRME / REINFORCE

$A = x^2 - 2x + y^2 + 4y \Rightarrow \min(A) = ?$

YANIT / ANSWER : –5

D. TERİM EKLEYİP ÇIKARARAK ÇARPANLARA AYIRMA FACTORIZATION by TERM ADDING or SUBTRACTING

Verilen bir ifade, terim ekleyip çıkartılarak bilinen bir özdeşliğin açılımına dönüştürülür.

By term adding or subtracting, we can convert the expression to a known expansion of an identity.

ÖRNEK / EXAMPLE 11

$x = \sqrt[3]{5} + 1 \Rightarrow x^3 - 3x^2 + 3x = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$x^3 - 3x^2 + 3x = (x-1)^3 + 1$$

$$x = \sqrt[3]{5} + 1 \Rightarrow x^3 - 3x^2 + 3x = (\sqrt[3]{5} + 1 - 1)^3 + 1 = 6$$

ÖRNEK / EXAMPLE 12

$$\frac{a^4 + a^2 + 1}{a^3 + 1} = ?$$

A) $\frac{a^2 + 2a + 1}{a + 1}$ B) $\frac{a^2 + 1}{a - 1}$ C) $\frac{a^2 - a}{a + 1}$

D) $\frac{a^2 - a + 1}{a - 1}$ E) $\frac{a^2 + a + 1}{a + 1}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{a^4 + a^2 + 1}{a^3 + 1} = \frac{a^4 + a^2 + 1}{(a+1)\cdot(a^2 - a + 1)}$$ [İki küp toplamı / *Sum of two cubes*]

$$= \frac{(a^2 + 1)^2 - a^2}{(a+1)\cdot(a^2 - a + 1)}$$ [Tam kare ifadeler / *Perfect square expression*]

$$= \frac{(a^2 - a + 1)\cdot(a^2 + a + 1)}{(a+1)\cdot(a^2 - a + 1)}$$ [İki kare farkı / *Difference of two squares*]

$$= \frac{a^2 + a + 1}{a + 1}$$ [Sonuç / *Conclusion*]

ÖRNEK / EXAMPLE 13

ANADOLU ÜNİ. YÖS 2011

$2x + 3y = 5,$

$a + b = 4$

$\Rightarrow \mathbf{2ax + 3by + 3ay + 2bx = ?}$

A) 10 B) 15 C) 20 D) 25 E) 30

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{aligned} 2ax + 2bx + 3ay + 3by &= 2x(a + b) + 3y(a + b) \\ &= (a + b)\cdot(2x + 3y) \\ &= 4\cdot 5 \\ &= 20 \end{aligned}$$

PEKİŞTİRME / REINFORCE

$$\frac{x^3 - y^3}{x^2 - y^2} : \frac{1}{x + y} = ?$$

YANIT / ANSWER : $x^2 + xy + y^2$

ÖRNEK / EXAMPLE 14

ANADOLU ÜNİ. YÖS 2011

$$x = 3 - y \Rightarrow \frac{(x+y)^2 - xy}{x^3 - y^3} : \frac{x+y}{x-y} = ?$$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) 0 E) $\frac{1}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{(x+y)^2 - xy}{x^3 - y^3} : \frac{x+y}{x-y} = \frac{x^2 + y^2 + xy}{(x-y)\cdot(x^2 + xy + y^2)} \cdot \frac{x-y}{x+y}$$

$$= \frac{1}{x+y} = \frac{1}{3}$$

ÖRNEK / EXAMPLE 15

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$\frac{a^2+3b-3a-b^2}{(a+b)^2-9}=?$$

A) $\frac{a}{b+3}$ B) $\frac{a+3}{b-3}$ C) $\frac{a-b}{a+b+3}$

D) $\frac{a+b}{a+b-3}$ E) $\frac{a+b}{a-b-3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{a^2+3b-3a-b^2}{(a+b)^2-9}=\frac{a^2-b^2+3(b-a)}{(a+b)^2-3^2}$$

$$=\frac{(a-b)(a+b)-3(a-b)}{(a+b-3)(a+b+3)}$$

$$=\frac{(a-b)(a+b-3)}{(a+b-3)(a+b+3)}$$

$$=\frac{a-b}{a+b+3}$$

ÖRNEK / EXAMPLE 16

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$\left(\frac{x+1+\frac{1}{x}}{x^3-1}\right)(-x^2+x)=?$$

A) $-x$ B) -1 C) 1 D) $x-1$ E) $\frac{1}{x}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left(\frac{x+1+\frac{1}{x}}{x^3-1}\right)(-x^2+x)=\frac{x^2+x+1}{x(x-1)(x^2+x+1)}\cdot(-x)\cdot(x-1)=-1$$

ÖRNEK / EXAMPLE 17

YILDIZ TEKNİK ÜNİ. YÖS 2013

$(\sqrt{2}-\sqrt{5})^3 = ?$

A) $5\sqrt{2}+10\sqrt{5}$ B) $17\sqrt{2}-11\sqrt{5}$ C) $7\sqrt{2}-3\sqrt{5}$

D) $12\sqrt{2}+7\sqrt{5}$ E) $7\sqrt{2}-11\sqrt{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned}(\sqrt{2}-\sqrt{5})^3 &= (\sqrt{2})^3 - 3(\sqrt{2})^2\sqrt{5} + 3\sqrt{2}(\sqrt{5})^2 - (\sqrt{5})^3\\ &= 2\sqrt{2} - 6\sqrt{5} + 15\sqrt{2} - 5\sqrt{5}\\ &= 17\sqrt{2} - 11\sqrt{5}\end{aligned}$$

ÖRNEK / EXAMPLE 18

İSTANBUL ÜNV. YÖS 2013

$$\frac{x^2+5x+6}{x^3+x^2-2x}\cdot\frac{x^3-x}{x^2+6x+5} = ?$$

A) $\frac{x+5}{x-1}$ B) $\frac{x-3}{x+1}$ C) $\frac{x-2}{x+2}$ D) $\frac{x+1}{x+3}$ E) $\frac{x+3}{x+5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{x^2+5x+6}{x^3+x^2-2x}\cdot\frac{x^3-x}{x^2+6x+5}$$

$$=\frac{(x+3)\cdot(x+2)}{x(x^2+x-2)}\cdot\frac{x(x-1)\cdot(x+1)}{(x+5)\cdot(x+1)}$$

$$=\frac{(x+3)\cdot(x+2)}{(x-1)(x+2)}\cdot\frac{(x-1)}{(x+5)}=\frac{x+3}{x+5}$$

PEKİŞTİRME / REINFORCE

$4^x + 4^{-x} = 14 \Rightarrow 2^x + 2^{-x} = ?$

YANIT / ANSWER : 4

ÖRNEK / EXAMPLE 19

MARMARA ÜNV. YÖS 2015

$$\frac{x^3+y^3+3xy\cdot(x+y)}{x^3\cdot y+2x^2\cdot y^2+xy^3}=?$$

A) $\frac{x+y}{xy}$ B) $\frac{xy}{x+y}$ C) $\frac{x}{x-y}$ D) $x+y$ E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer B

$$\frac{x^3+y^3+3xy\cdot(x+y)}{x^3\cdot y+2x^2\cdot y^2+xy^3}=\frac{x^3+y^3+3x^2y+3xy^2}{xy(x^2+2xy+y^2)}$$

$$=\frac{(x+y)^3}{xy(x+y)^2}$$

$$=\frac{x+y}{xy}$$

ÖRNEK / EXAMPLE 20

ANKARA ÜNİ. YÖS 2016

$a = 2013$

$b = 2015$

$\Rightarrow$ **$a^5 - 5a^4b + 10a^2b^3 + 5ab^4 - b^5 = ?$**

A) –32 B) –21 C) 0 D) 16 E) 20

ÇÖZÜM / SOLUTION

Yanıt / Answer B

$$a^5 - 5a^4b + 10a^2b^3 + 5ab^4 - b^5 = (a-b)^5$$

$$= (2013 - 2015)^5$$

$$= (-2)^5$$

$$= -32$$

1. $x > y,$

$$x + y - 2\sqrt{xy} = 4 \Rightarrow \frac{\sqrt{x} + \sqrt{y}}{x - y} = ?$$

A) $\frac{1}{2}$ B) 2 C) 4

D) 8 E) 16

2. $x = 4$

$y = 3 \Rightarrow \mathbf{x^3 - 6x^2y + 12xy^2 - 8y^3 = ?}$

A) –36 B) –27 C) –18

D) –9 E) –8

3.

$$\left.\begin{array}{l} x + \dfrac{1}{y} = \dfrac{10}{3} \\ x^2 - \dfrac{1}{y^2} = \dfrac{80}{9} \end{array}\right\} \Rightarrow x = ?$$

A) $\frac{3}{2}$ B) 3 C) 6

D) 12 E) 24

4. $\frac{1}{x} + \frac{1}{y} = 4$

$x \cdot y = 3 \Rightarrow \mathbf{(x - y)^2 = ?}$

A) 96 B) 108 C) 120

D) 132 E) 144

5. $x > y,$

$x + y = 8$

$x^2 + y^2 = 40 \Rightarrow \mathbf{x - y = ?}$

A) 1 B) 2 C) 3

D) 4 E) 5

6. $x - y = 5$

$x \cdot y = 2 \Rightarrow \mathbf{x^3 - y^3 = ?}$

A) 155 B) 160 C) 165

D) 170 E) 175

7. $x - \frac{5}{x} = 4 \Rightarrow \mathbf{x^3 - \frac{125}{x^3} = ?}$

A) 120 B) 124 C) 125

D) 126 E) 130

8. $\left.\begin{aligned} a^2+b^2+c^2 &= 29 \\ a+b+c &= 9 \end{aligned}\right\} \Rightarrow \mathbf{ab+ac+bc = ?}$

A) 26 B) 30 C) 38

D) 40 E) 45

9. $4a^2 + \frac{1}{4a^2} = 18 \Rightarrow \mathbf{2a - \frac{1}{2a} = ?}$

A) 1 B) 3 C) 4

D) 8 E) 12

10. $x, y \in Z^+$

$\left.\begin{aligned} x^2 &= 63 + xy \\ y^2 &= xy - 14 \end{aligned}\right\} \Rightarrow \mathbf{x = ?}$

A) 7 B) 8 C) 9

D) 10 E) 11

11. $\left.\begin{aligned} a &= \sqrt{5} - 1 \\ b &= \sqrt{5} + 1 \end{aligned}\right\} \Rightarrow \mathbf{\dfrac{(a+b)^2 - 2ab}{\frac{a}{b} + \frac{b}{a}}} = ?$

A) 4 B) 10 C) 18

D) 20 E) 24

12. $4^x + 4^{-x} = 3 \Rightarrow \mathbf{16^x + 16^{-x} = ?}$

A) 5 B) 7 C) 9

D) 11 E) 13

YANITLAR / ANSWERS

1. A	**2. E**	**3. B**	**4. D**	**5. D**	**6. A**	**7. B**	**8. A**
9. C	**10. C**	**11. A**	**12. B**				

1. $x+y-2\sqrt{xy}=(\sqrt{x}-\sqrt{y})^2=4$

$$x>y \Rightarrow \sqrt{x}-\sqrt{y}=2$$

$$\frac{\sqrt{x}+\sqrt{y}}{x-y}=\frac{\sqrt{x}+\sqrt{y}}{(\sqrt{x}+\sqrt{y})\cdot(\sqrt{x}-\sqrt{y})}$$

$$=\frac{1}{\sqrt{x}-\sqrt{y}}=\frac{1}{2}$$

Yanıt / The Answer is A

---

2. $x^3-6x^2y+12xy^2-8y^3=(x-2y)^3$

$$\left.\begin{array}{l} x=4 \\ y=3 \end{array}\right\} \Rightarrow (4-2\cdot 3)^3=-8$$

Yanıt / The Answer is E

---

3. $x^2-\frac{1}{y^2}=\left(x-\frac{1}{y}\right)\cdot\left(x+\frac{1}{y}\right)=\frac{80}{9}$

$$x+\frac{1}{y}=\frac{10}{3} \Rightarrow \left(x-\frac{1}{y}\right)\cdot\frac{10}{3}=\frac{80}{9}$$

$$x-\frac{1}{y}=\frac{8}{3}$$

$$\left.\begin{array}{l} x+\frac{1}{y}=\frac{10}{3} \\ x-\frac{1}{y}=\frac{8}{3} \end{array}\right\} \Rightarrow \begin{array}{l} 2x=6 \\ x=3 \end{array}$$

Yanıt / The Answer is B

---

4. $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}=4$

$x+y=4\cdot(xy) \Rightarrow x+y=4\cdot 3=12$

$(x-y)^2=(x+y)^2-4xy$

$(x-y)^2=144-12=132$

Yanıt / The Answer is D

---

5. $x^2+y^2=(x+y)^2-2xy=40$

$8^2-2xy=40$

$64-2xy=40$

$2xy=64-40$

$2xy=24$

$xy=12$

$(x-y)^2=x^2-2xy+y^2$

$(x-y)^2=40-24=16$

$x-y=4$

Yanıt / The Answer is D

---

6. $x^3-y^3=(x-y)^3+3xy(x-y)$

$x^3-y^3=5^3+3\cdot 2\cdot(5)$

$x^3-y^3=125+30=155$

Yanıt / The Answer is A

7.

$$x^3-\frac{125}{x^3}=\left(x-\frac{5}{x}\right)^3+3\cdot x\cdot\frac{5}{x}\cdot\left(x-\frac{5}{x}\right)$$

$$x^3-\frac{125}{x^3}=4^3+15\cdot 4$$

$$=64+60=124$$

Yanıt / The Answer is B

8.

$$a+b+c=9 \Rightarrow (a+b+c)^2=9^2$$

$$\underbrace{a^2+b^2+c^2}_{29}+2\cdot(ab+ac+bc)=81$$

$$29+2\cdot(ab+ac+bc)=81$$

$$\frac{2\cdot(ab+ac+bc)}{2}=\frac{52}{2}$$

$$ab+ac+bc=26$$

Yanıt / The Answer is A

9.

$$2a-\frac{1}{2a}=x$$

$$\left(2a-\frac{1}{2a}\right)^2=x^2 \Rightarrow 4a^2-2\cdot 2a\frac{1}{2a}+\frac{1}{4a^2}=x^2$$

$$4a^2-2+\frac{1}{4a^2}=x^2$$

$$18-2=x^2$$

$$16=x^2$$

$$x^2=16 \Rightarrow x=2a-\frac{1}{2a}=4$$

Yanıt / The Answer is C

10.

$$\left.\begin{array}{l} x^2=63+xy \\ y^2=xy-14 \end{array}\right\} \Rightarrow \quad x^2+y^2=49+2xy$$

$$x^2-2xy+y^2=49$$

$$(x-y)^2=49$$

$$x-y=7$$

$$\left.\begin{array}{l} x^2=63+xy \\ y^2=xy-14 \end{array}\right\} \Rightarrow \begin{array}{c} x^2-y^2=77 \\ (x-y)\cdot(x+y)=77 \end{array}$$

$$x-y=7 \Rightarrow 7\cdot(x+y)=77$$

$$x+y=11$$

$$\left.\begin{array}{l} x-y=7 \\ x+y=11 \end{array}\right\} \Rightarrow x=9$$

Yanıt / The Answer is C

11.

$$\frac{(a+b)^2-2ab}{\frac{a}{b}+\frac{b}{a}}=\frac{a^2+2ab+b^2-2ab}{\frac{a^2+b^2}{ab}}$$

$$=\frac{(a^2+b^2)\cdot ab}{a^2+b^2}=ab$$

$$\left.\begin{array}{l} a=\sqrt{5}-1 \\ b=\sqrt{5}+1 \end{array}\right\} \Rightarrow a\cdot b=(\sqrt{5}-1)\cdot(\sqrt{5}+1)=4$$

Yanıt / The Answer is A

12.

$$16^x+16^{-x}=4^{2x}+4^{-2x}$$

$$4^{2x}+4^{-2x}=(4^x+4^{-x})^2-2\cdot 4^x\cdot 4^{-x}$$

$$=3^2-2\cdot 1$$

$$=7$$

Yanıt / The Answer is B

1. $a + c = 4$

$c - b = 5 \Rightarrow ab - bc - ac + b^2 = ?$

A) 20

B) 5

C) 0

D) –5

E) –20

2. $\dfrac{a^3 + a^2b - ac^2 - bc^2}{a^2 - ac + ab - bc} = ?$

A) a + c

B) a + b

C) b + c

D) a – c

E) a – b

3. $\dfrac{x^2 + mx + n}{x^2 + 7x + 12} \cdot \dfrac{x^2 + 6x + 8}{x^2 - 4} = \dfrac{x + 5}{x + 3} \Rightarrow m \cdot n = ?$

A) –42

B) –30

C) –28

D) –24

E) –18

4. $\dfrac{x^2 - 1}{x^2 - 2x - 3} : \dfrac{x - 1}{x - 3} = ?$

A) x – 2

B) x – 3

C) 1

D) x + 1

E) x + 3

5. $\left.\begin{array}{l} 2a^2 + 3ab + b^2 = 10 \\ a + b = 2 \end{array}\right\} \Rightarrow b = ?$

A) –1

B) 0

C) 1

D) 2

E) 3

6. $\left(\dfrac{x^2 - 3x + 2}{x^2 - 1} : \dfrac{x^2 - x - 20}{x^2 + 4x} \right) : \dfrac{x - 2}{x^2 - 4x - 5} = ?$

A) x

B) x – 1

C) x + 1

D) x – 2

E) x + 2

7. $\left(x+\frac{1}{x^2}\right):\left(1-\frac{1}{x}+\frac{1}{x^2}\right)=?$

A) x

B) $x+1$

C) $x-1$

D) $\frac{1}{x}$

E) $\frac{1}{x+1}$

8. $\left(a-\frac{1}{a}\right):\left(\frac{a^2-2a-3}{3a-a^2}\right)=?$

A) $3-a$

B) $2-a$

C) $1-a$

D) $1+a$

E) $2+a$

9. $\frac{x^4-3x^2-4}{x^4-1}\cdot\frac{x^2-2x-3}{x^2+x-6}\cdot\frac{x-1}{x+2}=?$

A) $\frac{x+1}{x-1}$

B) $\frac{x-2}{x+2}$

C) $\frac{x-3}{x+3}$

D) $\frac{x-2}{x+1}$

E) $\frac{x+3}{x-1}$

10. $\dfrac{x-\frac{1}{x^2}}{\frac{1}{x}+x+1}:\frac{x-1}{x^2}=?$

A) $\frac{1}{x^2}$

B) $\frac{1}{x}$

C) 1

D) x

E) x^2

11. $\frac{5^{20}-3^{20}}{5^{15}+5^{10}\cdot 3^5+5^5\cdot 3^{10}+3^{15}}+3^5=x^5 \Rightarrow x=?$

A) 3

B) 4

C) 5

D) 6

E) 7

12. $\frac{(a+b)^2-11\cdot(a+b)+28}{a+b-4}=?$

A) $a+b-7$

B) $a+b+7$

C) $a-b-7$

D) $a+7$

E) $a-7$

YANITLAR / ANSWERS

1. B	2. A	3. B	4. C	5. A	6. A	7. B	8. C
9. C	10. D	11. C	12. A				

1. $ab - bc - ac + b^2 = ab - ac + b^2 - bc$

$= a(b - c) + b(b - c)$

$= (b - c) \cdot (a + b)$

$$\left.\begin{array}{r} a+c=4 \\ c-b=5 \end{array}\right\} \Rightarrow a+b=-1$$

$b-c=-5$

$\Rightarrow (b - c)\cdot(a + b) = -1 \cdot -5 = 5$

Yanıt / The Answer is B

2. $$\frac{a^3+a^2b-ac^2-bc^2}{a^2-ac+ab-bc}=\frac{a^2(a+b)-c^2(a+b)}{a(a-c)+b(a-c)}$$

$$=\frac{(a+b)\cdot(a-c)\cdot(a+c)}{(a-c)\cdot(a+b)}$$

$$=a+c$$

Yanıt / The Answer is A

3. $$\frac{x^2+mx+n}{x^2+7x+12}\cdot\frac{x^2+6x+8}{x^2-4}=\frac{x+5}{x+3}$$

$$\frac{x^2+mx+n}{x^2+7x+12}\cdot\frac{(x+2)\cdot(x+4)}{(x-2)\cdot(x+2)}=\frac{x+5}{x+3}$$

$$\frac{x^2+mx+n}{(x+4)\cdot(x+3)}\cdot\frac{x+4}{x-2}=\frac{x+5}{x+3}$$

$\Rightarrow x^2 + mx + n = x^2 + 3x - 10$

$\Rightarrow m = 3,\ n = -10 \Rightarrow m\cdot n = 3\cdot(-10) = -30$

Yanıt / The Answer is B

4. $$\frac{x^2-1}{x^2-2x-3}:\frac{x-1}{x-3}=\frac{(x-1)\cdot(x+1)}{(x-3)\cdot(x+1)}\cdot\frac{x-3}{x-1}$$

$$=1$$

Yanıt / The Answer is C

5. $2a^2 + 3ab + b^2 = 10$

$(2a + b)\cdot(a + b) = 10$

$a + b = 2 \Rightarrow 2a + b = 5$

$$\left.\begin{array}{r} 2a+b=5 \\ a+b=2 \end{array}\right\} \Rightarrow b=-1$$

Yanıt / The Answer is A

6. $$\left(\frac{x^2-3x+2}{x^2-1}:\frac{x^2-x-20}{x^2+4x}\right):\frac{x-2}{x^2-4x-5}$$

$$=\left(\frac{(x-2)\cdot(x-1)}{(x-1)\cdot(x+1)}\cdot\frac{x\cdot(x+4)}{(x-5)\cdot(x+4)}\right)\cdot\frac{(x-5)\cdot(x+1)}{(x-2)}$$

$$=x$$

Yanıt / The Answer is A

7. $\left(x+\frac{1}{x^2}\right):\left(1-\frac{1}{x}+\frac{1}{x^2}\right)=\left(\frac{x^3+1}{x^2}\right):\left(\frac{x^2-x+1}{x^2}\right)$

$=\frac{(x+1)\cdot(x^2-x+1)}{x^2}\cdot\frac{x^2}{x^2-x+1}$

$=x+1$

Yanıt / The Answer is B

8. $\left(a-\frac{1}{a}\right):\left(\frac{a^2-2a-3}{3a-a^2}\right)=\left(\frac{a^2-1}{a}\right):\left(\frac{(a-3)\cdot(a+1)}{a(3-a)}\right)$

$=\frac{(a-1)\cdot(a+1)}{a}\cdot\frac{a\cdot(3-a)}{(a-3)\cdot(a+1)}$

$=1-a$

Yanıt / The Answer is C

9. $\frac{x^4-3x^2-4}{x^4-1}\cdot\frac{x^2-2x-3}{x^2+x-6}\cdot\frac{x-1}{x+2}$

$=\frac{(x^2-4)\cdot(x^2+1)}{(x^2-1)\cdot(x^2+1)}\cdot\frac{(x-3)\cdot(x+1)}{(x+3)\cdot(x-2)}\cdot\frac{x-1}{x+2}$

$=\frac{(x-2)\cdot(x+2)}{(x-1)\cdot(x+1)}\cdot\frac{(x-3)\cdot(x+1)}{(x+3)\cdot(x-2)}\cdot\frac{x-1}{x+2}$

$=\frac{x-3}{x+3}$

Yanıt / The Answer is C

10. $\frac{x-\frac{1}{x^2}}{\frac{1}{x}+x+1}:\frac{x-1}{x^2}=\frac{\frac{x^3-1}{x^2}}{\frac{1+x^2+x}{x}}\cdot\frac{x^2}{x-1}$

$=\frac{(x-1)\cdot(x^2+x+1)}{x\cdot(x^2+x+1)}\cdot\frac{x^2}{x-1}$

$=x$

Yanıt / The Answer is D

11. $\frac{(5^5)^4-(3^5)^4}{5^{15}+5^{10}\cdot3^5+5^5\cdot3^{10}+3^{15}}+3^5=x^5$

$\frac{(5^5-3^5)\ (5^{15}+5^{10}\cdot3^5+5^5\cdot3^{10}+3^{15})}{5^{15}+5^{10}\cdot3^5+5^5\cdot3^{10}+3^{15}}+3^5=x^5$

$5^5-3^5+3^5=x^5\Rightarrow\ 5^5=x^5\Rightarrow x=5$

Yanıt / The Answer is C

12. $\frac{(a+b)^2-11\cdot(a+b)+28}{a+b-4}=\frac{(a+b-4)\ (a+b-7)}{a+b-4}$

$=a+b-7$

Yanıt / The Answer is A

1. $\dfrac{ab+ac-b^2-bc}{b+c}=?$

Yanıt / The Answer is: **a – b**

2. $x+1-2\sqrt{x+1}-3=0 \Rightarrow x=?$

Yanıt / The Answer is: **8**

3. $1+a=\dfrac{a}{b} \Rightarrow \dfrac{a^2b+ab^2}{b^2-a^2}=?$

Yanıt / The Answer is: **–1**

4. $\left(\dfrac{x^4-29x^2+100}{x^2-7x+10}\right):(x+5)=?$

Yanıt / The Answer is: **x + 2**

5. $\dfrac{x^2+3x-10}{2x^2+ax+b}=\dfrac{x-2}{2x-3} \Rightarrow a+b=?$

Yanıt / The Answer is: **–8**

6. $\dfrac{2}{x-2}-\dfrac{x}{x^2-4}\cdot\dfrac{x^2-x-6}{x-3}=?$

Yanıt / The Answer is: **–1**

7. $\dfrac{25^x - 64^x}{5^x - 8^x} = 625 + 8^x \Rightarrow x = ?$

Yanıt / The Answer is: **4**

8. $\left.\begin{array}{l} \dfrac{1}{x} + \dfrac{1}{y} = 5 \\ x \cdot y = 3 \end{array}\right\} \Rightarrow (x-y)^2 = ?$

Yanıt / The Answer is: **213**

9. $\dfrac{2x^2 + 7x - 4}{x^2 + 8x + 15} : \dfrac{2x^2 - 5x + 2}{x^2 + x - 6} = ?$

Yanıt / The Answer is: $\dfrac{x+4}{x+5}$

10. $\dfrac{a^2 - 3a + ab - 3b}{3 - a} + a = ?$

Yanıt / The Answer is: **–b**

11. $\left.\begin{array}{l} \dfrac{1}{x} - \dfrac{1}{y} = 3 \\ x \cdot y = \dfrac{1}{18} \end{array}\right\} \Rightarrow (x-y)^2 = ?$

Yanıt / The Answer is: $\dfrac{1}{36}$

12. $\dfrac{x^2 + 2x - 15}{x^2 + mx + n} = \dfrac{x+5}{x-4} \Rightarrow m + n = ?$

Yanıt / The Answer is: **5**

1. $\left(\frac{x+3}{3-x}\right):\left(1+\frac{6}{x-3}\right)=?$

A) –3 B) –2 C) –1
D) 1 E) 2

---

2. $\frac{x}{x^2+x^3}:\frac{1-\frac{1}{x}}{x-\frac{1}{x}}=?$

A) x^2 B) x C) $\frac{1}{x}$
D) $-\frac{1}{x}$ E) $-\frac{1}{x^2}$

---

3. $\left.\begin{array}{l}\sqrt{x}+\sqrt{y}=4\\ x-y=8\end{array}\right\}\Rightarrow x+y=?$

A) 3 B) 4 C) 6
D) 8 E) 10

---

4. $\left.\begin{array}{l}a=2{,}6\\ b=7{,}4\end{array}\right\}\Rightarrow \frac{(a-b)^2+4ab}{5a+5b}=?$

A) 1 B) $\frac{3}{2}$ C) 2
D) $\frac{5}{2}$ E) 3

---

5. $\frac{2x^2+8y^2}{(x-2y)^2+(x+2y)^2}=?$

A) 1 B) 2 C) 3
D) 4 E) 5

---

6. $x^2-12x+3=0 \Rightarrow \frac{x}{3}+\frac{1}{x}=?$

A) 3 B) 4 C) 5
D) 6 E) 8

7. $\left.\begin{matrix} x\cdot y = 3 \\ x-2y=2 \end{matrix}\right\} \Rightarrow \mathbf{x^3-8y^3=?}$

A) 41
B) 42
C) 43
D) 44
E) 45

8. $a-\dfrac{1}{a}=5 \;\Rightarrow\; \mathbf{a^2-\dfrac{1}{a^2}=?}$

A) $\sqrt{29}$
B) $2\sqrt{29}$
C) $3\sqrt{29}$
D) $4\sqrt{29}$
E) $5\sqrt{29}$

9. $\mathbf{\dfrac{x^3-y^3}{x^2+y^2+xy} : \dfrac{(x^2+y^2-xy)(x-y)}{x^3+y^3} = ?}$

A) x
B) $x+y$
C) $x-y$
D) $\dfrac{1}{x+y}$
E) $\dfrac{1}{x-y}$

10. $x+\dfrac{1}{x}=4\sqrt{2} \;\Rightarrow\; \mathbf{x-\dfrac{1}{x}=?}$

A) $5\sqrt{2}$
B) $\sqrt{6}$
C) $2\sqrt{6}$
D) $\sqrt{7}$
E) $2\sqrt{7}$

11. $9x^2-12x+1=0 \;\Rightarrow\; \mathbf{9x^2+\dfrac{1}{9x^2}=?}$

A) 14
B) 15
C) 18
D) 21
E) 24

12. $\left.\begin{matrix} a+b-c=4 \\ a^2+b^2+c^2=6 \end{matrix}\right\} \Rightarrow \mathbf{ac+bc-ab=?}$

A) –1
B) –2
C) –3
D) –4
E) –5

YANITLAR / ANSWERS

1. C 2. C 3. E 4. C 5. A 6. B 7. D 8. E
9. B 10. E 11. A 12. E

1. $(2x+1)^2-2(2x+1)-15=0 \Rightarrow \sum x = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

2. $x^2-4x+4y^2+12y+13=0 \Rightarrow x+y=?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

3. $\frac{x^2+mx+n}{x^2-7x+10}=\frac{x+1}{x-5} \Rightarrow m+n=?$

A) 0 B) –1 C) –2 D) –3 E) –4

4. $x^2+\frac{9}{x^2}=6 \Rightarrow x+\frac{3}{x}=?$

A) $-\sqrt{3}$ B) $\sqrt{3}$ C) $2\sqrt{3}$ D) $3\sqrt{3}$ E) $4\sqrt{3}$

5. $\left(\frac{a^3+a^2b+ab^2+b^3}{a^5b-ab^5}\right):\frac{a+b}{a^2-b^2}=?$

A) $\frac{1}{a}$ B) $\frac{1}{b}$ C) $\frac{1}{ab}$ D) $\frac{1}{a+b}$ E) $\frac{1}{a-b}$

6. $\left.\begin{array}{l} x\sqrt{x}+y\sqrt{y}=65 \\ x\sqrt{y}+y\sqrt{x}=20 \end{array}\right\} \Rightarrow \sqrt{x}+\sqrt{y}=?$

A) 3 B) 5 C) 6 D) 7 E) 9

7. $x^2 + x + 1 = 0 \Rightarrow \mathbf{x^6 + 1 = ?}$

A) 2 B) x C) x + 1

D) x – 1 E) 2x

8. $a, b \in Z$,

$a^2 - b^2 = 19 \Rightarrow \mathbf{a + 2b = ?}$

A) 18 B) 28 C) 38

D) 48 E) 58

9. $x + \frac{1}{x} = 2 \Rightarrow \mathbf{x^{64} + \frac{1}{x^{64}} = ?}$

A) 2 B) 2^2 C) 2^8

D) 2^{16} E) 2^{32}

10. $2a^2 + ab - 3b^2 = 8$,

$a - b = 2 \Rightarrow \mathbf{a \cdot b = ?}$

A) 10 B) 6 C) 3

D) 0 E) –3

11. $\mathbf{\frac{x + x^{-2}}{1 - x^{-1} + x^{-2}} : (1 + x^{-1}) = ?}$

A) x B) x + 1 C) x – 1

D) $x^2 + 1$ E) $x^2 + 2$

12. $3x^2 - 6x + 4 = 9 \Rightarrow \mathbf{x - \frac{5}{3x} = ?}$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1

D) 2 E) 3

YANITLAR / ANSWERS

1. C	**2. A**	**3. D**	**4. C**	**5. C**	**6. B**	**7. A**	**8. B**
9. A	**10. D**	**11. A**	**12. D**				

1. $5x^2 + 4x + 4 + 9y^2 + 12xy = 0 \Rightarrow \mathbf{x \cdot y = ?}$

A) $-\frac{10}{3}$

B) $-\frac{8}{3}$

C) $-\frac{5}{3}$

D) $-\frac{4}{3}$

E) $-\frac{2}{3}$

2. $\frac{x^2+mx+n}{x^2+x-12}=\frac{x+1}{x+4} \Rightarrow \mathbf{m \cdot n = ?}$

A) 6

B) 8

C) 9

D) 12

E) 16

3. $(x^2 + 7x)^2 - 2(x^2+7x) - 48 = (x + a)(x + b)(x + c)(x + d)$

$\Rightarrow \mathbf{a + b + c + d = ?}$

A) 11

B) 12

C) 13

D) 14

E) 15

4. $\left(\frac{x^2+4x}{x^2-x-20}:\frac{x^2-1}{x^2-3x+2}\right):\frac{x-2}{x^2-4x-5}=?$

A) $x^2 + 1$

B) $x - 1$

C) $x + 1$

D) x

E) 1

5. $\frac{x-y}{x+y+2\sqrt{xy}}=\frac{1}{2} \Rightarrow \frac{x}{y}=?$

A) 1

B) 2

C) 3

D) 8

E) 9

6. $\frac{a^3+(a-b)^5+(b-a)^5+b^3}{a^2-ab+b^2}=?$

A) $a + b$

B) $a - b$

C) $2a$

D) $2b$

E) $\frac{1}{a+b}$

7. $x = (2^{32}+1)\cdot(2^{16}+1)\cdot(2^{8}+1)\cdot(2^{4}+1) \Rightarrow \mathbf{2^{64} = ?}$

A) $4x + 4$
B) $6x + 1$
C) $8x + 5$
D) $12x + 3$
E) $15x + 1$

8. $x^2 + x + 1 = 0 \Rightarrow \mathbf{x^5 + 1 = ?}$

A) $-x - 2$
B) $-x - 1$
C) $-x$
D) x
E) $x + 1$

9. $4x^2 + y^2 - 4x + 8y + 17 = 0 \Rightarrow \mathbf{x \cdot y = ?}$

A) 3
B) 2
C) 1
D) –1
E) –2

10. $\left.\begin{array}{l} x^2 + y^2 = 292 \\ (x-2)^2 + (y-2)^2 = 260 \end{array}\right\} \Rightarrow \mathbf{x + y = ?}$

A) 6
B) 7
C) 8
D) 9
E) 10

11. $\left(\dfrac{x^2}{x+2} - 1\right)\cdot\left(\dfrac{y-2x}{2x^2+2x-xy-y}\right)\cdot\left(\dfrac{x^2-5x-14}{x-2}\right) = ?$

A) $7 - x$
B) $6 - x$
C) $5 - x$
D) $x - 6$
E) $x - 7$

12. $x, y \in \mathbb{R} \Rightarrow \mathbf{\min(x^2 + y^2 - 2x + 6y) = ?}$

A) –8
B) –9
C) –10
D) –11
E) –12

YANITLAR / ANSWERS

1. B 2. A 3. D 4. D 5. E 6. A 7. E 8. C
9. E 10. E 11. A 12. C

1. $3^x - 3^{-x} = 4 \Rightarrow \mathbf{27^x - 27^{-x} = ?}$

A) 76
B) 80
C) 81
D) 84
E) 90

2. $\left(\dfrac{a\sqrt{a}+1}{a-\sqrt{a}+1}-1\right)^2 = ?$

A) a
B) $a+1$
C) a^2
D) $(a+1)^2$
E) $(a-1)^2$

3. $a^2+5a-1=0 \Rightarrow a^2+\dfrac{1}{a^2}=?$

A) 25
B) 27
C) 30
D) 36
E) 40

4. $2^{\frac{1}{5}}+2=a \Rightarrow \dfrac{\left(2^{\frac{1}{10}}+1\right)\cdot\left(2^{\frac{1}{10}}-1\right)}{2^{\frac{2}{5}}-1}=?$

A) $\dfrac{1}{a+2}$
B) $\dfrac{1}{a+1}$
C) $\dfrac{1}{a}$
D) $\dfrac{1}{a-1}$
E) $\dfrac{1}{a-2}$

5. $x-\dfrac{3}{x}=5 \Rightarrow 3\cdot\left(\dfrac{x^2}{3}+\dfrac{3}{x^2}\right)=?$

A) 23
B) 25
C) 27
D) 29
E) 31

6. $x=6-y \Rightarrow$

$$\frac{x^2-y^2-6y-9}{x^2-y^2-3x-3y}=?$$

A) $\dfrac{1}{2}$
B) 1
C) $\dfrac{3}{2}$
D) 2
E) $\dfrac{5}{2}$

7. $x+y+2\sqrt{xy}=12 \Rightarrow \dfrac{x-y}{4\sqrt{x}-4\sqrt{y}}=?$

A) $\dfrac{\sqrt{3}}{2}$

B) $\dfrac{\sqrt{3}}{3}$

C) $\dfrac{\sqrt{3}}{4}$

D) $\dfrac{\sqrt{3}}{6}$

E) $\dfrac{\sqrt{3}}{8}$

8. $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{3}{4}$

$x+y=6 \Rightarrow x^2+y^2=?$

A) 15

B) 16

C) 18

D) 20

E) 24

9. $x=\sqrt[4]{7} \Rightarrow (x^4+1)\cdot(x^2+1)\cdot(x-1)\cdot(x+1)=?$

A) 48

B) 49

C) 50

D) 54

E) 63

10. $\dfrac{4^x-1}{2^x-1}\cdot\dfrac{3\cdot 2^x-1}{3\cdot 2^{2x}+2\cdot 2^x-1}=?$

A) 2^{x+3}

B) 2^{x+2}

C) 2^{x+1}

D) 2^x

E) 1

11. $\left.\begin{aligned} x^2+xyz&=18 \\ y+\dfrac{x}{z}&=\dfrac{1}{2}\end{aligned}\right\} \Rightarrow x\cdot z=?$

A) 9

B) 18

C) 36

D) 54

E) 72

12. $a+\sqrt{a}=2 \Rightarrow a+\dfrac{1}{\sqrt{a}}=?$

A) −1

B) 1

C) 2

D) 3

E) 4

YANITLAR / ANSWERS

1. A	2. A	3. B	4. D	5. E	6. C	7. A	8. D
9. A	10. E	11. C	12. C				

ORAN ve ORANTI
RATIOS and PROPORTIONB

BÖLÜM CHAPTER 2

Tanım: Aynı birimden iki çokluğun birbirine bölümüne *oran* denir.

Definition: *Divided by two quantities is called ratio each other.*

$$\frac{7\text{kg}}{2\text{kg}}=\frac{7}{2},\ \frac{9\text{cm}}{5\text{cm}}=\frac{9}{5},\ \frac{9\text{ lt}}{6\text{lt}}=\frac{3}{2}$$

Tanım: İki veya daha fazla oranın eşitliğine *orantı* denir.

Definition: *The equality of two and more than two ratio is called proportion.*

$\frac{a}{b}=k,\ \frac{c}{d}=k,\ \frac{e}{f}=k \Rightarrow \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

Yukarıdaki ifade bir orantı olup k, orantı sabitidir.

The expression as follow is a proportion and k is a proportion constant.

$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\ldots=\frac{2n}{3n}$ bir orantıdır. Orantı sabiti $k=\frac{2}{3}$ tür.

$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\ldots=\frac{2n}{3n}$ *is a proportion. The proportion constant of this proportion k is* $\frac{2}{3}$.

ORAN ve ORANTININ ÖZELLİKLERİ
PROPERTIES of RATIOS and PROPORTIONS

1. $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow\ a=b\cdot k$

$c=d\cdot k$

ÖRNEK / EXAMPLE 1

$\frac{a}{3}=\frac{b}{5}=4 \Rightarrow$ **a + b = ?**

A) 30 B) 32 C) 34 D) 36 E) 40

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left.\begin{array}{l} a=3\cdot 4 \\ b=5\cdot 4 \end{array}\right\} \Rightarrow a+b=12+20=32$$

PEKİŞTİRME / REINFORCE

$\frac{x}{2}=\frac{y}{3}=2 \Rightarrow x\cdot y=?$

YANIT / ANSWER : 24

2. $\frac{a}{b}=\frac{c}{d} \Leftrightarrow a:c=b:d$

$\Rightarrow b\cdot c=a\cdot d$

ÖRNEK / EXAMPLE 2

$\frac{x-2y}{x+y}=\frac{4}{5}$, $x-y=13 \Rightarrow$ **$y=?$**

A) 1 B) 3 C) 5 D) 7 E) 9

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$\frac{x-2y}{x+y}=\frac{4}{5}$	[Verilen eşitlik / *Given equality*]
$5x-10y=4x+4y$	[Çapraz çarpım / *Cross – multiply*]
$x=14y$	[Benzer terimleri topla. / *Combine like terms.*]
$x-y=14y-y=13$	[Verilen eşitlik / *Given equality*]
$y=1$	[Sonuç / *Conclusion*]

3. $\frac{a}{b}=\frac{c}{d}=k \Rightarrow \frac{a+c}{b+d}=k$

ÖRNEK / EXAMPLE 3

$\frac{a}{3}=\frac{b}{5}=k$, $a+b=24 \Rightarrow$ **$b=?$**

A) 12 B) 13 C) 14 D) 15 E) 16

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\frac{a}{3}=\frac{b}{5}=k$

$\Rightarrow \frac{a+b}{3+5}=\frac{24}{8}=3=k$

$k=3 \Rightarrow b=5\cdot 3=15$

PEKİŞTİRME / REINFORCE

$\frac{x}{2}=\frac{y}{3}$, $x+y=20 \Rightarrow x-y=?$

YANIT / ANSWER : –4

4. $m, t \in R$

$$\frac{a}{b}=\frac{c}{d}=k \Rightarrow \frac{m\cdot a+t\cdot c}{m\cdot b+t\cdot d}=k$$

ÖRNEK / EXAMPLE 4

$\frac{a}{2}=\frac{b}{3}=\frac{c}{5}=k$, $2a-b+3c=64 \Rightarrow$ **$a=?$**

A) 6 B) 7 C) 8 D) 9 E) 10

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{a}{2}=\frac{b}{3}=\frac{c}{5}=k \Rightarrow \frac{2a-b+3c}{2\cdot 2-3+3\cdot 5}=k$$

$$\frac{64}{16}=k=4$$

$a=2\cdot 4=8$

PEKİŞTİRME / REINFORCE

$\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$, $x+y-z=6 \Rightarrow z=?$

YANIT / ANSWER : 24

5. $\frac{a}{b}=\frac{c}{d}=k \Rightarrow \frac{a\cdot c}{b\cdot d}=k^2$

ÖRNEK / EXAMPLE 5

$\frac{a}{2}=\frac{b}{4}=\frac{c}{5}=k$, $a\cdot b\cdot c=320 \Rightarrow$ **$k=?$**

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{a}{2}=\frac{b}{4}=\frac{c}{5}=k \Rightarrow \frac{a\cdot b\cdot c}{2\cdot 4\cdot 5}=k^3$$

$$\frac{320}{40}=k^3$$

$$k^3=8$$

$$k=2$$

6. I. a ile b doğru orantılı ise $\frac{a}{b} = k$ dir.

If a and b are directly proportional, then $\frac{a}{b} = k$

II. a ile b ters orantılı ise, $a \cdot b = k$ dir.

If a and b are inversely proportional, then $a \cdot b = k$.

ÖRNEK / EXAMPLE 6

$(x - 2) \cdot (y^2 + 1) = k$

$x = 3 \Rightarrow y = 2$

$y = 1 \Rightarrow$ **x = ?**

A) $\frac{7}{2}$ B) 4 C) $\frac{9}{2}$ D) 5 E) $\frac{11}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$(x - 2) \cdot (y^2 + 1) = k$

$x = 3, y = 2 \Rightarrow (3 - 2) \cdot (2^2 + 1) = 5 = k$

$y = 1 \Rightarrow (x - 2) \cdot (1^2 + 1) = k = 5$

$x - 2 = \frac{5}{2} \Rightarrow x = \frac{9}{2}$

ÖRNEK / EXAMPLE 7

$$\left.\begin{aligned} a + \frac{1}{b} &= 5 \\ b + \frac{1}{a} &= 7 \end{aligned}\right\} \Rightarrow \frac{\mathbf{a+b}}{\mathbf{b-a}} = \mathbf{?}$$

A) 2 B) 3 C) 4 D) 5 E) 6

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$a + \frac{1}{b} = 5 \Rightarrow ab + 1 = 5b$

$b + \frac{1}{a} = 7 \Rightarrow ab + 1 = 7a$

$ab + 1 = ab + 1$

$\Rightarrow 5b = 7a \Rightarrow a = 5k, \ b = 7k$

$\frac{a+b}{b-a} = \frac{12k}{2k} = 6$

PEKİŞTİRME / REINFORCE

$$\left.\begin{aligned} x + \frac{1}{y} &= 3 \\ y + \frac{1}{x} &= 6 \end{aligned}\right\} \Rightarrow \frac{y}{x} = ?$$

YANIT / ANSWER : 2

ÖRNEK / EXAMPLE 8

YILDIZ TEKNİK ÜNV. YÖS 2013

$$\left.\begin{array}{l} \frac{1}{x}-3=y \\ \frac{1}{y}-5=x \end{array}\right\} \Rightarrow \frac{x+y}{x-y}=?$$

A) 1 B) $\frac{1}{8}$ C) $\frac{3}{5}$ D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{1}{x}-3=y \Rightarrow \frac{1-xy}{x}=3$$

$$\frac{1}{y}-5=x \Rightarrow \frac{1-xy}{y}=5$$

$$\frac{\frac{1-xy}{x}}{\frac{1-xy}{y}}=\frac{y}{x}=\frac{3}{5} \Rightarrow \frac{x+y}{x-y}=\frac{8}{2}=4$$

ÖRNEK / EXAMPLE 9

İSTANBUL ÜNV. YÖS 2013

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{1}{5}$$

$2a-3c+5e=-8$

$2b+5f=35$

$\Rightarrow$ **d = ?**

A) –25 B) –5 C) 1 D) 5 E) 25

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{2a-3c+5e}{2b+5f-3d}=\frac{1}{5}$$

$$-\frac{8}{35-3d}=\frac{1}{5}$$

$$35-3d=-40$$

$$75=3d$$

$$25=d$$

PEKİŞTİRME / REINFORCE

$x, y, z \in Z^+$,

$$\left.\begin{array}{l} \frac{x}{2}=\frac{y}{3}=\frac{z}{4} \\ x^2+y^2+z^2=29 \end{array}\right\} \Rightarrow x+y-z=?$$

YANIT / ANSWER : 1

PEKİŞTİRME / REINFORCE

$$\frac{a}{b}=\frac{2}{3} \Rightarrow \frac{2a-3b}{a+b}=?$$

YANIT / ANSWER : –1

PEKİŞTİRME / REINFORCE

$$\frac{x}{y}=\frac{z}{t}=\frac{3}{2} \Rightarrow \frac{x\cdot z}{y\cdot t}=?$$

YANIT / ANSWER : 9 / 4

ÖRNEK / EXAMPLE 10

ANKARA ÜNV. YÖS 2013

$$\frac{x}{3}=\frac{y}{5}=\frac{z}{8},$$

$$x-2y+z=3 \Rightarrow \mathbf{y=?}$$

A) 1 B) 5 C) 10 D) 15 E) 20

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$x = 3k$, $y = 5k$, $z = 8k$

$x - 2y + z = 3k - 10k + 8k$

$3 = k$

$y = 5k = 5\cdot 3 = 15$

ÖRNEK / EXAMPLE 11

KARADENİZ TEKNİK ÜNİV. 2016

$$x+\frac{1}{y}=2,$$

$$y+\frac{1}{x}=5 \Rightarrow \frac{\mathbf{x+2y}}{\mathbf{x-2y}}=\mathbf{?}$$

A) –1,5 B) –1 C) 0 D) 1 E) 1,5

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\left.\begin{array}{l} x+\dfrac{1}{y}=2 \Rightarrow xy+1=2y \\ \\ y+\dfrac{1}{x}=5 \Rightarrow xy+1=5x \end{array}\right\} \Rightarrow 2y=5x$$

$2y = 5x \Rightarrow y = 5k,\ x = 2k$

$$\frac{x+2y}{x-2y}=\frac{2k+10k}{2k-10k}=\frac{12}{-8}=-1{,}5$$

1. $\frac{x}{2}=\frac{y}{5}=\frac{z}{7}$,

$3x-2y+z=6 \Rightarrow \mathbf{x+y=?}$

A) 12 B) 13 C) 14 D) 15 E) 16

2. $\frac{a}{3}=2b=\frac{c}{4}$,

$a+b+c=75 \Rightarrow \mathbf{b=?}$

A) 3 B) 5 C) 12 D) 25 E) 30

3. $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$,

$a+b+c=36 \Rightarrow \mathbf{a+c=?}$

A) 22 B) 24 C) 26 D) 28 E) 30

4. $3x=5y=6z$,

$x+y+2z=520 \Rightarrow \mathbf{x=?}$

A) 100 B) 150 C) 200 D) 250 E) 300

5. $\frac{a+b}{4}=\frac{a^2-b^2}{8}=\frac{16}{a-b} \Rightarrow \mathbf{a=?}$

A) 12 B) 15 C) 17 D) 18 E) 20

6. $\frac{x}{y}=\frac{3}{5}$, $\frac{y}{z}=\frac{4}{3} \Rightarrow \mathbf{\frac{2x+y}{x+2z}=?}$

A) $\frac{20}{19}$ B) $\frac{21}{20}$ C) $\frac{22}{21}$ D) $\frac{23}{22}$ E) $\frac{24}{23}$

7. $$\left.\begin{aligned} x+\frac{3}{y}&=5 \\ y+\frac{3}{x}&=2 \end{aligned}\right\} \Rightarrow \frac{\mathbf{x+y}}{\mathbf{2y-x}}=\mathbf{?}$$

A) –7 B) –3 C) 1

D) 3 E) 7

8. $$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{3}{2} \Rightarrow \frac{\mathbf{a^2}}{\mathbf{d^2}}\cdot\frac{\mathbf{f}}{\mathbf{e}}\cdot\frac{\mathbf{c^2}}{\mathbf{b^2}}=\mathbf{?}$$

A) $\frac{1}{5}$ B) $\frac{27}{8}$ C) $\frac{1}{3}$

D) $\frac{20}{9}$ E) $\frac{9}{2}$

9. $$\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=\frac{5}{3}$$

$3x + y + 2z = 20$,

$b + 2c = 3 \quad \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

10. $3x = 5y = a\cdot z = 12$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1 \Rightarrow \mathbf{a=?}$$

A) 7 B) 6 C) 5

D) 4 E) 3

11. $$\frac{x+y}{4}=\frac{y+z}{5}=\frac{x+z}{7}$$

$y = 3 \Rightarrow$ **x = ?**

A) 4 B) 5 C) 6

D) 8 E) 9

12. $$\frac{x+y}{y}\cdot(x-1)=k$$

$x = 2 \Rightarrow y = 3$

$x = 3 \Rightarrow$ **y = ?**

A) –12 B) –15 C) –16

D) –18 E) –20

YANITLAR / ANSWERS

1. C	**2. B**	**3. B**	**4. C**	**5. C**	**6. C**	**7. A**	**8. B**
9. E	**10. D**	**11. E**	**12. D**				

1. $\frac{x}{2}=\frac{y}{5}=\frac{z}{7}=k$

$$\left.\begin{aligned}x&=2k\\y&=5k\\z&=7k\end{aligned}\right\}\Rightarrow \begin{aligned}3x-2y+z&=6\\6k-10k+7k&=6\\3k&=6\\k&=2\end{aligned}$$

$k=2\Rightarrow x+y=7k=7\cdot2=14$

Yanıt / The Answer is C

2. $\frac{a}{3}=2b=\frac{c}{4}=k$

$$\left.\begin{aligned}a&=3k\\b&=\frac{k}{2}\\c&=4k\end{aligned}\right\}\Rightarrow \begin{aligned}a+b+c&=75\\3k+\frac{k}{2}+4k&=75\\15k&=150\\k&=10\end{aligned}$$

$\Rightarrow b=\frac{k}{2}=\frac{10}{2}=5$

Yanıt / The Answer is B

3. $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}\Rightarrow a=2k,\ b=3k,\ c=4k$

$a+b+c=36$

$2k+3k+4k=36$

$9k=36$

$k=4$

$a+c=2k+4k=6k=6\cdot4=24$

Yanıt / The Answer is B

4. $3x=5y=6z\Rightarrow x=10k,\ y=6k,\ z=5k$

$x+y+2z=10k+6k+2\cdot5k$

$x+y+2z=26k=520$

$k=20$

$k=20\Rightarrow x=10\cdot20=200$

Yanıt / The Answer is C

5. $\frac{a+b}{4}=\frac{a^2-b^2}{8}=\frac{16}{a-b}=k$

$(a+b)\cdot(a-b)=4\cdot16$

$a^2-b^2=64$

$\frac{a^2-b^2}{8}=\frac{64}{8}=8=k$

$$\left.\begin{aligned}a+b&=32\\a-b&=2\end{aligned}\right\}\Rightarrow a=17$$

Yanıt / The Answer is C

6. $$\left.\begin{aligned}\frac{x}{y}&=\frac{3}{5}\\\frac{y}{z}&=\frac{4}{3}\end{aligned}\right\}\Rightarrow \frac{x}{y}=\frac{12}{20},\ \frac{y}{z}=\frac{20}{15}$$

$x=12k,\ y=20k,\ z=15k$

$\Rightarrow \frac{2x+y}{x+2z}=\frac{24k+20k}{12k+30k}=\frac{44k}{42k}=\frac{22}{21}$

Yanıt / The Answer is C

7. $x+\frac{3}{y}=5 \Rightarrow xy+3=5y$

$y+\frac{3}{x}=2 \Rightarrow xy+3=2x$

$x+y+3=5y=2x \Rightarrow x=5k, \quad y=2k$

$\frac{x+y}{2y-x}=\frac{5k+2k}{4k-5k}=-7$

Yanıt / The Answer is A

8. $\frac{a^2}{d^2}\cdot\frac{f}{e}\cdot\frac{c^2}{b^2}=\frac{a^2\cdot c^2\cdot f}{b^2\cdot d^2\cdot e}$

$=\frac{a^2}{b^2}\cdot\frac{c^2}{d^2}\cdot\frac{f}{e}$

$=\frac{9}{4}\cdot\frac{9}{4}\cdot\frac{2}{3}$

$=\frac{27}{8}$

Yanıt / The Answer is B

9. $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=\frac{5}{3} \Rightarrow \frac{3x+y+2z}{3a+b+2c}=\frac{5}{3}$

$\left.\begin{aligned}3x+y+2z&=20\\ b+2c&=3\end{aligned}\right\} \Rightarrow \frac{20}{3a+3}=\frac{5}{3}$

$60=15a+15$

$a=3$

$a=3 \Rightarrow x=5$

Yanıt / The Answer is E

10. $3x=5y=a\cdot z=12$

$\underset{(3)}{\frac{1}{x}}+\underset{(5)}{\frac{1}{y}}+\underset{(a)}{\frac{1}{z}}=\frac{3+5+a}{12}=1$

$3+5+a=12 \Rightarrow a=4$

Yanıt / The Answer is D

11. $\frac{x+y}{4}=\frac{y+z}{5}=\frac{x+z}{7}=k$

$\frac{x+y}{4}=\frac{y+z}{5}=\frac{-x-z}{-7}=k$

$\frac{2y}{2}=k$

$y=3 \Rightarrow k=3$

$\frac{x+y}{4}=k \Rightarrow x+y=4k$

$x+3=12$

$x=9$

Yanıt / The Answer is E

12. $x=2, y=3 \Rightarrow \frac{x+y}{y}\cdot(x-1)=\frac{5}{3}\cdot 1=\frac{5}{3}$

$k=\frac{5}{3}$

$x=3 \Rightarrow \frac{3+y}{y}\cdot(2)=\frac{5}{3}$

$\frac{6+2y}{y}=\frac{5}{3} \Rightarrow 18+6y=5y$

$y=-18$

Yanıt / The Answer is D

1. $\dfrac{3a-b}{2}=a+b \Rightarrow \mathbf{\dfrac{a+b}{a-b}=?}$

Yanıt / The Answer is: **2**

2. $\dfrac{a}{2}=\dfrac{b}{3}=\dfrac{c}{5},$

$a+b+c=130 \Rightarrow \mathbf{c=?}$

Yanıt / The Answer is: **65**

3. $\dfrac{a}{4}=\dfrac{b}{5}=\dfrac{c}{7}=k$

$\dfrac{2a+b+xc}{34}=k \Rightarrow \mathbf{x=?}$

Yanıt / The Answer is: **3**

4. $\dfrac{a}{b}=\dfrac{3}{7} \quad \dfrac{b}{c}=\dfrac{2}{5} \Rightarrow \mathbf{a:b:c=?}$

Yanıt / The Answer is: **6 : 14 : 35**

5. $\dfrac{a}{5}=\dfrac{b}{4}=2c,$

$a+2b-c=100 \Rightarrow \mathbf{a=?}$

Yanıt / The Answer is: **40**

6. $\left.\begin{array}{l}\dfrac{a-2b}{b}=3\\[2mm]\dfrac{b+c}{c}=4\end{array}\right\} \Rightarrow \mathbf{a:b:c=?}$

Yanıt / The Answer is: **15 : 3 : 1**

7. $a, b, c \in \mathbb{R}^-$

$4a = 5b = 3c \Rightarrow$ **$\max(a + b + c) = ?$**

Yanıt / The Answer is: **–47**

---

8. $\frac{x+y}{7} = \frac{x-y}{5} = \frac{xy}{12}$

$\Rightarrow$ **$x^2 + y^2 = ?$**

Yanıt / The Answer is: **148**

---

9. $2x = 5y = 4z$

$x + y + z = 76 \Rightarrow$ **$y = ?$**

Yanıt / The Answer is: **16**

---

10. $x, y, z \in \mathbb{R}^+$

$\frac{x}{5} = \frac{y}{3} = \frac{z}{7}$

$x^2 - 2y^2 + 3z^2 = 616 \Rightarrow$ **$z = ?$**

Yanıt / The Answer is: **14**

---

11. $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{5}{2}$

$3x + y + z = 15,$

$b + c = 2 \Rightarrow$ **$a = ?$**

Yanıt / The Answer is: $\mathbf{\frac{4}{3}}$

---

12. $x + \frac{2}{y} = 3, \ y + \frac{2}{x} = 4$

$x + 2y = 33 \Rightarrow$ **$x = ?$**

Yanıt / The Answer is: **9**

1. $\frac{a-b}{b}=\frac{2}{3} \Rightarrow \frac{\mathbf{a+b}}{\mathbf{b}}=\mathbf{?}$

A) $\frac{4}{3}$ B) $\frac{5}{3}$ C) 2
D) $\frac{7}{3}$ E) $\frac{8}{3}$

2. x, y, z ∈ N^+,

$\frac{x}{y}=\frac{4}{5},\ \frac{y}{z}=\frac{3}{5} \Rightarrow$ **min(x + y + z) = ?**

A) 34 B) 42 C) 48
D) 51 E) 52

3. $\left.\begin{matrix}2a=5b\\4b=c\end{matrix}\right\} \Rightarrow \frac{\mathbf{a}}{\mathbf{c}}=\mathbf{?}$

A) $\frac{2}{5}$ B) $\frac{3}{5}$ C) $\frac{3}{8}$
D) $\frac{5}{8}$ E) $\frac{7}{8}$

4. $x+\frac{1}{y}=\frac{4}{5}\ ,\ y+\frac{1}{x}=\frac{5}{9} \Rightarrow \frac{\mathbf{y}}{\mathbf{x}}=\mathbf{?}$

A) $\frac{9}{20}$ B) $\frac{25}{18}$ C) $\frac{18}{25}$
D) $\frac{36}{25}$ E) $\frac{25}{36}$

5. $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=2 \Rightarrow \frac{\mathbf{a}}{\mathbf{b}}\cdot\frac{\mathbf{d}}{\mathbf{c}}\cdot\frac{\mathbf{e}}{\mathbf{f}}=\mathbf{?}$

A) 1 B) 2 C) 4
D) 8 E) 16

6. ax = by = cz = 20,

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{5} \Rightarrow \mathbf{x+y+z=?}$

A) 4 B) 10 C) 20
D) 40 E) 80

7. $\dfrac{xy}{3}=\dfrac{yz}{4}=\dfrac{xz}{5}$,

$x+y+z=141 \Rightarrow \mathbf{z=?}$

A) 36 B) 42 C) 49 D) 54 E) 60

8. $\left.\begin{array}{l}\dfrac{a}{5}=\dfrac{b}{7}=\dfrac{c}{9}\\ \dfrac{3x-6}{9}=\dfrac{a+b}{c}\end{array}\right\} \Rightarrow \mathbf{x=?}$

A) 6 B) 8 C) 12 D) 15 E) 18

9. $\dfrac{a}{b}=\dfrac{c}{d}$, $a\cdot d\cdot b-b^2\cdot c+2\cdot a-6=0 \Rightarrow \mathbf{a=?}$

A) 2 B) 3 C) 4 D) 8 E) 12

10. $\dfrac{a}{2}=\dfrac{b}{3}=4c$,

$a+b+c=105 \Rightarrow \mathbf{a=?}$

A) 10 B) 20 C) 30 D) 40 E) 50

11. $\dfrac{a}{x}=\dfrac{b}{y}=\dfrac{c}{z}=5$

$3a-2b+c=60$

$3x-2y=7 \Rightarrow \mathbf{z=?}$

A) 5 B) 6 C) 8 D) 10 E) 15

12. $\dfrac{a}{b+1}\cdot(2b-1)=k$

$a=4 \Rightarrow b=1$

$b=3 \Rightarrow \mathbf{a=?}$

A) $\dfrac{8}{5}$ B) $\dfrac{9}{5}$ C) 2 D) $\dfrac{11}{5}$ E) $\dfrac{12}{5}$

YANITLAR / ANSWERS

1. E 2. E 3. D 4. E 5. B 6. A 7. E 8. A
9. B 10. D 11. A 12. A

1. $\frac{a}{2} = \frac{b}{3} = 4c$

$a + b + c = 420 \Rightarrow \mathbf{a = ?}$

A) 120 B) 130 C) 140 D) 150 E) 160

2. $x, y, z \in Z^-$

$\frac{x}{4} = \frac{y}{5}, \quad \frac{x}{z} = \frac{4}{3} \quad \Rightarrow \quad \mathbf{? < ? < ?}$

A) $x < y < z$ B) $y < x < z$ C) $z < y < x$ D) $x < z < y$ E) $y < z < x$

3. $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = 15$

$3x - y + z = 180$

$b - c = -6 \Rightarrow \mathbf{a = ?}$

A) 1 B) 2 C) 4 D) 5 E) 7

4. $\frac{x+y}{5} = \frac{x-z}{2} = \frac{x-y}{3} = k \quad \Rightarrow \quad \mathbf{x : y : z = ?}$

A) 3 : 2 : 4 B) 5 : 3 : 1 C) 2 : 1 : 6 D) 4 : 1 : 2 E) 3 : 1 : 4

5. $\frac{a}{b} = \frac{2}{3}, \quad \frac{b}{c} = \frac{4}{5}, \quad \frac{c}{d} = \frac{1}{2},$

$a + d = 76 \Rightarrow \mathbf{b = ?}$

A) 24 B) 25 C) 27 D) 30 E) 32

6. $ax = by = cz = 30$

$\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = \frac{1}{2} \quad \Rightarrow \quad \mathbf{a + b - c = ?}$

A) 5 B) 6 C) 10 D) 15 E) 60

7. $x:y:z = \frac{1}{2}:\frac{1}{3}:\frac{1}{4}$,

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{4} \quad \Rightarrow \quad \mathbf{y = ?}$

A) 4 B) 8 C) 12
D) 16 E) 20

8. $\frac{a-3}{4} = \frac{b+2}{2} = \frac{c-1}{5}$

$a - b + c = 27 \quad \Rightarrow \quad \mathbf{c = ?}$

A) 12 B) 14 C) 15
D) 16 E) 18

9. $\left.\begin{array}{l} a + \frac{4}{b} = 3 \\ b + \frac{4}{a} = 5 \end{array}\right\} \quad \Rightarrow \quad \mathbf{\frac{a+b}{b-a} = ?}$

A) 1 B) 2 C) 3
D) 4 E) 5

10. $a = \frac{5b}{2}$,

$c = 4b$,

$a + c = 26 \quad \Rightarrow \quad \mathbf{b = ?}$

A) 4 B) 6 C) 8
D) 10 E) 12

11. $\frac{xy}{3} = \frac{yz}{2} = \frac{xz}{5} \quad \Rightarrow \quad \mathbf{x:y:z = ?}$

A) 15 : 6 : 10 B) 9 : 7 : 12 C) 4 : 6 : 9
D) 12 : 5 : 18 E) 12 : 8 : 25

12. $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-3}{5}$

$x - y + z = 46 \quad \Rightarrow \quad \mathbf{y = ?}$

A) 39 B) 42 C) 45
D) 49 E) 52

YANITLAR / ANSWERS

1. E	**2. B**	**3. B**	**4. D**	**5. A**	**6. D**	**7. C**	**8. D**
9. D	**10. A**	**11. A**	**12. A**				

1. $$\left.\begin{array}{l} 3a+\dfrac{6}{b}=4 \\ b+\dfrac{2}{a}=5 \end{array}\right\} \Rightarrow \mathbf{\dfrac{a+b}{4a-b}=?}$$

A) 18 B) 19 C) 20

D) 21 E) 22

2. $$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{1}{2} \Rightarrow \mathbf{\frac{a\cdot d^2\cdot e}{b\cdot c^2\cdot f}=?}$$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$

D) $\frac{1}{8}$ E) $\frac{1}{16}$

3. $$\frac{x+y}{5}=\frac{y+z}{8}=\frac{x+z}{7}$$

$x-y=-6 \Rightarrow$ **$z=?$**

A) 24 B) 25 C) 27

D) 30 E) 32

4. $$\left.\begin{array}{l} \dfrac{3a}{a+b}=\dfrac{1}{2} \\ \dfrac{b}{b+c}=\dfrac{2}{3} \end{array}\right\} \Rightarrow \mathbf{a:b:c=?}$$

A) 4 : 7 : 2 B) 3 : 8 : 6 C) 2 : 10 : 5

D) 5 : 3 : 1 E) 4 : 1 : 5

5. $$\frac{1}{3x}=\frac{2}{5y}=\frac{3}{4z},$$

$x+y+z=89 \Rightarrow$ **$y-x=?$**

A) 1 B) 2 C) 3

D) 4 E) 5

6. $$\frac{x}{2}=\frac{y}{3}=\frac{x+y+z}{7},$$

$2x-y+z=9 \Rightarrow$ **$y=?$**

A) 2 B) 4 C) 6

D) 8 E) 9

7. $\frac{a-b+c}{3}=\frac{a+b-c}{5}=\frac{b+c-a}{9}$,

$a+b+c=51 \Rightarrow \mathbf{a=?}$

A) 6　B) 9　C) 12　D) 15　E) 18

8. $a, b, c \in R^+$

$\frac{3}{4ab}=\frac{2}{5bc}=\frac{4}{3ac} \Rightarrow \mathbf{?<?<?}$

A) $c<a<b$　B) $a<b<c$　C) $b<c<a$　D) $c<b<a$　E) $b<a<c$

9. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z+1}{6}$,

$x+y+z=182 \Rightarrow \mathbf{z=?}$

A) 83　B) 84　C) 86　D) 88　E) 90

10. $\frac{x+y}{8}=\frac{x+y-z}{5}=\frac{x+z}{4}$,

$z-x=12 \Rightarrow \mathbf{y=?}$

A) 30　B) 32　C) 36　D) 40　E) 42

11. $a, b, c \in R^+$

$\frac{a}{2}=\frac{b}{3}=\frac{c}{5}$,

$a^2+b^2-c^2=-48 \Rightarrow \mathbf{b+c=?}$

A) 10　B) 12　C) 14　D) 16　E) 18

12. $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=3 \Rightarrow \frac{\mathbf{a}}{\mathbf{d}}=\mathbf{?}$

A) 1　B) 3　C) 9　D) 27　E) 81

YANITLAR / ANSWERS

1. B	**2. A**	**3. D**	**4. C**	**5. D**	**6. E**	**7. C**	**8. C**
9. A	**10. E**	**11. D**	**12. D**				

1. $\left.\begin{array}{l} \dfrac{x}{4}=\dfrac{y}{5} \\ \dfrac{y}{3}=\dfrac{z}{2} \end{array}\right\}, 4x-3y+2z=23 \Rightarrow \mathbf{z=?}$

A) 4 B) 6 C) 7
D) 8 E) 10

2. $\left.\begin{array}{l} ax=by=cz=6 \\ x+y+z=24 \end{array}\right\} \Rightarrow \mathbf{\dfrac{7}{2a}+\dfrac{7}{2b}+\dfrac{7}{2c}=?}$

A) 7 B) 14 C) 21
D) 28 E) 35

3. a, b, c ∈ R^+,

$\dfrac{a}{4}=\dfrac{b}{3}=\dfrac{c}{6}$,

$\sqrt{a^2+b^2}=60 \Rightarrow \mathbf{c=?}$

A) 48 B) 60 C) 72
D) 84 E) 96

4. $\left.\begin{array}{l} \dfrac{a+b}{a-b}=\dfrac{x}{y} \\ \dfrac{3}{x}=\dfrac{4}{y} \end{array}\right\} \Rightarrow \mathbf{\dfrac{a+2b}{b}=?}$

A) –1 B) –2 C) –3
D) –4 E) –5

5. $\dfrac{a}{x}=\dfrac{b}{y}=\dfrac{c}{z}=5$,

$3a-b+2c=30$,

$3x-y=4 \Rightarrow \mathbf{z=?}$

A) 1 B) 4 C) 6
D) 8 E) 10

6. $\dfrac{3a+1}{x}=\dfrac{b-3}{y}=\dfrac{2c+1}{z}=2$

$6a+b+2c=40 \Rightarrow \mathbf{2x+y+z=?}$

A) 15 B) 20 C) 25
D) 30 E) 35

7. $\dfrac{x}{(y+2)} \cdot (x^2-1) = k$

$x = 3 \Rightarrow y = 6$

$x = 4 \Rightarrow$ **$y = ?$**

A) 12 B) 14 C) 16 D) 18 E) 20

8. $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c} = 3$,

$2x + 3y - z = 48$,

$2a - c = 10 \Rightarrow$ **$b = ?$**

A) 2 B) 4 C) 6 D) 8 E) 10

9. $\dfrac{a+2}{b} = \dfrac{b+3}{c} = \dfrac{c+4}{a} = 2 \Rightarrow$ **$a+b+c = ?$**

A) 9 B) 11 C) 12 D) 14 E) 15

10. $\dfrac{a}{xy} = \dfrac{b}{xz} = \dfrac{c}{yz}$,

$a+b+c = \dfrac{2}{x} + \dfrac{2}{y} + \dfrac{2}{z} \Rightarrow$ **$a \cdot z = ?$**

A) 2 B) 3 C) 4 D) 5 E) 6

11. $\dfrac{a}{b} = \dfrac{c}{d} = k$

$\dfrac{3a+16}{9b+12d} = \dfrac{k}{3} \Rightarrow$ **$c = ?$**

A) 2 B) 4 C) 5 D) 6 E) 8

12. $ax = by = cz = 36$

$xy + yz + xz = xyz$

$\Rightarrow$ **$a + b + c = ?$**

A) 24 B) 28 C) 30 D) 32 E) 36

YANITLAR / ANSWERS

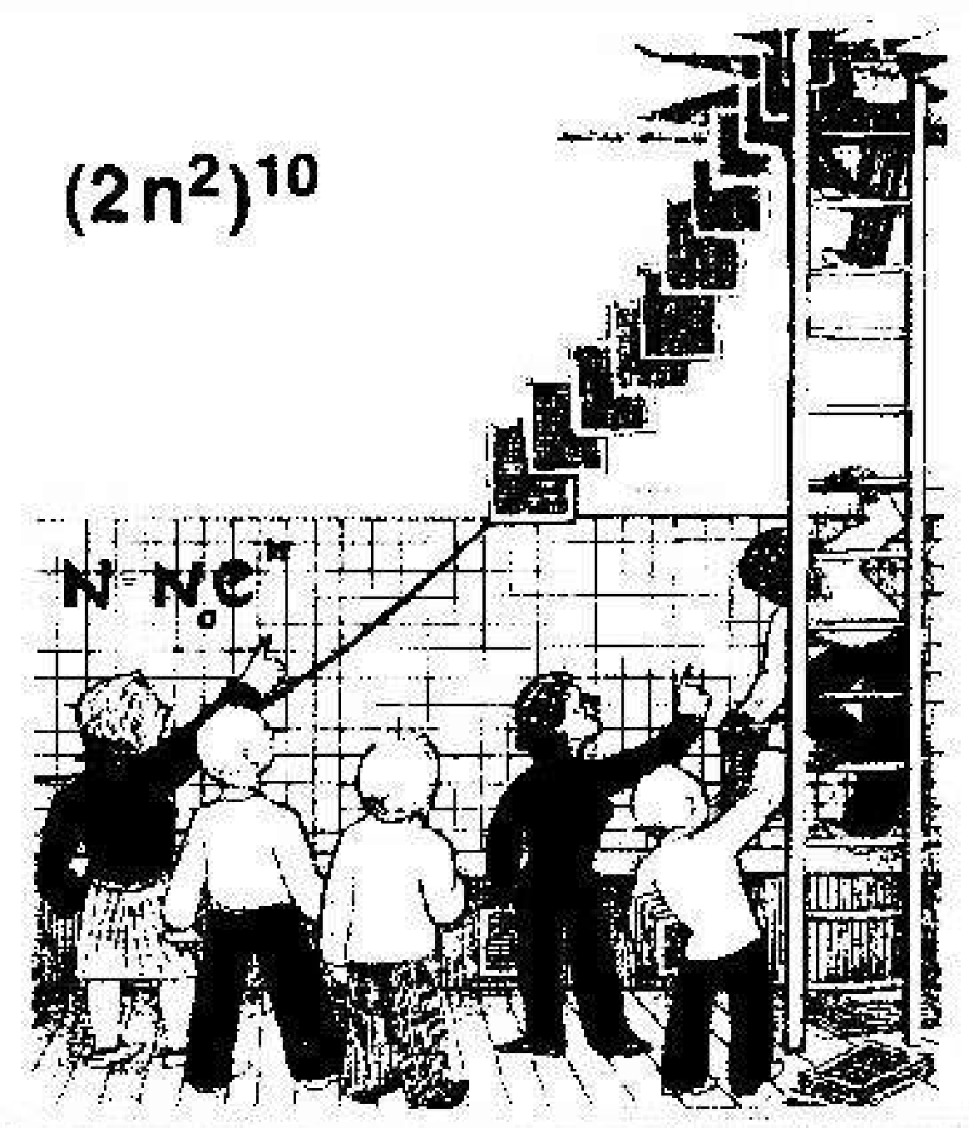
1. E	2. B	3. C	4. E	5. A	6. B	7. D	8. A
9. A	10. A	11. B	12. E				

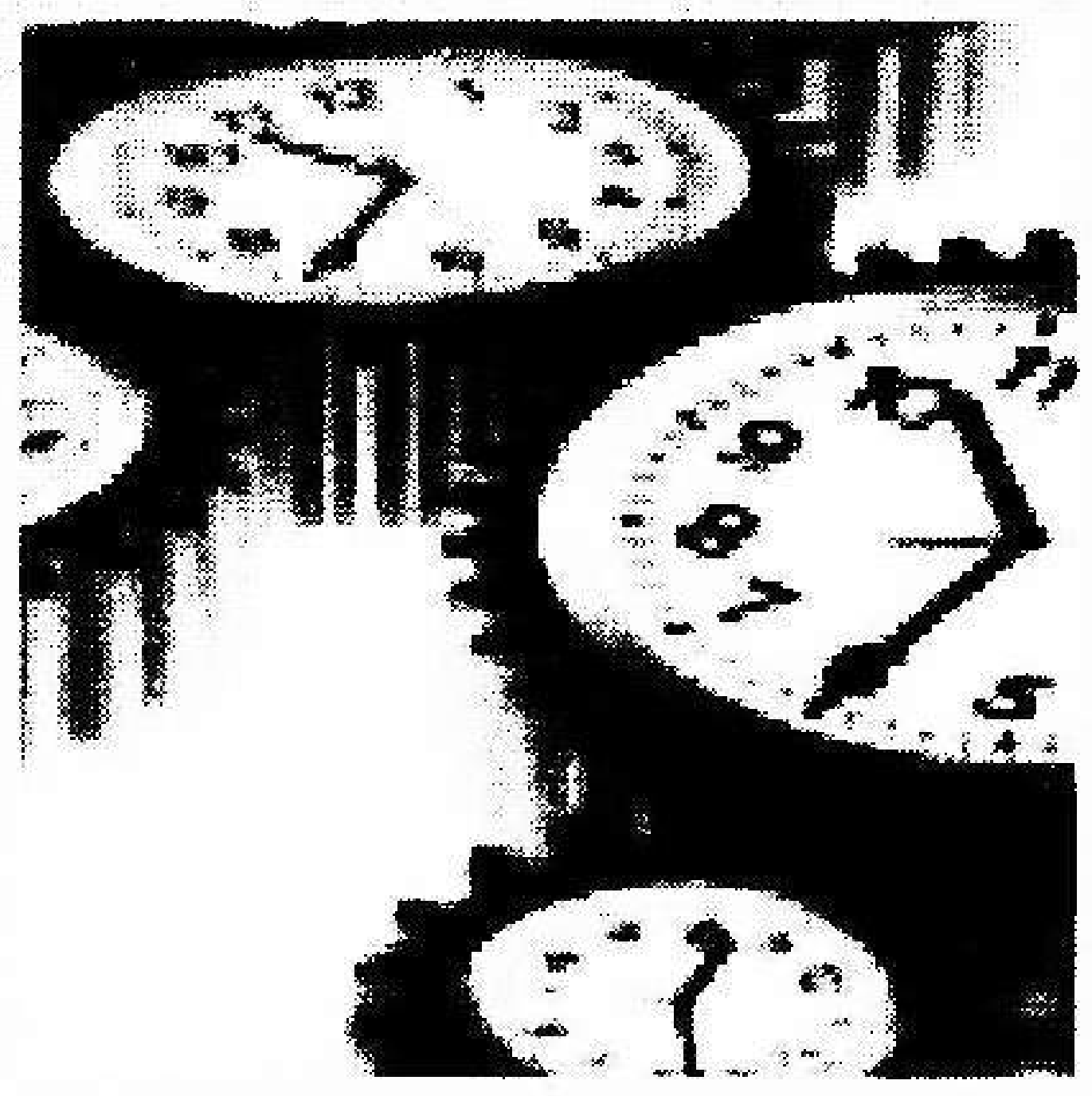
ÜNİTE 3

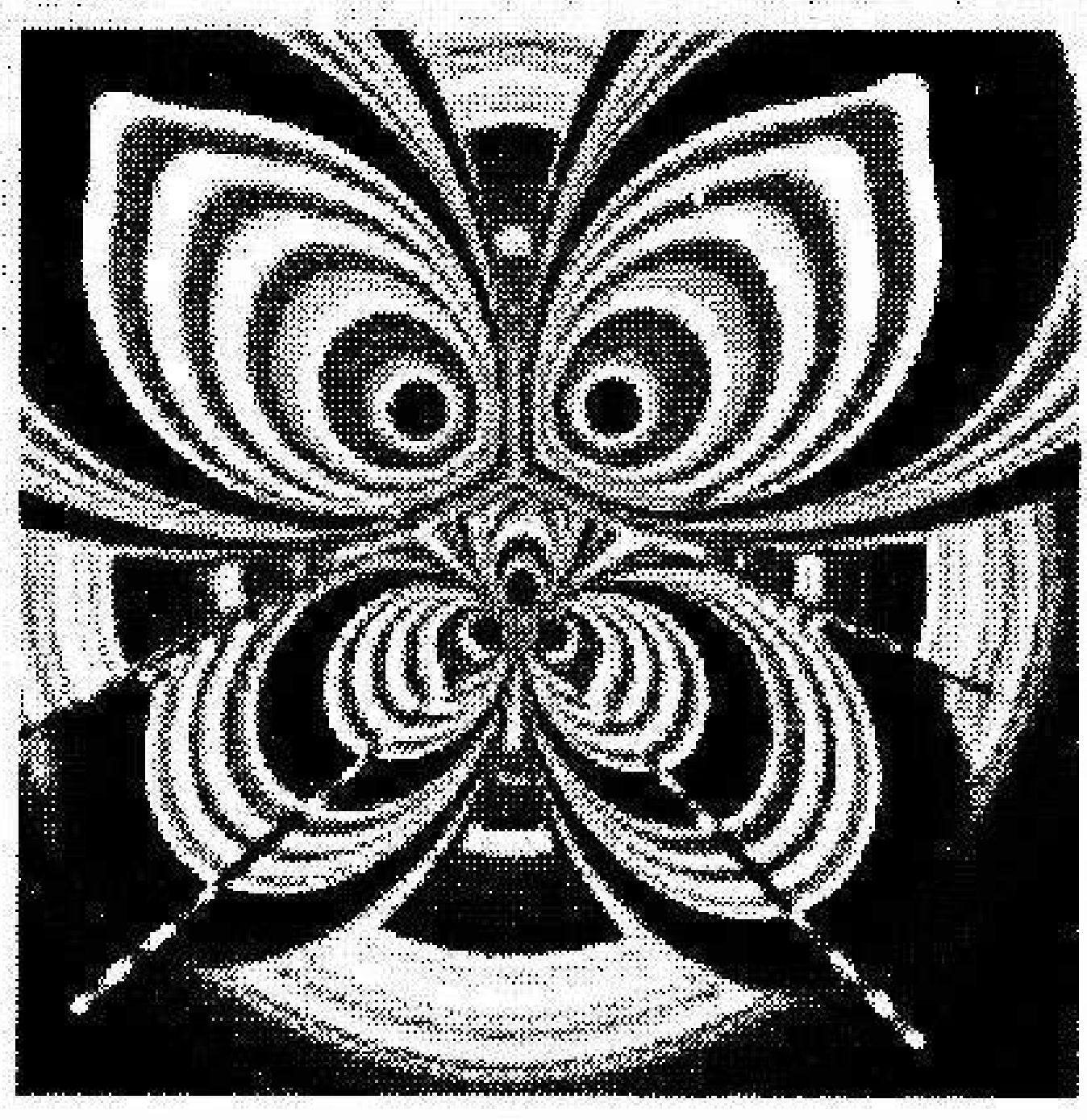
UNIT

Bu ünitede; üslü sayılar, köklü sayılar ve onların özellikleri ayrıntılı anlatılmıştır.
Bu ünite, 68 çözümlü örnek, 24 alıştırma, 96 test sorusu içermektedir.

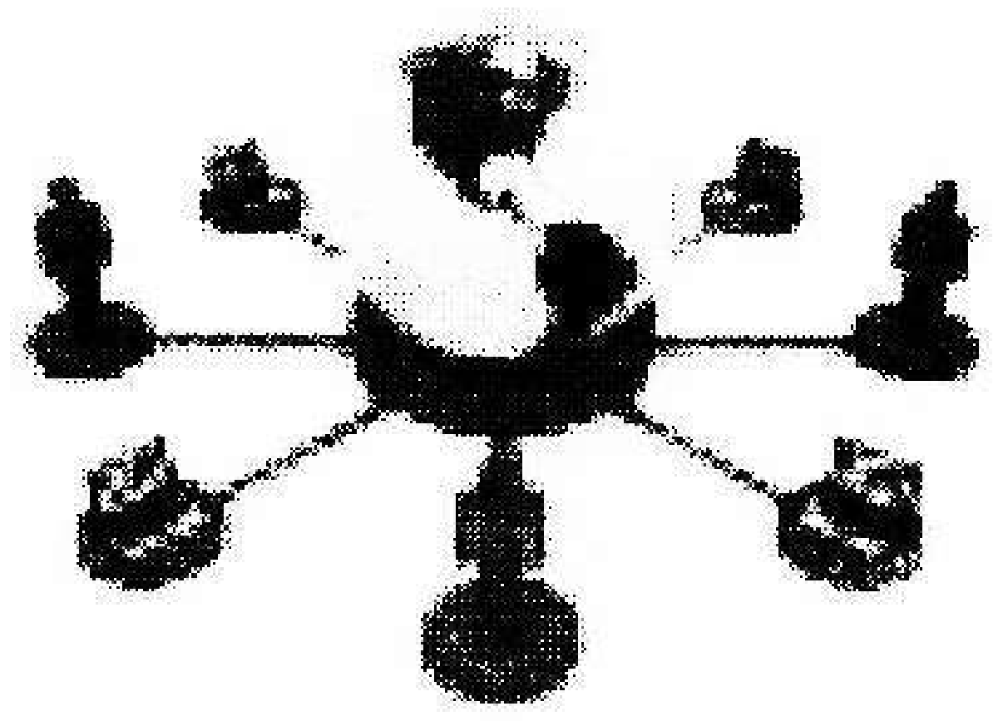
In this unit, exponentials, radicals and their properties are explained in detailed.
This unit includes; 68 examples, 24 exercises, and 96 test questions.







- **ÜSLÜ SAYILAR /** ***EXPONENTIALS*** . . . . 123 – 144
- **KÖKLÜ SAYILAR /** ***RADICALS*** . . . . 145 – 176



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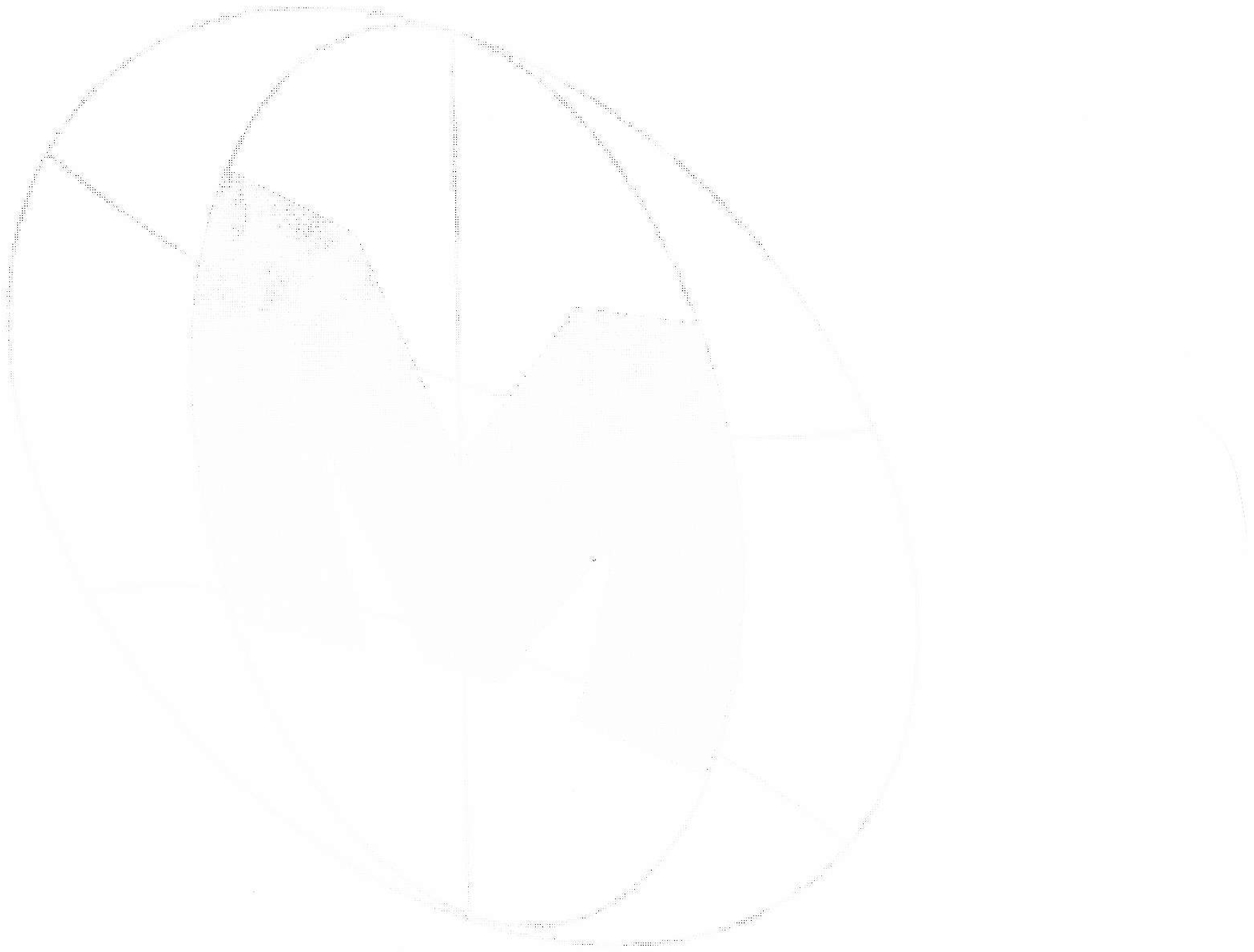

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ÜSLÜ SAYILAR
EXPONENTIALS

BÖLÜM 1
CHAPTER 1

Tanım: a sıfırdan farklı bir gerçel sayı ve n bir doğal sayı olmak üzere,

$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ çarpan}} = a^n$ ifadesine, *üslü ifade* denir.

n : Üs (kuvvet) , **a :** Taban

a^n ifadesi, a nın n. kuvveti olarak okunur.

Definition: *Provided that $n \in N$ and $a \in R - \{0\}$, the expression*

$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}} = a^n$ *is called exponential expression.*

n *: power ,* ***a*** *: base.*

The expression a^n is read as " a to the n^{th} power".

NOTLAR / NOTES

- ✓ Eğer kuvvet 0 ise, ifadenin değeri her zaman 1 dir.

  If the exponent is 0, the expression always has a value of 1.

- ✓ Eğer kuvvet 1 ise, ifadenin değeri aynı tabandaki sayıya eşittir.

  If the exponent is 1, the value of the expression is the number base.

TEMEL ÖZELLİKLER / BASIC PROPERTIES

A. Sıfırdan farklı bir sayının sıfırıncı kuvveti 1 dir.

Zeroth power of a number is 1. Here this number is different than zero.

$a \neq 0 \Rightarrow a^0 = 1$

B. Negatif bir sayının çift tamsayı kuvvetleri pozitif, tek tamsayı kuvvetleri ise negatiftir.

Even integer power of a negative number is positive, odd integer power of it is negative.

$a \in R^+$ ve (and) $n \in Z$

a. $(-a)^{2n} = a^{2n}$

b. $(-a)^{2n-1} = -a^{2n-1}$

ÖRNEK / EXAMPLE 1

$(x-4)^{x^2+x-2}=1 \Rightarrow \sum x = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$(x-4)^{x^2+x-2}=1 \Rightarrow$ I. $x^2+x-2=0$, $x-4 \neq 0$

II. $x-4=1$

III. $x-4=-1$ [x^2+x-2 çift sayı / *even number*]

I. $x^2+x-2=0$

$(x+2)\cdot(x-1)=0 \Rightarrow x=-2$, $x=+1$

II. $x-4=1 \Rightarrow x=5$

III. $x-4=-1 \Rightarrow x=3$

$x=3 \Rightarrow x^2+x-2=10$ [10 çift sayı / *10 is even number*]

$\Rightarrow \sum x = -2+1+3+5=7$

ÖRNEK / EXAMPLE 2

$(-3)^4-(-2)^2\cdot(-2)^3=?$

A) 35 B) 49 C) 65 D) 98 E) 113

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{aligned}(-3)^4-(-2)^2\cdot(-2)^3 &= 81-4\cdot(-8)\\ &= 81+32\\ &= 113\end{aligned}$$

PEKİŞTİRME / REINFORCE

$2^3-2^2:2+(-5)^2=?$

YANIT / ANSWER : 31

ÜSLÜ İFADELERDE İŞLEMLER / OPERATIONS in EXPONENTIALS

Dağılma özelliği kullanılarak üslü sayıların toplam veya farkı tek bir eşitlik ile ifade edilebilir.
Using the distributive property, you can express the sum and difference of exponents as a single form.

1. Toplama ve Çıkarma / Addition and Subtraction

$a \cdot x^n + b \cdot x^n - c \cdot x^n = (a + b - c) \cdot x^n$

2. Çarpma / Multiplication

1. Tabanları aynı olan üslü sayıların çarpımında üsler toplanır.
   When multiplying of exponentials with same base, add exponents.
   $$a^n \cdot a^m = a^{n+m}$$

2. Üsleri aynı olan üslü sayıların çarpımında, tabanlar çarpılarak ortak üs, üs olarak yazılır.
   In multiplication of exponentials with same power, multiply the base and write the power.
   $$a^m \cdot b^m = (a \cdot b)^m$$

ÖRNEK / EXAMPLE 3

$2^{x+2} + 5 \cdot 2^{x+1} + 6 \cdot 2^x = 80 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$2^{x+2} + 5 \cdot 2^{x+1} + 6 \cdot 2^x = 80$ [Verilen eşitlik / *Given equality*]

$2^x \cdot 4 + 5 \cdot 2^x \cdot 2 + 6 \cdot 2^x = 80$ [İndirge. / *Reduce.*]

$2^x \cdot (20) = 80$ [Ortak parantez / *Common paranthesis*]

$2^x = 4$ [Sadeleştir. / *Simplify.*]

$x = 2$ [Sonuç / *Conclusion*]

PEKİŞTİRME / REINFORCE

$3^x + 2 \cdot 3^x - 4 \cdot 3^x = -27 \Rightarrow x = ?$

YANIT / ANSWER : 3

3. Bölme / Division

1. Tabanları aynı olan iki üslü sayının bölümünde, ortak taban üzerinde payın üssünden paydanın üssü çıkarılır.

In the division of exponentials with same base subtract the power of denominator from that of numerator.

$$\frac{a^n}{a^m} = a^{n-m}$$

2. Üsleri aynı olan iki üslü sayının bölümünde, tabanların bölümüne ortak üs, üs olarak yazılır.

In the division of exponentials with same power, divide bases and write common power.

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n, \quad b \neq 0$$

PEKİŞTİRME / REINFORCE

$$\frac{3^4}{3^2} + \frac{2^5}{2^3} = ?$$

YANIT / ANSWER : 13

ÖRNEK / EXAMPLE 4

$$\left(\frac{5^x}{3^y}\right)^a \cdot \left(\frac{3^{y+1}}{5^x}\right)^a = ?$$

A) 5^x B) 5^a C) 3^x D) 3^a E) 15^a

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\left(\frac{5^x}{3^y}\right)^a \cdot \left(\frac{3^{y+1}}{5^x}\right)^a = \frac{5^{xa}}{3^{ya}} \cdot \frac{3^{ya+a}}{5^{xa}}$$

$$= 3^a$$

BİR ÜSLÜ İFADENİN KUVVETİ / POWER to A POWER

Kuvvetin kuvveti alınırken, üsler çarpılıp aynı taban üzerine üs olarak yazılır.

To raise the power to power, multiply the powers and write the same base.

$$(a^m)^n = (a^n)^m = a^{m \cdot n}$$

ÖRNEK / EXAMPLE 5

$x^x = 4^{32} \Rightarrow x = ?$

A) 2 B) 4 C) 8 D) 16 E) 32

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$x^x = 4^{32} \Rightarrow x^x = 2^{64}$

$x^x = (2^4)^{16}$

$x^x = (16)^{16}$

$x = 16$

NEGATİF KUVVET / NEGATIVE POWERS

$$a^{-1} = \frac{1}{a}, \quad a^{-n} = \frac{1}{a^n}$$

ONDALIK SAYILARIN ÜSLÜ GÖSTERİMİ
WRITING EXPONENTIAL FORM of DECIMAL NUMBERS

$\underbrace{0{,}000 \ldots 000}_{\text{n rakam/ digits}}a = a \cdot 10^{-n}$ [virgülden sonra n rakam / *n digits after comma*]

PEKİŞTİRME / REINFORCE

$0,00027 = 3^x \cdot 10^y \Rightarrow x + y = ?$

YANIT / ANSWER : -2

PEKİŞTİRME / REINFORCE

$2^x + 2^x + 2^x = 9 \Rightarrow 4^x = ?$

YANIT / ANSWER : 9

ÖRNEK / EXAMPLE 6

$$\frac{(0,6)^2}{0,03} : \frac{(0,3)^4}{(0,1)^2} = ?$$

A) $\frac{400}{37}$ B) $\frac{250}{17}$ C) $\frac{400}{27}$ D) $\frac{340}{27}$ E) $\frac{300}{13}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{(0,6)^2}{0,03} : \frac{(0,3)^4}{(0,1)^2} = \frac{\frac{36}{100}}{\frac{3}{100}} : \frac{\left(\frac{3}{10}\right)^4}{\left(\frac{1}{10}\right)^2}$$

$$= 12 \cdot \frac{100}{81} = \frac{400}{27}$$

ÜSLÜ DENKLEMLER / EXPONENTIALS EQUATIONS

▼ a, –1, 0 ya da 1 olmaması koşuluyla,

a is different than –1, 0, 1

$a^n = a^m \Rightarrow n = m$

▼ $\left.\begin{matrix} a^m = b^x \\ a^n = b^y \end{matrix}\right\} \Rightarrow \frac{m}{n} = \frac{x}{y}$

▼ $x^n = y^n \Rightarrow \begin{cases} x = y, & \text{n tek ise / n is odd} \\ x = y \vee x = -y, & \text{n çift ise / n is even} \end{cases}$

▼ x ve y aralarında asal sayılardır. / *x and y are relatively prime numbers.*

$x^a = y^b \Rightarrow a = b = 0$

ÖRNEK / EXAMPLE 7

$6^{x+1} = 3^x \Rightarrow 2^{x-2} = ?$

A) $\frac{1}{24}$ B) $\frac{1}{12}$ C) $\frac{1}{6}$ D) $\frac{3}{4}$ E) $\frac{3}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$6^{x+1} = 3^x \Rightarrow 3^{x+1} \cdot 2^{x+1} = 3^x$

$2^{x+1} \cdot 3 = 1$

$2^x = \frac{1}{6} \Rightarrow 2^{x-2} = \frac{1}{24}$

ÖRNEK / EXAMPLE 8

YILDIZ TEKNİK ÜNV. YÖS 2013

$$\left(\left(-\frac{2}{5}\right)^{-2}\right)^{3}\cdot\left(\left(\frac{5}{2}\right)^{3}\right)^{-2}=?$$

A) -1 B) 1 C) -2 D) 2^5 E) 2^3

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left(\left(-\frac{2}{5}\right)^{-2}\right)^{3}\cdot\left(\left(\frac{5}{2}\right)^{3}\right)^{-2}=\left(\frac{2}{5}\right)^{-6}\cdot\left(\frac{5}{2}\right)^{-6}$$

$$=\left(\frac{2}{5}\cdot\frac{5}{2}\right)^{-6}$$

$$=1$$

ÖRNEK / EXAMPLE 9

YILDIZ TEKNİK ÜNV. YÖS 2013

$(3^x)^{-3} = 25$

$$\Rightarrow \frac{100}{\left(\frac{1}{27}\right)^{x}}=?$$

A) 2 B) 3 C) 4 D) 5 E) 10

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$(3^x)^{-3} = 25 \Rightarrow$

$$\frac{100}{\left(\frac{1}{27}\right)^{x}}=\frac{100}{(3^x)^{-3}}$$

$$=\frac{100}{25}=4$$

PEKİŞTİRME / REINFORCE

$$\frac{2}{2^{-1}+3^{-1}}=?$$

YANIT / ANSWER : 12 / 5

ÖRNEK / EXAMPLE 10

ANKARA ÜNV. YÖS 2013

$a^2 \cdot b^3 > 0$,

$b \cdot c > 0$,

$a^3 \cdot c > 0$

$\Rightarrow$ **a = ? , b = ?, c = ?**

A) +, +, + B) +, –, + C) –, –, +

D) –, –, – E) +, +, –

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$a^2 \cdot b^3 > 0 \Rightarrow b = +$

$b \cdot c > 0 \Rightarrow c = +$

$a^3 \cdot c > 0 \Rightarrow a = +$

$\Rightarrow a = +, b = +, c = +$

ÖRNEK / EXAMPLE 11

EGE ÜNİV. YÖS 2016

16. $x, y, z \in R$

$x^2 \cdot y^3 \cdot z^4 > 0$

$x^5 \cdot y^2 \cdot z^6 < 0$

$x \cdot y \cdot z > 0$

Yukarıdaki koşulları sağlayan x, y, z sayılarının işaretleri sırasıyla aşağıdakilerden hangisidir?

What are the signs of, x, y, z number respectively which are satisfying the above conditions?

A) –, +, – B) –, –, – C) +, –, –

D) –, –, + E) +, +, +

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$x^2 \cdot y^3 \cdot z^4 > 0 \Rightarrow y^3 > 0 \Rightarrow y > 0$

$x^5 \cdot y^2 \cdot z^6 < 0 \Rightarrow x^5 < 0 \Rightarrow x < 0$

$x \cdot y \cdot z > 0 \Rightarrow z < 0$

1. $\dfrac{3^x+3^x+3^x+3^x}{12^x+12^x}=\dfrac{1}{32} \Rightarrow x=?$

A) –3

B) –2

C) 1

D) 2

E) 3

---

2. $\dfrac{2^{n+1}+2^{n-1}}{2^n}-\dfrac{3^{n+2}-3^n}{3^{n+1}}=?$

A) $-\dfrac{1}{2}$

B) $-\dfrac{1}{3}$

C) $-\dfrac{1}{4}$

D) $-\dfrac{1}{5}$

E) $-\dfrac{1}{6}$

---

3. $\left.\begin{matrix} a=3^x-1 \\ b=2^x-1 \end{matrix}\right\} \Rightarrow ab+a+b+1=?$

A) 6^x-2

B) 6^x-1

C) 6^x

D) 6^x+1

E) 6^x+2

---

4. $3^{a+2}+\dfrac{6}{3^{1-a}}=99 \Rightarrow a=?$

A) 2

B) 1

C) –1

D) –2

E) –3

---

5. $\dfrac{2^{2x+3}+24}{2^{2x+3}}=\dfrac{2^{x-2}}{16}+\dfrac{3}{4^x} \Rightarrow x=?$

A) 3

B) 4

C) 5

D) 6

E) 7

---

6. $\left.\begin{matrix} 5^x=27 \\ 3^y=125 \end{matrix}\right\} \Rightarrow x \cdot y=?$

A) 9

B) 15

C) 18

D) 21

E) 27

7. $\dfrac{2^{-8}-2^{-12}+2^{-9}}{2^{-11}+2^{-10}-2^{-14}}=?$

A) $\frac{1}{4}$

B) $\frac{2}{3}$

C) 2

D) 4

E) 8

8. $\dfrac{4}{5^{x-2}+1}+\dfrac{4}{5^{2-x}+1}=?$

A) 2

B) 4

C) 5

D) 10

E) 20

9. $a^3 \cdot b^2 = 1$

$a^{x+1} = b^{2x-1} \Rightarrow$ **8x = ?**

A) –2

B) –1

C) 1

D) 2

E) 3

10. $\left.\begin{array}{l} 2^a = 10 \\ 2^b = 5 \end{array}\right\} \Rightarrow 2^{a^2-b^2} = ?$

A) 50

B) 60

C) 80

D) 100

E) 120

11. $\left.\begin{array}{l} 2^x = 3 \\ 3^y = 2 \\ 5^z = 80 \end{array}\right\} \Rightarrow ? < ? < ?$

A) $z < x < y$

B) $x < z < y$

C) $y < x < z$

D) $y < z < x$

E) $x < y < z$

12. $\left.\begin{array}{l} 2^x = 9 \\ 5 \cdot 3^y = 80 \end{array}\right\} \Rightarrow x \cdot y = ?$

A) 8

B) 9

C) 10

D) 12

E) 16

YANITLAR / ANSWERS							
1. E	**2. E**	**3. C**	**4. A**	**5. D**	**6. A**	**7. D**	**8. B**
9. C	**10. A**	**11. C**	**12. A**				

1.
$$\frac{3^x+3^x+3^x+3^x}{12^x+12^x}=\frac{1}{32}$$
$$\frac{4\cdot 3^x}{2\cdot 4^x\cdot 3^x}=\frac{1}{32}$$
$$4^x=64$$
$$x=3$$

Yanıt / The Answer is E

2.
$$\frac{2^{n+1}+2^{n-1}}{2^n}-\frac{3^{n+2}-3^n}{3^{n+1}}=\frac{2\cdot 2^n+\frac{1}{2}\cdot 2^n}{2^n}-\frac{3^n\cdot 9-3^n}{3\cdot 3^n}$$
$$=\left(2+\frac{1}{2}\right)-\frac{8}{3}$$
$$=\frac{5}{2}-\frac{8}{3}$$
$$=\frac{15-16}{6}$$
$$=-\frac{1}{6}$$

Yanıt / The Answer is E

---

3. $a\cdot b+a+b+1=a(b+1)+b+1$
$$=(a+1)(b+1)$$
$$=3^x\cdot 2^x$$
$$=6^x$$

Yanıt / The Answer is C

4.
$$3^{a+2}+\frac{6}{3^{1-a}}=99$$
$$9\cdot 3^a+\frac{6}{3\cdot\frac{1}{3^a}}=99$$
$$9\cdot 3^a+\frac{6\cdot 3^a}{3}=99$$
$$33\cdot 3^a=3\cdot 99$$
$$a=2$$

Yanıt / The Answer is A

5.
$$\frac{2^{2x+3}+24}{2^{2x+3}}=\frac{2^{x-2}}{16}+\frac{3}{4^x}$$
$$\frac{2^{2x}\cdot 8+24}{2^{2x}\cdot 8}=\frac{2^x}{64}+\frac{3}{4^x}$$
$$\frac{8(4^x+3)}{8\cdot 4^x}-\frac{3}{4^x}=\frac{2^x}{64}$$
$$\frac{4^x+3-3}{4^x}=\frac{2^x}{64}$$
$$2^x=64$$
$$x=6$$

Yanıt / The Answer is D

6.
$$\left.\begin{aligned}5^x&=3^3\\5^3&=3^y\end{aligned}\right\}\Rightarrow\frac{x}{3}=\frac{3}{y}$$
$$\Rightarrow x\cdot y=3\cdot 3=9$$

Yanıt / The Answer is A

7. $$\frac{2^{-8}-2^{-12}+2^{-9}}{2^{-11}+2^{-10}-2^{-14}}=\frac{2^{-12}(2^{4}-1+2^{3})}{2^{-14}(2^{3}+2^{4}-1)}$$

$$=2^{2}$$

$$=4$$

Yanıt / The Answer is D

8. $$\frac{4}{5^{x-2}+1}+\frac{4}{5^{2-x}+1}=\frac{4}{5^{x}\cdot 5^{-2}+1}+\frac{4}{5^{2}\cdot 5^{-x}+1}$$

$$=\frac{4}{\frac{5^{x}+25}{25}}+\frac{4}{\frac{25+5^{x}}{5^{x}}}$$

$$=\frac{4\cdot 25+4\cdot 5^{x}}{25+5^{x}}$$

$$=4$$

Yanıt / The Answer is B

9. $$\left.\begin{array}{l} a^{3}=b^{-2} \\ a^{x+1}=b^{2x-1} \end{array}\right\} \Rightarrow \frac{3}{x+1}=\frac{-2}{2x-1}$$

$$3(2x-1)=-2(x+1)$$

$$6x-3=-2x-2$$

$$8x=1$$

Yanıt / The Answer is C

10. $$\left.\begin{array}{l} 2^{a}=10 \\ 2^{b}=5 \end{array}\right\} \Rightarrow \ 2^{a^{2}-b^{2}}=(2^{a-b})^{a+b}$$

$$=2^{a+b}=10\cdot 5=50$$

Yanıt / The Answer is A

11. $2<3<4 \Rightarrow 2^{1}<2^{x}<2^{2}$

$1<2<3 \Rightarrow 3^{0}<3^{y}<3^{1}$

$25<80<125 \Rightarrow 5^{2}<5^{z}<5^{3}$

$$\left.\begin{array}{l} 1<x<2 \\ 0<y<1 \\ 2<z<3 \end{array}\right\} \Rightarrow y<x<z$$

Yanıt / The Answer is C

12. $5\cdot 3^{y}=80 \Rightarrow 3^{y}=16$

$$\left.\begin{array}{l} 3^{y}=16 \\ 2^{x}=9 \end{array}\right\} \Rightarrow \frac{y}{2}=\frac{4}{x} \Rightarrow x\cdot y=8$$

Yanıt / The Answer is A

1. $[(-2^{-2})^{-3}]^{-1} : (-2^{2})^{-3} = ?$

Yanıt / The Answer is: **1**

2. $\left.\begin{matrix} 3^x = a \\ 2^x = b \end{matrix}\right\} \Rightarrow \mathbf{24^x = ?}$

Yanıt / The Answer is: **ab^3**

3. $\left[\left(\dfrac{1}{81}\right)^{-3}\right]^{0,25} = ?$

Yanıt / The Answer is: **27**

4. $3^{x+2} + 3^{x-2} + 3^x = \dfrac{182}{9} \Rightarrow \mathbf{3^{x+1} = ?}$

Yanıt / The Answer is: **6**

5. $2^{x-y} = 6^{-y} \Rightarrow \mathbf{3^{\frac{y}{x}} = ?}$

Yanıt / The Answer is: $\mathbf{\dfrac{1}{2}}$

6. $A = \underbrace{2 \cdot 2 \cdot 2 \ldots 2}_{x \text{ tane/times}}$

$B = \underbrace{2 + 2 + 2 + \ldots + 2}_{2^x \text{ tane/times}}$

$A + B = 96 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **5**

7. $(x-3)^6 = 64x^6 \Rightarrow \boldsymbol{\sum x = ?}$

Yanıt / The Answer is: **–2**

8. $\dfrac{3^x+9^x+27^x}{2^x+6^x+18^x}=\dfrac{8}{27} \Rightarrow \boldsymbol{x=?}$

Yanıt / The Answer is: **–3**

9. $2^{x-1} = 5 \Rightarrow \boldsymbol{2^{x+1} + 2^{2-x} = ?}$

Yanıt / The Answer is: $\boldsymbol{\dfrac{102}{5}}$

10. $9^x = 3^{x-1} \Rightarrow \boldsymbol{2^{-x+3} = ?}$

Yanıt / The Answer is: **16**

11. $\left.\begin{array}{l} 3^x = 5 \\ 5^y = 8 \\ 2^z = 9 \end{array}\right\} \Rightarrow \boldsymbol{x \cdot y \cdot z = ?}$

Yanıt / The Answer is: **6**

12. $(x+2)^{x^2-x-6} = 1 \Rightarrow \boldsymbol{\sum x = ?}$

Yanıt / The Answer is: **–1**

1. $\dfrac{a^{-2}\cdot(-a)^3\cdot(-a^4)}{-a^{-4}\cdot(-a)^3}=?$

A) $-a^6$ B) $-a^4$ C) a^2
D) a^4 E) a^6

2. $\dfrac{2^{-10}+2^{-14}-2^{-12}}{2^{-5}-2^{-7}+2^{-9}}=?$

A) $\dfrac{1}{32}$ B) $\dfrac{1}{8}$ C) 2
D) 8 E) 32

3. $\dfrac{4^{x+2}-4^{x-1}}{2\cdot4^{x+1}-4^{x}}=?$

A) $\dfrac{3}{4}$ B) $\dfrac{5}{4}$ C) $\dfrac{7}{4}$
D) $\dfrac{9}{4}$ E) $\dfrac{11}{4}$

4. $81^{x+2}=27^{2x-3} \Rightarrow x=?$

A) $\dfrac{17}{2}$ B) $\dfrac{19}{2}$ C) $\dfrac{21}{2}$
D) $\dfrac{23}{2}$ E) $\dfrac{25}{2}$

5. $(3x-2)^4=(x+5)^4 \Rightarrow \sum x=?$

A) $\dfrac{7}{2}$ B) $\dfrac{7}{4}$ C) $\dfrac{11}{4}$
D) $\dfrac{15}{4}$ E) $\dfrac{19}{4}$

6. $2^{x+y-7}=3^{x-y+1} \Rightarrow x\cdot y=?$

A) 12 B) 14 C) 16
D) 18 E) 20

7. $5^x = 3 \Rightarrow 9^{\frac{x+1}{x}} = ?$

A) 125 B) 150 C) 175 D) 200 E) 225

8. $3^{x+1} = 15^x \Rightarrow 25^x = ?$

A) 3 B) 9 C) 12 D) 18 E) 25

9. $\left.\begin{array}{l} 2^{x+1} = 5 \\ 4^y = 20 \end{array}\right\} \Rightarrow x = ?$

A) $2y + 1$ B) $2y - 3$ C) $y + 5$ D) $y - 4$ E) $5 - y$

10. $\left.\begin{array}{l} 2^x = 3^{x+1} \\ 2^{y+1} = 3^y \end{array}\right\} \Rightarrow x + y = ?$

A) −3 B) −2 C) −1 D) 0 E) 1

11. $(x+1)^{x-3} = 1 \Rightarrow \sum x = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

12. $x = (-5^3)^4, \quad y = (-5)^{(3^4)}, \quad z = (-25^2)^3$

$\Rightarrow ? < ? < ?$

A) $x < z < y$ B) $x < y < z$ C) $y < x < z$ D) $y < z < x$ E) $z < x < y$

YANITLAR / ANSWERS

1. E 2. A 3. D 4. A 5. C 6. A 7. E 8. B

9. B 10. C 11. D 12. D

1. $\left(\frac{5}{4}\right)^{3-x} = \frac{4}{5}, \quad 8^{3y} = x \quad \Rightarrow \quad 2^{9y+1} = ?$

A) 2 B) 4 C) 8 D) 16 E) 32

2. $\left.\begin{array}{l} x^{a+b} = 150 \\ x^{a-b} = 6 \end{array}\right\} \Rightarrow \frac{x^a + x^b}{x^a - x^b} = ?$

A) $\frac{1}{5}$ B) $\frac{3}{5}$ C) $\frac{7}{5}$ D) $\frac{9}{5}$ E) $\frac{11}{5}$

3. $5^{2n-1} = 2 \quad \Rightarrow \quad 25^{3n-2} = ?$

A) $\frac{4}{5}$ B) $\frac{6}{5}$ C) $\frac{8}{5}$ D) $\frac{16}{5}$ E) $\frac{32}{5}$

4. $\frac{(-3^{-4})^{-3} \cdot (-9^{-2})^{-3}}{(-9)^{-1} \cdot (-81^{-4})^{-1}} = ?$

A) 3^{10} B) 3^{5} C) 3^{-5} D) 3^{-10} E) 3^{-15}

5. $3^{x+y} = 6^x \quad \Rightarrow \quad 4^{\frac{x}{y}} = ?$

A) 3 B) 9 C) 27 D) 81 E) 243

6. $2 \cdot 3^{x+1} - 4 \cdot 3^{x-1} + 5 \cdot 3^x = 87 \quad \Rightarrow \quad x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

7. $\left.\begin{array}{l} 2^x \cdot 3^y = 4 \\ 2^y \cdot 3^x = 9 \end{array}\right\} \Rightarrow \mathbf{x+y=?}$

A) 1 B) 2 C) 3 D) 4 E) 5

8. $15^x = 9^y \cdot 5^x \Rightarrow \dfrac{\mathbf{x+y}}{\mathbf{x-y}} = \mathbf{?}$

A) –1 B) 1 C) 2 D) 3 E) 4

9. $\left.\begin{array}{l} x = 2^{100} \\ y = 3^{125} \\ z = 4^{75} \end{array}\right\} \Rightarrow \mathbf{? < ? < ?}$

A) $z < x < y$ B) $y < z < x$ C) $y < x < z$ D) $x < y < z$ E) $x < z < y$

10. $75^{x+1} = 3^{x-1} \cdot 5^{2x+1} \cdot A \Rightarrow \mathbf{A = ?}$

A) 9 B) 15 C) 30 D) 45 E) 75

11. $3^{2x-y+4} = 5^{x+y-1} \Rightarrow \mathbf{x \cdot y = ?}$

A) –6 B) –5 C) –4 D) –3 E) –2

12. $2^1 \cdot 2^2 \cdot 2^3 \cdot \ldots \cdot 2^n = (0{,}5)^{-45} \Rightarrow \mathbf{n = ?}$

A) 5 B) 6 C) 7 D) 8 E) 9

YANITLAR / ANSWERS

1. C	**2. C**	**3. C**	**4. A**	**5. B**	**6. B**	**7. B**	**8. D**
9. E	**10. D**	**11. E**	**12. E**				

1. $[(-0,5)^{-3} + (3^{-2})^{-1} + (-7)^{0}]^{-1} = ?$

A) -1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$

D) 1 E) $\frac{3}{2}$

2. $\left.\begin{array}{l} 2^{a+1} = b \\ 2^{a-2} = x \cdot b \end{array}\right\} \Rightarrow x = ?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) 2

D) 4 E) 8

3. $48^{x+2} = 4^{2x+5} \Rightarrow 3^{x} = ?$

A) $\frac{1}{9}$ B) $\frac{4}{9}$ C) $\frac{5}{9}$

D) $\frac{16}{9}$ E) $\frac{32}{9}$

4. $\left.\begin{array}{l} x = \underbrace{5^{3} \cdot 5^{3} \cdot 5^{3} \cdot \ldots \cdot 5^{3}}_{\text{n tane (n times)}} \\ y = \underbrace{5^{n} + 5^{n} + \ldots + 5^{n}}_{5^{n} \text{ tane } (5^{n} \text{ times})} \end{array}\right\} \Rightarrow \frac{x}{y} = ?$

A) 5^{-n} B) 1 C) 5^{n}

D) $5 \cdot 2^{n}$ E) $5 \cdot 3^{n}$

5. $2^{x} = 3 \Rightarrow 4^{x+1} - 8^{x} = ?$

A) 3 B) 6 C) 9

D) 12 E) 15

6. $2^{x+3} - 3 \cdot 2^{x-1} + 2^{x} = 60 \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

7. $\left.\begin{array}{l} 2^x = 125 \\ 5^y = 16 \end{array}\right\} \Rightarrow x \cdot y = ?$

A) 6 B) 8 C) 9 D) 10 E) 12

8. $\left.\begin{array}{l} a = \left((-2)^{-2}\right)^{-1} \\ b = \left((-2)^{3}\right)^{1} \\ c = \left(2^{-2}\right)^{2} \end{array}\right\} \Rightarrow ? < ? < ?$

A) $a < b < c$ B) $b < c < a$ C) $c < b < a$ D) $a < c < b$ E) $c < a < b$

9. $\left.\begin{array}{l} a = 2^{45} \\ b = 3^{27} \\ c = 5^{18} \end{array}\right\} \Rightarrow ? < ? < ?$

A) $a < b < c$ B) $b < a < c$ C) $a < c < b$ D) $c < b < a$ E) $c < a < b$

10. $\left.\begin{array}{l} a^{x-1} = 5 \\ a^x - 2 = a \end{array}\right\} \Rightarrow a = ?$

A) $-\frac{3}{2}$ B) -1 C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) 1

11. $\dfrac{(-3)^x \cdot (-3^x)^4 \cdot (-3)^{-x}}{-3^x \cdot 3^{3x}} = ?$

A) -3^x B) -1 C) 1 D) 3^x E) 3^{2x}

12. $\dfrac{2^x \cdot 2^x \cdot 2^x}{2^x + 2^x + 2^x + 2^x} = \dfrac{1}{16} \Rightarrow x = ?$

A) -2 B) -1 C) 1 D) 2 E) 3

YANITLAR / ANSWERS

1. C	**2. A**	**3. B**	**4. C**	**5. C**	**6. C**	**7. E**	**8. B**
9. D	**10. D**	**11. B**	**12. B**				

1. $\left.\begin{array}{l}2^{a}=5\\ 25^{b}=27\end{array}\right\}\Rightarrow 2^{4ab}=?$

A) 3^2 B) 3^4 C) 3^6

D) 3^8 E) 3^{10}

2. $\dfrac{3^{n+2}+3^{n+1}}{5^{n+1}-5^{n}}=\dfrac{25}{3}\Rightarrow n=?$

A) –2 B) –1 C) 1

D) 2 E) 3

3. $x, y \in Z$,

$\left(\dfrac{1}{x^{3}}\right)^{2y}=64 \Rightarrow x+y=?$

A) –2 B) –1 C) 1

D) 2 E) 3

4. $\left.\begin{array}{l}(0{,}2)^{x}=27\\ (0{,}04)^{y}=9\end{array}\right\}\Rightarrow \dfrac{x+3y}{x-2y}=?$

A) 2 B) 3 C) 4

D) 5 E) 6

5. $x \in Z$,

$\left(\dfrac{1}{2}\right)^{x+2}<\left(\dfrac{1}{4}\right)^{x-2}\Rightarrow \max(x)=?$

A) 2 B) 3 C) 4

D) 5 E) 6

6. $x \in Z$,

$\left(\dfrac{2}{5}\right)^{x+1}>\left(\dfrac{25}{4}\right)^{x-1}\Rightarrow \max(x)=?$

A) 1 B) 0 C) –1

D) –2 E) –3

7. $\dfrac{4}{5^x+1}+\dfrac{4}{5^{-x}+1}=\,?$

A) 1 B) 2 C) 3
D) 4 E) 5

8. $48^x = 12 \;\Rightarrow\; \mathbf{2^{4x-1}\cdot 3^{x+1}=\,?}$

A) 6 B) 8 C) 12
D) 15 E) 18

9. $\left.\begin{aligned} 2^x &= 7 \\ 3^y &= 2 \\ 5^z &= 10 \end{aligned}\right\} \Rightarrow \mathbf{?<?<?}$

A) x < y < z B) y < z < x C) z < x < y
D) y < x < z E) x < z < y

10. $\left.\begin{aligned} x &= 2+3^{2-2a} \\ y &= 9^{a+1}-5 \end{aligned}\right\} \Rightarrow \mathbf{(x-2)(y+5)=\,?}$

A) 27 B) 36 C) 48
D) 75 E) 81

11. $\left.\begin{aligned} 3^x\cdot 5^y &= 375 \\ 3^y\cdot 5^x &= 135 \end{aligned}\right\} \Rightarrow \mathbf{x+y=\,?}$

A) 4 B) 5 C) 6
D) 7 E) 8

12. $a^x > 0,$

$\left.\begin{aligned} a^{x+y} &= 45 \\ a^{x-y} &= 5 \end{aligned}\right\} \Rightarrow \mathbf{a^x=\,?}$

A) 3 B) 5 C) 9
D) 15 E) 27

YANITLAR / ANSWERS

1. C	**2. A**	**3. C**	**4. E**	**5. D**	**6. B**	**7. D**	**8. E**
9. B	**10. E**	**11. A**	**12. D**				

KÖKLÜ SAYILAR
RADICALS

BÖLÜM 2
CHAPTER

Tanım: $n \in Z^+$ olmak üzere,

$x^n = a$ eşitliğini sağlayan x sayısına, *a'nın n'inci dereceden kökü* denir ve $x = \sqrt[n]{a}$ ile gösterilir.

n = 2 için $\sqrt{a}$ ifadesi karekök a,

n = 3 için $\sqrt[3]{a}$ ifadesi küpkök a diye okunur.

Definition:

For any positive integer n, if $x^n = a$, then x is the n^{th} root of the number a. The symbol $\sqrt[n]{a}$ is used to denote the n^{th} root of a.

For n = 2, $\sqrt{a}$ the square root of a,

For n = 3, $\sqrt[3]{a}$ the cube root of a.

▼ $\sqrt{a} + \sqrt{b} = 0 \Rightarrow a = 0, b = 0$

ÖRNEK / EXAMPLE 1

$\sqrt{x-3} + \sqrt{x \cdot y - 12} = 0 \Rightarrow y = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$\sqrt{x-3} + \sqrt{x \cdot y - 12} = 0$

$x - 3 = 0 \qquad x \cdot y - 12 = 0$

$x = 3 \qquad xy = 12$

$x = 3 \Rightarrow y = 4$

NOT / NOTE

Çift dereceli köklerin sonucu negatif olamayacağından $\sqrt{x} = -4$ şeklindeki denklemler reel sayılarda tanımsızdır.

The results of even roots can not be negative and equations like $\sqrt{x} = -4$ are undefined in real numbers.

NOT / NOTE

Reel sayılarda bir negatif sayının karekökü yoktur.

A negative number does not have a square root in R.

PEKİŞTİRME / REINFORCE

$\sqrt[4]{a-b+4} + \sqrt{a+b-2} = 0$

$\Rightarrow a \cdot b = ?$

YANIT / ANSWER : –3

KÖKLÜ SAYILARIN ÜSLÜ BİÇİMDE YAZILMASI
EXPONENTIAL FORM of RADICALS

▼ p tek sayı ise / *if p is an odd number,* $\sqrt[p]{a^n} = a^{\frac{n}{p}}$

▼ p ve n çift sayı ise / *If p and n are even numbers,* $\sqrt[p]{a^n} = |a|^{\frac{n}{p}}$

▼ **a.** x tek doğal sayı ve a > 0 ise,

If x is an odd natural number and a > 0, then

$\sqrt[n]{a^n} = \sqrt[nx]{a^{nx}}$

b. a < 0 ve x çift doğal sayı ise,

If a < 0 and x is an even natural number, then

$\sqrt[n]{a^n} \neq \sqrt[nx]{a^{nx}}$

ÖRNEK / EXAMPLE 2

$\sqrt[5]{4^x} = 16 \Rightarrow x = ?$

A) 3 B) 4 C) 10 D) 12 E) 14

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\sqrt[5]{4^x} = 16$ [Verilen eşitlik / *Given equality*]

$2^{\frac{2x}{5}} = 2^4$ [Üslü biçim / *Exponential form*]

$\frac{2x}{5} = 4$ [Üslü denklem / *Exponential equation*]

$2x = 20$ [Çapraz çarpım / *Cross – multiply*]

$x = 10$ [Sonuç / *Conclusion*]

KÖKLÜ SAYILARIN SADALEŞTİRİLMESİ / SIMPLIFYING RADICALS

- n çift sayı ise / *if n is an even number,*

$$\sqrt[n]{a^n \cdot b} = |a| \cdot \sqrt[n]{b}$$

- n tek sayı ise / *if n is an odd number,*

$$\sqrt[n]{a^n \cdot b} = a \cdot \sqrt[n]{b}$$

ÖRNEK / EXAMPLE 3

$$\sqrt[3]{60+\sqrt{18-\sqrt[3]{13-\sqrt{25}}}} = ?$$

A) 2 B) 4 C) 5 D) 6 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned}\sqrt[3]{60+\sqrt{18-\sqrt[3]{13-\sqrt{25}}}} &= \sqrt[3]{60+\sqrt{18-\sqrt[3]{13-5}}}\\ &= \sqrt[3]{60+\sqrt{18-\sqrt[3]{8}}}\\ &= \sqrt[3]{60+\sqrt{18-2}}\\ &= \sqrt[3]{60+\sqrt{16}}\\ &= \sqrt[3]{64}\\ &= 4\end{aligned}$$

KÖKLÜ SAYILARDA TOPLAMA ve ÇIKARMA İŞLEMİ
ADDITION and SUBTRACTION of RADICALS

$a, x, y \in R,\ n \in N^+$

$$x\sqrt[n]{a} \pm y\sqrt[n]{a} = (x \pm y)\sqrt[n]{a}$$

NOTLAR / NOTES

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$

PEKİŞTİRME / REINFORCE

$\sqrt{8}+\sqrt{72}-\sqrt{18}=?$

YANIT / ANSWER : $5\sqrt{2}$

ÖRNEK / EXAMPLE 4

$\sqrt{48}-\sqrt{75}+2\cdot\sqrt{108}=?$

A) $10\sqrt{3}$ B) $11\sqrt{3}$ C) $12\sqrt{3}$ D) $13\sqrt{3}$ E) $14\sqrt{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\sqrt{48}-\sqrt{75}+2\cdot\sqrt{108}=4\sqrt{3}-5\sqrt{3}+2\cdot 6\sqrt{3}$$
$$=11\sqrt{3}$$

KÖKLÜ SAYILARDA ÇARPMA İŞLEMİ MULTIPLICATION of RADICALS

1. İki veya daha fazla köklü ifade aynı indise sahipse birlikte çarpılabilir.

   If two or more radicals have the same index, they can be multiplied together.

$$a, b \in R^+$$
$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab}$$
$$\sqrt{a}\cdot\sqrt{a}=\sqrt{a^2}$$

ÖRNEK / EXAMPLE 5

$\sqrt{2}\cdot\sqrt[3]{3}=?$

A) $\sqrt[6]{6}$ B) $\sqrt[6]{36}$ C) $\sqrt[6]{48}$ D) $\sqrt[6]{54}$ E) $\sqrt[6]{72}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\sqrt{2}\cdot\sqrt[3]{3}=\sqrt[6]{2^3}\cdot\sqrt[6]{3^2}$$
$$=\sqrt[6]{8}\cdot\sqrt[6]{9}$$
$$=\sqrt[6]{72}$$

▼ $a \in R^+,\ n \in N^+ \Rightarrow \left(\sqrt{a}\right)^n = \sqrt{a^n}$

ÖRNEK / EXAMPLE 6

$$\frac{\sqrt[3]{4^{x+1}}}{\sqrt{8^{x-1}}} = 16 \Rightarrow x = ?$$

A) $-\frac{13}{5}$ B) $-\frac{11}{5}$ C) $-\frac{9}{5}$ D) $\frac{11}{5}$ E) $\frac{13}{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{\sqrt[3]{4^{x+1}}}{\sqrt{8^{x-1}}} = 16$$
[Verilen eşitlik / *Given equality*]

$$\frac{2^{\frac{2x+2}{3}}}{2^{\frac{3x-3}{2}}} = 2^4$$
[Üstel form / *Exponential form*]

$$2^{\frac{2x+2}{3}-\frac{3x-3}{2}} = 2^4$$
[Üsler çıkarılır. / *Subtract exponential.*]

$$\frac{2x+2}{3} - \frac{3x-3}{2} = 4$$
[Üstel denklem / *Exponential equation*]

$$4x + 4 - 9x + 9 = 24$$
[6 ile çarp. / *Multiply by 6.*]

$$-11 = 5x$$
[Benzer terimleri topla. / *Combine like terms.*]

$$x = -\frac{11}{5}$$
[Sonuç / *Conclusion*]

KÖKLÜ SAYILARDA BÖLME İŞLEMİ / DIVISION of RADICALS

Dereceleri aynı olan ve kök içinde verilen sayılar, ortak bir kök içinde yazılır, daha sonra sadeleştirme işlemi yapılır.

The numbers inside roots are written in a root and then simplification is done.

$a, b \in R^+,\ b \neq 0$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

▼ **$\sqrt{a \mp 2\sqrt{b}}$ nin $\sqrt{m} \mp \sqrt{n}$ biçiminde yazılışı**

$\sqrt{a \mp 2\sqrt{b}}$ ifadesi için a, b ∈ R^+ olmak üzere,

b = m·n , a = m + n koşulunu sağlayan m ve n gibi iki doğal sayı bulunabiliyorsa,

m > n olmak üzere, $\sqrt{a \mp 2\sqrt{b}} = \sqrt{m} \mp \sqrt{n}$ şeklinde yazılabilir.

Writing in the form $\sqrt{m} \mp \sqrt{n}$ of $\sqrt{a \mp 2\sqrt{b}}$

For the expression $\sqrt{a \mp 2\sqrt{b}}$,

you can find b = m·n and a = m + n when m > n, m,n ∈ N^+.

We can write as follows: $\sqrt{a \mp 2\sqrt{b}} = \sqrt{m} \mp \sqrt{n}$.

ÖRNEK / EXAMPLE 7

$(2-\sqrt{3})\cdot\sqrt{7+4\sqrt{3}} = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$
\begin{aligned}
(2-\sqrt{3})\cdot\sqrt{7+4\sqrt{3}} &= (2-\sqrt{3})\cdot\sqrt{7+2\sqrt{12}} \\
&= (2-\sqrt{3})\cdot(\sqrt{4}+\sqrt{3}) \\
&= (2-\sqrt{3})\cdot(2+\sqrt{3}) \\
&= (2)^2-(\sqrt{3})^2 \\
&= 4-3 = 1
\end{aligned}
$$

SONSUZA GİDEN ZİNCİRLEME KÖKLÜ İFADELER

▼ a ∈ R^+, $y=\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}}$ ve(*and*) $y=\sqrt{a-\sqrt{a-\sqrt{a-\ldots}}}$

$a = x\cdot(x+1) \Rightarrow \sqrt{a+\sqrt{a+\sqrt{a+\ldots}}} = x+1$

$a = x\cdot(x+1) \Rightarrow \sqrt{a-\sqrt{a-\sqrt{a-\ldots}}} = x$

▼ $y=\sqrt[n]{a\sqrt[n]{a\sqrt[n]{a\ldots}}}$ köklü ifadesinin değeri her iki tarafın n. kuvveti alınarak veya aşağıdaki formülle bulunur.

$$\sqrt[n]{a\sqrt[n]{a\sqrt[n]{a\ldots}}}=\sqrt[n-1]{a}$$

Take the n^{th} power of both sides of expression $y=\sqrt[n]{a\sqrt[n]{a\sqrt[n]{a\ldots}}}$ or use following formula.

$$\sqrt[n]{a\sqrt[n]{a\sqrt[n]{a\ldots}}}=\sqrt[n-1]{a}$$

ÖRNEK / EXAMPLE 8

$\sqrt[4]{27\cdot\sqrt[4]{27\cdot\sqrt[4]{27\ldots}}}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\sqrt[4]{27\cdot\sqrt[4]{27\cdot\sqrt[4]{27\ldots}}}=\sqrt[4-1]{27}$$

$$=\sqrt[3]{27}=3$$

PEKİŞTİRME / REINFORCE

$\sqrt{2\cdot\sqrt{2\cdot\sqrt{2\ldots}}}=?$

YANIT / ANSWER : 2

▼ $y=\sqrt[n]{a:\sqrt[n]{a:\sqrt[n]{a\ldots}}}$ köklü ifadesinin değeri her iki tarafın n. kuvveti alınarak veya aşağıdaki formülle çözülür.

$$y=\sqrt[n]{a:\sqrt[n]{a:\sqrt[n]{a\ldots}}}=\sqrt[n+1]{a}$$

Take the n^{th} power of both sides of expression $y=\sqrt[n]{a:\sqrt[n]{a:\sqrt[n]{a\ldots}}}$ or use following formula.

$$y=\sqrt[n]{a:\sqrt[n]{a:\sqrt[n]{a\ldots}}}=\sqrt[n+1]{a}$$

PEKİŞTİRME / REINFORCE

$\sqrt{27:\sqrt{27:\ldots}} = ?$

YANIT / ANSWER : 3

ÖRNEK / EXAMPLE 9

$\sqrt[3]{81:\sqrt[3]{81:\sqrt[3]{81\ldots}}} = ?$

A) 3 B) 5 C) 7 D) 9 E) 11

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\sqrt[3]{81:\sqrt[3]{81:\sqrt[3]{81\ldots}}} = \sqrt[3+1]{81}$$

$$= \sqrt[4]{3^4} = 3$$

İÇ İÇE GEÇMİŞ SONLU KÖKÜ İFADELER
FINITE RADICALS INSIDE EACH OTHER

1. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n\cdot m]{a}$
2. $\sqrt[n]{a\sqrt[k]{b\sqrt[x]{c}}} = \sqrt[n\cdot k\cdot x]{c\cdot b^x\cdot a^{k\cdot x}}$

ÖRNEK / EXAMPLE 10

$\sqrt[3]{a\sqrt{b}} = \sqrt[6]{432} \Rightarrow \min(a+b) = ?$

A) 5 B) 7 C) 9 D) 12 E) 15

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\sqrt[3]{a\sqrt{b}} = \sqrt[6]{432}$$

$$\sqrt[3]{\sqrt{a^2b}} = \sqrt[6]{432}$$

$$\sqrt[6]{a^2b} = \sqrt[6]{432}$$

$$a^2b = 432$$

$$a^2b = 144\cdot 3$$

$$a = 12,\quad b = 3$$

$$\Rightarrow\ a + b = 12 + 3 = 15$$

EŞLENİKLER / CONJUGATES

	ifade / *Expression*	Eşlenik / *Conjugate*
1.	$\sqrt{a}$	$\sqrt{a}$
2.	$\sqrt[n]{a^t}$	$\sqrt[n]{a^{n-t}}$
3.	$\sqrt{a}+\sqrt{b}$	$\sqrt{a}-\sqrt{b}$
4.	$\sqrt[3]{a}-\sqrt[3]{b}$	$\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}$
5.	$\sqrt{a}+\sqrt{b}+\sqrt{c}$	$\sqrt{a}+\sqrt{b}-\sqrt{c}$, $\sqrt{a}-\sqrt{b}+\sqrt{c}$, $-\sqrt{a}+\sqrt{b}+\sqrt{c}$

ÖRNEK / EXAMPLE 11

$$\frac{2}{\sqrt[7]{8}}=?$$

A) $\sqrt[7]{2}$ B) $\sqrt[7]{4}$ C) $\sqrt[7]{8}$ D) $\sqrt[7]{16}$ E) $\sqrt[7]{32}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{2}{\sqrt[7]{8}}=\frac{2}{\sqrt[7]{2^3}}$$

$$(\sqrt[7]{2^4})$$

$$=\frac{2\cdot\sqrt[7]{2^4}}{\sqrt[7]{2^7}}$$

$$=\frac{2\cdot 2^{\frac{7}{4}}}{2}$$

$$=2^{\frac{4}{7}}=\sqrt[7]{16}$$

PEKİŞTİRME / REINFORCE

$$\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{1}{\sqrt{3}-\sqrt{2}}=?$$

YANIT / ANSWER : $-2\sqrt{2}$

ÖRNEK / EXAMPLE 12

$$\left.\begin{array}{l} a=\sqrt{3}-1\\ b=\sqrt{3}+1\end{array}\right\}\Rightarrow\sqrt{\frac{a}{b}+\frac{b}{a}}=?$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\sqrt{\frac{a}{b}+\frac{b}{a}}=\sqrt{\frac{a^2+b^2}{ab}}$$

$$=\sqrt{\frac{(a+b)^2-2ab}{ab}}$$

$$=\sqrt{\frac{\left((\sqrt{3}-1)+(\sqrt{3}+1)\right)^2-2\cdot(\sqrt{3}-1)\cdot(\sqrt{3}+1)}{(\sqrt{3}-1)\cdot(\sqrt{3}+1)}}$$

$$=\sqrt{\frac{12-4}{2}}=2$$

ÖRNEK / EXAMPLE 13

$$\frac{\sqrt{3}+1}{\sqrt{5}+1}=x\Rightarrow\frac{\sqrt{5}-1}{\sqrt{3}-1}=?$$

A) $\frac{x}{4}$ B) $\frac{x}{2}$ C) x D) 2x E) 4x

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{\sqrt{5}-1}{\sqrt{3}-1}=y\Rightarrow\frac{1}{y}=\frac{\sqrt{3}-1}{\sqrt{5}-1}$$

$$x\cdot\frac{1}{y}=\frac{\sqrt{3}+1}{\sqrt{5}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{5}-1}$$

$$\frac{x}{y}=\frac{2}{4}$$

$$\frac{x}{y}=\frac{1}{2}\Rightarrow y=2x$$

ÖRNEK / EXAMPLE 14

YILDIZ TEKNİK ÜNV. YÖS 2011

$a, b \in R^+$

$$\frac{a\sqrt{b}-b\sqrt{a}}{\sqrt{ab}}+\sqrt{b}=?$$

A) $\sqrt{a}$ B) $\sqrt{b}$ C) $\frac{a}{\sqrt{b}}$ D) $\frac{b}{\sqrt{a}}$ E) $\frac{a}{b}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{a\sqrt{b}-b\sqrt{a}}{\underset{(\sqrt{ab})}{\sqrt{ab}}}+\sqrt{b}=\frac{ab\sqrt{a}-ba\sqrt{b}}{ab}+\sqrt{b}$$

$$=\frac{ab(\sqrt{a}-\sqrt{b})}{ab}+\sqrt{b}$$

$$=\sqrt{a}-\sqrt{b}+\sqrt{b}$$

$$=\sqrt{a}$$

ÖRNEK / EXAMPLE 15

YILDIZ TEKNİK ÜNV. YÖS 2011

$9^x = 7$

$3^{3x} = ?$

A) $\sqrt{7}$ B) $7\sqrt{7}$ C) $\sqrt{3}$ D) 21 E) 27

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$9^x = 7 \Rightarrow 3^{2x} = 7$$

$$3^x = \sqrt{7}$$

$$3^{3x} = (3^x)^3$$

$$= (\sqrt{7})^3$$

$$= 7\sqrt{7}$$

PEKİŞTİRME / REINFORCE

$\sqrt{x-1}+\sqrt{4x-4}=6 \Rightarrow x=?$

YANIT / ANSWER : 5

ÖRNEK / EXAMPLE 16

ANKARA ÜNV. YÖS 2011

$\sqrt{128x+64}-36=\sqrt{32x+16} \Rightarrow x=?$

A) 12 B) 20 C) 30 D) 40 E) 80

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\sqrt{128x+64}-36=\sqrt{32x+16}$$
$$\sqrt{64(2x+1)}-\sqrt{16(2x+1)}=36$$
$$(8-4)\sqrt{2x+1}=36$$
$$\sqrt{2x+1}=9$$
$$2x+1=81$$
$$2x=80$$
$$x=40$$

ÖRNEK / EXAMPLE 17

ANKARA ÜNV. YÖS 2011

$\dfrac{6}{\sqrt{10}-\sqrt{11}}+\dfrac{6}{\sqrt{10}-3}+\dfrac{66}{\sqrt{11}}=?$

A) 1 B) 6 C) 12 D) 16 E) 18

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\underset{(\sqrt{10}+\sqrt{11})}{\frac{6}{\sqrt{10}-\sqrt{11}}}+\underset{(\sqrt{10}+3)}{\frac{6}{\sqrt{10}-3}}+\underset{(\sqrt{11})}{\frac{66}{\sqrt{11}}}=\frac{6(\sqrt{10}+\sqrt{11})}{10-11}+\frac{6(\sqrt{10}+3)}{10-9}+\frac{66\sqrt{11}}{11}$$
$$=-6\sqrt{10}-6\sqrt{11}+6\sqrt{10}+18+6\sqrt{11}$$
$$=18$$

ÖRNEK / EXAMPLE 18

İSTANBUL ÜNİ. YÖS 2013

$$\sqrt[3]{2\sqrt{21-\sqrt{23+\sqrt{4}}}}=?$$

A) 6 B) 5 C) 4 D) 3 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\sqrt[3]{2\sqrt{21-\sqrt{23+\sqrt{4}}}}=\sqrt[3]{2\sqrt{21-\sqrt{25}}}$$

$$=\sqrt[3]{2\sqrt{21-5}}$$

$$=\sqrt[3]{2\cdot 4}$$

$$=2$$

ÖRNEK / EXAMPLE 19

ON DOKUZ MAYIS ÜNİ. YÖS 2013

$$\frac{\sqrt{21}-\sqrt{15}+\sqrt{10}-\sqrt{14}}{\sqrt{7}-\sqrt{5}}(\sqrt{3}+\sqrt{2})=?$$

A) 1 B) $\sqrt{6}$ C) 3 D) 5 E) $5+2\sqrt{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{\sqrt{21}-\sqrt{15}+\sqrt{10}-\sqrt{14}}{\sqrt{7}-\sqrt{5}}(\sqrt{3}+\sqrt{2})$$

$$=\frac{\sqrt{3}(\sqrt{7}-\sqrt{5})+\sqrt{2}(\sqrt{5}-\sqrt{7})}{\sqrt{7}-\sqrt{5}}\cdot(\sqrt{3}+\sqrt{2})$$

$$=\frac{(\sqrt{7}-\sqrt{5})\cdot(\sqrt{3}-\sqrt{2})\cdot(\sqrt{3}+\sqrt{2})}{\sqrt{7}-\sqrt{5}}$$

$$=(\sqrt{3}-\sqrt{2})\cdot(\sqrt{3}+\sqrt{2})=3-2=1$$

ÖRNEK / EXAMPLE 20

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$x\sqrt{0{,}08} = 2 \quad \Rightarrow x = ?$

A) $\sqrt{2}$ B) $3\sqrt{2}$ C) $5\sqrt{2}$ D) $7\sqrt{2}$ E) $8\sqrt{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$x \cdot \sqrt{\frac{8}{100}} = 2$$

$$x \cdot \frac{2\sqrt{2}}{10} = 2$$

$$x = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

ÖRNEK / EXAMPLE 21

MARMARA ÜNV. YÖS 2013

$\sqrt{3\sqrt[3]{3\sqrt[3]{3^2\sqrt{3}}}} = ? \qquad$ **x = ?**

A) $\frac{8}{9}$ B) $\frac{31}{36}$ C) $\frac{5}{6}$ D) $\frac{29}{36}$ E) $\frac{14}{18}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\sqrt{\sqrt[3]{3^4\sqrt[3]{\sqrt{3^5}}}} = 3^x$$

$$\sqrt{\sqrt[3]{3^4\sqrt[6]{3^5}}} = 3^x$$

$$\sqrt{\sqrt[3]{\sqrt[6]{3^{29}}}} = 3^x$$

$$\sqrt[36]{3^{29}} = 3^x$$

$$3^{\frac{29}{36}} = 3^x$$

$$x = \frac{29}{36}$$

1. $A \in \mathbb{R},\ x \in \mathbb{Z}$,

$$\sqrt{x+5}+\sqrt[3]{x-1}+\sqrt[4]{2-x}=A \;\Rightarrow\; \sum x = ?$$

A) 4 B) 1 C) –5 D) –9 E) –12

2. $A \in \mathbb{R}$

$$\frac{\sqrt{x-2}+3x}{2x-\sqrt{2-x}}=A \;\Rightarrow\; \mathbf{A=?}$$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

3. $$\sqrt[6]{(-3)^6}-\sqrt{(-5)^2}=?$$

A) –8 B) –3 C) –2 D) 2 E) 8

4. $$\sqrt{1-\frac{25}{169}}\cdot\sqrt{1+\frac{9}{16}}=?$$

A) $\frac{4}{13}$ B) $\frac{9}{13}$ C) $\frac{12}{13}$ D) $\frac{14}{13}$ E) $\frac{15}{13}$

5. $$\sqrt{1{,}44}-\sqrt[3]{0{,}027}+\sqrt[4]{0{,}0016}=?$$

A) 0,11 B) 1,1 C) 1,4 D) 2,1 E) 3,2

6. $$\sqrt[3]{-60-\sqrt{19-\sqrt[3]{8-(-19)}}}=?$$

A) –4 B) –2 C) 3 D) 4 E) 5

7. $\left.\begin{aligned} a &= \sqrt{3} \\ b &= \sqrt[3]{9} \\ c &= \sqrt[4]{2} \end{aligned}\right\} \Rightarrow \mathbf{? < ? < ?}$

A) a < c < b　　B) b < c < a　　C) c < a < b

D) c < b < a　　E) a = b < c

8. $A \in N$

$\sqrt{2000} \cdot x = A \quad \Rightarrow \quad \mathbf{min(x) = ?}$

A) $\sqrt{2}$　　B) $\sqrt{3}$　　C) $\sqrt{5}$

D) $\sqrt{7}$　　E) $\sqrt{11}$

9. $\sqrt[4]{2\sqrt{\frac{1}{2}\sqrt[3]{4}}} = ?$

A) $\sqrt[3]{4}$　　B) $\sqrt[6]{32}$　　C) $\sqrt[15]{16}$

D) $\sqrt[18]{32}$　　E) $\sqrt[24]{32}$

10. $\dfrac{\sqrt[3]{4}\cdot\sqrt[15]{2}}{\sqrt[5]{8}} = \sqrt[15]{2^x} \quad \Rightarrow \quad \mathbf{x = ?}$

A) 2　　B) 4　　C) 6

D) 8　　E) 16

11. $\dfrac{1}{\sqrt{3}} + \dfrac{4}{\sqrt{27}} - \dfrac{4}{\sqrt{12}} = ?$

A) $3\sqrt{3}$　　B) $2\sqrt{3}$　　C) $\dfrac{\sqrt{3}}{3}$

D) $\dfrac{\sqrt{3}}{9}$　　E) $\dfrac{\sqrt{3}}{27}$

12. $\dfrac{\sqrt{5}}{\sqrt{10}+\sqrt{5}} + 1 = ?$

A) $\sqrt{2}$　　B) $\sqrt{3}$　　C) $\sqrt{5}$

D) $\sqrt{10}$　　E) $\sqrt{15}$

YANITLAR / ANSWERS

1. E	**2. C**	**3. C**	**4. E**	**5. B**	**6. A**	**7. C**	**8. C**
9. E	**10. A**	**11. D**	**12. A**				

1. $\sqrt{x+5}+\sqrt[3]{x-1}+\sqrt[4]{2-x}=A$, $A\in R$

$x+5\geq 0$ $2-x\geq 0$

$x\geq -5$ $2\geq x$

$\Rightarrow -5\leq x\leq 2$ $(x\in Z)$

$\Rightarrow \sum x=-5+-4+\ldots+1+2=-12$

Yanıt / The Answer is E

---

2. $\dfrac{\sqrt{x-2}+3x}{2x-\sqrt{2-x}}=A$, $A\in R$

$\sqrt{x-2}\in R\Rightarrow x-2\geq 0$, $x\geq 2$ ...... **(1)**

$\sqrt{2-x}\in R\Rightarrow 2-x\geq 0$, $x\leq 2$ ...... **(2)**

1 ve (and) 2

$x=2\Rightarrow \dfrac{\sqrt{2-2}+3\cdot 2}{2\cdot 2-\sqrt{2-2}}=\dfrac{6}{4}=\dfrac{3}{2}=A$

Yanıt / The Answer is C

---

3. $\sqrt[6]{(-3)^6}-\sqrt{(-5)^2}=|-3|-|-5|$

$=3-5$

$=-2$

Yanıt / The Answer is C

---

4. $\sqrt{1-\dfrac{25}{169}}\cdot\sqrt{1+\dfrac{9}{16}}=\sqrt{\dfrac{169-25}{169}}\cdot\sqrt{\dfrac{16+9}{16}}$

$=\sqrt{\dfrac{144}{169}}\cdot\sqrt{\dfrac{25}{16}}$

$=\dfrac{12}{13}\cdot\dfrac{5}{4}$

$=\dfrac{15}{13}$

Yanıt / The Answer is E

---

5. $\sqrt{1{,}44}-\sqrt[3]{0{,}027}+\sqrt[4]{0{,}0016}$

$=\sqrt{\dfrac{144}{100}}-\sqrt[3]{\dfrac{27}{1000}}+\sqrt[4]{\dfrac{16}{10000}}$

$=\dfrac{12}{10}-\dfrac{3}{10}+\dfrac{2}{10}$

$=\dfrac{11}{10}=1{,}1$

Yanıt / The Answer is B

---

6. $\sqrt[3]{-60-\sqrt{19-\sqrt[3]{8-(-19)}}}=\sqrt[3]{-60-\sqrt{19-\sqrt[3]{27}}}$

$=\sqrt[3]{-60-\sqrt{19-3}}$

$=\sqrt[3]{-60-4}$

$=\sqrt[3]{-64}=-4$

Yanıt / The Answer is A

7. $$\left.\begin{array}{l} a=\sqrt{3} \\ b=\sqrt[3]{9} \\ c=\sqrt[4]{2} \end{array}\right\} \Rightarrow \begin{array}{l} a^{12}=3^{6} \\ b^{12}=9^{4}=3^{8} \\ c^{12}=2^{3} \end{array}$$

$\Rightarrow b>a>c$

Yanıt / The Answer is C

8. $\sqrt{2000}\cdot x=A$

$20\sqrt{5}\cdot x=A \quad (A\in N)$

$\Rightarrow \min(x)=\sqrt{5}$

Yanıt / The Answer is C

9. $$\sqrt[4]{2\sqrt{\frac{1}{2}\sqrt[3]{4}}}=\sqrt[4]{\sqrt{4\cdot\frac{1}{2}\sqrt[3]{4}}}$$

$$=\sqrt[4]{\sqrt{2\sqrt[3]{4}}}$$

$$=\sqrt[4]{\sqrt{\sqrt[3]{32}}}$$

$$=\sqrt[24]{32}$$

Yanıt / The Answer is E

10. $$\frac{\sqrt[3]{4}\cdot\sqrt[15]{2}}{\sqrt[5]{8}}=\sqrt[15]{2^{x}}$$

$$\frac{2^{\frac{2}{3}}\cdot 2^{\frac{1}{15}}}{2^{\frac{3}{5}}}=2^{\frac{x}{15}} \Rightarrow \frac{2}{3}+\frac{1}{15}-\frac{3}{5}=\frac{x}{15}$$

$$\frac{10+1-9}{15}=\frac{x}{15}$$

$$x=2$$

Yanıt / The Answer is A

11. $$\frac{1}{\sqrt{3}}+\frac{4}{\sqrt{27}}-\frac{4}{\sqrt{12}}=\frac{1}{\sqrt{3}}+\frac{4}{3\sqrt{3}}-\frac{4}{2\sqrt{3}}$$

$$=\frac{\sqrt{3}}{3}+\frac{4\sqrt{3}}{9}-\frac{4\sqrt{3}}{6}$$

$$=\frac{6\sqrt{3}+8\sqrt{3}-12\sqrt{3}}{18}$$

$$=\frac{2\sqrt{3}}{18}=\frac{\sqrt{3}}{9}$$

Yanıt / The Answer is D

12. $$\frac{\sqrt{5}}{\sqrt{10}+\sqrt{5}}+1=\frac{\sqrt{5}(\sqrt{10}-\sqrt{5})}{5}+1$$

$$=\frac{5(\sqrt{2}-1)}{5}+1$$

$$=\sqrt{2}-1+1$$

$$=\sqrt{2}$$

Yanıt / The Answer is A

1. $\dfrac{\sqrt[4]{(-16)^2}-\sqrt[3]{-27}}{\sqrt{0,49}}=?$

A) 4 B) 5 C) 7 D) 8 E) 10

2. $\dfrac{\sqrt{a^5}\,\sqrt{b^3}}{\sqrt{ab}}=?$

A) a^2b B) a^3b C) ab^2 D) ab^3 E) a^2b^2

3. $\dfrac{2}{3-\sqrt{3}}+\dfrac{2}{3+\sqrt{3}}=?$

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

4. $\sqrt[x]{0,\overline{3}}=\sqrt[3]{81^{-y}} \Rightarrow x\cdot y=?$

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{5}{4}$ D) $\frac{7}{4}$ E) $\frac{9}{4}$

5. $x<0<y \Rightarrow$

$$\frac{\sqrt[4]{(x-y)^4}+\sqrt[3]{x^3}}{\sqrt[5]{(x-y)^5}+\sqrt{y^2}}=?$$

A) $x-y$ B) $-x$ C) $-y$ D) $\frac{y}{x}$ E) $\frac{x}{y}$

6. $\sqrt[3]{\dfrac{5^x+5^x+5^x+5^x+5^x}{4^x+4^x+4^x+4^x}}=\dfrac{64}{125} \Rightarrow x=?$

A) –11 B) –10 C) –9 D) –8 E) –7

7. $\sqrt{2x+1-2\sqrt{2x}} = 11 \Rightarrow x = ?$

A) 72 B) 76 C) 81 D) 90 E) 96

8. $\sqrt{209 \cdot 239 + 225} = ?$

A) 220 B) 221 C) 222 D) 223 E) 224

9. $x > 0,$

$$\sqrt[3]{(x+12)\sqrt[3]{(x+12)\sqrt[3]{(x+12)\ldots}}} = x \Rightarrow x = ?$$

A) 2 B) 4 C) 6 D) 9 E) 12

10. $\sqrt{x+\sqrt{1+\sqrt{x+\sqrt{1+\ldots}}}} = 3 \Rightarrow x = ?$

A) 7 B) 8 C) 9 D) 10 E) 11

11. $\sqrt{7-4\cdot\sqrt{8-2\sqrt{7}}} + 3 - \frac{7}{\sqrt{7}} = ?$

A) 1 B) 2 C) 4 D) 7 E) 8

12. $\left.\begin{array}{l} x = \sqrt{2} \\ y = \sqrt[3]{5} \\ z = \sqrt[6]{10} \end{array}\right\} \Rightarrow ? < ? < ?$

A) x < y < z B) y < z < x C) z < x < y D) z < y < x E) x < z < y

YANITLAR / ANSWERS

1. E	2. A	3. E	4. B	5. D	6. B	7. A	8. E
9. B	10. A	11. A	12. E				

1. $$\frac{\sqrt[4]{(-16)^2}-\sqrt[3]{-27}}{\sqrt{0,49}}=\frac{4-(-3)}{0,7}=\frac{7}{0,7}=10$$

Yanıt / The Answer is E

2. $$\frac{\sqrt{a^5}\,\sqrt{b^3}}{\sqrt{ab}}=\frac{\sqrt{a^4\cdot a}\cdot\sqrt{b^2\cdot b}}{\sqrt{ab}}$$

$$=\frac{a^2\sqrt{a}\cdot b\cdot\sqrt{b}}{\sqrt{ab}}$$

$$=a^2b$$

Yanıt / The Answer is A

3. $$\frac{2}{3-\sqrt{3}}+\frac{2}{3+\sqrt{3}}=\frac{2\cdot(3+\sqrt{3})+2\cdot(3-\sqrt{3})}{(3+\sqrt{3})\cdot(3-\sqrt{3})}$$

$$=\frac{6+2\sqrt{3}+6-2\sqrt{3}}{6}$$

$$=\frac{12}{6}=2$$

Yanıt / The Answer is E

4. $$\sqrt[x]{0,\overline{3}}=\sqrt[3]{81^{-y}}$$

$$3^{-\frac{1}{x}}=3^{-\frac{4y}{3}}$$

$$\frac{1}{x}=\frac{4y}{3}$$

$$4xy=3$$

$$xy=\frac{3}{4}$$

Yanıt / The Answer is B

5. $x<0<y\ \Rightarrow$

$$\frac{\sqrt[4]{(x-y)^4}+\sqrt[3]{x^3}}{\sqrt[5]{(x-y)^5}+\sqrt{y^2}}=\frac{|x-y|+x}{x-y+|y|}$$

$$=\frac{-(x-y)+x}{x-y+y}$$

$$=\frac{y}{x}$$

Yanıt / The Answer is D

6. $$\sqrt[3]{\frac{5^x+5^x+5^x+5^x+5^x}{4^x+4^x+4^x+4^x}}=\frac{64}{125}$$

$$\sqrt[3]{\frac{5^{x+1}}{4^{x+1}}}=\frac{64}{125}$$

$$\left(\frac{5}{4}\right)^{\frac{x+1}{3}}=\left(\frac{5}{4}\right)^{-3}$$

$$\frac{x+1}{3}=-3$$

$$x=-10$$

Yanıt / The Answer is B

7. $\sqrt{2x+1-2\sqrt{2x}} = \sqrt{2x}-1$

↙ ↘

2x 1

$$\sqrt{2x+1-2\sqrt{2x}} = 11$$

$$\sqrt{2x}-1 = 11$$

$$\sqrt{2x} = 12$$

$$2x = 144$$

$$x = 72$$

Yanıt / The Answer is A

8. $t = 209 \Rightarrow \sqrt{t(t+30)+225} = \sqrt{t^2+30t+225}$

$$= t+15$$

$$= 209+15$$

$$= 224$$

Yanıt / The Answer is E

9. $\sqrt[3]{(x+12)\sqrt[3]{(x+12)\sqrt[3]{(x+12)\ldots}}} = x$

$$\sqrt{x+12} = x$$

$$x+12 = x^2$$

$\Rightarrow x^2 - x - 12 = 0$

$(x-4)\cdot(x+3) = 0$

$x_1 = 4, \ x_2 = -3$

$x > 0 \Rightarrow x = 4$

Yanıt / The Answer is B

10. $\sqrt{x+\sqrt{1+\sqrt{x+\sqrt{1+\ldots}}}} = 3$

$$\sqrt{x+\sqrt{1+\underbrace{\sqrt{x+\sqrt{1+\ldots}}}_{3}}} = 3$$

$$\sqrt{x+\sqrt{1+3}} = 3$$

$$\sqrt{x+2} = 3$$

$$x+2 = 9$$

$$x = 7$$

Yanıt / The Answer is A

11. $\sqrt{7-4\cdot\sqrt{8-2\sqrt{7}}} + 3 - \dfrac{7}{\sqrt{7}}$

$$= \sqrt{7-4(\sqrt{7}-1)} + 3 - \sqrt{7}$$

$$= \sqrt{7-4\sqrt{7}+4} + 3 - \sqrt{7}$$

$$= \sqrt{11-2\sqrt{28}} + 3 - \sqrt{7}$$

$$= \sqrt{7} - 2 + 3 - \sqrt{7}$$

$$= 1$$

Yanıt / The Answer is A

12. $$\left.\begin{aligned} x &= \sqrt{2} \\ y &= \sqrt[3]{5} \\ z &= \sqrt[6]{10} \end{aligned}\right\} \Rightarrow \begin{aligned} x^6 &= 2^3 = 8 \\ y^6 &= 5^2 = 25 \\ z^6 &= 10 \end{aligned}$$

$\Rightarrow x < z < y$

Yanıt / The Answer is E

1. $\sqrt{1{,}6}+\sqrt{0{,}9}-\sqrt{0{,}4}=?$

Yanıt / The Answer is: $\frac{\sqrt{10}}{2}$

2. $\frac{\sqrt[3]{3^2}\cdot\sqrt{3}\cdot\sqrt[4]{3}}{\sqrt[6]{3}}=?$

Yanıt / The Answer is: $3\sqrt[4]{3}$

3. $\frac{3}{\sqrt{2}+\sqrt{3}}-\frac{2}{\sqrt{2}-\sqrt{3}}=a\sqrt{3}+b\sqrt{2}\ \Rightarrow\ \mathbf{a\cdot b=?}$

Yanıt / The Answer is: **–5**

4. $\frac{1}{\sqrt[3]{5}-\sqrt[3]{4}}-\left(\sqrt[3]{25}+\sqrt[3]{16}\right)=?$

Yanıt / The Answer is: $\sqrt[3]{20}$

5. $\sqrt[3]{2\sqrt{4\sqrt[3]{2\sqrt{4\ldots}}}}=?$

Yanıt / The Answer is: $\sqrt[5]{2^4}$

6. $\sqrt[3]{\sqrt{\frac{3}{x}}\cdot\frac{x}{9}}=\sqrt[6]{3}\ \Rightarrow\ \mathbf{x=?}$

Yanıt / The Answer is: **81**

7. $\sqrt{\frac{9}{4^{x-3}}+4^{5-x}}=40 \Rightarrow x=?$

Yanıt / The Answer is: 0

8. $\frac{\sqrt{\sqrt{20}-\sqrt{5}+1}\cdot\sqrt{\sqrt{20}-\sqrt{5}-1}}{\sqrt{2}}=?$

Yanıt / The Answer is: $\sqrt{2}$

9. $\sqrt{x+3}-\sqrt{x-5}=A \Rightarrow \sqrt{x+3}+\sqrt{x-5}=?$

Yanıt / The Answer is: $\frac{8}{A}$

10. $\sqrt{9-4\sqrt{5}}-\sqrt{8-\sqrt{60}}-\frac{3}{\sqrt{3}}=?$

Yanıt / The Answer is: –2

11. $\frac{\sqrt[3]{x\sqrt{x^5}}}{\sqrt{x\sqrt[3]{x}}}=\sqrt{2} \Rightarrow x=?$

Yanıt / The Answer is: 2

12. $\sqrt{5^{x-1}+\frac{100}{5^{3-x}}}=\sqrt[4]{125} \Rightarrow x=?$

Yanıt / The Answer is: $\frac{3}{2}$

1. $\frac{2}{\sqrt{3}} - \frac{3}{1+\sqrt{3}} + \frac{5}{2\sqrt{3}} = ?$

A) $\frac{3}{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$

D) $\frac{3\sqrt{3}}{2}$ E) $\frac{5\sqrt{3}}{2}$

---

2. $\sqrt{\frac{\sqrt{3}-1}{\sqrt{3}+1}} \cdot \left(\sqrt{3}+1\right) = ?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 3

D) $3\sqrt{3}$ E) 9

---

3. $\frac{6\sqrt{3}+6}{\sqrt{15}+\sqrt{6}+\sqrt{5}+\sqrt{2}} = x\cdot\left(\sqrt{5}-\sqrt{2}\right) \Rightarrow x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

---

4. $2^x = \sqrt[4]{2\sqrt{\sqrt[3]{16}}} \Rightarrow 12\cdot x = ?$

A) 1 B) 2 C) 3

D) 4 E) 5

---

5 $\sqrt[5]{16\sqrt[5]{16\sqrt[5]{16\ldots}}} + \sqrt{27:\sqrt{27:\sqrt{27:\ldots}}} = ?$

A) 3 B) 5 C) 8

D) 9 E) 12

---

6. $\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}} = \sqrt{12-\sqrt{12-\sqrt{12\ldots}}} \Rightarrow x = ?$

A) 3 B) 4 C) 6

D) 9 E) 12

7. $\left(\sqrt{30-\sqrt{30-\sqrt{30-\ldots}}}\right)\cdot\left(\sqrt{20+\sqrt{20+\sqrt{20+\ldots}}}\right)=?$

A) 9 B) 16 C) 25

D) 36 E) 49

8. $\sqrt{8}\cdot\sqrt{9}\cdot\sqrt{10}\ \ldots\sqrt{x}=24\sqrt{165}\ \Rightarrow\ \mathbf{x=?}$

A) 11 B) 12 C) 13

D) 14 E) 15

9. $\sqrt{4-2\sqrt{3}}\cdot(\sqrt{3}+1)=?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2

D) 3 E) 4

10. $\sqrt{\frac{1}{25}+\frac{1}{30}+\frac{1}{144}}=?$

A) $\frac{7}{60}$ B) $\frac{11}{60}$ C) $\frac{13}{60}$

D) $\frac{17}{60}$ E) $\frac{19}{60}$

11. $a<0<b\ \Rightarrow$

$$\frac{\sqrt{(a-b)^2}-\sqrt[4]{a^4}}{\sqrt[3]{b^3}}=?$$

A) 1 B) a C) b

D) $\frac{a}{b}$ E) $\frac{b}{a}$

12. $\frac{\sqrt{x}\cdot\sqrt[4]{y}+\sqrt[4]{x}\cdot\sqrt{y}}{\sqrt[4]{x}+\sqrt[4]{y}}=3\ \Rightarrow\ \mathbf{x\cdot y=?}$

A) $\sqrt{3}$ B) 3 C) 9

D) 27 E) 81

YANITLAR / ANSWERS

1. A	2. A	3. B	4. E	5. B	6. C	7. C	8. B
9. C	10. D	11. A	12. E				

1. $\sqrt{11+2\sqrt{30}}\cdot\sqrt{11-2\sqrt{30}}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $\sqrt{x+\sqrt{x}}+\sqrt{x-\sqrt{x}}=4 \Rightarrow x=?$

A) $\frac{9}{4}$ B) $\frac{21}{8}$ C) $\frac{36}{11}$ D) $\frac{64}{15}$ E) $\frac{71}{18}$

3. $\sqrt{9+\sqrt{56}}-\sqrt{9-\sqrt{56}}=?$

A) $2\sqrt{2}$ B) $2\sqrt{7}$ C) $2\sqrt{14}$ D) $2\sqrt{21}$ E) $2\sqrt{42}$

4. $A \in R$,

$\sqrt{x-4}+\sqrt[5]{5-2x}+\sqrt[8]{7-x}=A \Rightarrow \sum x=?$

A) 11 B) 15 C) 22 D) 30 E) 33

5. $\sqrt[3]{8\sqrt[4]{8^3\sqrt{8^{-5}}}}=\sqrt[8]{2^x} \Rightarrow x=?$

A) 3 B) 4 C) 6 D) 8 E) 9

6. $\left(\frac{1}{5}\right)^{x+2}=\sqrt[3]{5^{9-6y}} \Rightarrow 2y-x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

7. $\sqrt[4]{7-2\sqrt{10}}\cdot\sqrt{\sqrt{5}+\sqrt{2}}=?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2 D) $\sqrt{5}$ E) $\sqrt{7}$

8. $\frac{1}{2\sqrt{x}}+\frac{\sqrt{x}+1}{x+\sqrt{x}}=3 \Rightarrow x=?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

9. $\sqrt{9+4\sqrt{5}}-\sqrt{7-2\sqrt{10}}=?$

A) $2-\sqrt{5}$
B) $1+\sqrt{5}$
C) $2+\sqrt{2}$
D) $2+\sqrt{5}$
E) $\sqrt{2}-2\sqrt{5}$

10. $\frac{\sqrt{10+\sqrt{84}}+\sqrt{10-\sqrt{84}}}{\sqrt{7}}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

11. $\sqrt{x-1}+\sqrt{x+1}=\sqrt{2x+2} \Rightarrow x=?$

A) $\frac{\sqrt{2}}{5}$ B) $\frac{\sqrt{2}}{4}$ C) $\frac{\sqrt{2}}{3}$ D) $\frac{\sqrt{2}}{2}$ E) $\sqrt{2}$

12. $(\sqrt{3}-2)^{12}\cdot(\sqrt{3}+2)^{11}+\sqrt{3}=?$

A) 5 B) 4 C) 3 D) 2 E) 1

YANITLAR / ANSWERS

1. A	2. D	3. A	4. C	5. E	6. E	7. B	8. C
9. C	10. B	11. E	12. D				

1. $\sqrt{3+x}+\sqrt{3-x}=\sqrt{10} \Rightarrow$ ÇK(SS) = ?

A) $\{-\sqrt{2},\sqrt{2}\}$

B) $\{-\sqrt{3},\sqrt{3}\}$

C) $\{-\sqrt{5},\sqrt{5}\}$

D) $\{-\sqrt{7},\sqrt{7}\}$

E) $\{-\sqrt{10},\sqrt{10}\}$

2. $\sqrt{7-\sqrt{48}}+\sqrt{7+\sqrt{48}}=?$

A) 1

B) 2

C) 3

D) 4

E) 5

3. $\left(\sqrt{2}+\sqrt{5}+\sqrt{7}\right)^2\cdot\left(\sqrt{2}+\sqrt{5}-\sqrt{7}\right)^2=?$

A) 10

B) 20

C) 30

D) 40

E) 50

4. $\sqrt[4]{5-2\sqrt{6}}\cdot\sqrt[4]{5+2\sqrt{6}}=?$

A) $4\sqrt{3}$

B) $\sqrt[4]{2}$

C) 1

D) $\sqrt{2}$

E) $\sqrt{3}$

5. $\sqrt[3]{\sqrt[4]{x}}+\sqrt{\sqrt[6]{x}}=2\sqrt[3]{3} \Rightarrow x=?$

A) 3

B) 6

C) 9

D) 27

E) 81

6. $\sqrt{4+2\sqrt{3}}-1=2\sqrt{x} \Rightarrow x=?$

A) $\frac{1}{4}$

B) $\frac{3}{4}$

C) $\frac{5}{4}$

D) $\frac{9}{4}$

E) $\frac{15}{4}$

7. $\dfrac{\sqrt{\sqrt{3\sqrt{3}}}}{\sqrt[3]{\sqrt{3}}} = 243^{x} \Rightarrow x = ?$

A) $\dfrac{1}{12}$

B) $\dfrac{1}{18}$

C) $\dfrac{1}{24}$

D) $\dfrac{1}{36}$

E) $\dfrac{1}{48}$

8. $\dfrac{\sqrt{\sqrt{5}+\sqrt{2+\sqrt{5}}} \cdot \sqrt{\sqrt{5}-\sqrt{2+\sqrt{5}}} \cdot \sqrt{3+\sqrt{5}}}{\sqrt{2}} = ?$

A) 1

B) $\sqrt{2}$

C) $\sqrt{3}$

D) $2\sqrt{2}$

E) $\sqrt{5}$

9. $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{2}-1} \Rightarrow \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1} = ?$

A) 3x

B) 2x

C) x

D) $\dfrac{1}{x}$

E) $\dfrac{2}{x}$

10. $\sqrt{5+2\sqrt{3-\sqrt{8}}} - 1 = ?$

A) $\sqrt{2}$

B) $\sqrt{3}$

C) $2\sqrt{2}$

D) $\sqrt{5}$

E) $2\sqrt{5}$

11. $\sqrt{25^{3x+1}} = 125\sqrt{5^{6x-2y}} \Rightarrow y = ?$

A) –1

B) 1

C) 2

D) 4

E) 6

12. $\sqrt[3]{5\sqrt{3\sqrt[3]{5\sqrt{3\ldots}}}} = ?$

A) $\sqrt[5]{75}$

B) $\sqrt[5]{45}$

C) $\sqrt[5]{30}$

D) $\sqrt[5]{15}$

E) $\sqrt[5]{5}$

YANITLAR / ANSWERS

1. C	2. D	3. D	4. C	5. E	6. B	7. C	8. B
9. D	10. A	11. C	12. A				

1. $\sqrt[3]{-16} - \sqrt[3]{54} - \sqrt[3]{-128} = ?$

A) $-\sqrt[3]{2}$

B) $\sqrt[3]{2}$

C) $2\sqrt[3]{2}$

D) $3\sqrt[3]{2}$

E) $4\sqrt[3]{2}$

2. $\dfrac{\sqrt{1,2} + \sqrt{7,5}}{\sqrt{4,8}} = ?$

A) $\frac{3}{4}$

B) $\frac{5}{4}$

C) $\frac{7}{4}$

D) $\frac{9}{4}$

E) $\frac{11}{4}$

3. $\dfrac{\sqrt{0,75} - \sqrt{0,12} + \sqrt{2,43}}{\sqrt{0,48}} = ?$

A) –1

B) 2

C) 3

D) 6

E) 9

4. $\sqrt[3]{3\sqrt{2}} \cdot \sqrt[3]{2\sqrt{3}} = ?$

A) 1

B) $\sqrt{2}$

C) $\sqrt{3}$

D) $\sqrt{5}$

E) $\sqrt{6}$

5. $\left.\begin{array}{l} \sqrt[3]{2^x} = b \\ \sqrt{2^y} = b^3 \end{array}\right\} \Rightarrow 2^{2x-y} = ?$

A) $\frac{1}{4}$

B) $\frac{1}{2}$

C) 1

D) 2

E) 4

6. $\sqrt[x]{3} = \dfrac{3^x \cdot \sqrt[3]{3}}{27} \Rightarrow x = ?$

A) 1

B) 2

C) 3

D) 4

E) 5

7. $\sqrt{x+9-6\sqrt{x}}=2 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{1, -\frac{5}{7}\right\}$

B) $\left\{\frac{5}{7}\right\}$

C) $\left\{-1, \frac{5}{7}\right\}$

D) $\left\{1, \frac{5}{7}\right\}$

E) $\{1, 25\}$

8. $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=?$

A) 7

B) 8

C) 9

D) 10

E) 11

9. $\frac{3}{\sqrt{3}-2}+\frac{5}{2+\sqrt{3}}+\frac{24}{\sqrt{3}}=?$

A) 1

B) 2

C) 3

D) 4

E) 5

10. $\frac{\sqrt{24}-5}{\sqrt{3}-\sqrt{2}}+\sqrt{3}=?$

A) $\sqrt{2}$

B) $2\sqrt{2}$

C) $\sqrt{3}$

D) $2\sqrt{3}$

E) $3\sqrt{3}$

11. $\left.\begin{array}{l} x=\sqrt[3]{4} \\ y=\sqrt[4]{8} \\ z=\sqrt{6} \end{array}\right\} \Rightarrow$ **? < ? < ?**

A) $z<y<x$

B) $y<z<x$

C) $z<x<y$

D) $y<x<z$

E) $x<y<z$

12. $\frac{2}{\sqrt{2}+1}+\frac{3}{\sqrt{3}+\sqrt{2}}+\frac{3}{2+\sqrt{3}}+\frac{2}{\sqrt{2}}=?$

A) 4

B) 6

C) 8

D) 10

E) 12

YANITLAR / ANSWERS

1. A	2. C	3. C	4. E	5. C	6. C	7. E	8. B
9. D	10. A	11. E	12. A				

ÜNİTE 4

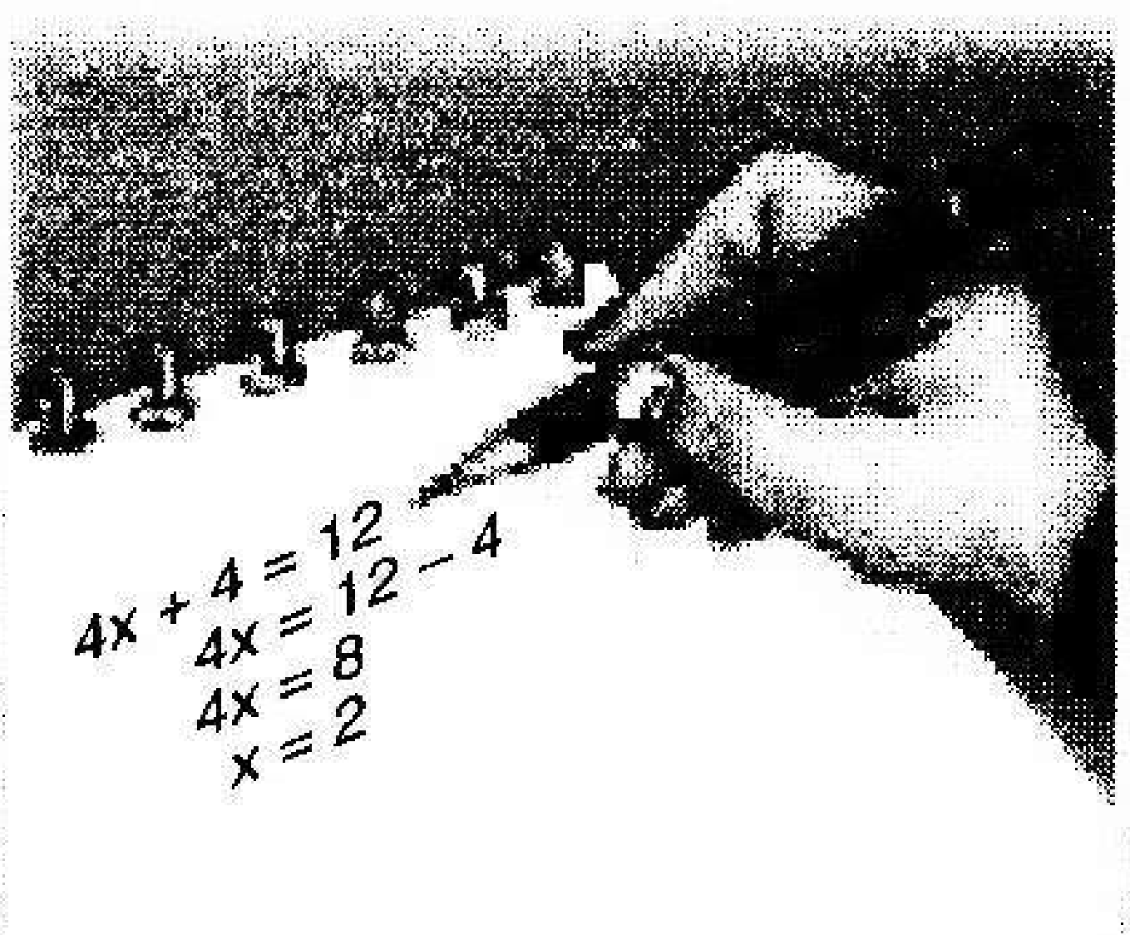
UNIT

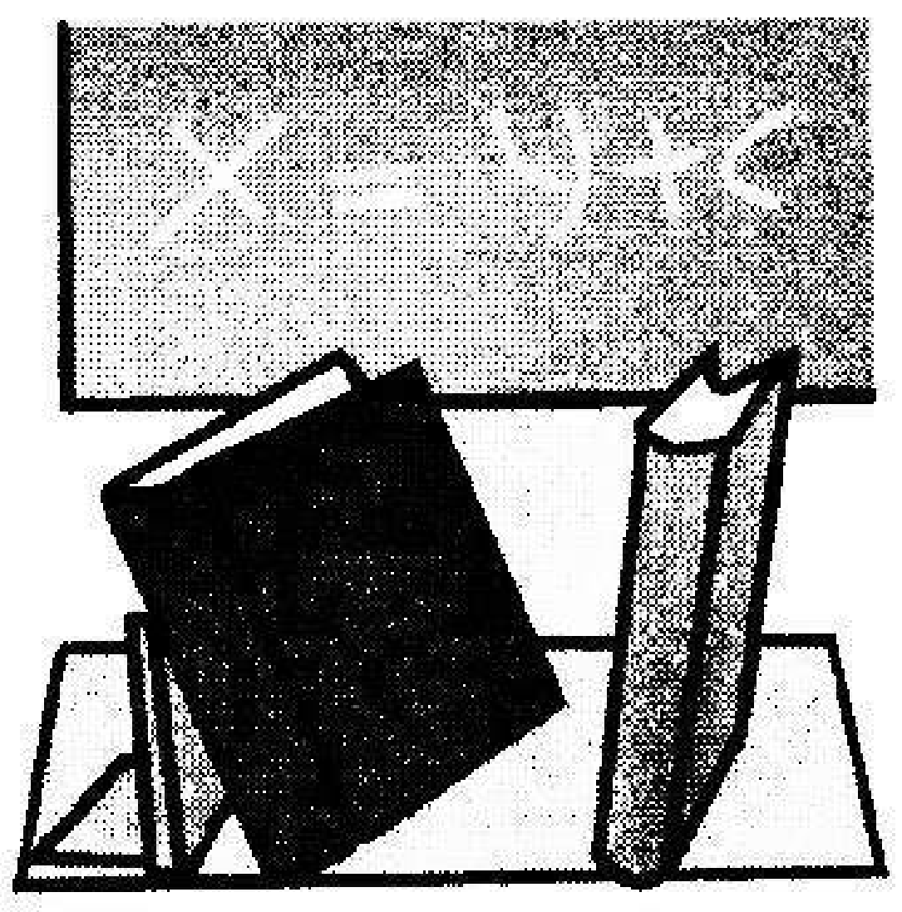
Bu ünitede; birinci dereceden denklem, birinci dereceden eşitsizlik ve mutlak değer konuları ayrıntılı anlatılmıştır. Bu ünitede anlatılan konular ikinci dereceden denklem ve eşitsizliklerin temelini oluşturacağından öğrencilerin, bu ünitede anlatılan konuları iyi öğrenmeleri gerekmektedir.

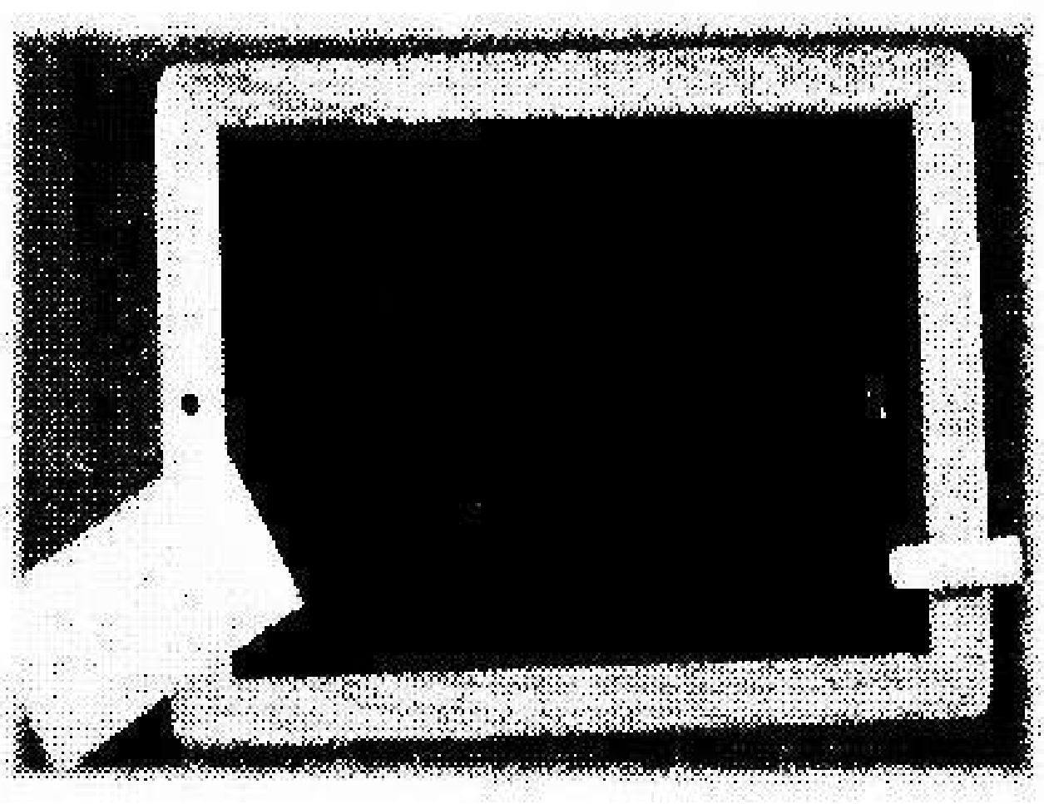
Bu ünite, 76 çözümlü örnek, 36 alıştırma, 96 test sorusu içermektedir.

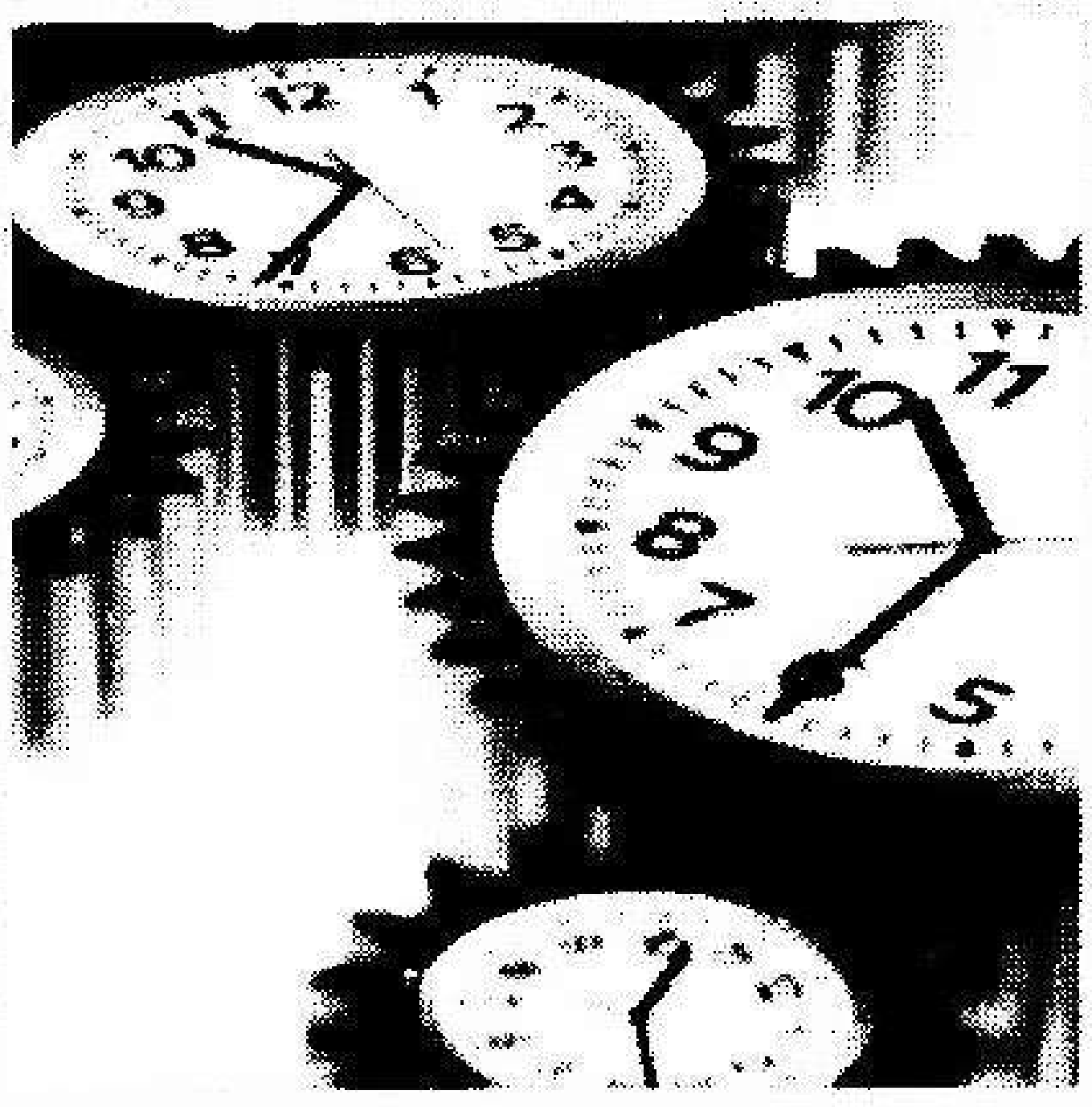
In this unit, first order equation, first order inequality, and absolute value are explained in detailed. These concepts are very important. Because inequalities are necessary in quadratic equations and inequalities.

This unit includes; 76 examples, 36 exercises, and 96 test questions.





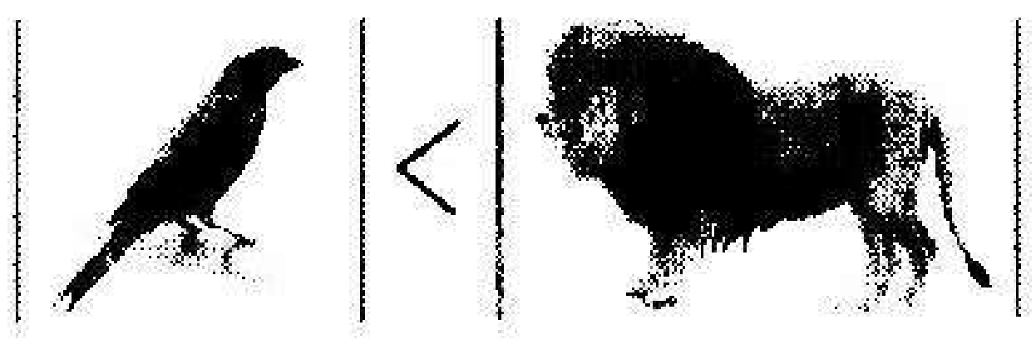




◆ **BİRİNCİ DERECEDEN DENKLEMLER / DOĞRUSAL DENKLEMLER**
FIRST ORDER EQUATIONS / LINEAR EQUATIONS .... 179 – 199

◆ **BİRİNCİ DERECEDEN EŞİTSİZLİKLER / DOĞRUSAL EŞİTSİZLİKLER**
FIRST ORDER INEQUALITIES / LINEAR INEQUALITIES .... 199 – 214

◆ **MUTLAK DEĞER**
ABSOLUTE VALUE .... 215 – 236



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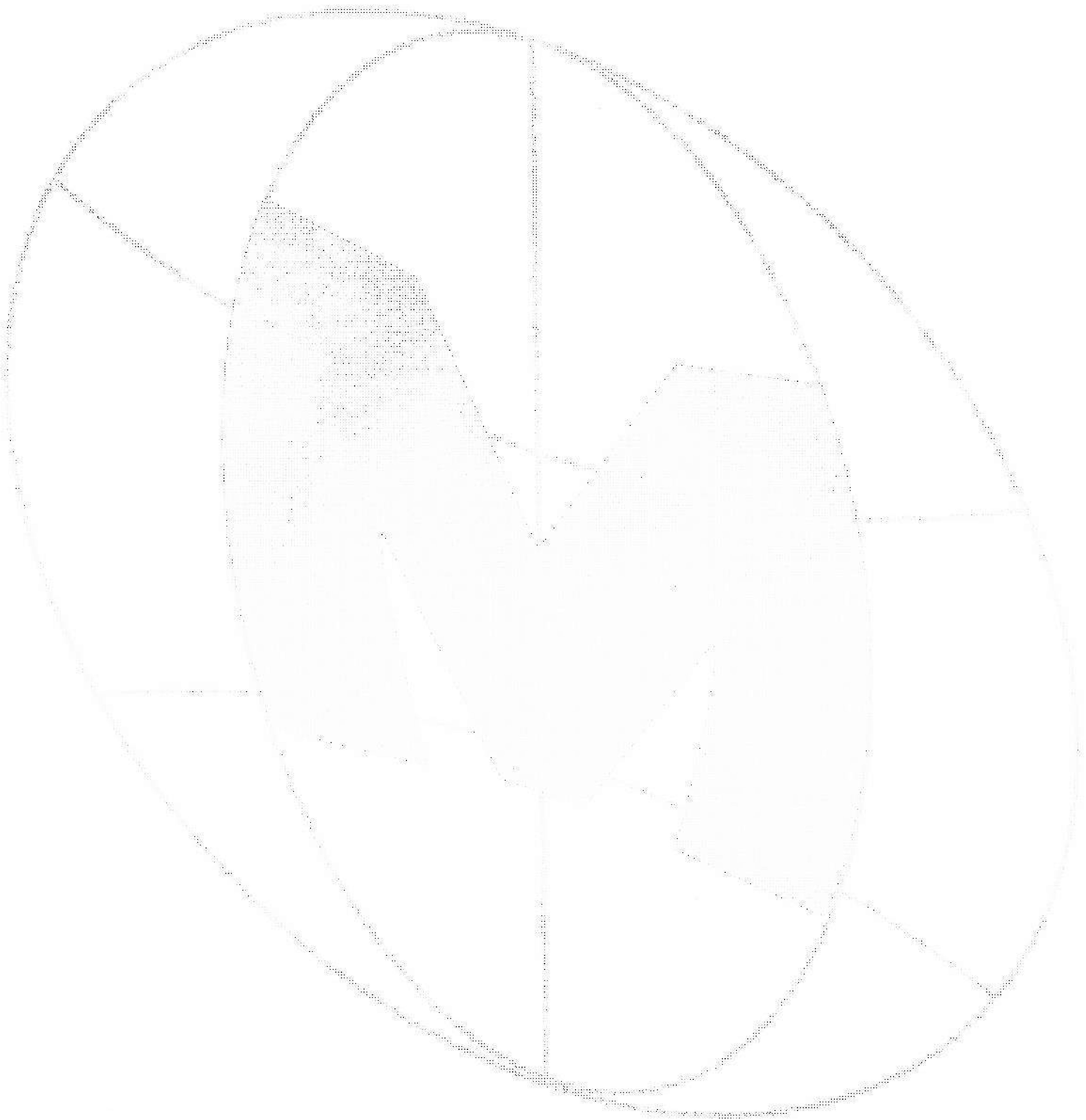

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BİRİNCİ DERECEDEN DENKLEMLER
FIRST ORDER EQUATIONS

BÖLÜM
CHAPTER **1**

Tanım: a ve b reel sayı ve $a \neq 0$ olmak üzere,
$ax + b = 0$ ifadesine *birinci dereceden bir bilinmeyenli denklem* denir. Bu denklemi sağlayan, x reel sayısına da *denklemin kökü* denir. Denklemi sağlayan x değerlerinin kümesine *çözüm kümesi* denir ve *ÇK* ile gösterilir.

Definition: *A first order equation in one variable x is an equation that can be written in the form $ax + b = 0$, where a and b are real numbers, and $a \neq 0$. The real number x which satisfy the equation is called the root of this equation. The set of the values of x which satisfy the equation is called solution set and showen by SS.*

$$ax + b = 0 \Rightarrow ax = -b$$

$$x = -\frac{b}{a} \Rightarrow \text{ÇK(SS)} = \left\{-\frac{b}{a}\right\}$$

ÖRNEK / EXAMPLE 1

$5x + 11 = 4x - 3 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-10\}$ B) $\{-12\}$ C) $\{-14\}$ D) $\{-16\}$ E) $\{-18\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$5x + 11 = 4x - 3$

$5x - 4x = -11 - 3$

$x = -14 \Rightarrow \text{ÇK(SS)} = \{-14\}$

ÖRNEK / EXAMPLE 2

$2(x + 3) + 3x = 4(3 - x) + 2 + x \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{1}{8}\right\}$ B) $\left\{\frac{1}{4}\right\}$ C) $\left\{\frac{1}{3}\right\}$ D) $\left\{\frac{1}{2}\right\}$ E) $\{1\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$2(x + 3) + 3x = 4(3 - x) + 2 + x$

$2x + 6 + 3x = 12 - 4x + 2 + x$

$8x = 8$

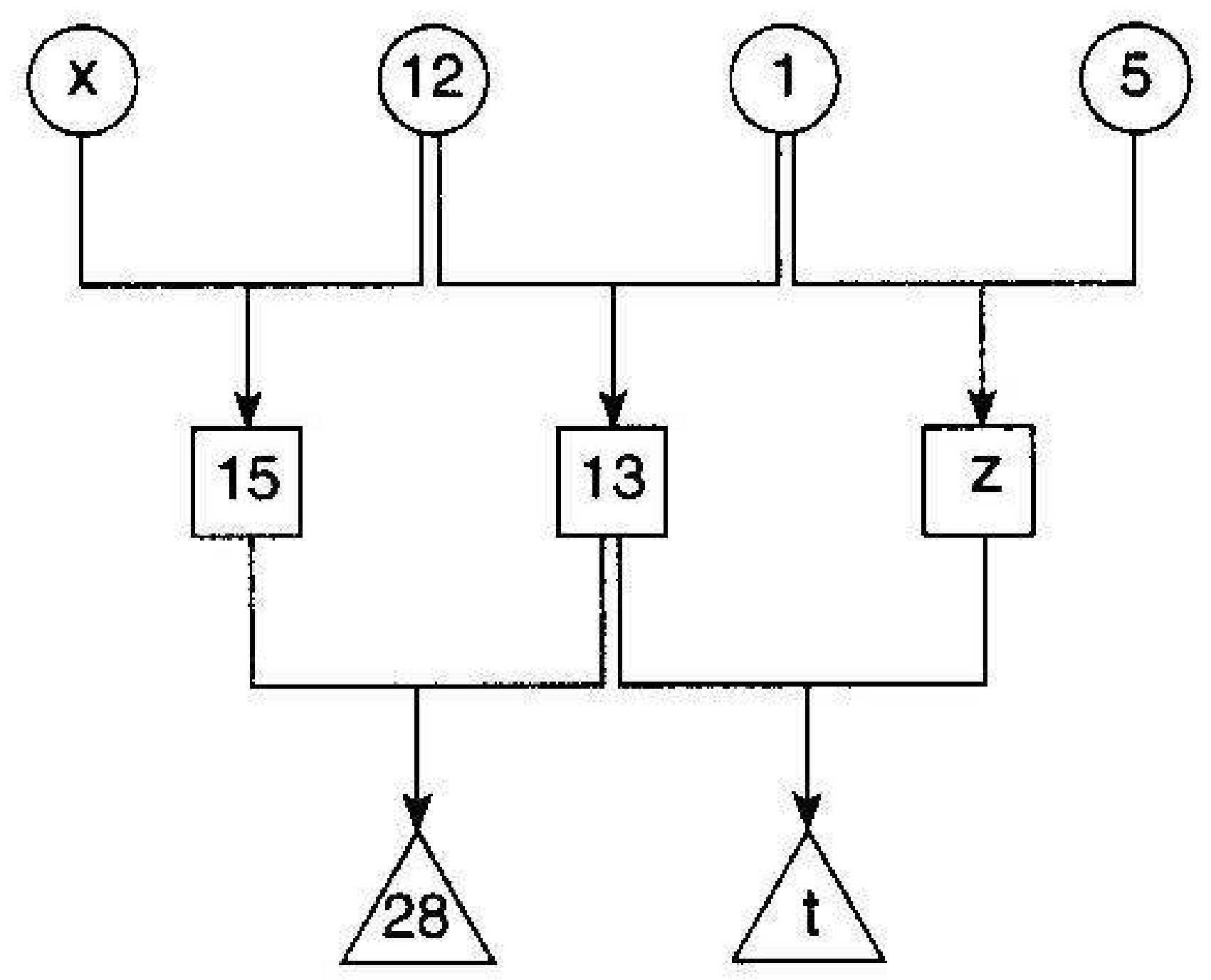
$x = 1 \Rightarrow \text{ÇK(SS)} = \{1\}$

PEKİŞTİRME / REINFORCE

$2(x + 1) + 3 = 3(x - 1) \Rightarrow \text{ÇK(SS)} = ?$

YANIT / ANSWER : 8

Düşünün / Think about it



$\Rightarrow t - x - z = ?$

ÖRNEK / EXAMPLE 3

$\frac{2x+2}{3} - \frac{x+1}{2} = x - \frac{14}{6} \Rightarrow$ **ÇK(SS) = ?**

A) {1} B) {2} C) {3} D) {4} E) {5}

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{2x+2}{\underset{(2)}{3}} - \frac{x+1}{\underset{(3)}{2}} = \frac{x}{\underset{(6)}{1}} - \frac{14}{\underset{(1)}{6}}$$

$$\frac{4x+4-3x-3}{6} = \frac{6x-14}{6}$$

$$\frac{x+1}{6} = \frac{6x-14}{6}$$

$$15 = 5x$$

$$x = 3 \Rightarrow \text{ÇK(SS)} = \{3\}$$

▼ **Birinci Dereceden Bir Bilinmeyenli Denklemlerde Çözüm Kümesi için Özel Durumlar / *Special Cases of First Order Equations with an unknown***

1. $a, b, c, d \in R$, $a \neq 0$, $c \neq 0$
$ax + b = cx + d$
$a = c,\ b = d \Rightarrow$ ÇK(SS) = R

2. $ax + b = cx + d$
$a = c,\ b \neq d \Rightarrow$ ÇK(SS) = ∅

ÖRNEK / EXAMPLE 4

$4x + 5 = 3(x + 2) + x - 1 \Rightarrow$ **ÇK(SS) = ?**

A) R B) {4} C) {5} D) {12} E) ∅

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$4x + 5 = 3(x + 2) + x - 1$
$4x + 5 = 3x + 6 + x - 1$
$4x + 5 = 4x + 5 \Rightarrow$ ÇK(SS) = R

Yanıt / Answer

10

ÖRNEK / EXAMPLE 5

$2x+3=\frac{4x-5}{2} \Rightarrow$ ÇK(SS) = ?

A) ∅ B) {–5} C) {4} D) {6} E) R

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$2x+3=\frac{4x-5}{2}$

$4x+6=4x-5 \Rightarrow$ ÇK(SS) = ∅

BİRİNCİ DERECEDEN İKİ BİLİNMEYENLİ DENKLEM SİSTEMLERİ
SYSTEMS of FIRST ORDER EQUATIONS with TWO UNKNOWNS

Tanım: a, b, c ∈ R ve a ≠ 0 ve b ≠ 0 olmak üzere, ax + by + c = 0 biçiminde verilen denklemlere *birinci dereceden iki bilinmeyenli denklem* denir.

Definition: *Provided that a, b, c ∈ R, a ≠ 0, b ≠ 0, the equations in the ax + by + c = 0 form are called first order equations with two unknowns.*

Tanım: Denklemi sağlayan x ve y değerlerine *denklemin kökleri* denir. Birinci dereceden iki bilinmeyenli denklemlerin çözüm kümesi ikililerden oluşur. Birinci dereceden iki bilinmeyenli denklem, koordinat sisteminde bir doğru belirtir.

$a \neq 0,\ b \neq 0,\ a, b, c, d, e, f \in R \Rightarrow$

$ax + by = c$

$dx + ey = f$

denklemlerine *birinci dereceden iki bilinmeyenli denklem sistemi* denir.

Definition: *x and y values which satisfy the equation are called the roots of equation. The equations in that form have infinity roots and define a line in coordinate system.*

$a \neq 0,\ b \neq 0,\ a, b, c, d, e, f \in R \Rightarrow$

$ax + by = c$

$dx + ey = f$

System in a form as shown above are called system of first order equations with two unknowns.

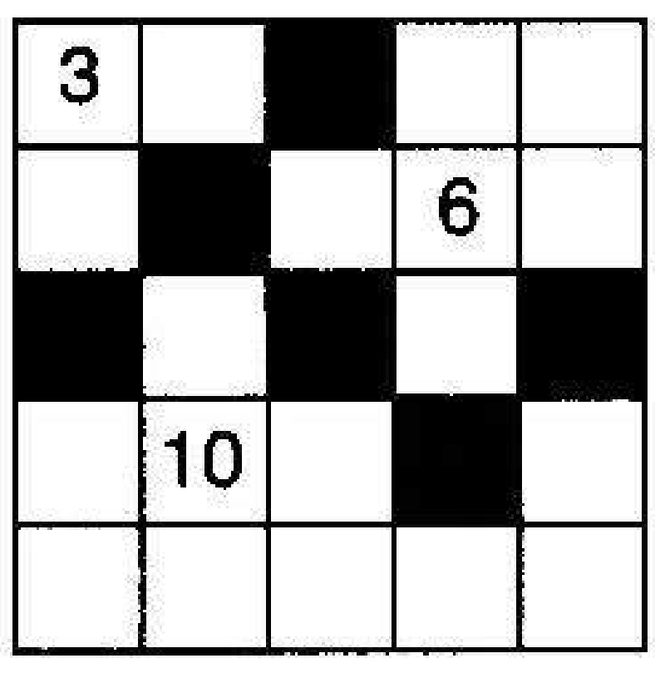
Bu denklem sisteminde,

In this equation system,

1. $\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$ ilişkisi varsa, çözüm kümesi sonsuz elemanlıdır.

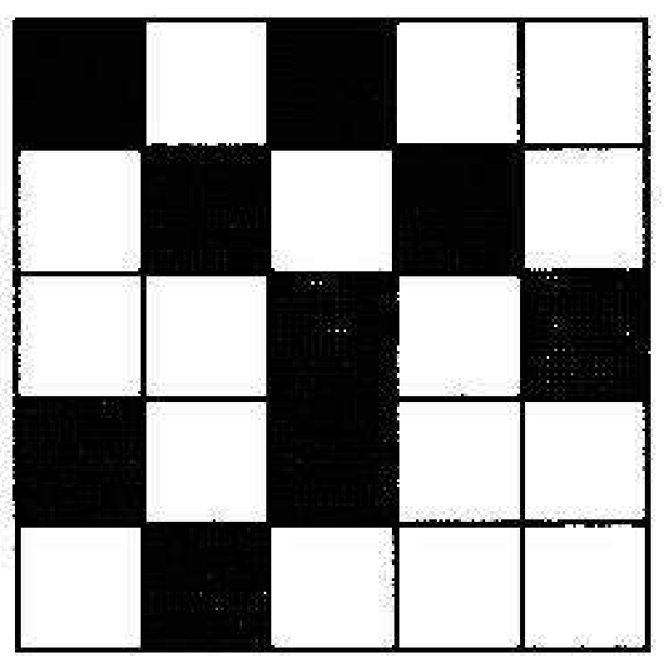
   If $\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$, *then the solution set has infinite elements.*

Düşünün / Think about it



= 3·6·10

⇒



= ?

Yanıt / Answer

72

ÖRNEK / EXAMPLE 6

$ax + 3y = 5,$

$3x + by = 15,$

ÇK(SS) = R ⇒ **a + b = ?**

A) 8 B) 10 C) 12 D) 14 E) 16

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$ax + 3y = 5$

$3x + by = 15$

$$ÇK(SS) = R \Rightarrow \frac{a}{3} = \frac{3}{b} = \frac{5}{15}$$

$$a = 1,\ b = 9$$

$$\Rightarrow a + b = 1 + 9 = 10$$

2. $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$ orantısı varsa, çözüm kümesi boş kümedir.

If $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$*, then the solution set is an empty set.*

ÖRNEK / EXAMPLE 7

$3x + 2y = 4$

$mx + (m - 1)y = 5,$

ÇK(SS) = ∅ ⇒ **m = ?**

A) –1 B) 1 C) 2 D) 3 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$3x + 2y = 4$

$mx + (m - 1)y = 5,$

$$ÇK(SS) = \varnothing \Rightarrow \frac{3}{m} = \frac{2}{m-1} \neq \frac{4}{5}$$

$$3m - 3 = 2m$$

$$m = 3$$

3. $\frac{a}{d} \neq \frac{b}{e}$ ise çözüm kümesi bir elemanlıdır.

If $\frac{a}{d} \neq \frac{b}{e}$, *then the solution set has one element.*

ÖRNEK / EXAMPLE 8

$$\left.\begin{array}{l} 2x-3y=-8 \\ x+y=1 \end{array}\right\} \Rightarrow (x,y)=?$$

A) (3, –1) B) (–1, 2) C) (1, –1) D) (2, –3) E) (1, 3)

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left.\begin{array}{r} 2x-3y=-8 \\ x+y=1 \end{array}\right\} \Rightarrow \begin{array}{r} 2x-3y=-8 \\ 3x+3y=3 \\ \hline 5x=-5 \\ x=-1 \end{array}$$

$$x=-1 \Rightarrow -1+y=1$$
$$y=2$$
$$(x,y)=(-1,2)$$

PEKİŞTİRME / REINFORCE

$x+2y=5,\ 2x+3y=9 \Rightarrow x \cdot y = ?$

YANIT / ANSWER : 3

BİRİNCİ DERECEDEN İKİ BİLİNMEYENLİ DENKLEM SİSTEMLERİNİN ÇÖZÜMÜ
SOLUTION of SYSTEMS of FIRST ORDER EQUATIONS with TWO UNKNOWNS

A. Yok Etme Yöntemi / The Elimination Method

Bu yöntemde değişkenlerden birini, diğer denklemde yer alan aynı değişkenin aynı büyüklükte kat sayılı, fakat zıt işaretli değeri ile çarpılarak değişkenlerden biri yok edilir.

In the method of elimination, we eliminate (remove) one of the variables by making the coefficients of x (or y) the same size but opposite in sign and then adding the equations.

B. Yerine Koyma Yöntemi / The Substitution Method

Bu yöntemde değişkenlerden biri, diğer değişken cinsinden yazılır. Seçilen değişken diğer denklemde yazılarak bir bilinmeyenli denklem haline getirilir.

The key to this method of solving a system of equations is the elimination of one of the variables. In this method the variable selected is eliminated by substitution.

Düşünün / Think about it

	x, y	y, z	x, z
+	16		
–	4		
x			40

$\Rightarrow z = ?$

Yanıt / Answer

4

ÖRNEK / EXAMPLE 9

$$\left.\begin{aligned} 3x - y &= -7 \\ 2x + 3y &= 10 \end{aligned}\right\} \Rightarrow \text{ÇK(SS)} = ?$$

A) $\{(-1, 3)\}$ B) $\{(2, 3)\}$ C) $\{(-1, 4)\}$ D) $\{(-1, 5)\}$ E) $\{(2, 4)\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{aligned} 3x - y = -7 &\Rightarrow 3x + 7 = y \\ 2x + 3y = 10 &\Rightarrow 2x + 3(3x + 7) = 10 \\ &\qquad 11x + 21 = 10 \\ &\qquad x = -1 \\ x = -1 &\Rightarrow y = 3\cdot(-1) + 7 \\ &\qquad y = 4 \end{aligned}$$

$\Rightarrow$ ÇK(SS) $= \{(-1, 4)\}$

ÖRNEK / EXAMPLE 10

$$\left.\begin{aligned} a - b + 2 &= 0 \\ 2a + b + 7 &= 0 \\ ax + b &= 2 \end{aligned}\right\} \Rightarrow x = ?$$

A) –1 B) 1 C) 2 D) 3 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\begin{array}{r} a - b + 2 = 0 \\ +\ \ 2a + b + 7 = 0 \\ \hline 3a + 9 = 0 \end{array} \Rightarrow a = -3$$

$$\begin{aligned} a = -3 &\Rightarrow 2a + b + 7 = 0 \\ &\qquad -6 + b + 7 = 0 \\ &\qquad b = -1 \end{aligned}$$

$$\left.\begin{aligned} a &= -3 \\ b &= -1 \end{aligned}\right\} \Rightarrow \begin{aligned} ax + b &= 2 \\ -3x - 1 &= 2 \\ x &= -1 \end{aligned}$$

ÖRNEK / EXAMPLE 11

YILDIZ TEKNİK ÜNV. YÖS 2013

$$\left.\begin{aligned} \frac{1}{x}-3=y \\ \frac{1}{y}-5=x \end{aligned}\right\} \Rightarrow \frac{x+y}{x-y}=?$$

A) 1 B) $\frac{1}{8}$ C) $\frac{3}{5}$ D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{1}{x}-3=y \Rightarrow \frac{1-xy}{x}=3$$

$$\frac{1}{y}-5=x \Rightarrow \frac{1-xy}{y}=5$$

$$\frac{\dfrac{1-xy}{x}}{\dfrac{1-xy}{y}}=\frac{y}{x}=\frac{3}{5} \Rightarrow \frac{x+y}{x-y}=\frac{8}{2}=4$$

ÖRNEK / EXAMPLE 12

MARMARA ÜNV. YÖS 2013

$x + y = 9$

$y + z = 12$

$x + z = 17 \Rightarrow \mathbf{x = ?}$

A) –4 B) 7 C) 10 D) 12 E) 13

PEKİŞTİRME / REINFORCE

$0{,}02x + 15 = 0{,}01x + 19 \Rightarrow x = ?$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{array}{rl} & x + y = 9 \\ - / & y + z = 12 \\ + & x + z = 17 \\ \hline & 2x = 14 \\ & x = 7 \end{array}$$

YANIT / ANSWER : 400

ÖRNEK / EXAMPLE 13

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$$\left.\begin{aligned}\frac{3}{x}+\frac{2}{y}&=2\\ \frac{1}{x}-\frac{1}{y}&=4\end{aligned}\right. \Rightarrow x=?$$

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{1}{2}$ E) $-\frac{1}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\left.\begin{aligned}\frac{3}{x}+\frac{2}{y}&=2\\ \frac{2}{x}-\frac{2}{y}&=8\end{aligned}\right\} \Rightarrow \frac{5}{x}=10 \Rightarrow x=\frac{1}{2}$$

ÖRNEK / EXAMPLE 14

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$$\frac{3}{1-\frac{2}{a-1}}=1 \qquad \Rightarrow a=?$$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$1-\frac{2}{a-1}=3$$

$$\frac{2}{a-1}=-2$$

$$a-1=-1$$

$$a=0$$

1. $4(3x-2)+5(2-3x)=x-6 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3
D) 4 E) 5

2. $3(x-2)+5=3x+7 \Rightarrow$ **ÇK(SS) = ?**

A) {1} B) {3} C) {5}
D) ∅ E) R

3. $\dfrac{2x+1}{x-2}+\dfrac{x-3}{2-x}=2 \Rightarrow$ **x = ?**

A) 0 B) 2 C) 4
D) 6 E) 8

4. $\dfrac{5}{2x}-\dfrac{4}{3x}+\dfrac{3}{x}=\dfrac{5}{12} \Rightarrow$ **x = ?**

A) –10 B) –5 C) 0
D) 5 E) 10

5. $2-\dfrac{x}{3+\dfrac{x-1}{2}}=\dfrac{5}{3} \Rightarrow$ **x = ?**

A) –7 B) –5 C) –2
D) 1 E) 3

6. a, b ∈ R,

$\left.\begin{aligned} ax-by&=-11\\ bx+ay&=10 \end{aligned}\right\}$, $ÇK(SS)=\{(-1,4)\} \Rightarrow$ **a + b = ?**

A) –3 B) –1 C) 3
D) 5 E) 7

7. $\left.\begin{array}{l} x+y=9 \\ y+z=7 \\ x+z=4 \end{array}\right\} \Rightarrow$ **$x=?$**

A) 1 B) 2 C) 3 D) 5 E) 6

8. $(2x+y+4)^2+(3x-y+1)^2=0 \Rightarrow$ **$x+y=?$**

A) –6 B) –5 C) –4 D) –3 E) –2

9. $\left.\dfrac{x^2-25}{x+5}=x-5\right\} \Rightarrow$ **ÇK(SS) = ?**

A) ∅ B) {–1, 1} C) R D) {–5, 5} E) R – {–5}

10. $\left.\begin{array}{l} 2a+b-c=5 \\ -a+b+5c=2 \end{array}\right\} \Rightarrow$ **$a+b+c=?$**

A) 2 B) 4 C) 5 D) 6 E) 8

11. $\left.\begin{array}{l} \dfrac{4}{x}-\dfrac{2}{y}=-\dfrac{1}{5} \\ \dfrac{3}{x}+\dfrac{1}{y}=\dfrac{11}{10} \end{array}\right\} \Rightarrow$ **$x+y=?$**

A) 1 B) 3 C) 5 D) 7 E) 9

12. $a, b, c \in R^+$

$\left.\begin{array}{l} a\cdot b=27 \\ b\cdot c=6 \\ a\cdot c=8 \end{array}\right\} \Rightarrow$ **$c=?$**

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{4}{3}$ D) $\frac{5}{3}$ E) $\frac{7}{3}$

YANITLAR / ANSWERS

1. B	**2. D**	**3. E**	**4. E**	**5. D**	**6. D**	**7. C**	**8. D**
9. E	**10. B**	**11. D**	**12. C**				

1.

$$\begin{aligned} 4(3x-2)+5(2-3x) &= x-6 \\ 12x-8+10-15x &= x-6 \\ -3x+2 &= x-6 \\ 8 &= 4x \\ x &= 2 \end{aligned}$$

Yanıt / The Answer is B

2.

$$\begin{aligned} 3(x-2)+5 &= 3x+7 \\ 3x-6+5 &= 3x+7 \\ 3x-1 &= 3x+7 \\ 0 &\neq 8 \end{aligned}$$

$$\Rightarrow ÇK(SS)=\varnothing$$

Yanıt / The Answer is D

3.

$$\begin{aligned} \frac{2x+1}{x-2}+\frac{x-3}{2-x} &= 2 \\ \frac{2x+1}{x-2}-\frac{x-3}{x-2} &= 2 \\ \frac{2x+1-x+3}{x-2} &= 2 \\ x+4 &= 2x-4 \\ x &= 8 \end{aligned}$$

Yanıt / The Answer is E

4.

$$\begin{aligned} \frac{5}{2x}-\frac{4}{3x}+\frac{3}{x} &= \frac{5}{12} \\ \frac{15-8+18}{6x} &= \frac{5}{12} \\ \frac{25}{6x} &= \frac{5}{12} \\ x &= 10 \end{aligned}$$

Yanıt / The Answer is E

5.

$$\begin{aligned} 2-\frac{x}{3+\frac{x-1}{2}} &= \frac{5}{3} \\ 2-\frac{x}{\frac{x+5}{2}} &= \frac{5}{3} \\ 2-\frac{2x}{x+5} &= \frac{5}{3} \\ 2-\frac{5}{3} &= \frac{2x}{x+5} \\ \frac{1}{3} &= \frac{2x}{x+5} \\ x+5 &= 6x \\ 5x &= 5 \\ x &= 1 \end{aligned}$$

Yanıt / The Answer is D

6.

$$\left.\begin{aligned} ax-by &= -11 \\ bx+ay &= 10 \\ ÇK(SS) &= \{(-1,4)\} \end{aligned}\right\} \Rightarrow \begin{aligned} -a-4b &= -11 \\ -b+4a &= 10 \end{aligned}$$

$$\left.\begin{aligned} -a-4b &= -11 \\ -b+4a &= 10 \end{aligned}\right\} \Rightarrow \begin{aligned} -17b &= -34 \\ b &= 2 \end{aligned}$$

$$b=2 \Rightarrow -a-4\cdot 2=-11 \Rightarrow a=3$$

$$\Rightarrow a+b=5$$

Yanıt / The Answer is D

7.

$$\left.\begin{array}{l} x+y=9\\ y+z=7\\ x+z=4 \end{array}\right\} \Rightarrow \begin{array}{r} x+y=9\\ -y-z=-7\\ +\quad x+z=4\\ \hline 2x=6 \end{array}$$

$$\frac{2x}{2}=\frac{6}{2}$$

$$x=3$$

Yanıt / The Answer is C

8. $(2x+y+4)^2+(3x-y+1)^2=0$

$$\begin{array}{r} 2x+y+4=0\\ +\quad 3x-y+1=0\\ \hline 5x=-5\\ x=-1 \end{array}$$

$\Rightarrow x+y=-3$

Yanıt / The Answer is D

9. $\dfrac{x^2-25}{x+5}=x-5$

$x^2-25=(x-5)\cdot(x+5)$

$(x-5)\cdot(x+5)=(x-5)\cdot(x+5)$

$1=1 \Rightarrow ÇK(SS)=R-\{-5\}$

Yanıt / The Answer is E

10. $2a+b-c=5 \Rightarrow 4a+2b-2c=10$

$$\begin{array}{r} -a+b+5c=2\\ +\quad 4a+2b-2c=10\\ \hline 3a+3b+3c=12\\ a+b+c=4 \end{array}$$

Yanıt / The Answer is B

11. $\dfrac{3}{x}+\dfrac{1}{y}=\dfrac{11}{10} \Rightarrow \dfrac{6}{x}+\dfrac{2}{y}=\dfrac{11}{5}$

$$\left.\begin{array}{l} \dfrac{4}{x}-\dfrac{2}{y}=-\dfrac{1}{5}\\ \dfrac{6}{x}+\dfrac{2}{y}=\dfrac{11}{5} \end{array}\right\} \Rightarrow \frac{10}{x}=2 \Rightarrow x=5$$

$x=5 \Rightarrow \dfrac{4}{5}-\dfrac{2}{y}=-\dfrac{1}{5} \Rightarrow y=2$

$\Rightarrow x+y=5+2=7$

Yanıt / The Answer is D

12. $a, b, c \in R^+$

$$\left.\begin{array}{l} a\cdot b=27\\ b\cdot c=6\\ a\cdot c=8 \end{array}\right\} \Rightarrow \begin{array}{c} a^2\cdot b^2\cdot c^2=9^2\cdot 4^2\\ abc=36 \end{array}$$

$a\cdot b=27 \Rightarrow c=\dfrac{36}{27}=\dfrac{4}{3}$

Yanıt / The Answer is C

1. $\frac{x-2}{4} - 3 = \frac{1}{4} \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **15**

2. $\left.\begin{array}{l} 4x - y + z = 10 \\ 2x - 3y - z = 18 \end{array}\right\} \Rightarrow \mathbf{x + y + z = ?}$

Yanıt / The Answer is: **–4**

3. $\left.\begin{array}{l} x + y = 12 \\ y + z = 9 \\ x + z = 7 \end{array}\right\} \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **5**

4. $\left.\begin{array}{l} 3a - b + 4c = 12 \\ 6a - 3b + 8c = 4 \end{array}\right\} \Rightarrow \mathbf{b = ?}$

Yanıt / The Answer is: **20**

5. $2x + 1 - [x - (4x - 2)] + 3 = 17 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **3**

6. $x - [x - (3x + 2)] = 3x + 9 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: $\varnothing$

7. $$\left.\begin{array}{l}\dfrac{15}{x+y}-\dfrac{6}{x-y}=9\\ \dfrac{5}{x+y}+\dfrac{4}{x-y}=-3\end{array}\right\} \Rightarrow \mathbf{x=?}$$

Yanıt / The Answer is: **2**

8. $$\left.\begin{array}{l}3x-y+z=12\\ x+3y+2z=4\end{array}\right\} \Rightarrow \mathbf{x+y+z=?}$$

Yanıt / The Answer is: **4**

9. $$\left.\begin{array}{l}x-y+z=3\\ 2x+y+z=5\\ 6x+3y+2z=10\end{array}\right\} \Rightarrow \mathbf{y=?}$$

Yanıt / The Answer is: $\mathbf{\frac{4}{3}}$

10. $x^2-[(x-2)x-2(3-x)]=5 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: $\varnothing$

11. $$\left.\begin{array}{l}\dfrac{2}{x-y}+\dfrac{3}{x+y}=2\\ \dfrac{1}{x-y}+\dfrac{2}{x+y}=\dfrac{7}{6}\end{array}\right\} \Rightarrow \mathbf{x^2-y^2=?}$$

Yanıt / The Answer is: **6**

12. $$\left.\begin{array}{l}\dfrac{1}{x}+\dfrac{1}{y}=5\\ \dfrac{1}{x}+\dfrac{1}{z}=3\\ \dfrac{1}{y}+\dfrac{1}{z}=4\end{array}\right\} \Rightarrow \mathbf{x=?}$$

Yanıt / The Answer is: $\mathbf{\frac{1}{2}}$

1. $5(3x-1)-3(2x+5)=7 \Rightarrow \mathbf{x=?}$

A) 1

B) 2

C) 3

D) 4

E) 5

2. $x-[4-3(x-2)]=10 \Rightarrow \mathbf{x=?}$

A) 3

B) 5

C) 7

D) 9

E) 11

3. $\frac{3x}{2}-20=\frac{2x}{3}-5 \Rightarrow \mathbf{x=?}$

A) 17

B) 18

C) 19

D) 20

E) 21

4. $\frac{2x-1}{3}-\frac{4x+1}{2}=-\frac{45}{6} \Rightarrow \mathbf{x=?}$

A) 2

B) 5

C) 8

D) 11

E) 14

5. $\frac{2}{3x}+\frac{1}{x}-\frac{3}{4x}=\frac{11}{24} \Rightarrow \mathbf{x=?}$

A) –3

B) –2

C) –1

D) 1

E) 2

6. $1+\frac{x}{2+\frac{x-1}{3}}=\frac{17}{8} \Rightarrow \mathbf{x=?}$

A) 3

B) 4

C) 5

D) 6

E) 7

7. $1+\dfrac{6}{1+\dfrac{4}{2+\dfrac{x-1}{5}}}=4 \Rightarrow$ **$x=?$**

A) 4

B) 5

C) 7

D) 8

E) 11

8. $1+\dfrac{3}{2x+1}=2-\dfrac{x}{2x+1} \Rightarrow$ **$x=?$**

A) –4

B) –2

C) 2

D) 4

E) 6

9. $\dfrac{x}{a}+1=\dfrac{x-b}{2} \Rightarrow$ **$x=?$**

A) $\dfrac{a(b+2)}{a-2}$

B) $\dfrac{b(a-2)}{a+2}$

C) $\dfrac{b(a+2)}{a-2}$

D) $\dfrac{ab}{b+2}$

E) $\dfrac{ab}{b-2}$

10. $a \in R$,
$3(x-2)+4(x+a)=28$,
ÇK(SS) = {2} $\Rightarrow$ **$a=?$**

A) –4

B) –1

C) 3

D) 5

E) 6

11. $(a-2)x+(b+3)x=0$,
ÇK(SS) = R $\Rightarrow$ **$a \cdot b=?$**

A) –6

B) –3

C) 0

D) 3

E) 6

12. $(2a+b-1)x+(3a-b-4)y=0$,
ÇK(SS) = R $\Rightarrow$ **(a, b) = ?**

A) (1, –1)

B) (2, 0)

C) (–2, 1)

D) (2, 1)

E) (–1, –3)

YANITLAR / ANSWERS

1. C **2. B** **3. B** **4. B** **5. E** **6. A** **7. E** **8. C**
9. A **10. D** **11. A** **12. A**

1. $x + 2 - 3\,[4x - (2 - 3x)] = 6\,(4 - 3x) \;\Rightarrow\; \mathbf{x = ?}$

A) –8

B) –6

C) –4

D) –2

E) 2

2. $\left.\begin{array}{l} x - y - z = 4 \\ x + y - z = 8 \\ x + 2y + z = 13 \end{array}\right\} \Rightarrow \mathbf{x + y + z = ?}$

A) 5

B) 7

C) 9

D) 11

E) 13

3. $\dfrac{3x - 1}{2x} + \dfrac{4x + 3}{3x} = 3 \;\Rightarrow\; \mathbf{x = ?}$

A) 1

B) 2

C) 3

D) 4

E) 5

4. $\dfrac{x}{3} + \dfrac{y}{4} = 2x - y = 2 \;\Rightarrow\; \mathbf{x \cdot y = ?}$

A) 6

B) 8

C) 9

D) 12

E) 16

5. $a, b \in \mathbb{R}$,

$\left.\begin{array}{l} ax - by = 8 \\ bx + ay = -2 \end{array}\right\},\; \text{ÇK(SS)} = \{\,(1, -1)\,\} \;\Rightarrow\; \mathbf{a + 2b = ?}$

A) 7

B) 11

C) 13

D) 15

E) 17

6. $\left.\begin{array}{l} \dfrac{2}{x} + \dfrac{4}{y} = -\dfrac{1}{3} \\ \dfrac{5}{x} + \dfrac{1}{y} = \dfrac{13}{6} \end{array}\right\} \Rightarrow \mathbf{x \cdot y = ?}$

A) –6

B) –4

C) –3

D) 2

E) 4

7. $\left.\begin{array}{l} \dfrac{3}{x}+\dfrac{1}{y}=\dfrac{14}{15} \\ \dfrac{2}{x}-\dfrac{3}{y}=-\dfrac{3}{5} \end{array}\right\} \Rightarrow \mathbf{x+y=?}$

A) 7

B) 8

C) 9

D) 10

E) 11

8. $\left.\begin{array}{l} \dfrac{a}{a-b}=x+1 \\ \dfrac{3a}{a-b}=2x-3 \end{array}\right\} \Rightarrow \mathbf{x=?}$

A) –6

B) –3

C) 1

D) 3

E) 6

9. $x, y \in Z$,

$\dfrac{1}{x+5}+\dfrac{1}{y-4}=1 \quad \Rightarrow \quad \mathbf{x+y=?}$

A) –2

B) –1

C) 0

D) 1

E) 3

10. $\left.\begin{array}{l} ax+by=4 \\ 3x-2y=1 \end{array}\right\}, \text{ ÇK(SS)} = R \quad \Rightarrow \quad \mathbf{a+b=?}$

A) 4

B) 8

C) 12

D) 16

E) 20

11. $\left.\begin{array}{l} a-4b+5c=8 \\ 2a+b-3c=10 \\ 4a-3b+2c=14 \end{array}\right\} \Rightarrow \mathbf{a\ =\ ?}$

A) –2

B) –3

C) –4

D) –7

E) –10

12. $\left.\begin{array}{l} \dfrac{5}{x}-y=3 \\ x-\dfrac{5}{y}=4 \end{array}\right\} \Rightarrow \mathbf{\dfrac{x}{y}=?}$

A) $\dfrac{4}{3}$

B) 1

C) $\dfrac{3}{4}$

D) $-\dfrac{3}{4}$

E) $-\dfrac{4}{3}$

YANITLAR / ANSWERS

1. A 2. D 3. C 4. D 5. B 6. A 7. B 8. A
9. E 10. A 11. C 12. E

1. $$\left.\begin{array}{l} 3x-2y+z=5 \\ x-y+3z=9 \\ -4x+5y-6z=13 \end{array}\right\} \Rightarrow \mathbf{x+y+z=?}$$

A) 36
B) 38
C) 41
D) 43
E) 44

2. $$\left.\begin{array}{l} x-y+4z=8 \\ 2x-y+2z=9 \\ 2x-2y-z=5 \end{array}\right\} \Rightarrow \mathbf{x+y+z=?}$$

A) 3
B) 5
C) 6
D) 7
E) 8

3. $$\frac{2x+1}{3}+\frac{4x+2}{4}=\frac{8x+1}{6} \quad \Rightarrow \quad \mathbf{x=?}$$

A) –6
B) –4
C) –2
D) 2
E) 4

4. $$\frac{3x-1}{4x}-\frac{x+2}{6x}=\frac{1}{2} \quad \Rightarrow \quad \mathbf{x=?}$$

A) 3
B) 4
C) 5
D) 6
E) 7

5. $$\frac{2x-1}{x-2}+\frac{6}{x}=\frac{3}{x-2}+\frac{x-4}{4-x} \quad \Rightarrow \quad \mathbf{x=?}$$

A) –9
B) –5
C) –3
D) –2
E) 1

6. $$\frac{3}{x+1}+\frac{1}{x-1}=\frac{x}{x-1} \quad \Rightarrow \quad \mathbf{x=?}$$

A) –3
B) 2
C) 5
D) 9
E) 12

7. $\left.\begin{array}{l} x+3y=7 \\ x-z=2 \\ 3y-z=-3 \end{array}\right\} \Rightarrow$ **x = ?**

A) 2

B) 4

C) 6

D) 8

E) 10

8. $\left.\begin{array}{l} \dfrac{2}{x}+\dfrac{5}{y}=\dfrac{7}{12} \\ \dfrac{3}{x}-\dfrac{2}{y}=-\dfrac{3}{2} \end{array}\right] \Rightarrow$ **x+y = ?**

A) –1

B) 0

C) 1

D) 2

E) 3

9. $a \in R$,

$\dfrac{3}{x-1}+\dfrac{a}{x+2}=\dfrac{a+2}{x+1}$, ÇK(SS) = {2} $\Rightarrow$ **a = ?**

A) 22 B) 24 C) 26

D) 28 E) 30

10. $2+\dfrac{2}{4-\dfrac{1}{x-1}}=3 \Rightarrow$ **x = ?**

A) $\frac{1}{2}$

B) $\frac{3}{2}$

C) $\frac{5}{2}$

D) $\frac{7}{2}$

E) $\frac{9}{2}$

11. $\left.\begin{array}{l} 3x+2y=0 \\ x-y=-1 \end{array}\right\} \Rightarrow$ **x+y = ?**

A) $\frac{1}{5}$

B) $\frac{2}{5}$

C) $\frac{4}{5}$

D) 1

E) $\frac{8}{5}$

12. $x=\dfrac{3y-1}{y-1} \Rightarrow$ **y = ?**

A) $\dfrac{x-1}{3x+2}$ B) $\dfrac{x+1}{3x-2}$ C) $\dfrac{x-1}{x-3}$

D) $\dfrac{x+1}{x-3}$ E) $\dfrac{x+2}{x-5}$

YANITLAR / ANSWERS

1. A 2. B 3. C 4. E 5. D 6. B 7. C 8. C

9. D 10. B 11. A 12. C

BİRİNCİ DERECEDEN EŞİTSİZLİKLER
FIRST ORDER INEQUALITIES

BÖLÜM
CHAPTER **2**

Tanım: $a \neq 0$, a ve b birer reel sayı olmak üzere,
$ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, $ax + b \leq 0$ biçimindeki ifadelere birinci dereceden eşitsizlikler denir. Eşitsizliği sağlayan sayıların oluşturduğu kümeye çözüm kümesi denir. Eşitsizliklerin çözüm kümeleri reel sayı aralıkları ile gösterilir.

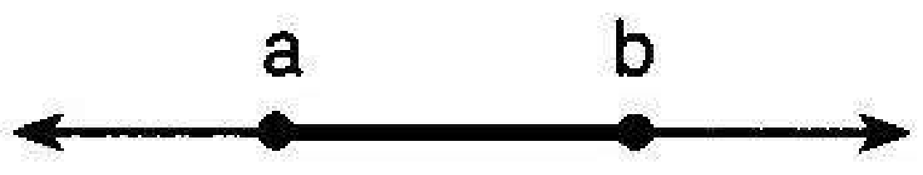
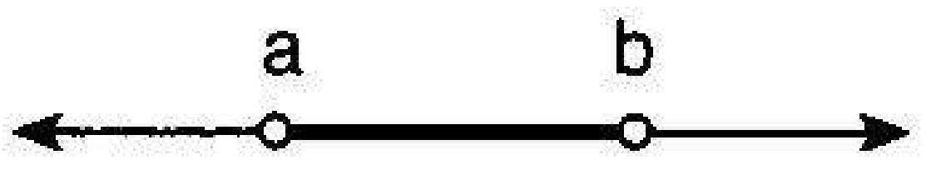
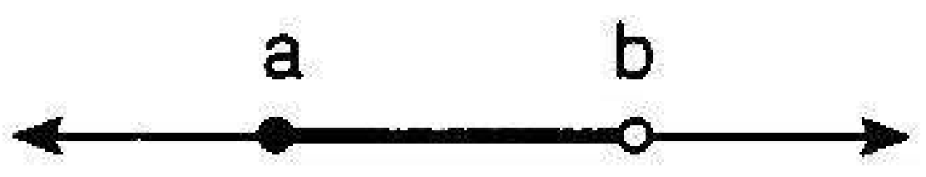
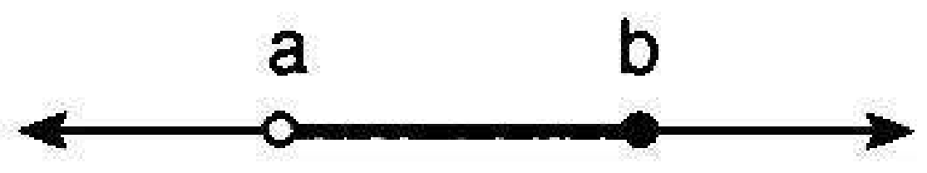
Definition: *Provided that $a \neq 0$, a and b are real numbers, the expressions in $ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, $ax + b \leq 0$ forms are called first order inequalities. The set of all real numbers that are solutions of an inequality is the solution set of the inequality. The solution set of inequalities is shown by intervals.*

Sembol / *Symbol*	Anlamı / *Meaning*
$=$	eşit / *is equal to*
$\neq$	eşit değil / *is not equal to*
$<$	küçük / *is less than*
$>$	büyük / *is greater than*
$\leq$	küçük veya eşit / *is less than or equal*
$\geq$	büyük veya eşit / *is greater than or equal*

Reel Sayı Doğrusu Üzerinde Sınırlı Aralıklar
Bounded Intervals on the Reel Number Line

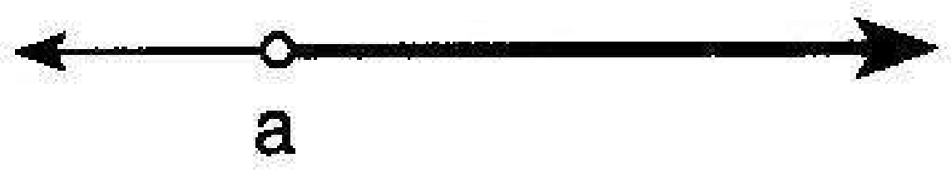
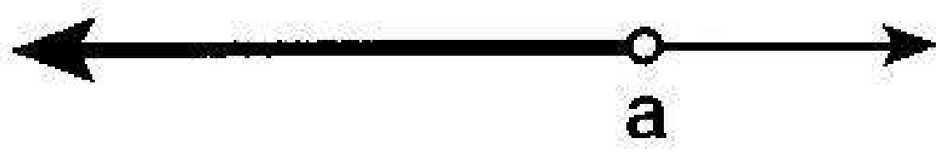
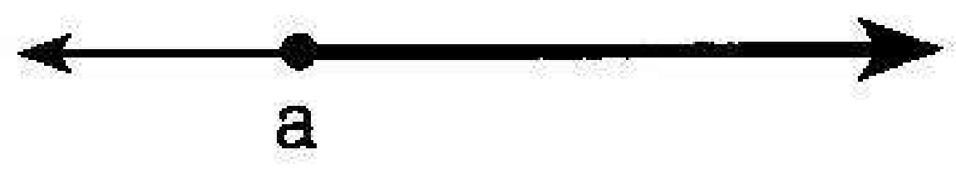
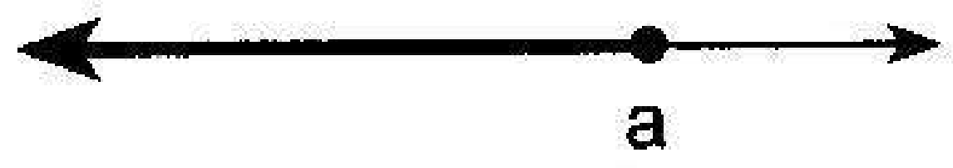
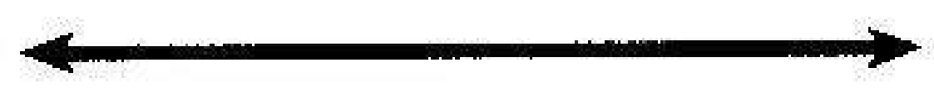
a < b olacak şekilde a ve b reel sayılarını alalım. Aşağıda verilen aralıklar, reel sayı doğrusu üzerindeki sınırlı aralıklardır. a ve b sayıları, her bir aralığın bitim noktasıdır.

Let a and b be real numbers such that a < b. The following intervals on the real number line are bounded. The numbers a and b are the endpoints of each interval.

Gösterim *Notation*	Aralık türü *Interval Type*	Eşitsizlik *Inequality*	Şekilsel Gösterim *Representation*
[a, b]	Kapalı / *Closed*	$a \leq x \leq b$	
(a, b)	Açık / *Open*	$a < x < b$	
[a, b)	yarı – açık / *half – open*	$a \leq x < b$	
(a, b]	yarı – açık / *half – open*	$a < x \leq b$	

Reel Sayı Doğrusu Üzerinde Sınırsız Aralıklar
Unbounded Intervals on the Reel Number Line

a reel sayısını ele alalım. Reel sayı doğrusu üzerinde aşağıdaki aralıklar sınırsızdır.
Let a be real number. The following intervals on the real number line are unbounded.

Gösterim *Notation*	Aralık türü *Interval Type*	Eşitsizlik *Inequality*	Şekilsel Gösterim *Representation*
(a, ∞)	Açık / *Open*	$a < x$	
$(-\infty, a)$	Açık / Open	$x < a$	
$[a, \infty)$	yarı – açık / *half – open*	$a \leq x$	
$(-\infty, a]$	yarı – açık / half – open	$x \leq a$	
$(-\infty, \infty)$	tüm reel eksen / *entire real line*		

EŞİTSİZLİKLERİN ÖZELLİKLERİ / PROPERTIES of INEQUALITIES

$a, b, c \in R$

1. **Toplama – Çıkarma Özelliği / *The Addition – Subtraction property***

   $a > b \Rightarrow a + c > b + c$

   $a < b \Rightarrow a + c < b + c$

2. **Çarpma – Bölme Özelliği / *The Multiplication – Division property***

   1. kural / *Rule 1*: $a > b$ ve (and) $c > 0 \Rightarrow a \cdot c > b \cdot c$

   $a < b$ ve (and) $c > 0 \Rightarrow a \cdot c < b \cdot c$

   2. kural / *Rule 2*: $a > b$ ve (and) $c < 0 \Rightarrow a \cdot c < b \cdot c$

   $a < b$ ve (and) $c < 0 \Rightarrow a \cdot c > b \cdot c$

3. $\left.\begin{array}{l} a < b \\ c < d \end{array}\right\} \Rightarrow a + c < b + d$

4. **Geçişme özelliği / *The Transitive property:*** $a < b$ ve(and) $b < c \Rightarrow a < c$

5. $a \cdot b > 0,\ a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

   $a \cdot b < 0,\ a < b \Rightarrow \frac{1}{a} < \frac{1}{b}$

6. $n \in Z^+,\ a < b \Rightarrow a^{2n+1} < b^{2n+1}$

7. $n > 1,\ n \in Z$

   $0 < a < 1 \Rightarrow 0 < a^n < a < 1$

8. $n \in Z^+$

   $0 < a < b \Rightarrow 0 < a^n < b^n$

ÖRNEK / EXAMPLE 1

$9 \leq \frac{2x-3}{7} < 13 \Rightarrow$ **ÇK(SS) = ?**

A) (25, 33]
B) (27, 42)
C) [31, 48)
D) [33, 47)
E) [35, 43]

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$9 \leq \frac{2x-3}{7} < 13$

$63 \leq 2x - 3 < 91$

$66 \leq 2x < 94$

$33 \leq x < 47 \Rightarrow$ ÇK(SS) = [33, 47)

ÖRNEK / EXAMPLE 2

$x \in Z$,

$4x - 5 < 5x + 2 < 2x + 3 \Rightarrow \sum x = ?$

A) –23
B) –22
C) –21
D) –20
E) –19

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$4x - 5 < 5x + 2 < 2x + 3$$

$4x - 5 < 5x + 2$

$-7 < x$

$5x + 2 < 2x + 3$

$3x < 1$

$x < \frac{1}{3}$

$\Rightarrow -7 < x < \frac{1}{3}$

$\Rightarrow \sum x = (-6) + (-5) + (-4) + \ldots + (-1) = -21$

PEKİŞTİRME / REINFORCE

$x \in Z$,

$3x - 1 < 2x + 4 < x - 2 \Rightarrow \max(x) = ?$

YANIT / ANSWER : –7

ÖRNEK / EXAMPLE 3

$y \in Z$,

$3 < x < 8$,

$2x + y = 10 \Rightarrow \sum y = ?$

A) 1 B) –3 C) –5 D) –7 E) –9

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$2x + y = 10 \Rightarrow y = 10 - 2x$$

$$3 < x < 8 \Rightarrow -6 > -2x > -16$$

$$4 > 10 - 2x > -6$$

$$4 > y > -6$$

$$y \in Z \Rightarrow \sum y = -9$$

ÖRNEK / EXAMPLE 4

$x \in Z$,

$$\left(\frac{16}{9}\right)^{2x-4} > \left(\frac{3}{4}\right)^{x-2} \Rightarrow \min(x) = ?$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\left(\frac{16}{9}\right)^{2x-4} > \left(\frac{3}{4}\right)^{x-2} \Rightarrow \left(\frac{4}{3}\right)^{4x-8} > \left(\frac{3}{4}\right)^{x-2}$$

$$\left(\frac{3}{4}\right)^{8-4x} > \left(\frac{3}{4}\right)^{x-2}$$

$$8 - 4x < x - 2$$

$$10 < 5x$$

$$2 < x \Rightarrow \min(x) = 3$$

ÖRNEK / EXAMPLE 5

$(x - 2y) \in Z$,

$3 < x < 7$, $-1 < y < 6 \Rightarrow$ **$\max(x - 2y) = ?$**

A) 1　　B) 3　　C) 4　　D) 7　　E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{array}{l} 3 < x < 7 \\ \underline{-12 < -2y < 2} \\ -9 < x - 2y < 9 \Rightarrow \max(x - 2y) = 8 \end{array}$$

ÖRNEK / EXAMPLE 6

$y \in Z$,

$x^2 < x$,

$y = 3x - 4 \Rightarrow$ **$\max(y) = ?$**

A) –4　　B) –3　　C) –2　　D) –1　　E) 0

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{array}{l} x^2 < x \Rightarrow 0 < x < 1 \\ 0 < 3x < 3 \\ -4 < 3x - 4 < -1 \\ -4 < y < -1 \\ \Rightarrow \max(y) = -2 \end{array}$$

ÖRNEK / EXAMPLE 7

YÖS / 2008

$a < b < 0$

$c = \frac{3a-b}{a}$

? < c < ?

A) $1 < c < 2$　B) $2 < c < 3$　C) $3 < c < 4$　D) $4 < c < 5$　E) $5 < c < 6$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$c = \frac{3a-b}{a} = \frac{3a}{a} - \frac{b}{a} = 3 - \frac{b}{a}$$

$$a < b < 0 \Rightarrow 0 < \frac{b}{a} < 1$$

$$-1 < -\frac{b}{a} < 0$$

$$-1+3 < 3 - \frac{b}{a} < 0+3$$

$$2 < c < 3$$

ÖRNEK / EXAMPLE 8

İSTANBUL ÜNV. YÖS 2013

$\frac{2}{5} < x < \frac{3}{4} \Rightarrow$ **x = ?**

A) $\frac{3}{10}$　B) $\frac{1}{3}$　C) $\frac{1}{2}$　D) $\frac{4}{5}$　E) $\frac{5}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\underset{(4)}{\frac{2}{5}} < x < \underset{(5)}{\frac{3}{4}} \Rightarrow \frac{8}{20} < x < \frac{15}{20}$$

$$x = \frac{10}{20} = \frac{1}{2}$$

1. $x \in Z,$

$$\frac{x}{3}-\frac{x+2}{2}<5 \Rightarrow \mathbf{min(x)=?}$$

A) –38 B) –37 C) –36

D) –35 E) –34

2. $x \in Z,$

$$2 \le \frac{4x+1}{3}<5 \Rightarrow \sum \mathbf{x}=\mathbf{?}$$

A) 5 B) 8 C) 12

D) 20 E) 22

3. $x \in Z,$

$$\frac{3-x}{2}-\frac{2+x}{3}<3 \Rightarrow \mathbf{min(x)=?}$$

A) –4 B) –3 C) –2

D) –1 E) 0

4. $x \in Z,$

$$2x-3<x+7 \le 3x+5 \Rightarrow \sum \mathbf{x=?}$$

A) 45 B) 48 C) 50

D) 55 E) 57

5. $-x-1<3x+7 \le 2x+8 \Rightarrow$ **ÇK(SS) = ?**

A) (–1, 2) B) (–2, 1] C) (–2, 3]

D) (–3, 4] E) (–3, 5)

6. $y \in Z,$

$$\left.\begin{array}{l} -3 \le x<10 \\ 2x+5y=24 \end{array}\right\} \Rightarrow \sum \mathbf{y}=\mathbf{?}$$

A) 20 B) 21 C) 22

D) 23 E) 24

7. $\dfrac{x^2+1}{x-3}<0 \;\Rightarrow\;$ **ÇK(SS) = ?**

A) $(-\infty, 3)$ B) $(-\infty, 2)$ C) $(-\infty, 1)$

D) $(-\infty, 0)$ E) $(-\infty, -1)$

8. $x \in Z$,

$$\frac{3}{5}<\frac{2}{x}<\frac{6}{5} \;\Rightarrow\; \boldsymbol{\sum x = ?}$$

A) 4 B) 5 C) 6

D) 7 E) 8

9. $x, y, z \in Z$,

$$\left.\begin{array}{r} x+y>4 \\ x+z<5 \\ y+z>3 \end{array}\right\} \Rightarrow \mathbf{min(y) = ?}$$

A) 0 B) 1 C) 2

D) 3 E) 4

10. $x, y \in Z$,

$$\left.\begin{array}{r} -3<x<4 \\ 1<y<5 \end{array}\right\} \Rightarrow \mathbf{min(2x-y) = ?}$$

A) -10 B) -8 C) -6

D) -4 E) -2

11. $(2x-y) \in Z$,

$$\left.\begin{array}{r} -3<x<4 \\ 1<y<5 \end{array}\right\} \Rightarrow \mathbf{min(2x-y) = ?}$$

A) -10 B) -8 C) -6

D) -4 E) -2

12. $$\left.\begin{array}{r} 2x+3\le 1 \\ 5<3x+11 \end{array}\right\} \Rightarrow \mathbf{ÇK(SS) = ?}$$

A) $[-3, -2]$ B) $(-2, -1]$ C) $[0, 1)$

D) $(1, 2)$ E) $(2, 3]$

YANITLAR / ANSWERS

1. D	**2. A**	**3. C**	**4. A**	**5. B**	**6. B**	**7. A**	**8. B**
9. C	**10. B**	**11. A**	**12. B**				

1. $\frac{x}{3}-\frac{x+2}{2}<5$

$2x-3x-6<30$

$-x<36$

$x>-36$

$\Rightarrow \min(x)=-35$

Yanıt / The Answer is D

2. $2\le\frac{4x+1}{3}<5$

$6\le 4x+1<15$

$5\le 4x<14$

$\frac{5}{4}\le x<\frac{14}{4}$

$x\in Z\ \Rightarrow \sum x=2+3=5$

Yanıt / The Answer is A

3. $\frac{3-x}{2}-\frac{2+x}{3}<3$

$9-3x-4-2x<18$

$5-5x<18$

$5x>-13$

$x>-\frac{13}{5}$

$x\in Z\ \Rightarrow \min(x)=-2$

Yanıt / The Answer is C

4.

$$\begin{tikzcd}
 & 2x-3<x+7\le 3x+5 \arrow[dl] \arrow[dr] & \\
\begin{matrix}2x-3<x+7\\ x<10\end{matrix} \arrow[dr] & & \begin{matrix}x+7\le 3x+5\\ 2\le 2x\end{matrix} \arrow[dl] \\
 & 1\le x<10 &
\end{tikzcd}$$

$\sum x=1+2+\ldots+9=45$

Yanıt / The Answer is A

5.

$$\begin{tikzcd}
 & -x-1<3x+7\le 2x+8 \arrow[dl] \arrow[dr] & \\
\begin{matrix}-x-1<3x+7\\ -8<4x\end{matrix} \arrow[dr] & & \begin{matrix}3x+7\le 2x+8\\ x\le 1\end{matrix} \arrow[dl] \\
 & -2<x\le 1 &
\end{tikzcd}$$

ÇK(SS) = $(-2, 1]$

Yanıt / The Answer is B

6. $2x+5y=24\ \Rightarrow\ y=\frac{24-2x}{5}$

$-3\le x<10 \Rightarrow 6\ge -2x>-20$

$30\ge 24-2x>4$

$6\ge\frac{24-2x}{5}>\frac{4}{5}$

$6\ge y>\frac{4}{5}$

$y\in Z\ \Rightarrow \sum y=1+2+\ldots+6=21$

Yanıt / The Answer is B

7. $0 \leq x^2 \Rightarrow 0 < x^2 < 1$

$$\left.\begin{array}{l} 0 < x^2 + 1 \\ \dfrac{x^2+1}{x-3} < 0 \end{array}\right\} \Rightarrow x - 3 < 0$$

$\Rightarrow$ ÇK(SS) $= (-\infty, 3)$

Yanıt / The Answer is A

---

8. $\dfrac{3}{5} < \dfrac{2}{x} < \dfrac{6}{5} \Rightarrow 3 < \dfrac{10}{x} < 6$

$\dfrac{1}{3} > \dfrac{x}{10} > \dfrac{1}{6} \Rightarrow 10 > 3x > 5$

$\dfrac{10}{3} > x > \dfrac{5}{3}$

$x \in Z \Rightarrow \sum x = 2 + 3 = 5$

Yanıt / The Answer is B

---

9. $x, y, z \in Z,$

$$\left.\begin{array}{l} x+y>4 \\ x+z<5 \\ y+z>3 \end{array}\right\} \Rightarrow \begin{array}{r} x+y>4 \\ -x-z>-5 \\ +\quad y+z>3 \\ \hline 2y>2 \\ y>1 \end{array}$$

$y \in Z,\ y > 1 \Rightarrow \min(y) = 2$

Yanıt / The Answer is C

---

10. $$\left.\begin{array}{l} x, y \in Z \\ \min(x) = -2 \\ \max(y) = 4 \end{array}\right\} \Rightarrow \min(2x - y) = -4 - 4 = -8$$

Yanıt / The Answer is B

---

11. $-3 < x < 4 \Rightarrow -6 < 2x < 8$

$1 < y < 5 \Rightarrow -5 < -y < -1$

$-11 < 2x - y < 7$

$(2x - y) \in Z \Rightarrow \min(2x - y) = -10$

Yanıt / The Answer is A

---

12. $2x + 3 \leq 1$

$2x \leq -2$

$x \leq -1$ .... **(1)**

$5 < 3x + 11$

$-6 < 3x$

$-2 < x$ .... **(2)**

1 ve (and) 2

$-2 < x \leq -1$

$\Rightarrow$ ÇK(SS) $= (-2, -1]$

Yanıt / The Answer is B

1. $x \in Z$,

$2x + 1 < x + 8 < 3x + 2 \Rightarrow \sum x = ?$

Yanıt / The Answer is: **15**

2. $x \in Z$,

$x - 5 \leq 2x + 6 < x + 7 \Rightarrow \sum x = ?$

Yanıt / The Answer is: **–66**

3. $x \in Z$,

$\frac{1}{2} - \frac{3}{x+2} \leq \frac{1}{4} \Rightarrow \sum x = ?$

Yanıt / The Answer is: **54**

4. $x < 0$,

$\frac{3x+8}{x} < 1 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: **(–4, 0)**

5. $\frac{3-x}{2} - \frac{x+4}{3} \leq 1 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: **[–1, ∞)**

6. $x \in Z$,

$\frac{x-5}{2} - \frac{2x+4}{4} < x - \frac{2x-1}{3} \Rightarrow$ **min(x) = ?**

Yanıt / The Answer is: **–11**

7. $x \in Z$,

$$\frac{2}{3}-\frac{x}{5}\geq\frac{x}{2}-\frac{4}{5} \Rightarrow \mathbf{max(x)=?}$$

Yanıt / The Answer is: **2**

8. $x \in Z$,

$$\frac{x-3}{5}-4>\frac{x}{6} \Rightarrow \mathbf{min(x)=?}$$

Yanıt / The Answer is: **139**

9. $x \in Z$,

$$\left.\begin{array}{r} -3<2y+1<4 \\ 3x-y=5 \end{array}\right\} \Rightarrow \mathbf{x=?}$$

Yanıt / The Answer is: **2**

10. $x \in Z$,

$$-5<\frac{4x-3}{7}<8 \Rightarrow \mathbf{min(x)+max(x)=?}$$

Yanıt / The Answer is: **7**

11. $x, y \in Z$,

$$\left.\begin{array}{r} -2<x\leq 3 \\ -6<y\leq 4 \end{array}\right\} \Rightarrow \mathbf{max(2x+y)=?}$$

Yanıt / The Answer is: **10**

12. $(x \cdot y) \in Z$,

$$\left.\begin{array}{r} -5<x<4 \\ -6<y<6 \end{array}\right\} \Rightarrow \mathbf{min(x \cdot y)\ +\ max(x \cdot y)=?}$$

Yanıt / The Answer is: **0**

1. $x \in Z^-$,

$3x - 1 < 4x + 8 \Rightarrow$ **max(x) + min(x) = ?**

A) –9 B) –8 C) –7

D) –6 E) –5

---

2. $x \in Z$,

$$\frac{4}{5} < \frac{x}{10} \leq \frac{6}{5} \Rightarrow \sum x = ?$$

A) 36 B) 38 C) 42

D) 44 E) 46

---

3. $x \in Z$,

$$\frac{2x-5}{4} - \frac{x+2}{8} < \frac{3x+1}{2} \Rightarrow \mathbf{min(x) = ?}$$

A) –2 B) –1 C) 0

D) 1 E) 2

---

4. $x \in Z^+$,

$$\frac{8}{2x+5} > \frac{1}{2} \Rightarrow \sum x = ?$$

A) 12 B) 13 C) 14

D) 15 E) 16

---

5. $x \in Z$,

$3x - 7 < x - 9 \leq 2x - 3 \Rightarrow \sum x = ?$

A) –22 B) –21 C) –20

D) –19 E) –18

---

6. $b \in Z$,

$$\left.\begin{aligned} a+b&<5 \\ b+c&<-2 \\ a+c&>3 \end{aligned}\right\} \Rightarrow \mathbf{max(b) = ?}$$

A) –3 B) –2 C) –1

D) 0 E) 1

7. $y \in Z$,

$-3 < x < 5,\ 2x - y + 4 = 0 \Rightarrow \sum y = ?$

A) 70 B) 75 C) 80

D) 85 E) 90

8. $a, b, c \in R^+$,

$$\left.\begin{array}{r} a \cdot b = 4 \\ b \cdot c = 2 \\ a \cdot c = 3 \end{array}\right\} \Rightarrow \ ? < ? < ?$$

A) $b < a < c$ B) $c < b < a$ C) $a < b < c$

D) $c < a < b$ E) $b < c < a$

9. $a, b \in Z$,

$$\left.\begin{array}{r} -3 < a \le 5 \\ 4 \le b < 8 \end{array}\right\} \Rightarrow \min(3b - 2a) = ?$$

A) –3 B) 2 C) 6

D) 10 E) 15

10. $(3x - 2y) \in Z$,

$$\left.\begin{array}{r} -2 < x \le 5 \\ -3 < y < 4 \end{array}\right\} \Rightarrow \max(3x - 2y) = ?$$

A) 19 B) 20 C) 21

D) 22 E) 23

11. $a, b \in R$, $(2a - 3b) \in Z$,

$$\left.\begin{array}{r} -1 \le a < 5 \\ 3 < b < 7 \end{array}\right\} \Rightarrow \sum (2a - 3b) = ?$$

A) –264 B) –260 C) –255

D) –253 E) –250

12. $a, b \in R$,

$a^2 + b^2 \in Z$

$$\left.\begin{array}{r} -3 < a \le 4 \\ 2 < b < 5 \end{array}\right\} \Rightarrow \max(a^2 + b^2) = ?$$

A) 30 B) 31 C) 34

D) 37 E) 40

YANITLAR / ANSWERS

1. A	**2. C**	**3. B**	**4. D**	**5. C**	**6. C**	**7. E**	**8. B**
9. B	**10. B**	**11. D**	**12. E**				

1. $x \in Z^+$,

$$\frac{2}{x+2} > \frac{4}{9} \Rightarrow \sum x = ?$$

A) 2 B) 3 C) 4 D) 5 E) 6

2. $(x + y + z) \in Z$,

$$\left.\begin{array}{l} 3x+2y>-10 \\ 2y+z>-5 \\ x+3z>-9 \end{array}\right\} \Rightarrow \min(x+y+z)=?$$

A) –8 B) –7 C) –6 D) –5 E) –4

3.

$$\left.\begin{array}{l} -3 \le x < 5 \\ 2 < y \le 4 \end{array}\right\} \Rightarrow 2x-y \in (?,?)$$

A) [–10, 8) B) [–8, 5) C) [–4, 4] D) (–2, 10) E) (–1, 5)

4. $y \in Z$,

$$\left.\begin{array}{l} -3 < x \le 4 \\ 3x-y=1 \end{array}\right\} \Rightarrow \sum y = ?$$

A) 17 B) 18 C) 19 D) 20 E) 21

5. $x, y \in Z$,

$$\left.\begin{array}{l} -4<x\le 5 \\ -3<y\le 8 \end{array}\right\} \Rightarrow \min(x \cdot y)+\max(x \cdot y)=?$$

A) 9 B) 11 C) 15 D) 16 E) 18

6. $x, y \in Z$,

$$\left.\begin{array}{l} -3<x<2 \\ 1<y<7 \end{array}\right\} \Rightarrow \max(2x-3y)=?$$

A) –8 B) –7 C) –6 D) –5 E) –4

7. $(x^2 + 5) \in Z$,

$-3 < x < 2 \Rightarrow \max(x^2 + 5) + \min(x^2 + 5) = ?$

A) 16 B) 17 C) 18 D) 19 E) 20

8. $(a^3 + b^2) \in Z$,

$$\left.\begin{array}{l} -2 < a < 3 \\ -3 < b < 1 \end{array}\right\} \Rightarrow \max(a^3 + b^2) = ?$$

A) 35 B) 38 C) 40 D) 42 E) 44

9. $x, y, z \in R$,

$$\left.\begin{array}{l} x^2 \cdot y^3 < 0 \\ z^3 \cdot x^4 > 0 \\ 5z = 4x \end{array}\right\} \Rightarrow ? < ? < ?$$

A) x < y < z B) y < z < x C) z < x < y D) y < x < z E) x < z < y

10.

$$\left.\begin{array}{l} b > 0 \\ a \cdot b < 0 \\ a^3 > a \end{array}\right\} \Rightarrow a \in (?,?)$$

A) (–4, –3) B) (–3, –2) C) (–2, ,1) D) (–1, 0) E) (0, 1)

11. $x > y > 5$

$$\left.\begin{array}{l} a = \dfrac{x}{y} \\ b = \dfrac{5}{y} \\ c = \dfrac{x}{5} \end{array}\right\} \Rightarrow ? < ? < ?$$

A) b < c < a B) a < b < c C) c < a < b D) c < b < a E) b < a < c

12. $x, y, z \in Z$,

$$\left.\begin{array}{l} 1 \le x < 6 \\ 1 \le y \le 2 \\ 1 < z \le 6 \end{array}\right\} \Rightarrow \min\left(\frac{10}{x} - \frac{2}{y} + z\right) = ?$$

A) 0 B) 1 C) 2 D) 3 E) 4

YANITLAR / ANSWERS

1. B	2. D	3. A	4. E	5. D	6. E	7. C	8. A
9. B	10. D	11. E	12. C				

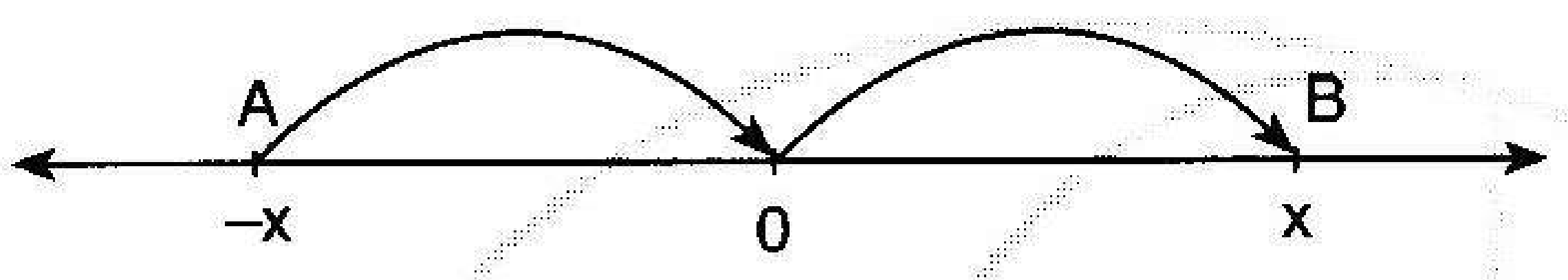
MUTLAK DEĞER
ABSOLUTE VALUE

BÖLÜM CHAPTER 3

Tanım: Sayı doğrusu üzerinde, x sayısının sıfıra olan uzaklığına, x sayısının *mutlak değeri* denir ve $|x|$ ile gösterilir.

Definition: *The absolute value of a number is its distance from zero on the number line. The absolute value of the number x is denoted by $|x|$.*

$$|OB| = |x|, \quad |OA| = |-x|$$



Buna göre,

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$

olarak tanımlanır. *According to that, $|x|$ is defined as below.*

Temel Özellikler / Basic Properties

A. Herhangi bir reel sayının mutlak değeri ya pozitiftir ya da sıfırdır.

The absolute value of any real number is either positive or zero.

Etkinlikler / Activities

a. $|4| = 4$

b. $|-5| = 5$

ÖRNEK / EXAMPLE 1

$2 < x < 5 \Rightarrow$ $|x - 5| + |x - 2| = ?$

A) 1 B) 3 C) 5 D) 7 E) 9

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$2 < x < 5 \Rightarrow x - 5 < 0$

$2 < x < 5 \Rightarrow 0 < x - 2$

$$\Rightarrow |x - 5| + |x - 2| = 5 - x + x - 2$$
$$= 3$$

PEKİŞTİRME / REINFORCE

$x < 0 \Rightarrow |x| + |-x + 3| = ?$

YANIT / ANSWER : $3 - 2x$

ÖRNEK / EXAMPLE 2

$a < b < c \Rightarrow |a - b| + |b - c| = ?$

A) $a - c$ B) $b - c$ C) $c - b$ D) $c - a$ E) $a - b$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$a < b < c \Rightarrow a - b < 0$

$b - c < 0$

$|a - b| + |b - c| = b - a + c - b$

$= c - a$

B. $a \in R^-$,

n tek bir doğal sayı ise $|a^n| \neq a^n$, n çift bir doğal sayı ise $|a^n| = a^n$ dir.

if n is odd number, then $|a^n| \neq a^n$, *if n is even number, then* $|a^n| = a^n$.

Etkinlikler / Activities

a. $|(x - 3)^3| \neq (x - 3)^3$

b. $|(x - 2)^2| = (x - 2)^2$

C. $a, b \in R^+$, $a > b$,

$|a - b| = |b - a| = a - b$

Etkinlikler / Activities

a. $|3 - 12| = 12 - 3 = 9$

b. $|12 - 15| = 15 - 12 = 3$

D. Bir sayının mutlak değeri daima sıfırdan büyük veya sıfıra eşittir. $|a|$ ifadesinin en küçük değeri sıfırdır.

Absolute value of a number is always greater or equal than zero. The smallest value of $|a|$ *is zero.*

$$|a| = 0 \Leftrightarrow a = 0$$

E. $a, b, c \in R$

$|a| + |b| + |c| = 0$

$\Rightarrow a = 0,\ b = 0,\ c = 0$

ÖRNEK / EXAMPLE 3

$|x - 2y - 7| + |x - y - 1| = 0 \Rightarrow \mathbf{x + y = ?}$

A) –14 B) –13 C) –12 D) –11 E) –10

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$|x - 2y - 7| + |x - y - 1| = 0$

$x - 2y - 7 = 0 \Rightarrow x - 2y = 7$

$x - y - 1 = 0 \Rightarrow x - y = 1$

$$\left.\begin{array}{r} x-2y=7 \\ x-y=1 \end{array}\right\} \Rightarrow x=-5,\ y=-6$$

$\Rightarrow x + y = (-5) + (-6) = -11$

PEKİŞTİRME / REINFORCE

$|a - b + 5| + |a + b - 3| = 0 \Rightarrow a \cdot b = ?$

YANIT / ANSWER : –4

▼ $\forall x \in \mathbf{R},\ \sqrt{x^2} = |x|,\ \sqrt{x^4} = x^2,\ \sqrt{x^8} = x^4 \ldots.$

ÖRNEK / EXAMPLE 4

$x < 0 < y \Rightarrow$

$\sqrt{9x^2} + \sqrt{4y^2} - |x - y| = ?$

A) $-2x - 3y$ B) $y - 2x$ C) $3y - 4x$ D) $2x + 3y$ E) $x - 2y$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$\sqrt{9x^2} + \sqrt{4y^2} - |x - y| = |3x| + |2y| - |x - y|$

$x < 0 < y$

$\Rightarrow |3x| + |2y| - |x - y| = -3x + 2y + x - y$

$= y - 2x$

Mutlak Değerli Denklemler / Equations of Absolute Value

Mutlak değer sembolü içeren denklemlere, **mutlak değer denklemleri** denir.

An equation that contains an absolute value symbol is called **an absolute value equation.**

1. $a > 0$,

$|f(x)| = a \Rightarrow \mathbf{f(x) = a}\ ,\ \mathbf{f(x) = -a}$

ÖRNEK / EXAMPLE 5

$|2x - 5| = 7 \Rightarrow$ **ÇK(SS) = ?**

A) {–1, 6} B) {–2, 5} C) {–1, 7} D) {0, 9} E) {–2, 3}

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$|2x - 5| = 7$$

$$2x - 5 = 7 \qquad 2x - 5 = -7$$

$$2x = 12 \qquad 2x = -2$$

$$x = 6 \qquad x = -1$$

$$\Rightarrow \text{ÇK(SS)} = \{-1, 6\}$$

PEKİŞTİRME / REINFORCE

$|x - 1| = -2 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : $\varnothing$

2. $|f(x)| = |g(x)| \Rightarrow \mathbf{f(x) = g(x)} \vee \mathbf{f(x) = -g(x)}$

ÖRNEK / EXAMPLE 6

$|2x + 5| = |x - 2| \Rightarrow \sum x = ?$

A) –12 B) –8 C) –6 D) –4 E) –2

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$|2x + 5| = |x - 2|$$

$$2x + 5 = x - 2 \qquad 2x + 5 = -x + 2$$

$$x = -7 \qquad 3x = -3$$

$$x = -1$$

$$\Rightarrow \sum x = -7 + (-1) = -8$$

3. $|f(x)| = g(x) \Rightarrow \begin{cases} f(x) = g(x), & g(x) \geq 0 \\ f(x) = -g(x), & g(x) < 0 \end{cases}$

ÖRNEK / EXAMPLE 7

$|2x - 1| - x = 4 \Rightarrow \sum x = ?$

A) −10 B) −8 C) −7 D) −1 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$|2x - 1| - x = 4$$

$$|2x - 1| = x + 4$$

$2x - 1 = x + 4$

$2x - x = 4 + 1$

$x = 5$

$2x - 1 = -x - 4$

$3x = -3$

$x = -1$

$$\Rightarrow \sum x = 5 - 1 = 4$$

PEKİŞTİRME / REINFORCE

$|x + 1| = 2x - 3 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : {4}

Mutlak Değerli Eşitsizlikler / Inequalities of Absolute Value

1. $a \in R^+$

$|x| < a \Leftrightarrow -a < x < a$

ÖRNEK / EXAMPLE 8

$|x - 3| < 4 \Rightarrow$ **ÇK(SS) = ?**

A) (−1, 7) B) [−1, 7) C) [−1, 7] D) (−1, 7] E) (−∞, 7)

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

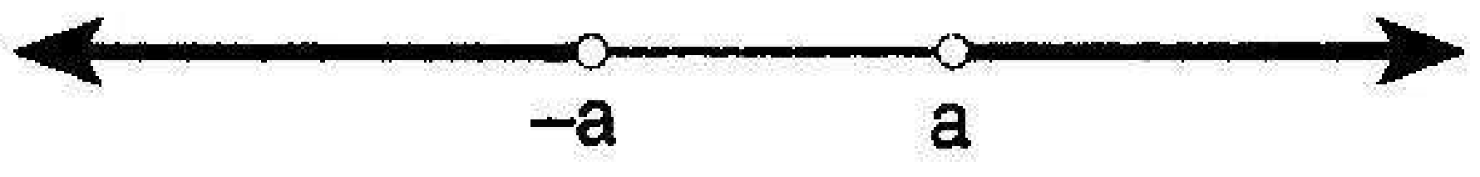
$|x - 3| < 4 \Rightarrow -4 < x - 3 < 4$

$-1 < x < 7$

$\Rightarrow$ ÇK(SS) = (−1, 7)

2. $a \in R^+$

$$|x| > a \Rightarrow \begin{cases} x > a \\ x < -a \end{cases}$$



PEKİŞTİRME / REINFORCE

$|x - 1| > -5 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : R

ÖRNEK / EXAMPLE 9

$x \in Z$,

$|3x - 5| \geq 4 \Rightarrow \sum x = ?$

A) –2 B) –3 C) –4 D) –5 E) –6

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$|3x - 5| \geq 4$$

$$3x - 5 \geq 4 \qquad 3x - 5 \leq -4$$

$$3x \geq 9 \qquad 3x \leq 1$$

$$x \geq 3 \qquad x \leq \frac{1}{3}$$

$$x \in Z \Rightarrow \sum x = (-1) + (-2) = -3$$

3. $a, b \in R^+$, $a < b$

$$a < |x| < b$$

$$a < x < b \qquad -b < x < -a$$

ÖRNEK / EXAMPLE 10

$x \in Z$,

$3 < |x + 4| < 7 \Rightarrow \sum x = ?$

A) –22 B) –24 C) –26 D) –28 E) –30

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$3 < |x + 4| < 7$$

$$3 < x + 4 < 7 \qquad -7 < x + 4 < -3$$

$$-1 < x < 3 \qquad -11 < x < -7$$

$$x \in Z \Rightarrow \sum x = -10 - 9 - 8 + 1 + 2 = -24$$

4. $a, b \in R - \{0\}, |a| < |b| \Rightarrow a^2 < b^2$

ÖRNEK / EXAMPLE 11

$|x+1| > |x-2| \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty,1)$ B) $\left(-\infty,\frac{3}{2}\right)$ C) $\left(\frac{1}{2},\infty\right)$ D) $\left(\frac{5}{2},\infty\right)$ E) $(9,\infty)$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$|x+1| > |x-2| \Rightarrow$$

$$x^2+2x+1 > x^2-4x+4$$

$$6x > 3$$

$$x > \frac{1}{2}$$

$$\Rightarrow \text{ÇK(SS)} = \left(\frac{1}{2},\infty\right)$$

ÖRNEK / EXAMPLE 12

$x > y$,

$x + y = 0$,

$$\frac{|x^2-2xy|+|y-2x|}{2\cdot|x|+|y|} = 4 \Rightarrow \mathbf{x-y=?}$$

A) –1 B) 0 C) 3 D) 6 E) 7

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left.\begin{array}{l} x > y \\ x+y=0 \end{array}\right\} \Rightarrow x=-y \Rightarrow x>0>y$$

$$\frac{|x^2-2xy|+|y-2x|}{2\cdot|x|+|y|} = 4$$

$$\frac{|x^2+2x^2|+|-x-2x|}{2x+|-x|} = 4$$

$$\frac{3x^2+3x}{3x} = 4 \Rightarrow x+1=4$$

$$x=3$$

$$x=3 \Rightarrow y=-3$$

$$x-y=3+3=6$$

ÖRNEK / EXAMPLE 13

$|x-2| + |x-3| = 7 \Rightarrow$ **ÇK (SS) = ?**

A) {0, 1, 2} B) {1, 6} C) {−1, 6}
D) {−1} E) {6}

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x-2=0 \Rightarrow x=2$

$x-3=0 \Rightarrow x=3$

x		2		3	
x − 2	−	○	+		+
x − 3	−		−	○	+
	I.		II.		III.

I. $x \in (-\infty, 2) \Rightarrow |x-2| + |x-3| = 7$

$$2 - x + 3 - x = 7$$

$$x = -1 \in (-\infty, 2)$$

II. $x \in [2, 3] \Rightarrow |x-2| + |x-3| = 7$

$$x - 2 + 3 - x = 7$$

$$1 = 7 \Rightarrow \text{ÇK(SS)} = \varnothing$$

III. $x \in (3, \infty) \Rightarrow |x-2| + |x-3| = 7$

$$x - 2 + x - 3 = 7$$

$$x = 6 \in (3, \infty)$$

$\Rightarrow$ ÇK(SS) = {−1, 6}

ÖRNEK / EXAMPLE 14

YILDIZ TEKNİK ÜNV. YÖS 2013

$|x-1| < 2$

$\Rightarrow$ **$|x+2| + |x-3| + |x-4| = ?$**

A) $x-3$ B) $-x+9$ C) $x+1$ D) $2x-1$ E) $x-1$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$|x-1| < 2$

$$-2 < x-1 < 2$$

$$-1 < x < 3$$

$$\Rightarrow |x+2| + |x-3| + |x-4| = x + 2 - x + 3 - x + 4$$

$$= -x + 9$$

ÖRNEK / EXAMPLE 15

İSTANBUL ÜNV. YÖS 2013

$x < \frac{5}{3}$

$\Rightarrow |5-3x| - \sqrt{9x^2-30x+25} + 3x - 1 = ?$

A) 4 B) $3x-1$ C) $9-3x$ D) $3x-9$ E) $1-3x$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$|5-3x| - \sqrt{9x^2-30x+25} + 3x - 1$

$= |3x-5| - \sqrt{(3x-5)^2} + 3x - 1$

$= \underbrace{|3x-5| - |3x-5|}_{0} + 3x - 1$

$= 3x - 1$

PEKİŞTİRME / REINFORCE

$\sqrt{x-2y+3} + |x-y-4| = 0 \Rightarrow x \cdot y = ?$

YANIT / ANSWER : 77

ÖRNEK / EXAMPLE 16

ON DOKUZ MAYIS ÜNV. YÖS 2013

$xy > 0$ ve $x^2y < 0$

$\frac{|x| - |x+y| - 1}{1-y} = ?$

A) $-x$ B) x C) -1 D) 0 E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x^2y < 0 \Rightarrow y < 0$

$xy > 0 \Rightarrow x < 0$

$\frac{|x| - |x+y| - 1}{1-y} = \frac{-x + x + y - 1}{1-y} = -1$

ÖRNEK / EXAMPLE 17

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$a < 0 < b$

$$\frac{a \cdot |b - a|}{||a| + b|} = ?$$

A) $-a$ B) a C) b D) $-b$ E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$b - a > 0 \Rightarrow \frac{a \cdot |b - a|}{||a| + b|} = \frac{a \cdot (b - a)}{b - a} = a$$

ÖRNEK / EXAMPLE 18

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$x + 3|x - 1| + 2 = 0$

denkleminin çözüm kümesi aşağıdakilerden hangisidir?

Which one of the following is the solution set of the equation $x + 3|x - 1| + 2 = 0$?

A) $\left(-\frac{1}{4}, -\frac{5}{2}\right)$ B) $\left(\frac{1}{4}\right)$ C) $\left(\frac{5}{2}\right)$

D) $\left(-\frac{1}{4}, -\frac{5}{2}\right)$ E) $\varnothing$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$3|x - 1| = -x - 2$$

$$\begin{tikzcd} & {|x-1| = \frac{-x-2}{3}} \arrow[dl] \arrow[dr] & \\ {x - 1 = \frac{-x-2}{3}} & & {x - 1 = \frac{x+2}{3}} \\ {\cancel{x = \frac{1}{4}}} & & {\cancel{x = \frac{5}{2}}} \end{tikzcd}$$

$$ÇK(SS) = \varnothing$$

1. $x > 3 \Rightarrow |3 - x| - |x - 3| - x = ?$

A) 6
B) 3
C) –3
D) x
E) –x

2. $2 < x < 4$,

$\sqrt{x^2 - 32x + 256} = 4|x| + 3 \Rightarrow x = ?$

A) $\frac{9}{5}$
B) $\frac{11}{5}$
C) $\frac{13}{5}$
D) $\frac{17}{5}$
E) $\frac{19}{5}$

3. $a < 0$,

$|3a - |a - 3|| = 7 \Rightarrow a = ?$

A) –1
B) –2
C) –3
D) –4
E) –5

4. $-2 < x < \frac{3}{2} \Rightarrow$

$|x + 2| - |2x - 3| + |x - 5| = ?$

A) x – 3
B) 2x – 5
C) 2x + 1
D) 2x + 4
E) 2x + 7

5. $\min(|3x - 4y|) \Rightarrow \frac{2x + y}{x - y} = ?$

A) 7
B) 9
C) 11
D) 13
E) 15

6. $|2x - 7| + 8 = 5 \Rightarrow$ **ÇK(SS) = ?**

A) ∅
B) {3}
C) {2, 5}
D) {5}
E) R

7. $||x + 2| - 5| = 6 \Rightarrow \sum x = ?$

A) −3 B) −4 C) −5

D) −6 E) −7

8. $|2x - 1| = x + 1 \Rightarrow \sum x = ?$

A) −1 B) 0 C) 1

D) 2 E) 3

9. $y \in Z$,

$|x + 2| < 4,\ 2x - y = 5 \Rightarrow \sum y = ?$

A) −133 B) −134 C) −135

D) −136 E) −137

10. $x \in Z$,

$$\frac{3}{|x - 2|} < \frac{1}{2} \Rightarrow \sum x = ?$$

A) −27 B) −26 C) −25

D) −24 E) −23

11. $-5 < |7x - 3| \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$ B) $\left(-\frac{2}{7}, \infty\right)$ C) $\left(\frac{3}{7}, \infty\right)$

D) $\left[\frac{5}{7}, \infty\right)$ E) R

12.
$$\left.\begin{array}{l} |x| > x \\ y > |z| \\ x \cdot z > x^2 \end{array}\right\} \Rightarrow\ ? < ? < ?$$

A) $z < x < 0 < y$ B) $x < z < y < 0$

C) $y < x < 0 < z$ D) $0 < x < y < z$

E) $y < 0 < x < z$

YANITLAR / ANSWERS

1. E	2. C	3. A	4. D	5. C	6. A	7. B	8. D
9. C	10. B	11. E	12. A				

1. $x > 3 \Rightarrow 3 - x < 0$

$x > 3 \Rightarrow x - 3 > 0$

$$\begin{aligned} |3 - x| - |x - 3| - x &= x - 3 - (x - 3) - x \\ &= x - 3 - x + 3 - x \\ &= -x \end{aligned}$$

Yanıt / The Answer is E

2.

$$\begin{aligned} \sqrt{x^2 - 32x + 256} &= 4 \cdot |x| + 3 \\ \sqrt{(x-16)^2} &= 4 \cdot |x| + 3 \\ |x - 16| &= 4 \cdot |x| + 3 \end{aligned}$$

$2 < x < 4 \Rightarrow 16 - x = 4x + 3$

$13 = 5x$

$x = \dfrac{13}{5}$

Yanıt / The Answer is C

3. $|3a - |a - 3|| = 7$

$a < 0 \Rightarrow a - 3 < 0$

$|3a - (3 - a)| = 7$

$|3a - 3 + a| = 7$

$a < 0 \Rightarrow 4a - 3 < 0$

$3 - 4a = 7$

$4a = -4$

$a = -1$

Yanıt / The Answer is A

4. $-2 < x < \dfrac{3}{2} \Rightarrow x + 2 > 0,\ 2x - 3 < 0,\ x - 5 < 0$

$$\begin{aligned} |x + 2| - |2x - 3| + |x - 5| &= x + 2 - (3 - 2x) + 5 - x \\ &= x + 2 - 3 + 2x + 5 - x \\ &= 2x + 4 \end{aligned}$$

Yanıt / The Answer is D

5. $\min(|3x - 4y|) = 0 \Rightarrow 3x - 4y = 0$

$3x = 4y$

$$3\underset{\substack{\downarrow \\ 4k}}{x} = 4\underset{\substack{\downarrow \\ 3k}}{y} \Rightarrow \frac{2x + y}{x - y} = \frac{2 \cdot (4k) + 3k}{4k - 3k} = 11$$

Yanıt / The Answer is C

6. $|2x - 7| + 8 = 5$

$|2x - 7| = -3 \Rightarrow$ ÇK(SS) $= \varnothing$

Yanıt / The Answer is A

7. $||x+2|-5| = 6$

$|x+2|-5 = 6$

$|x+2| = 11$

$x+2 = 11$ $\quad$ $x+2 = -11$

$x = 9$ $\quad$ $x = -13$

$ÇK(SS) = \{-13, 9\}$

$\Rightarrow \sum x = -13 + 9 = -4$

$|x+2|-5 = -6$

$|x+2| = -1$

$ÇK(SS) = \varnothing$

Yanıt / The Answer is B

---

8. $|2x-1| = x+1$

$2x-1 = x+1$

$2x - x = 1+1$

$x = 2$

$2x-1 = -x-1$

$3x = 0$

$x = 0$

$\Rightarrow \sum x = 2 + 0 = 2$

Yanıt / The Answer is D

---

9. $|x+2| < 4 \Rightarrow -4 < x+2 < 4$

$-6 < x < 2$

$2x - y = 5 \Rightarrow y = 2x - 5$

$y = 2x - 5 \Rightarrow -17 < 2x - 5 < -1$

$-17 < y < -1$

$y \in Z \Rightarrow \sum y = -135$

Yanıt / The Answer is C

---

10. $\dfrac{3}{|x-2|} < \dfrac{1}{2} \Rightarrow \dfrac{|x-2|}{3} > 2$

$|x-2| > 6$

$|x-2| > 6$

$x-2 > 6$ $\quad$ $x-2 < -6$

$x > 8$ $\quad$ $x < -4$

$x \in Z \Rightarrow \sum x = -26$

Yanıt / The Answer is B

---

11. $-5 < |7x-3|$

$|a| \geq 0 \Rightarrow 0 \leq |7x-3|$

$0 \leq |7x-3| \Rightarrow -5 < |7x-3|$

$\Rightarrow ÇK(SS) = R$

Yanıt / The Answer is E

---

12. $|x| > x \Rightarrow x < 0$

$y > |z| \Rightarrow y > 0$

$$\left.\begin{array}{l} x \cdot z > x^2 \\ x < 0 \end{array}\right\} \Rightarrow z < 0,\ z < x$$

$\Rightarrow z < x < 0 < y$

Yanıt / The Answer is A

1. $-2 < x < 5 \Rightarrow |x+2| + |x-5| = ?$

Yanıt / The Answer is: **7**

2. $3 < x \Rightarrow |2x + |3 - x|| = ?$

Yanıt / The Answer is: **3x – 3**

3. $a < 0,\ 0 < b \Rightarrow$

$\sqrt{a^2} + \sqrt[3]{-b^3} - |b - a| = ?$

Yanıt / The Answer is: **–2b**

4. $a < 0 < b < c \Rightarrow$

$\dfrac{|b-c| + |b-a|}{|a| + |c|} = ?$

Yanıt / The Answer is: **1**

5. $x \in Z$,

$\dfrac{|x-3|}{4} < 2 \Rightarrow \sum x = ?$

Yanıt / The Answer is: **45**

6. $x + 4 - |x - 4| = 0 \Rightarrow x = ?$

Yanıt / The Answer is: **0**

7. $\left.\begin{array}{l} a<0<b \\ |a|<b \end{array}\right\} \Rightarrow |b-a|+|b+a|-2|b|=?$

Yanıt / The Answer is: **0**

8. $y \in Z$,

$1 < |2x-1| < 2$

$3x + y = 5 \quad \Rightarrow \sum y = ?$

Yanıt / The Answer is: **7**

9. $|5 - |x-3|| = 7 \Rightarrow \sum x = ?$

Yanıt / The Answer is: **6**

10. $||3x-1| + 7| = 10 \Rightarrow \sum x = ?$

Yanıt / The Answer is: $\mathbf{\frac{2}{3}}$

11. $|3x+7| = 2x + 13 \Rightarrow \sum x = ?$

Yanıt / The Answer is: **2**

12. $x \in Z$,

$$\left|\frac{3}{x+2}\right| \geq \frac{1}{2} \quad \Rightarrow \sum x = ?$$

Yanıt / The Answer is: **–24**

1. $3 < x < 7 \Rightarrow \mathbf{|x - 10| + |x + 1| = ?}$

A) 3
B) 5
C) 7
D) 9
E) 11

2. $3 < x < 6 \Rightarrow$

$\mathbf{\sqrt{x^2 - 6x + 9} + \sqrt{x^2 - 14x + 49} = ?}$

A) –10
B) –4
C) 1
D) 4
E) 10

3. $1 < x < 5,$

$2|x - 1| + |11 - 2x| = 4 - y \Rightarrow \mathbf{y = ?}$

A) –9
B) –7
C) –5
D) –3
E) –1

4. $a < 0 < b \Rightarrow$

$\mathbf{\frac{|a|}{a} + \frac{|b|}{b} - \frac{|a - b|}{b - a} = ?}$

A) 2
B) 1
C) 0
D) –1
E) –2

5. $x < y < z$

$|x - y| - |y - z| + |x - z| = 8 \Rightarrow \mathbf{x - y = ?}$

A) –4
B) –2
C) 0
D) 2
E) 4

6. $|3x - 5y| + |y - 6| = 0 \Rightarrow \mathbf{x + y = ?}$

A) 12
B) 15
C) 16
D) 18
E) 21

7. $3|x+1|-5=|2x+2| \Rightarrow \boldsymbol{\sum x = ?}$

A) -4

B) -3

C) -2

D) -1

E) 0

8. $x \in Z$,

$\dfrac{|5x-2|-7}{|x-3|+4} \le 0 \Rightarrow \sum x = ?$

A) -2

B) -1

C) 0

D) 1

E) 2

9. $||2x-3|-5|=9 \Rightarrow \boldsymbol{\sum x = ?}$

A) -3

B) -1

C) 1

D) 3

E) 5

10. $A=\dfrac{30}{|x+1|+|x+2|} \Rightarrow \mathbf{max(A)=?}$

A) 5

B) 6

C) 10

D) 15

E) 30

11. $x \in Z$,

$3<|2x+1|<7 \Rightarrow \boldsymbol{\sum x = ?}$

A) -1

B) 0

C) 1

D) 2

E) 3

12. $a, b \in Z$,

$$\left.\begin{matrix} 5<|a|<8 \\ 2<|b|\le 3 \end{matrix}\right\} \Rightarrow \mathbf{max(a-b)=?}$$

A) 1

B) 4

C) 5

D) 8

E) 10

YANITLAR / ANSWERS

1. E	**2. D**	**3. C**	**4. D**	**5. A**	**6. C**	**7. C**	**8. C**
9. D	**10. E**	**11. A**	**12. E**				

1. $y \in Z$,

$|x - 2| < 3$

$3x + 2y = 15 \Rightarrow \min(y) = ?$

A) –1

B) 1

C) 3

D) 4

E) 6

2. $|x^2 - 9| - |x - 3| = 0 \Rightarrow \sum x = ?$

A) 3

B) 1

C) 0

D) –1

E) –3

3. $|x| = -x \Rightarrow$

$$\frac{|-x+2|x||}{|x|} = ?$$

A) –3

B) –2

C) 1

D) 2

E) 3

4. $x \in Z$,

$|x - 3| + |3 - x| < 10 \Rightarrow \max(x) = ?$

A) 3

B) 6

C) 7

D) 9

E) 10

5. $$\left.\begin{array}{l} |x+4| = x+4 \\ |x-3| = 3-x \end{array}\right\} \Rightarrow ÇK(SS) = ?$$

A) [–4, –3)

B) [–3, 3]

C) [–4, 4]

D) [–4, 3)

E) [–3, 4]

6. $x < |x| = -y$

$|x + y| + |x - y| + |x \cdot y| = 8 \Rightarrow x = ?$

A) –4

B) –2

C) 0

D) 2

E) 4

7. $x \in Z$,

$||x+1|-5| < 1 \Rightarrow \sum x = ?$

A) –2

B) 0

C) 3

D) 4

E) 7

8. $x \in Z$,

$-16 < |x^2 - 1| < 14 \Rightarrow \sum x = ?$

A) –4

B) –1

C) 0

D) 2

E) 3

9. $|x-5| + |5-x| = |3x+5| \Rightarrow \sum x = ?$

A) –14

B) –9

C) –6

D) –4

E) –1

10. $\left|-3+\frac{|x|}{x}\right|+2=y \Rightarrow \sum y = ?$

A) 7

B) 8

C) 9

D) 10

E) 11

11. $|3x-6| + |x-2| = 16 \Rightarrow \sum x = ?$

A) 4

B) 6

C) 8

D) 10

E) 12

12. $\min(|3x-6|-12) \Rightarrow |x^2-3x| = ?$

A) 0

B) 1

C) 2

D) 3

E) 4

YANITLAR / ANSWERS

1. B	**2. E**	**3. E**	**4. C**	**5. D**	**6. B**	**7. A**	**8. C**
9. A	**10. D**	**11. A**	**12. C**				

1. $$\left.\begin{array}{l}|x| > x\\ |y| < z\\ xy > x^2\end{array}\right\} \Rightarrow \mathbf{? < ? < ?}$$

A) $x < y < z < 0$

B) $y < x < 0 < z$

C) $x < 0 < z < y$

D) $x < z < 0 < y$

E) $z < 0 < x < y$

2. $|x - 3| = |3x - 5| \Rightarrow \sum x = ?$

A) –4

B) –2

C) 0

D) 1

E) 3

3. $A = |x - 2| + |x + 3| \Rightarrow$ **min(A) = ?**

A) 1

B) 2

C) 3

D) 4

E) 5

4. $1 - 2|3x - 4| = 5 \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$

B) $\{1, 2\}$

C) $\{1, 3\}$

D) $\{-1, 4\}$

E) R

5. $$\left.\begin{array}{l}|x+2| = x+2\\ |x-4| = 4-x\end{array}\right\} \Rightarrow$$ **ÇK(SS) = ?**

A) $[-2, 3]$

B) $[1, 5]$

C) $[-4, 2]$

D) $[-2, 4)$

E) $[2, 4)$

6. $||x + 1| - 2x| = 6 \Rightarrow \sum x = ?$

A) $\frac{4}{3}$

B) $\frac{5}{3}$

C) 2

D) $\frac{8}{3}$

E) $\frac{14}{3}$

7. $x^2 - 2|x| - 15 = 0 \Rightarrow \prod x = ?$

A) -25

B) -15

C) -9

D) 15

E) 25

8. $x < 0 < y$

$$\Rightarrow \frac{|x-2y|-|y-x|-|y|}{|y|-|x|} = ?$$

A) $-x$

B) $2y$

C) -2

D) 1

E) 0

9. $|2x - |x|| = 2x + 10 \Rightarrow \sum x = ?$

A) -4

B) -3

C) -2

D) -1

E) 0

10. $x \in Z$,

$||4x + 3| + |5 - x|| = 7 \Rightarrow x = ?$

A) -2

B) -1

C) 0

D) 2

E) 9

11. $|x - 2| \le |x + 1| \Rightarrow$ **ÇK(SS) = ?**

A) $\left[\frac{1}{2}, \infty\right)$

B) $\left[\frac{3}{2}, \infty\right)$

C) $\left[\frac{7}{2}, \infty\right)$

D) $\left[\frac{9}{2}, \infty\right)$

E) $\left[\frac{11}{2}, \infty\right)$

12. $x \cdot |x| \le 2 \Rightarrow$ **ÇK (SS) = ?**

A) $(-\infty, -2)$

B) $(\sqrt{2}, \infty)$

C) $(-\infty, -\sqrt{2})$

D) $(0, \sqrt{2})$

E) $(-\infty, \sqrt{2})$

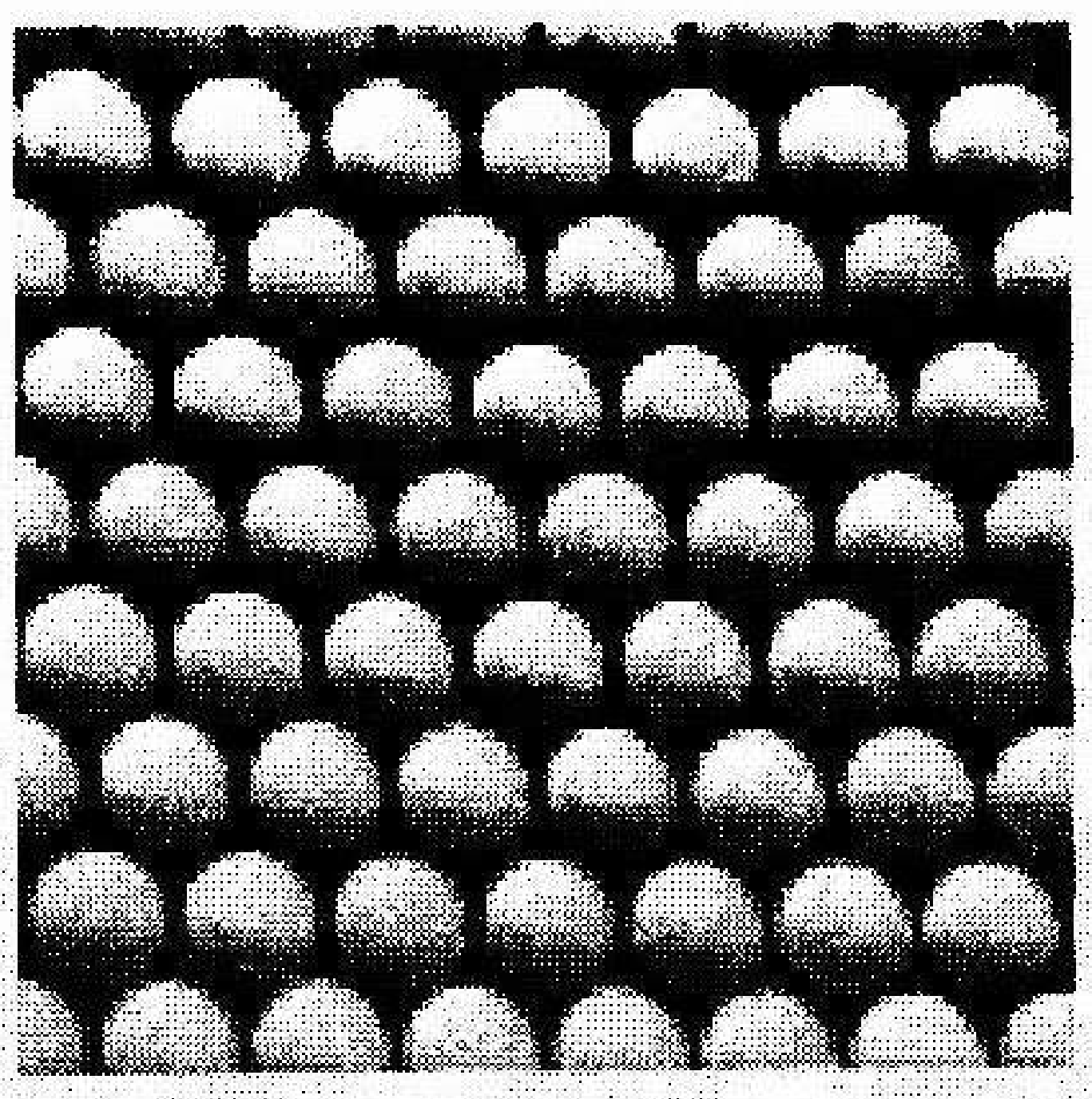
YANITLAR / ANSWERS

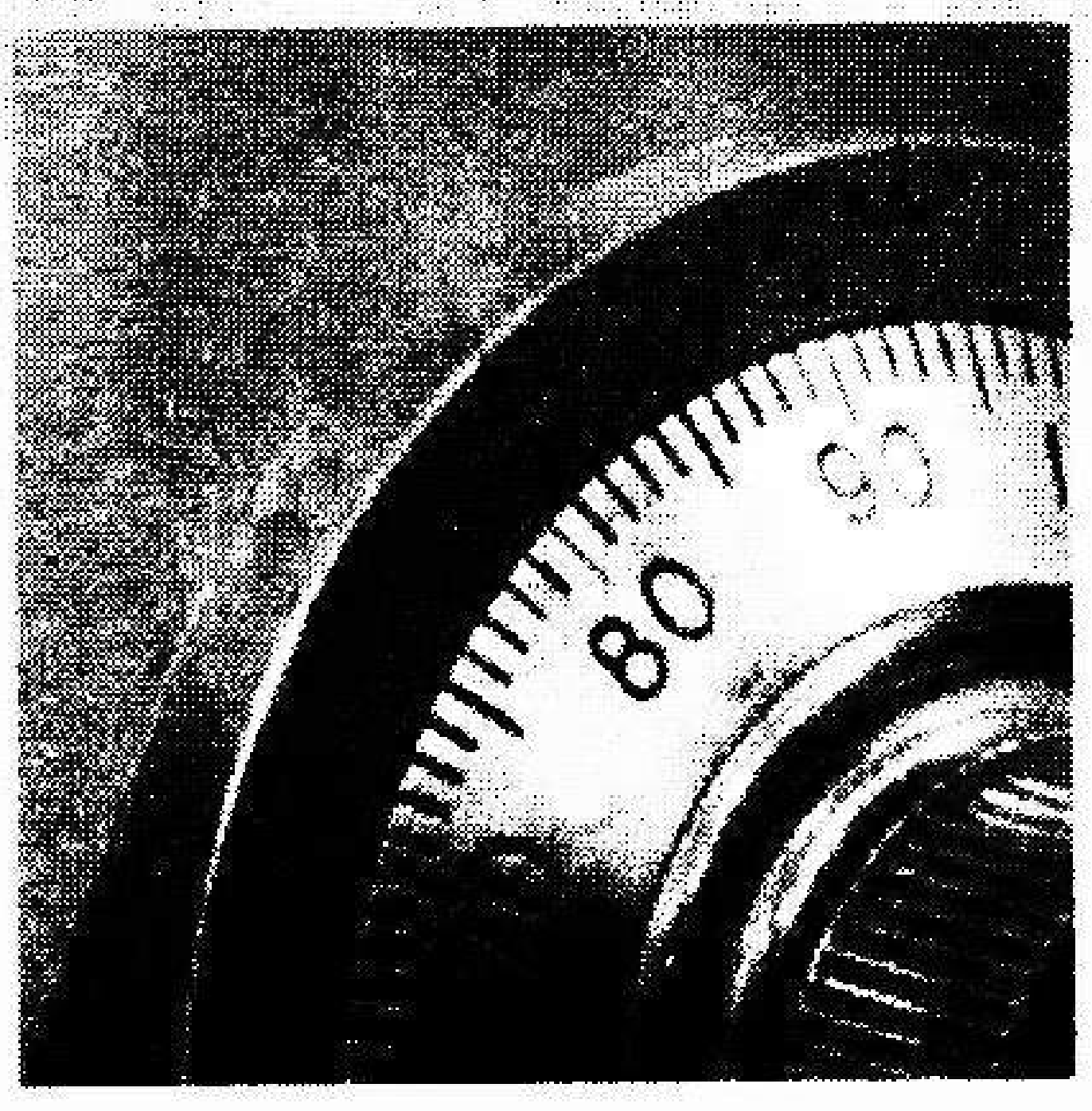
1. B	**2. E**	**3. E**	**4. A**	**5. D**	**6. E**	**7. A**	**8. E**
9. C	**10. B**	**11. A**	**12. E**				

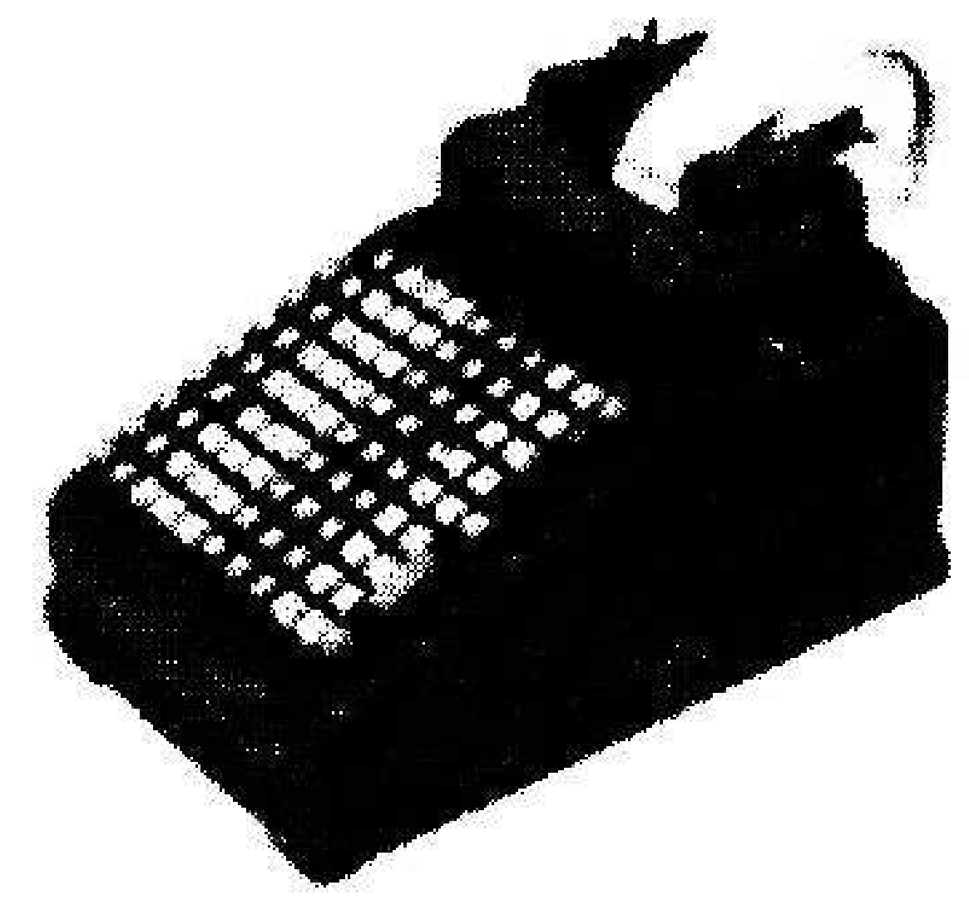
ÜNİTE 5
UNIT

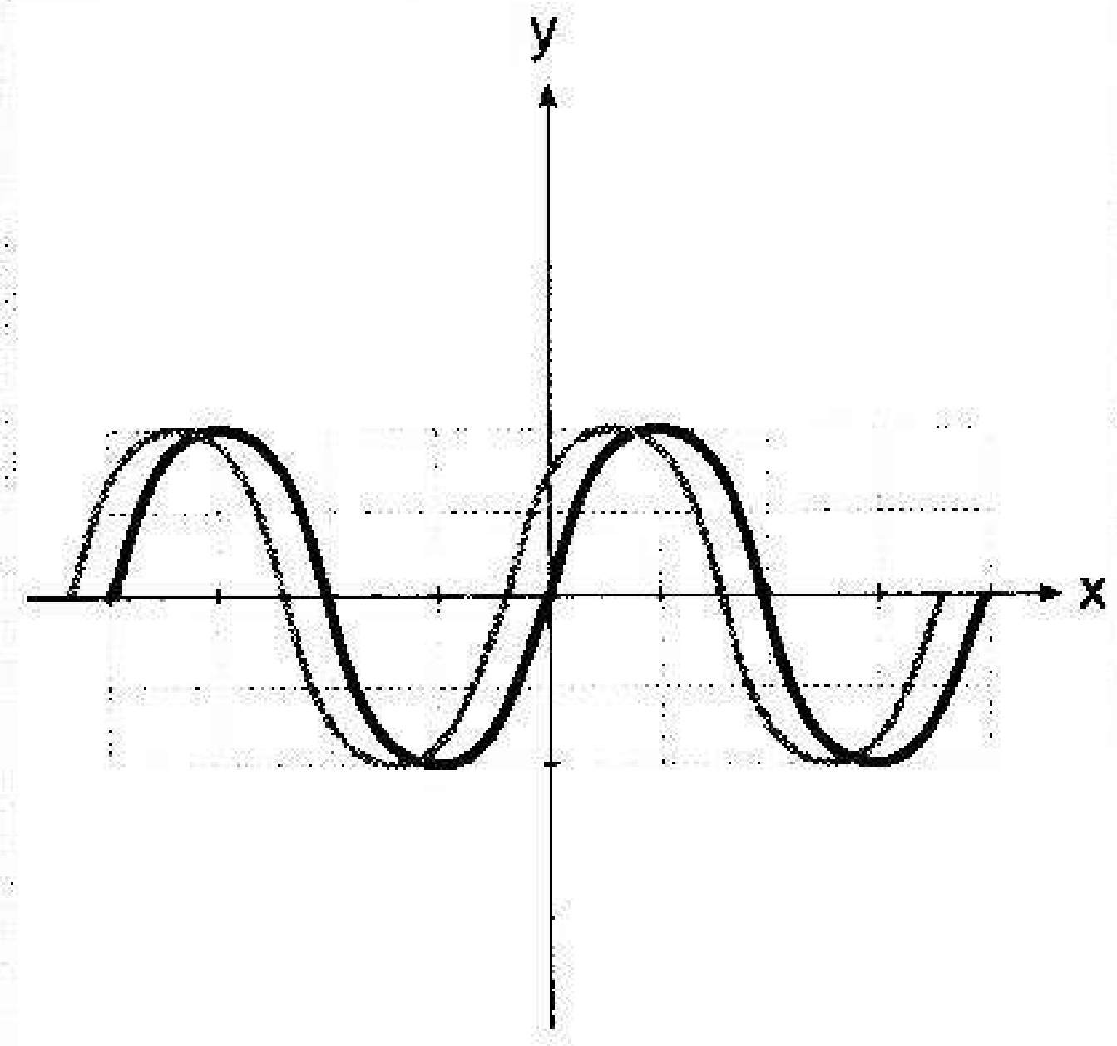
Bu ünitede; kümeler, fonksiyonlar, işlem ve modüler aritmetik kavramları ayrıntılı anlatılmıştır.
Bu ünite, 78 çözümlü örnek, 46 alıştırma, 96 test sorusu içermektedir.

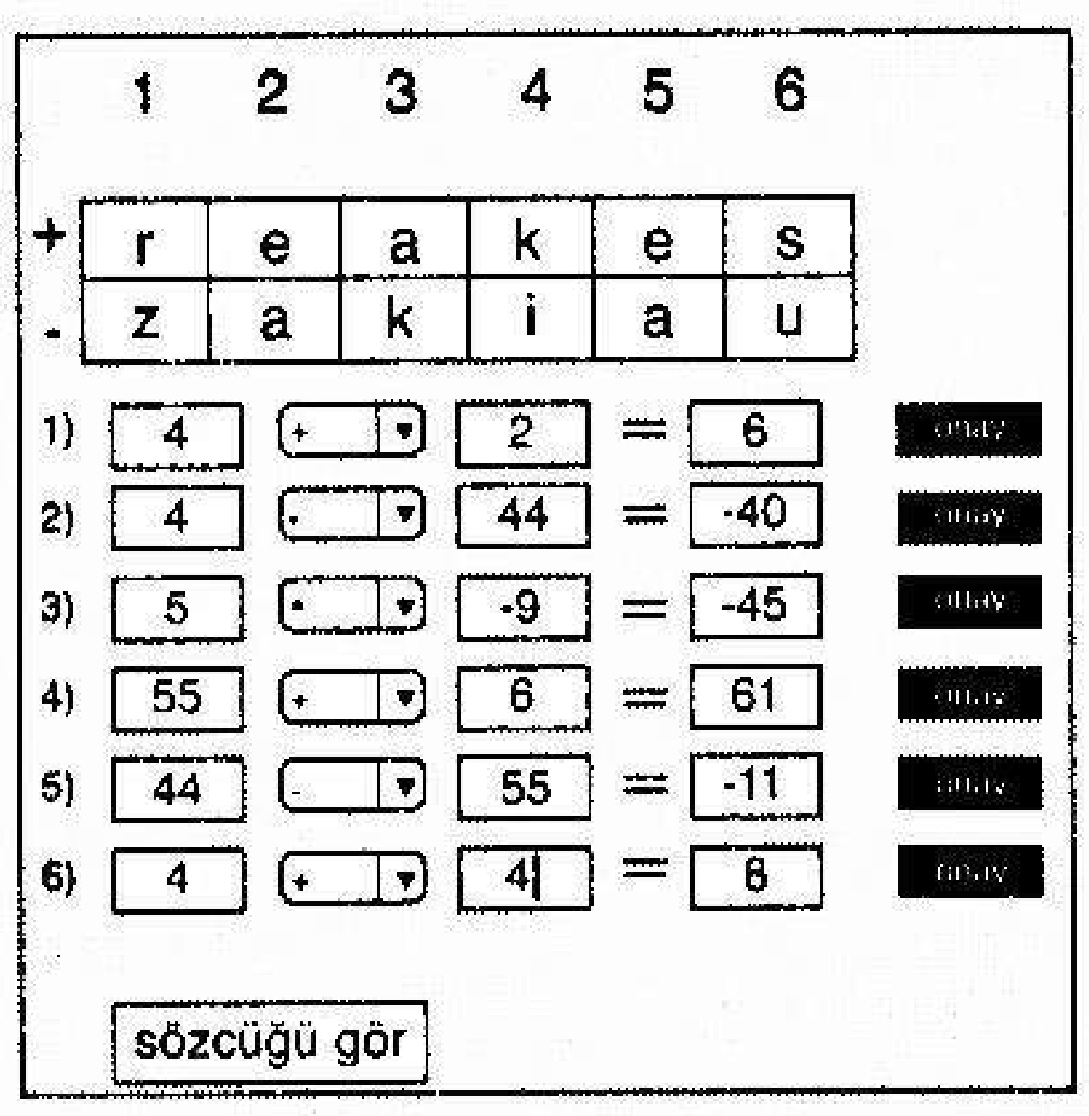
In this unit, sets, functions, operation and modular systems are explained in detailed.
This unit includes; 78 examples, 46 exercises, and 96 test questions.











	1	2	3	4	5	6
+	r	e	a	k	e	s
-	z	a	k	i	a	u

1) 4 + 2 = 6
2) 4 - 44 = -40
3) 5 · -9 = -45
4) 55 + 6 = 61
5) 44 - 55 = -11
6) 4 + 4 = 8

◆ **PERMÜTASYON, KOMBİNASYON VE OLASILIK**
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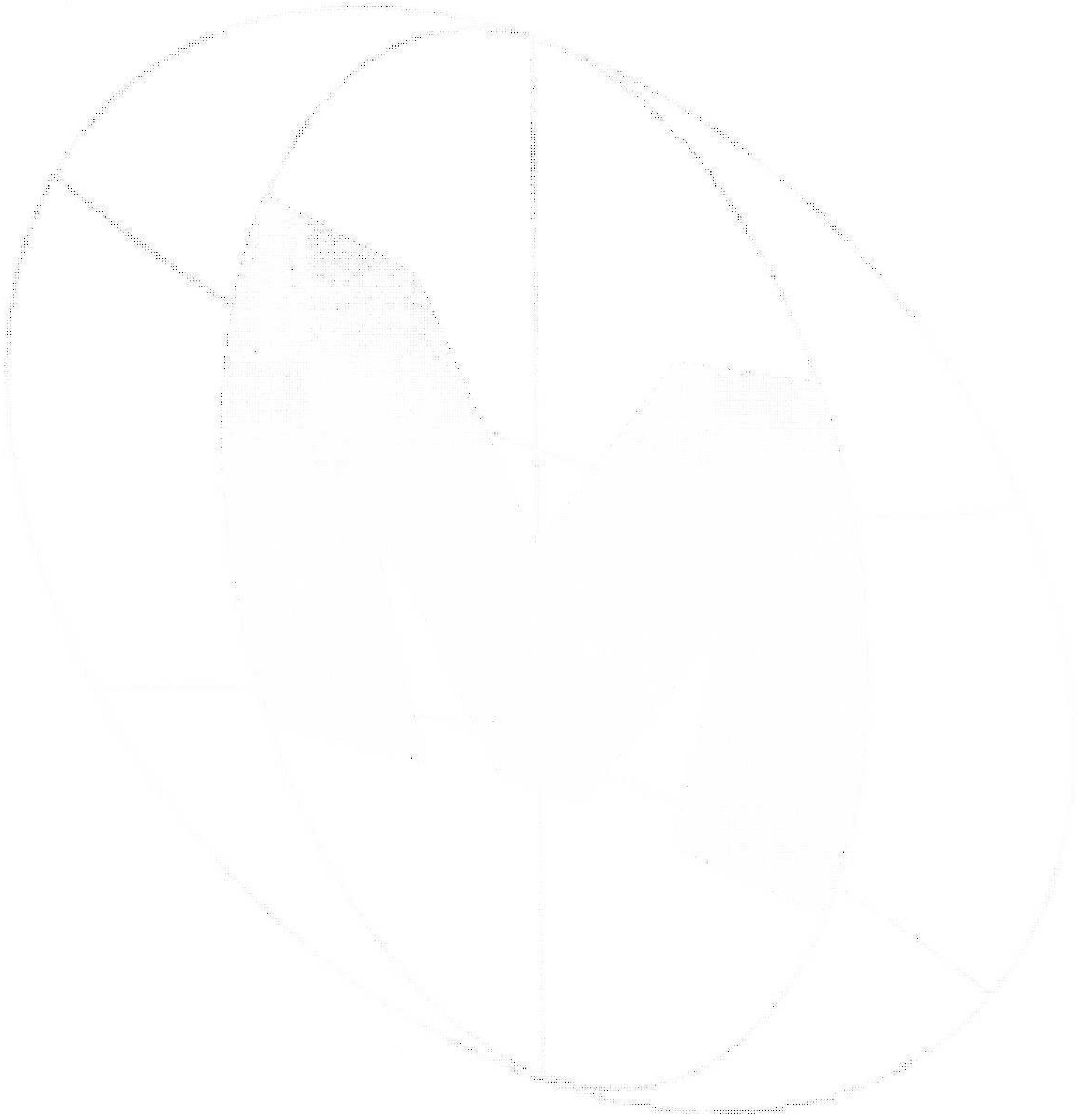
BU SAYFADA KONU MATERYALİ YOKTUR.



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SAYMA İLKELERİ (Permütasyon, Kombinasyon) ve OLASILIK
COUNTING PRINCIPLES (Permutations, Combinations) & PROBABILITY

BÖLÜM
CHAPTER 1

SAYMA ve SAYMANIN İLKELERİ / COUNTING and COUNTING PRINCIPLES

Tanım: Sayma, nesnelerin ve olayların sayılarla birebir eşlenmesidir. Saymanın temel ilkeleri, **toplama** ve **çarpmadır.**

Definition: *Counting is matching the objects and the events with the numbers. The fundamental principles of counting are the principle of* ***addition*** *and the principle of* ***multiplication****.*

TOPLAMA İLKESİ / PRINCIPLE of ADDITION

Birincisi n_1 yolla, ikincisi n_2 yolla yapılabilen iki işlem göz önüne alındığında, iki işlemden ancak birinden biri yapılabiliyorsa, bu işlemlerden biri **veya** diğeri $n_1 + n_2$ yolla yapılabilir. Bu ilke, sonlu sayıda işlemi içine alan durumlar için genelleştirilebilir.

Two events can occur. The first event can occur in n_1 different ways and the second event can occur in n_2 different ways, then this operation can occur in first or second event can occur in $n_1 + n_2$ ways. This principle can be generalize infinite number of operations.

ÖRNEK / EXAMPLE 1

Ankara'dan İstanbul'a üç farklı hava yolu, on farklı kara yolu şirketi ile gidilebilmektedir. Buna göre, Ankara'dan İstanbul'a gitmek isteyen bir kişi kaç farklı yolculuk gerçekleştirebilir?

There are 3 different airline companies and 10 different highway transportation companies that travels from Ankara to İstanbul. In how many different ways can a person travel from Ankara to Istanbul?

A) 13 B) 15 C) 20 D) 26 E) 30

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

3 hava yolu şirketinden 1'i 3 farklı şekilde seçilebilecektir. 10 farklı kara yolu şirketinden 1'i 10 farklı şekilde seçilebilir. 3 hava yolu, 10 kara yolu şirketinden 1 tane hava yolu **veya** 1 tane kara yolu toplama ilkesine göre 3 + 10 = 13 farklı şekilde seçilebilir.

One from 3 different airline companies can be chosen in 3 different ways. One from 10 different highway transportation companies can be chosen in 10 different ways. According to the principle of addition; one highway transportation company ***or*** *one airline company within 3 airline companies and 10 highway transportation companies can be chosen in 10 + 3 = 13 different ways.*

PEKİŞTİRME / REINFORCE

Asude'nin 2 beyaz, 3 siyah, 4 kırmızı elbisesi vardır. Buna göre, Asude elbise seçimini kaç farklı şekilde yapabilir?

Asude has 2 white, 3 black and 4 red dresses. In how many different ways Asude can choose a dress?

YANIT / ANSWER : 9

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NOT / NOTE

Toplama yoluyla sayma yapılırken **"veya"** ile **"ya da"**, çarpma yoluyla sayma yapılırken ise "ve" bağlacının kullanıldığına dikkat edilmelidir.

We are taken into consideration, when counting by addition rule "either" and "or" is used, and when counting by multiplication rule "and" is used.

ÇARPMA İLKESİ / THE PRINCIPLE of MULTIPLICATION

n_1 farklı yolla ortaya çıkan bir olay gözönüne alındığında, her n_1 yolu için, bu yolu izleyen ikinci olay n_2 farklı yolla ortaya çıkarsa, tüm olayın farklı biçimde ortaya çıkma sayısı $n_1 \cdot n_2$ olur. Eğer n_2 olayının her biri için bunu izleyen üçüncü olay n_3 yolla ortaya çıkarsa, tüm olayın ortaya çıkma sayısı $n_1 \cdot n_2 \cdot n_3$ olur. Bu şekilde devam edilirse, $n_1 \cdot n_2 \cdot n_3 ....n_k$ yazılabilir. Buna, **çarpma ilkesi** denir.

The first event can occur in n different ways. After the first event has occurred. The second event can occur in n_2 different ways and the third event can occur in n_3 different ways. The number of ways that the three events can occur is $n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$.

The most important of these is the Fundamental Counting Principle.

n_1	n_2	n_3	n_4	....	n_{r-1}	n_r

$n_1 \cdot n_2 \cdot n_3 \cdot n_4 \ldots n_{r-1} \cdot n_r$

Örneğin; Bir okuldaki 2 matematik, 3 Türkçe, 4 tarih öğretmeni arasından 1 matematik, 1 Türkçe ve 1 tarih öğretmeni, $2 \cdot 3 \cdot 4 = 24$ farklı yolla seçilebilir.

For example; There are 2 Mathematics, 3 Literature and 4 History teachers in the school. 1 Mathematics, 1 Literature and 1 History teachers can be chosen in $2 \cdot 3 \cdot 4 = 24$ different ways.

ÖRNEK / EXAMPLE 2

7 farklı gömleği, 5 farklı pantolonu olan bir kişi giymek için bir gömlek ve bir pantolon seçecektir. Buna göre, bu kişi kaç farklı seçim yapabilir?

A person who has 7 different shirts and 5 different pants will choose one shirt and one pant from among them. In how many different ways this choise can be made?

A) 35 B) 40 C) 48 D) 60 E) 70

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

7 gömlekten birini 7 farklı şekilde, 5 pantolondan birini 5 farklı şekilde yapabilir.

7 tane gömlek, 5 tane pantolondan 1 gömlek ve 1 pantolon çarpma ilkesine göre $7 \cdot 5 = 35$ farklı şekilde seçilebilir.

A shirt from among 7 different shirts can be chosen in 7 different ways and a pant from among 5 different pants in 5 different ways. According to the principle of multiplication; a shirt and a pant can be chosen from among 7 shirts and 5 pants in $7 \cdot 5 = 35$ different ways.

PEKİŞTİRME / REINFORCE

8 kız, 12 erkek öğrenci arasından bir kız ve bir erkek öğrenci kaç farklı şekilde seçilebilir?

In how many different ways one male student can be chosen from among 8 female and 12 male students?

YANIT / ANSWER: 96

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Uygulamalar / Activities

A = {1, 2, 3, 4, 5} kümesi elemanlarıyla 3 basamaklı

By using the members of A = {1, 2, 3, 4,5}

1. **Kaç farklı sayı yazılabilir?**

   How many different three-digit numbers can be written?

   Çözüm / Solution:

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
5	5	5

Yüzler, onlar ve birler basamağının her birine 5 farklı rakamdan biri gelecektir. O halde, 5·5·5 = 125 farklı sayı yazılabilir.

5 will be placed into the ones, tens and hundreds places and 5·5·5 = 125 different numbers can be written.

2. **Rakamları tekrarsız kaç farklı sayı yazılabilir?**

   How many different three-digit numbers can be written with no repetition allowed?

   Çözüm / Solution:

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
5	4	3

Yüzler basamağına 5 farklı rakamdan biri, onlar basamağına 4 farklı rakamdan biri, birler basamağına 3 farklı rakamdan biri gelebilir. O halde, 5 · 4 · 3 = 60 farklı sayı yazılabilir.

5 different numbers is placed in hudreds place, 4 different numbers in tens place and 3 different numbers in ones place. According to that 5·4·3 = 60 different numbers can be written.

3. **Rakamları tekrarsız kaç farklı çift sayı yazılabilir?**

   How many different three-digit even numbers can be written with no repetition allowed?

   Çözüm / Solution:

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
		4

{2, 4}

Bir sayının çift olabilmesi için birler basamağının çift olması gerekir. O halde, birler basamağına 2 veya 4 rakamlarından biri gelmelidir.

If the number is an even number, then ones place must be even. The numbers 2 or 4 will be chosen.

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
4	3	2

Bu iki sayıdan biri birler basamağına yazıldığında geriye kalan 4 sayıdan biri yüzler, 3 sayıdan biri onlar basamağına yazılabilir. O halde, 4 · 3 · 2 = 24 farklı sayı yazılabilir.
The hundreds place is to be 4 tens place is 3 and ones place is to be 2.

4. 300 den büyük kaç farklı sayı yazılabilir?
How many different three-digit numbers greater than 300 can be written?

Çözüm / Solution:

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
3	5	5

{3, 4, 5}

Sayının 300 den büyük olması için yüzler basamağına 3, 4 veya 5 rakamlarından biri gelmelidir.
If the numbers greater than 300, in hundreds place is to be 3 or 4.

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
3	5	5

Rakamların tekrarsız koşulu olmadığından onlar ve birler basamağına 4 rakamdan da biri gelebilir. Bu nedenle, 3 · 5 · 5 = 75 farklı sayı yazılabilir.
Since there is no repetition, tens and ones places are to be 4 in four numbers in the set.

5. 300 den büyük rakamları tekrarsız kaç farklı sayı yazılabilir?
How many different three-digit numbers greater than 300 can be written with no repetition allowed?

Çözüm / Solution:

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
3	4	3

4. sorudaki gibi yüzler basamağına 3 farklı sayı gelebilir. Fakat rakamlar tekrarsız olacağından onlar basamağına 4, birler basamağına 3 farklı sayı gelebilir.
O halde, 3 · 4 · 3 = 36 farklı sayı yazılabilir.
There is no repetition, tens place is to be 3 ones place is to be 4 and hundreds place is to be 3.

PERMÜTASYON / PERMUTATIONS

Tanım: n farklı elemanın bir elemanı birinci, diğeri ikinci bir diğeri de üçüncü olacak şekilde sıralanarak devam etmesine **permütasyon** denir.

n farklı nesnenin (her biri bir kez kullanılması koşuluyla) r tanesi ($r \le n$) sıralanırsa, elde edilecek farklı sıralanışların sayısı P(n, r) ile gösterilirse,

Definition: *A permutation of n different elements is an ordering of the elements such that one element is first, one is second, one is third and so on.*

The number of permutations (arrangements, orders) of n elements taken at a time. It is denoted by P(n, r).

$$P(n, r) = \frac{n!}{(n-r)!}$$

yazılır. Burada, P (n, r), n elemanlı bir kümenin r li permütasyonlarının sayısını göstermektedir.

P(n,r) denotes the number of permutations of n elements r at a time.

ÖRNEKLER / EXAMPLES:

a. $P(5, 3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2!}{2!} = 60$

b. $P(7, 4) = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 840$

ÖRNEK / EXAMPLE 3

$x \cdot P(6, 2) = y \cdot P(5, 3) \Rightarrow \frac{x}{y} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$x \cdot P(6, 2) = y \cdot P(5, 3) \Rightarrow$

$x \cdot \frac{6!}{4!} = y \cdot \frac{5!}{2!}$

$x \cdot 30 = y \cdot 60$

$x = 2y$

$\frac{x}{y} = 2$

PEKİŞTİRME / REINFORCE

a. P(3, 1) = ?
b. P(4, 2) = ?
c. P(6, 0) = ?
d. P(10, 1) = ?

YANIT / ANSWER :

a. 3 **b.** 12 **c.**1 **d.**10

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NOT / NOTE

Permütasyon tanımına göre birbirinden farklı diziliş olayları permütasyonla çözülebilir. Her problem çarpma kuralı ile de çözülebilir.

According to the definition of permutation is in determining the number of ways that elements can be arranged (in order).

PEKİŞTİRME / REINFORCE

4 kişilik bir koltuğa 4 kişi kaç farklı şekilde oturabilir?

In how many different ways can 4 person be seated on a couch for 4 person?

YANIT / ANSWER : 24

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PEKİŞTİRME / REINFORCE

G, Ö, Z, D, E harfleri kullanılarak harfleri birbirinden farklı beş harfli anlamlı ya da anlamsız kaç farklı sözcük yazılabilir?

How many meaningful or meaninless words such that its letters are different from each other can be written by wing the letters G, Ö, Z, D, E?

YANIT / ANSWER : 120

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ÖRNEK / EXAMPLE 4

16 futbol takımının bulunduğu bir ligde birinci, ikinci ve üçüncü kaç farklı şekilde belirlenebilir?

In how many different ways can first, second and third be chosen from among a football leauge that contains 16 teams?

A) 3030 B) 3360 C) 3480 D) 3620 E) 4000

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$P(16,\ 3) = \frac{16!}{(16-3)!} = \frac{16\cdot15\cdot14\cdot13!}{13!} = 3360$$

ÖRNEK / EXAMPLE 5

5 matematik, 4 fizik, 3 geometri kitabı aynı branş kitapları bir arada olmak koşuluyla, yan yana kaç farklı şekilde sıralabilir?

In how many different ways can 5 Mathematics, 4 Physics and 3 Geometry books be put in order in condition of putting the books belong to same branch side by side?

A) $5!\cdot4\cdot3!$ B) $3!\cdot5!\cdot4\cdot3!$ C) $3!\cdot5!\cdot4\cdot3!$

D) $2!\cdot5!\cdot4\cdot3!$ E) $6!\cdot5!\cdot4!$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

Matematik kitapları $P(5,\ 5) = 5!$, Fizik kitapları $P(4,\ 4) = 4!$ ve Geometri kitapları ise $P(3,\ 3) = 3!$ şekilde sıralanabilir.

Bu üç branş kitapları kendi aralarında da 3! kadar sıralanabileceğinden; $3!\cdot5!\cdot4!\cdot3!$ şeklinde sıralabilir.

Mathematics books, Physics books and Geometry books can be put in order in $P(5,5) = 5!$, $P(4,\ 4) = 4!$ *and* $P(3,\ 3) = 3!$ *different ways, respectively. 3 branches can be put in order between each other in 3! different ways. In conclusion, ordering can be made in* $3!\cdot5!\cdot4!\cdot3!$ *different ways.*

TEKRARLI (YİNELEMELİ) PERMÜTASYON / REPEATING PERMUTATION

n tane nesnenin n_1 tanesi bir türden, n_2 tanesi ikinci türden, ... n_k tanesi de k ninci türden olmak üzere n nesnenin n tanesi sıralandığında, $\dfrac{n!}{n_1! \cdot n_2! \ldots n_k!}$ tane farklı düzen elde edilebilir.

The number of permutation P of n things taken all at a time, of which n_1 are alike, n_2 others are alike, n_3 others are alike.

Uygulamalar / Activities

1. **MALATYA sözcüğünün harfleri ile anlamlı veya anlamsız kaç sözcük yazılabilir?**

   Çözüm / Solution:

   M, A, L, T, Y

   $n_1 = 1,\ n_2 = 3,\ n_3 = 1,\ n_4 = 1,\ n_5 = 1$

   $$\frac{7!}{1! \cdot 3! \cdot 1! \cdot 1! \cdot 1!} = 840$$

2. **A ile başlayıp A ile biten anlamlı veya anlamsız kaç sözcük yazılabilir?**

   How many arrangements can be made by beginning with A and ending with A?

   Çözüm / Solution:

   A lardan 2 tanesinden birini başa, diğeri sona yazılırsa geriye kalan 5 harfle 5! = 120 farklı sayı yazılabilir.

   There are 2 A's on the letters of the word. One A is written at the beginning of letter and the other A is written to the end of letter 5 letters can be written in 5! = 120 different ways.

PEKİŞTİRME / REINFORCE

11223345 sayısının rakamları kullanılarak 8 basamaklı kaç farklı sayı yazılabilir?

How many different 8-digit numbers can be written by using 11223345?

YANIT / ANSWER : 5040

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DAİRESEL PERMÜTASYON / CIRCULAR PERMUTATION

n tane farklı nesnenin bir daire etrafında farklı sıralanmasında nesnelerden birisi bir yere yerleştirilir ve diğer (n–1) nesnenin sıralanışı, $(n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1 = (n-1)!$ olur. Bu da, n nesnenin dairesel sıralanış sayısını verir.

The number of ways of arranging n different objects around a circle object is $(n-1)!$.

In other words, it is written by $(n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1 = (n-1)!$. It is the number of ways of arranging n different objects around a circle object.

PEKİŞTİRME / REINFORCE

5 kişilik bir aile yuvarlak bir masa etrafında kaç farklı biçimde oturabilir?
In how many different ways can five people in a family be seated around a table?

YANIT / ANSWER : 24

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ÖRNEK / EXAMPLE 6

7 kişilik bir aile yuvarlak bir masa etrafında kaç değişik biçimde oturabilirler?
In how many different ways can seven people in a family be seated around a table?

A) 49 B) 70 C) 140 D) 520 E) 720

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

n = 7, n – 1 = 6
(n – 1)! = 6! = 720

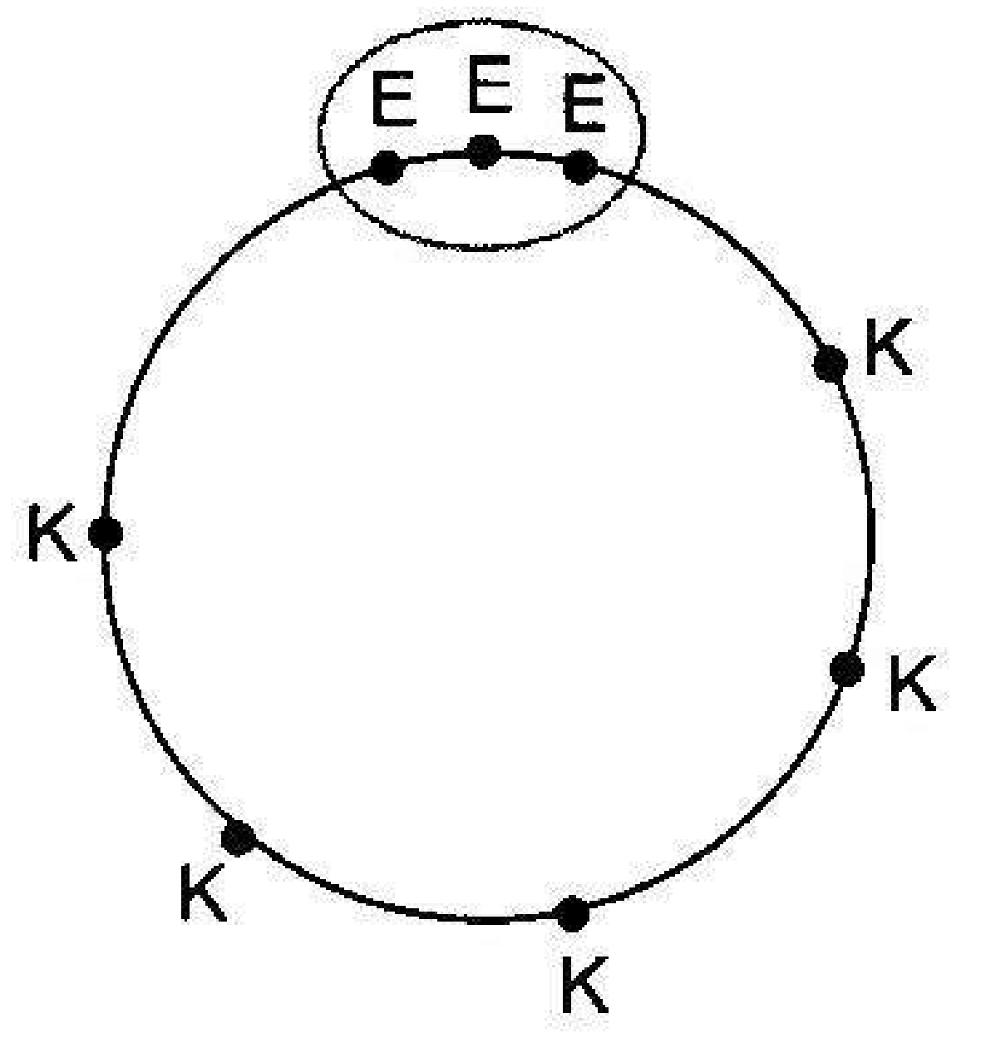
ÖRNEK / EXAMPLE 7

5 kız, 3 erkek öğrenci erkekler yan yana olmak koşuluyla yuvarlak bir masa etrafında kaç farklı biçimde oturabilir?
In how many different ways can 5 female 3 male students be seated around a table in condition of male students seating adjcently?

A) 3 · 5! B) 3! · 5! C) 5 · 3!
D) 3! · 7! E) 3! · 8!

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**



Erkekler yanyana 3! farklı şekilde oturabilir. Erkek öğrenciler, yan yana oldukları için üçü bir kişi gibi düşünülür ve bu grup toplamda 6 kişi kabul edilir.

Male students can be seated side by side in 3! different ways since male students are seating adjacently altogether, they are considered as one person and the group is considered as, 6 people.

n = 6 ⇒ 3!(n – 1)! = 3!·5!

KOMBİNASYON / COMBINATIONS

Tanım: n tane farklı nesne verilmiş ise, bu nesnelerden herhangi $r \le n$ (r pozitif bir tamsayı) tanesi seçilip alınır ve bu r nesne herhangi bir sırada yazılırsa, buna **r ninci mertebeden kombinasyon** denir.

Definition: *The number of combinations of a set of n different objects taken r at a time $r \le n$, r and n are non negative integers.*

Permütasyon, sıralama problemlerinin çözümünü içerir ve sıralama önemlidir. Kümenin elemanlarından alt küme seçme problemleri kombinasyon problemi olup sıralama önemli değildir. A = {a, b, c} kümesinin 2 li permütasyonları ve kombinasyonları aşağıda gösterilmiştir.

In the permutations order is important but in the combinations order is not important.

Let us consider the set {a, b, c} and the permutations and combinations taken two at a time.

Permütasyonlar / *Permutations*	Kombinasyonlar / *Combinations*
ab	{a, b}
ba	{a, c}
ac	{b, c}
ca	
bc	
cb	

Farklı üç nesnenin 6 tane 2 li permütasyonu varken, 2 li kombinasyonları 3 tanedir. n elemanlı bir kümenin r elemanlı ($n \ge r$) kombinasyonlarının sayısı,

3 different objects have 6 permutations with 2 elements while they have 3 combinations with 2 elements. It gives the number of combinations with n different objects taken r at a time ($n \ge r$).

$$C(n, r) = \binom{n}{r} = \frac{n!}{r! \cdot (n-r)!} = \frac{P(n, r)}{r!}$$

NOT / NOTE

n elemanlı bir kümenin r elemanlı ($n \ge r$) alt kümelerinden her birine n'in r'li kombinasyonu denir.

Selecting subsets of r elements of a set of n elements in which order is not important, is called combinations of n elements taken r at a time.

ÖZELLİKLER / PROPERTIES

1. $C(n, 0) = \binom{n}{0} = 1$
2. $C(n, 1) = \binom{n}{1} = \binom{n}{n-1} = n$
3. $C(n, 0) + C(n, 1) + C(n, 2) + \ldots + C(n, n) = 2^n$
4. $C(n, r) = C(n, k) \Rightarrow$ **i.** $n = r + k$ **ii.** $r = k$

PEKİŞTİRME / REINFORCE

a. $C(7, 3) = ?$

b. $\binom{10}{8} = ?$

c. $\binom{3}{0}+\binom{3}{1}+\binom{3}{2}+\binom{3}{3} = ?$

YANIT / ANSWER :

a. 35 **b.** 45 **c.**8

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PEKİŞTİRME / REINFORCE

$\binom{n}{4}=\binom{n}{5} \Rightarrow n = ?$

YANIT / ANSWER : 9

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ÖRNEK / EXAMPLE 8

$C(x, 1) = C(7, 6) + \binom{10}{0} \Rightarrow x = ?$

A) 6 B) 8 C) 9 D) 10 E) 12

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$C(x, 1) = C(7, 6) + \binom{10}{0}$$

$$\frac{x!}{(x-1)!\cdot 1!} = \frac{7!}{(7-6)!\cdot 6!} + \frac{10!}{(10-0)!\cdot 0!}$$

$$x = 7 + 1 = 8$$

ÖRNEK / EXAMPLE 9

$\binom{n}{2}+\binom{n}{1} = 6 \Rightarrow C(6,n) = ?$

A) 1 B) 6 C) 15 D) 20 E) 30

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\binom{n}{2}+\binom{n}{1} = 6 \Rightarrow \frac{n!}{(n-2)\cdot 2!} + \frac{n!}{(n-1)!\cdot 1!} = 6$$

$$\Rightarrow \frac{n(n-1)!}{2} + n = 6$$

$$\Rightarrow n^2 + n - 12 = 0$$

$$\Rightarrow (n+4)\cdot(n-3) = 0$$

$$\Rightarrow n = 3$$

$$n = 3 \Rightarrow C(6, 3) = \frac{6!}{(6-3)!\cdot 3!} = 20$$

ÖRNEK / EXAMPLE 10

n kişilik bir topluluktan oluşturulabilecek 3 kişilik grupların sayısı 10 olduğuna göre, n kaçtır?

If the number of groups which can be constituted by choosing 3 people from among 10 people is n, what is n?

A) 4 B) 5 C) 6 D) 7 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$C(n,\ 3) = 10$

$$\frac{n!}{(n-3)!\cdot 3!} = \frac{n(n-1)\cdot(n-2)}{6} = 10$$

$$= n(n-1)\cdot(n-2) = 60$$

$$\Rightarrow n = 5$$

PEKİŞTİRME / REINFORCE

10 kişilik bir öğrenci topluluğundan bilgi yarışmasına katılmak üzere 3 öğrenci seçilecektir. Bu seçim kaç farklı biçimde gerçekleşebilir?

3 students will be chosen from among 10 students in order to participate in a knowledge contest. In how many different ways this choise can be made?

YANIT / ANSWER : 120

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ÖRNEK / EXAMPLE 11

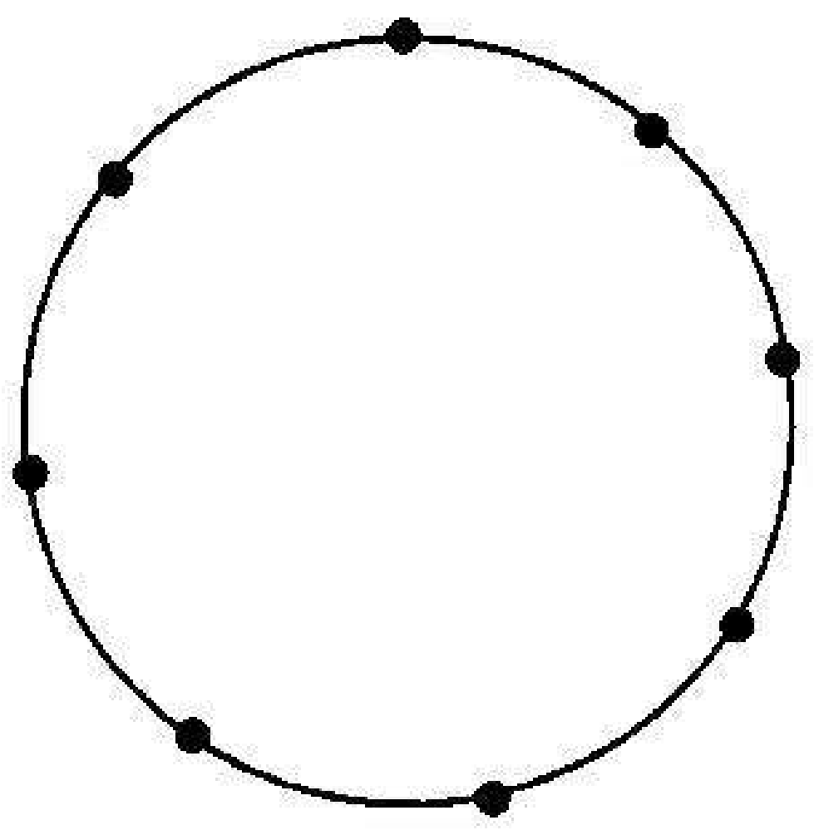
Bir çember üzerinde verilen 8 noktanın herhangi ikisinden geçen kaç doğru çizilebilir?

How many lines that pass through any of two points from among 8 points located on a circle can be drawn?

A) 27 B) 28 C) 29 D) 30 E) 31

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**



8 noktada 2 nokta seçmek gerektiğinden;

Since 2 points must be chosen from among 8 points;

$$C(8,2) = \frac{8!}{(8-2)!\cdot 2!} = \frac{8\cdot 7}{2} = 28$$

NOT

Bir madeni paranın havaya atılması olayında, paranın üst yüzünün yazı veya tura geleceği bilinir, fakat hangi oranda yazı, hangi oranda tura geleceği olasılık hesaplamalarıyla bulunabilir.

AÇIKLAMA

Bir zarın atılması deneyinde çıktılar 1, 2, 3, 4, 5, 6 bir paranın iki kere havaya atılması deneyinde çıktılar YY, YT, TY, TT dır (T: Tura, Y: Yazı).

AÇIKLAMA

a. İki madeni paranın atılması olayında örnek uzay, E = {(YY), (YT), (TY), (TT)} dır.

b. Bir zarın atılması deneyinde altı mümkün sonuç olup
E = {1, 2, 3, 4, 5, 6} dır.

AÇIKLAMA

a. Bir zarın atılması deneyinde 3 gelmesi olayı

b. İki zarın atılması deneyinde üst yüze gelen sayıların toplamının 4 olması olayı

NOT

E örnek uzayında A ve B kümeleri için küme işlemleri geçerlidir.

$A \cap B$, A ve B olayı

$A \cup B$, A veya B olayı

A', A olayının tümleyenidir.

$A \cup A' = E$

OLASILIK

Tanım: Olabilecek sonuçların belli fakat hangisinin olacağı kesin olmayan olayların matematiksel sayılarla ifade edilmesine **olasılık** denir.

1. **Deney:** Sonucun gözlenmesi için yapılan her işe **olasılıkta deney** denir.
2. **Çıktı :** Yapılan deneyin sonuçlarından her birine **çıktı** denir.
3. **Örnek Uzay:** Tüm çıktıların oluşturduğu kümeye **örnek uzay** denir ve E ile gösterilir.
4. **Olay:** Bir örnek uzayın her alt kümesine **olay** denir.
5. **Olanaksız (İmkansız), Kesin Olaylar:** E örnek uzayının bir alt kümesi A olsun. Eğer $A \neq \emptyset$ ise olanaksız, $A = E$ ise kesin olaydır.
   Örneğin;
   a. Bir zarın atılması deneyinde gelen sayının 6 dan büyük olması olayı olanaksız olaydır.
   b. Bir zarın atılması deneyinde gelen sayının 6 dan küçük veya eşit gelmesi olayı kesin olaydır.
6. **Ayrık Olaylar:** A ve B kümeleri, E örnek uzayın birer alt kümesi olmak üzere, $A \cap B = \emptyset$ ise A ve B olaylarına **ayrık olaylar** denir. Örneğin; bir zarın atılması deneyinde gelen sayının 2 den küçük gelmesi olayı ile 4 ten büyük gelmesi olayları ayrık olaylardır.

Uygulamalar

Bir zar atılması deneyinde,

1. Örnek uzayını bulunuz.
2. Çift gelmesi olayını bulunuz.
3. Tek gelmesi olayını bulunuz.
4. 7 gelmesi olayını bulunuz.
5. 4 ten küçük gelmesi olayını bulunuz.
6. 2 den büyük gelmesi olayını bulunuz.
7. 2 gelmemesi olayını bulunuz.

Çözümler:

1. Örnek uzayının elemanları, E = {1, 2, 3, 4, 5, 6} dır.
2. Çift gelmesi olayı Ç = {2, 4 6} dır.
3. Tek gelmesi olayı T = {1, 3, 5} dir. Ç ve T kümelerinin kesişimi boş küme olduğundan, ayrık olaylardır.
4. 7 gelmesi olayı imkansız olaydır.
5. 4 ten küçük gelmesi olayı A = {1, 2, 3} tür.
6. 2 den büyük gelmesi olayı B = {3, 4, 5, 6} dır.
7. 2 gelmemesi olayı A = {2} kümesinin tümleyeni A′ = {1, 3, 4, 5, 6} dir.

PROBABILITY

Definition: *The numeric expression of the results that might happen but not certain which one will happen is called probability.*

1. **Experiment:** *Any happening to observe the result is called an experiment.*
2. **Output:** *Every result of the experiment is called output.*
3. **Sample Space:** *The set that consists of all the outputs is called sample space. It is represented as E.*
4. **Event:** *Every subset of a sample space is called event.*
5. **Impossible Event, Certain Event:** *A is a subset of E. If $A = \emptyset$, then A is an impossible event. If $A = E$, then A is a certain event.*
   For example;
   - **a.** *The event of getting a number greater than 6 on the experiment of rolling a dice is an impossible event.*
   - **b.** *The event of getting a number equal to 6 or less than 6 on the experiment of rolling a dice is a certain event.*
6. **Mutually Exclusive Events:** *A and B are the subsets of E. If $A \cap B = \emptyset$, then the event A and the event B are called mutually exclusive events. For instance the event of getting a number less than 2 and the event of getting a number more than 4 are independent events when a dice is rolled.*

Activities

On an experiment of rolling a dice,

1. *Find the sample space.*
2. *Find the event of getting an even integer.*
3. *Find the event of getting an odd integer.*
4. *Find the event of getting 7.*
5. *Find the event of getting a number less than 4.*
6. *Find the event of getting a number greater than 2.*
7. *Find the event of not getting 2.*

Solutions:

1. *The sample space is $E = \{1, 2, 3, 4, 5, 6\}$.*
2. *The event of getting an even integer is $Ç = \{2, 4 6\}$.*
3. *The event of getting an odd integer is $T = \{1, 3, 5\}$. Since $Ç \cap T = \emptyset$.*
4. *The event of getting 7 is an impossible event.*
5. *The event of getting a number less than 4 is $A = \{1, 2, 3\}$.*
6. *The event of getting a number greater than 2 is $B = \{3, 4, 5, 6\}$.*
7. *The complement of the set $A = \{2\}$, the event of getting 2, is $A' = \{1, 3, 4, 5, 6\}$.*

NOTE

For instance, when a coin is tossed, we know the surface of the coin is either heads or tails but we use probability calculations to find out what ratio of the event is heads and what ratio of the event is tails.

EXPLANATION

The outputs of the experiment of rolling a dice are 1, 2, 3, 4, 5, 6. The outputs of the experiment of tossing a coin for two times are YY, YT, TY, TT. (Y = tails, T: heads)

EXPLANATION

- **a.** *The sample space of the event of tossing two coins is $E = \{(YY), (YT), (TY), (TT)\}$.*
- **b.** *The sample space of the event of rolling a dice is $E = \{1, 2, 3, 4, 5, 6\}$.*

NOTE

- **a.** *The event of getting 3 on the experiment of rolling a dice.*
- **b.** *The event of the sum of the surface, of two dices is 4 on the experiment of rolling two dices.*

NOTE

The operations of sets are obtained for the sets A and B in the sample space E.
The event of A and B $A \cap B$,
The event of A or B is $A \cup B$,
The complement of A' is A.
$A \cup A' = E$

NOT / NOTE

Bir olayın olma olasılığı 0 dan küçük, 1 den büyük olamaz.

The probability of an event can not be less than 0 and greater than 1.

i. $P(A) + P(A') = 1$

ii. Kesin olay için $\mathbf{P(E) = 1}$

For a certain event; $P(E) = 1$

İmkansız olay için $\mathbf{P(\emptyset) = 0}$ dır.

For impossible event; $P(\emptyset) = 0$

iii. $A, B \in S$ için $A \cap B = \emptyset$ (A ve B ayrık olaylar) ise $\mathbf{P(A \cup B) = P(A) + P(B)}$ dir.

For $A, B \in S$, *if* $A \cap B = \emptyset$ *(A and B are independent events) then* $P(A \cup B) = P(A) + P(B)$.

OLASILIK FONKSİYONU / THE PROBABILITY FUNCTION

Tanım: E örnek uzayının tüm alt kümelerinin oluşturduğu kümeye S denirse, $P: S \rightarrow [0, 1]$ şeklinde tanımlanan fonksiyona **olasılık fonksiyonu** denir ve $A \in S$ olmak üzere, P(A) sayısına da **A olayının olasılığı** denir.

Definition: *If S is a set that consists of all the subsets of E, the function P that defined from S to (0, 1) is called the probability function. If* $A \in S$, *P(A) is the probability of A.*

Olasılıkla ilgili, aşağıdaki aksiyom verilebilir:

An axiom is given below about the probability:

i. Olasılık fonksiyonunda görüntü kümesi [0, 1] olduğundan, $A \in S$ için $0 \leq P(A) \leq 1$ dir.

Since the range of the probability function is [0, 1], for $A \in S$, $0 \leq P(A) \leq 1$.

ÖRNEK / EXAMPLE 12

Ali, Seven ve Nazlı'nın katıldığı bir yarışmada Ali'nin birinci olma olasılığı 1/6, Seven"in birinci olma olasılığı 1/3 olduğuna göre, yarışta Nazlı'nın birinci olma olasılığı kaçtır?

In a contest that Ali, Seven and Nazlı participated, if the probabilty of Ali comes, in first is 1/6, and the probabilty of Seven comes in first 1/3, what is the probability of Nazlı coines in first?

A) $\frac{1}{6}$ B) $\frac{3}{8}$ C) $\frac{2}{3}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

Örnek uzay, $E = \{A, S, N\}$ / *Sample space is* $E = \{A, S, N\}$. $P(A) = \frac{1}{6}$, $P(S) = \frac{1}{3}$

$$P(A) + P(S) + P(N) = 1 \Rightarrow \frac{1}{6} + \frac{1}{3} + P(N) = 1$$

$$P(N) = \frac{1}{2}$$

OLASILIK FONKSİYONUNUN ÖZELLİKLERİ
PROPERTIES OF THE PROBABILITY FUNCTION

1. $A \cap B \neq \emptyset$ olmak üzere, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ dir.
2. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(P(A \cap B \cap C)$
3. A olayının tümleyeni A′ olmak üzere, $P(A) = 1 - P(A')$ dır.

   If the complement of A is A′, then $P(A) = 1 - P(A')$.
4. $A \subset B$ ise $P(A) \leq P(B)$ dir. / *If* $A \subset B$, *then* $P(A) \leq P(B)$.

ÖRNEK / EXAMPLE 13

Büşra, içinde 10 sarı, 7 kırmızı top bulunan bir torbadan elinde bir sarı top olana kadar art arda top çekiyor. Büşra, en çok kaç top çekmiştir?

Büşra picks balls successively from a pouch that contains 10 yellow 7 red balls until she has 1 yellow ball. What is the maximum number of balls that she picked?

A) 6 B) 7 C) 8 D) 9 E) 10

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

Önce bütün kırmızı topları birer birer çektiğinden, 7. çekilişte sadece sarı toplar kalır. 8. çekilişte sarı top çekilir. Yanıt 8'dir.

Since she picked all the red balls one by one, at the 7th picking only yellow ball remains. At the 8th time, she picks a yellow ball. The answer is 8.

EŞ OLUMLU ÖRNEK UZAYDA OLASILIK HESAPLARI
EQUALLY LIKELY SAMPLE SPACE

Tanım: Bir deneyde bütün sonuçların olabilme olasılığı eşit ise bu şekildeki örnek uzaylara **eş olumlu örnek uzay** denir. Örneğin; bir zar atılması olayında 6 sayının da gelme olasılığı aynıdır. O halde, zar atma deneyinde örnek uzay bir eş olumlu örnek uzaydır.

Definition: *If the possibilities of all the events are equal then this kind of space are called equally likely sample space. For instance the probability of each number, from 1 to 6 inclusively, is equal, when a dice is tossed. Because of that, for the experiment of rolling a dice the sample is an equally likely sample space.*

NOT / NOTE

$E = (A_1, A_2, \ldots, A_n\}$ örnek uzayında $P(A_1) = P(A_2) = \ldots = P(A_n)$ ise E eş olumlu bir örnek uzaydır.

If $P(A_1) = P(A_2) = \ldots = P(A_n)$, *then* $E = (A_1, A_2, \ldots, A_n\}$ *is an equally likely sample space.*

Eş olumlu bir örnek uzayında, $A \subset E$ olmak üzere,

$$P(A) = \frac{\text{İlgilenilen sonuç sayısı}(\textit{desired results})}{\text{Tüm olası sonuç sayısı}(\textit{all results})} = \frac{s(A)}{s(E)}$$

Örneğin; bir madeni paranın atılması deneyinde, E = {Y, T} olup tura gelme olasılığı,

For instance the sample space of the experiment of tossing a coin is E = {Y, T}.

The probability of getting a heads is

$$P(T) = \frac{s(T)}{s(E)} = \frac{1}{2}$$

Aynı şekilde bir zar atma deneyinde 2 gelme olasılığı 1/6 dır.

With the same logic, the probability of getting 2 when a dice is rolled is 1/6.

PEKİŞTİRME / REINFORCE

Bir zarın atılması deneyinde üst yüze gelen sayının asal olma olasılığı kaçtır?

What is the probability of getting a prime number when a dice is rolled?

YANIT / ANSWER: 1/2

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PEKİŞTİRME / REINFORCE

Bir çift zar atıldığında, üst yüze gelen sayıların toplamı 9 olduğu bilindiğine göre bu sayılardan birinin 3 olma olasılığı kaçtır?

When two dice is rolled, the sum of the numbers on the surfaces is 9. What is the probability of one of tne numbers is 3?

YANIT / ANSWER: 1/2

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ÖRNEK / EXAMPLE 14

Bir zar atma deneyinde üst yüze gelen sayının 4 ten küçük olma olasılığı kaçtır?

What is the probability of getting a number smaller than 4, when a dice is rolled?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$E = \{1, 2, 3, 4, 5, 6\} \Rightarrow s(E) = 6$

Zarın 4 ten küçük gelme olayı, $C = \{1, 2, 3\}$ olduğundan, $s(C) = 3$ tür.

Since the event of getting a number smaller than is $C = \{1, 2, 3\}$, $s(C) = 3$.

$$P(C) = \frac{s(C)}{s(E)} = \frac{3}{6} = \frac{1}{2}$$

KOŞULLU OLASILIK (BAĞIMLI OLAYLAR) CONDITIONAL PROBABILITY (DEPENDENT EVENTS)

Tanım: Bir olayın gerçekleşmesi başka bir olayın gerçekleşmesine bağlı ise bu iki olaya **bağımlı olaylar** denir.

Definition: *Two events are called dependent events if the occurrance of one has effect on the occurrance of the other.*

Bir örnek uzayının iki olayı A ve B olup A nın gerçekleşmesi, B olayının gerçekleşmesine bağlı ise A olayının, B olayına bağlı koşullu olasılığı $P(A \setminus B)$ ile gösterilir.

A and B are two events of a sample space and if the occurrance of A has an effect on occurrance of B, then the conditional probability of A conditioned to B is shown as P(A/B).

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)} \quad , \quad P(A \setminus B) = \frac{s(A \cap B)}{s(B)}$$

ÖRNEK / EXAMPLE 15

Bir zar atma deneyinde üst yüze gelen sayının tek olduğu bilindiğine göre **bu sayının 5 olma olasılığı kaçtır?**

If getting an odd integer is known when a dice is rolled, what is the probability of getting 5?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

Tek gelme olayına T denirse, $T = \{1, 3, 5\}$ olup, $s(T) = 3$ olur.

If the probability of getting an odd integer is T, $T = \{1, 3, 5\}$ then, $s(T) = 3$.

5 gelme olayına A denirse, $A = \{5\}$ ve $s(A) = 1$ olur.

If the probability of getting 5 is A, $A = \{5\}$, then $s(A) = 1$.

$A \cap T = \{5\}$, $s(A \cap T) = 1$

$$\Rightarrow P(A \setminus T) = \frac{s(A \cap T)}{s(T)} = \frac{1}{3}$$

BAĞIMSIZ OLAYLAR / INDEPENDENT EVENTS

Tanım: A ve B, E örnek uzayının iki olayı olmak üzere bu iki olaydan birinin olması diğerinin olmasını etkilemiyorsa A ve B olaylarına **bağımsız olaylar** denir.

Definition: *Two events are called as independent events if the occurance of one has no effect on the occurrance of the other.*

$$P(A) \neq 0,\ P(B) \neq 0 \Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

ÖRNEK / EXAMPLE 16

Bir zar ile madeni para aynı anda havaya atılıyor. **Paranın yazı, zarın üst yüzündeki sayının 5 ten büyük olması olasılığı kaçtır?**

A dice is rolled and a coin is tossed at the same time. What is the possibility of getting tail and getting a number greater than 5?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

Bir zar ile paranın atılması olayları bağımsız olaylardır. Zarın 5'ten küçük gelmesi olayına A denirse, $P(A) = \frac{4}{6} = \frac{2}{3}$ olur.

Tossing a coin and rolling a dice are independent events. If the event of getting numbers smaller then 5 when dice is rolled is named as A, then $P(A) = \frac{4}{6} = \frac{2}{3}$.

Paranın yazı gelmesi olayına B denirse, P(B) = 1/2 olur.

If the event of getting tails when the coin is tossed is named as B, then P(B) = 1/2.

$$P(A \cap B) = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$$

PEKİŞTİRME / REINFORCE

Bir zar ile madeni para aynı anda havaya atılıyor. Paranın tura, zarın üst yüzeyinin 4 olma olasılığı kaçtır?

A coin is tossed and a dice is rolled at the same time. What is the probability of getting heads and getting 4?

YANIT / ANSWER: 1/12

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ÖRNEK / EXAMPLE 17

1'den 6'ya kadar numaralandırılmış bir zar düz bir zemine atılıyor. Buna göre, üst yüze gelen sayının çift veya 3'ten büyük olma olasılığı kaçtır?

What is the probability of getting an even number or a number greater than 3 when a dice is rolled?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{3}{5}$ E) $\frac{5}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

E = {1, 2, 3, 4, 5, 6},

Ç = {2, 4, 6}

$A \cap Ç = \{6\}$

$P(Ç) = \frac{1}{2}$, $P(A) = \frac{1}{2}$, $P(A \cap Ç) = \frac{1}{6}$

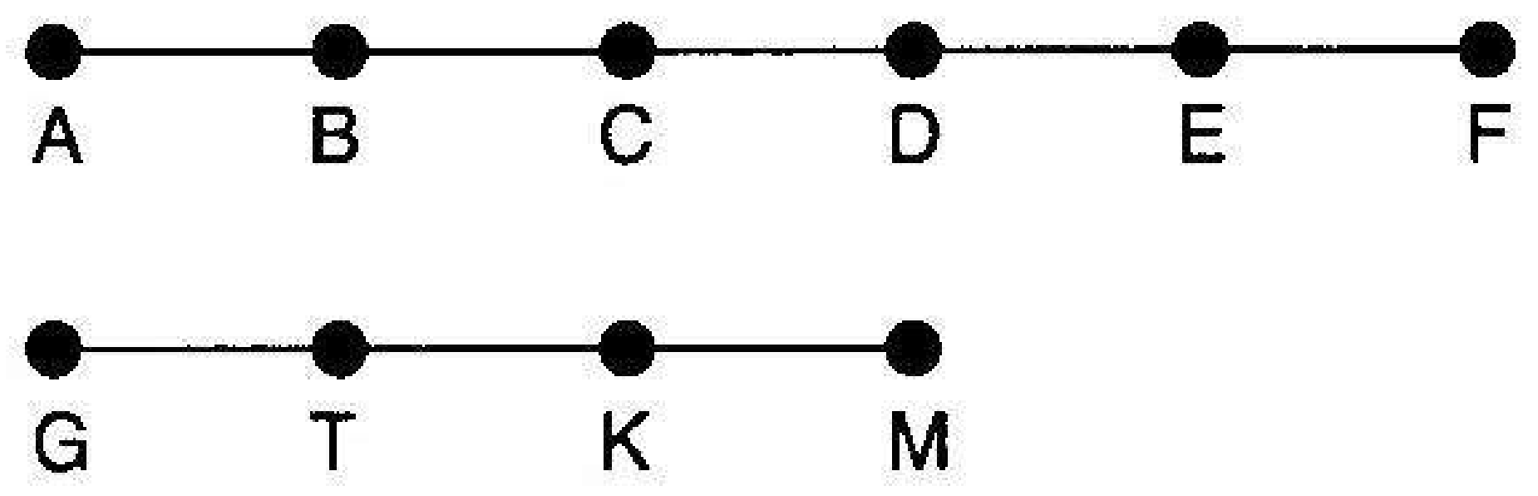
3'ten büyük; A = {4, 5, 6}

The event of getting a number greater than 3 is A={4,5,6}.

$P(A \cup B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6}$

ÖRNEK / EXAMPLE 18

KARADENİZ TEKNİK ÜNİV. YÖS 2016



Şekildeki noktalar kullanılarak kaç tane üçgen oluşturulabilir?

How many triangles can be constructed by using the dots in the figure?

A) 105 B) 135 C) 120 D) 90 E) 96

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\binom{6}{2} \cdot \binom{4}{1} + \binom{4}{2} \cdot \binom{6}{1}$$

$15 \cdot 4 + 6 \cdot 6 = 60 + 36 = 96$

1. $\binom{n}{2} = \frac{P(n,4)}{24} \Rightarrow$ **n = ?**

A) 4

B) 5

C) 6

D) 7

E) 8

2. $A = \binom{20}{n}$, max(A), $\binom{n}{a+2} = \binom{n}{4a-7} \Rightarrow$ **$\sum a$ = ?**

A) 2

B) 3

C) 4

D) 5

E) 6

3. **Mehran ve Katya'nın bulunduğu 10 kişilik bir grupta Mehran ve Katya'nın farklı gruplarda olması koşuluyla 5 er kişilik 2 grup kaç farklı şekilde oluşturulabilir?**

From among a group of 10 people that includes Mehran and Katya, 2 groups of 5 people will be chosen. In how many different ways this choise can be made if Mehran and Katya must be in different groups?

A) 70

B) 120

C) 140

D) 180

E) 210

4. **Bir sınıfın öğrencileri arasından 6 kişilik bir basketbol takımı oluşturulacaktır. Sınıfın 12 uygun öğrencisi arasından belirli iki öğrenci, oluşacak takımda birlikte bulunmayacağına göre, bu takım kaç değişik biçimde oluşturulabilir?**

A basketball team of 6 people will be constituted from among the students in a classroom. In how many different ways the team can be constituted if certain 2 students, will not participate in the team?

A) 210

B) 420

C) 630

D) 704

E) 714

5. **6 kız, 3 erkek arasından 2'si kız 1'i erkek olması koşuluyla 3 kişilik bir grup kaç farklı şekilde seçilebilir?**

In how many different ways a group of 3 people that consists of 2 girls and 1 boy be constituted from among 6 girls and 3 boys?

A) 25

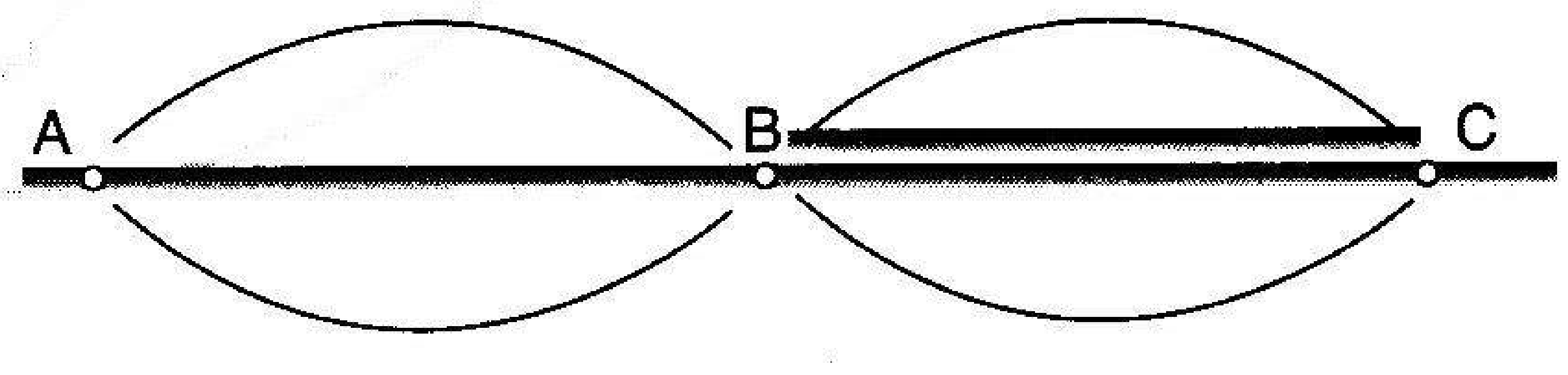
B) 30

C) 35

D) 40

E) 45

6.



Şekilde A ve B noktaları arasında 3, B ve C noktaları arasında 4 farklı yol vardır. Buna göre bir kişi A ile C arasında kaç farklı gidiş-dönüş yapabilir?

There are 3 different roads between Point A and Point B and 4 different roads between Point B and Point C. According to that, how many different round-trips can be made between A and C?

A) 7 B) 24 C) 49

D) 64 E) 144

7. Üç basamaklı, kaç tek doğal sayı yazılabilir?

How many three digit odd numbers can be written?

A) 270

B) 280

C) 300

D) 370

E) 450

---

8. 6 kişiden seçilen 4 kişi yuvarlak bir masa etrafında kaç farklı şekilde oturabilir?

In how many different ways can 4 people that are chosen from among 6 people be seated around a table?

A) 40

B) 46

C) 50

D) 60

E) 90

---

9. Bir kutudaki siyah kalemlerin sayısı, kırmızı kalemlerin sayısından 4 fazladır. Kutudan rastgele seçilen bir kalemin siyah olma olasılığı 4/7 olduğuna göre, bu kutuda toplam kaç kalem vardır?

In a box, the number of black pens exceeds the number of red pens by 4. If the probability of getting a black pen when a pen is chosen randomly, which of the following is the number of pens in this box?

A) 28

B) 26

C) 24

D) 22

E) 20

10. Bir grupta 5 kız, 6 erkek öğrenci vardır. Buna göre, gruptan seçilen 3 öğrencinin de kız olma olasılığı kaçtır?

There are 5 girls and 6 boys in a group. According to that, what is the probability of choosing 3 girls from among the group randomly?

A) $\frac{2}{121}$ B) $\frac{1}{22}$ C) $\frac{2}{33}$

D) $\frac{4}{121}$ E) $\frac{3}{11}$

---

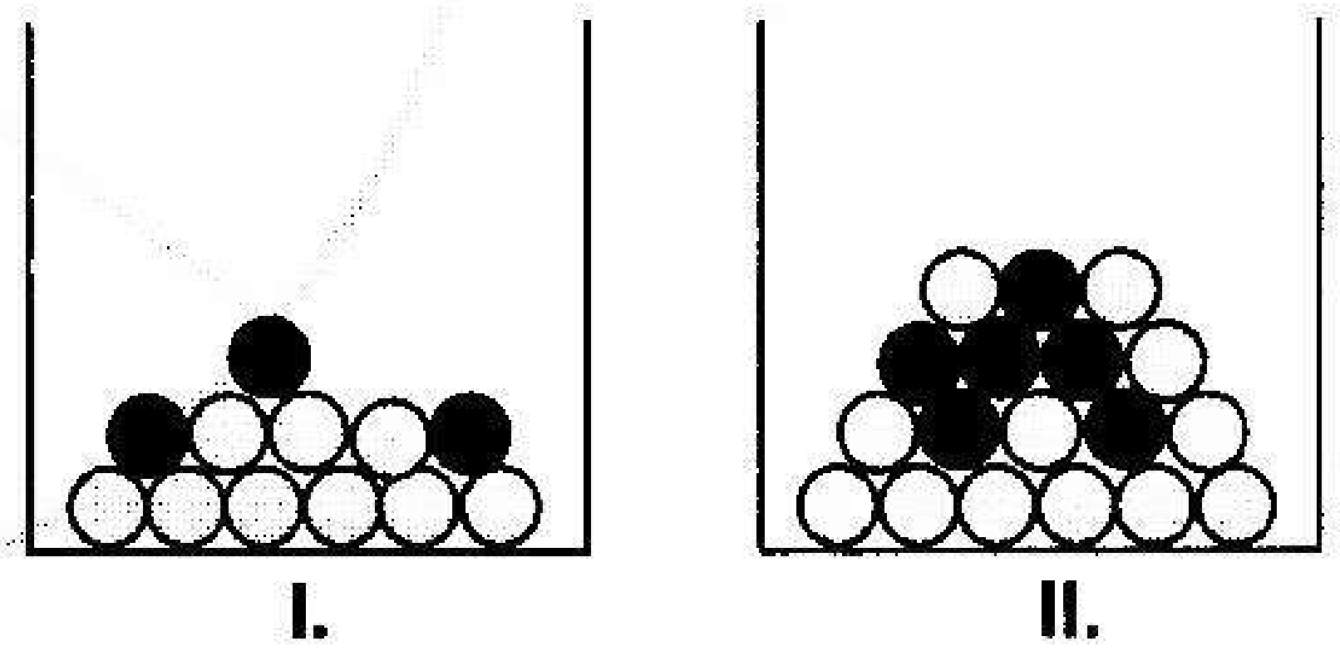
11. A, A, H, T, Y harfleri aynı özellikteki 5 karta yazılarak bir torbaya atılıyor. Kartlar art arda rastgele çekilerek soldan sağa dizildiğinde HATAY sözcüğünü oluşturma olasılığı kaçtır?

The letters of A, A, H, T, Y are written on the some sized and shaped cards and put in a pouch. What is the probability of forming the word HATAY when the cards have picked adjacently and ordered from left to right?

A) $\frac{1}{5}$ B) $\frac{1}{25}$ C) $\frac{1}{30}$

D) $\frac{1}{40}$ E) $\frac{1}{60}$

12.



Şekildeki I. kutuda 3 mavi, 9 yeşil top, II. kutuda is 6 mavi, 12 yeşil top vardır. Buna göre, kutulardan rastgele çekilen 1'er topun aynı renkte olma olasılığı kaçtır?

In the first box, there are 3 blue and 12 green balls and in the second box, there are 6 blue and 12 green balls. According to that, what is the probability of picking the same cobored balls from each of the pouches?

A) 28 B) 26 C) 24

D) 22 E) 20

YANITLAR / ANSWERS

1. C	2. B	3. C	4. E	5. E	6. E	7. E	8. E
9. A	10. C	11. E	12. A				

1. $$\frac{n!}{(n-2)!\cdot 2!}=\frac{n!}{(n-4)!\cdot 24}$$

$$\Rightarrow\ 2\cdot(n-2)!=24\cdot(n-4)!$$

$$2\cdot(n-2)\cdot(n-3)\cdot(n-4)!=24\cdot(n-4)!$$

$$2\cdot(n-2)\cdot(n-3)=24$$

$$n=6$$

Yanıt / The Answer is C

---

2. $$A=\binom{20}{n}\ \Rightarrow \max(A)=n=10$$

$$\binom{10}{a+2}=\binom{10}{4a-7}\ \Rightarrow$$

$a+2=4a-7$ veya(or) $a+2+4a-7=10$

$a+2=4a-7$ $\qquad$ $5a-5=10$

$3a=9$ $\qquad$ $5a=15$

$a=3$ $\qquad$ $a=3$

$$\Rightarrow a=3$$

Yanıt / The Answer is B

---

3. 1. grupta Mehran ile 4 kişi bulunacak, geri kalanlar ise Katya ile birlikte ikinci grubu oluşturacaktır.
Mehran ve Katya'nın yerdeğiştirmesi göz önüne alındığında,

$$2\cdot C(8,4)=2\cdot\frac{8!}{(8-4)!(4!)}=2\cdot 70=140$$

farklı şekilde grup oluşturulabilir.

In the first group; there will be 4 people including Mehran, and the rest will constitute the second group including Katya.

$$2\cdot C(8,4)=2\cdot\frac{8!}{(8-4)!(4!)}=2\cdot 70=140$$

is the number of groups that can be formed ,when the circumstance of switching places of Mehran and Katya is considered.

Yanıt / The answer is C

---

4. Takımda bulunan iki öğrenci A ve B ise, 6 kişi kalan 10 kişi arasından seçilir. Böylece

$C(10,\ 6)=\dfrac{10\cdot 9\cdot 8\cdot 7}{24}=210$ değişik takım oluşturulur.

A katılır B katılmaz veya B katılır A katılmaz.
Bu iki durumda da takıma girecek 1 kişi ile, takıma girmeyecek 1 kişi bellidir. 5 kişi kalan 10 kişi arasından seçilecektir.

$$C(10,\ 5)=2\cdot\frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{120}=2\cdot\frac{9\cdot 8\cdot 7}{2}=72\cdot 7=504$$

O halde, oluşturulacak değişik takım sayısı, toplama ilkesine göre, 210 + 504 = 714 tür.

6 people will be chosen from among 10 people because 2 people in the team is already considered as A and B. According to that;

$C(10,\ 6)=\dfrac{10\cdot 9\cdot 8\cdot 7}{24}=210$ *is the number of teams that can be formed.*
If we consider that when A is in the team B will not participate and when B is in the team A wil not participate, then in both cases, one who will participate in the team and one who will not is certain. According to that 5 people in the team will be chosen from among 10 people.

$$C(10,\ 5)=2\cdot\frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{120}=2\cdot\frac{9\cdot 8\cdot 7}{2}=72\cdot 7=504$$

According to the addition law, the number of teams that can be formed is 210 + 504 = 714.

Yanıt / The answer is E

---

5. 6 kızdan 2 si, $\dbinom{6}{2}=\dfrac{6!}{(6-2)!\cdot 2!}=15$, 3 erkekten 1 i

$\dbinom{3}{1}=\dfrac{3!}{(3-1)!\cdot 1!}=3$ farklı şekilde seçileceğinden çarpma ilkesinden15· 3 = 45 farklı grup oluşturulabilir.

2 girls from among 6 girls can be chosen in $\dbinom{6}{2}=\dfrac{6!}{(6-2)!\cdot 2!}=15$ *ways and 1 boy from among 3 boys can be chosen in*

$\dbinom{3}{1}=\dfrac{3!}{(3-1)!\cdot 1!}=3$ *ways. According to the multiplication law,*

15·3 = 45 different groups can be formed.

Yanıt / The answer is E

6. A ile B arasında 3, B ile C arasında 4 farklı yol olduğuna göre, çarpma ilkesine göre kişi; 3·4 = 12 farklı şekilde A'dan C'ye ulaşabilir. Gidiş-dönüş yapılacağına göre, 12·12 = 144 farklı gidiş-dönüş yapılabilir.

Since there are 3 different roads between A and B, and there are 4 different roads between B and C, according to the multiplication law; a person can travel from A to be in 3·4 =12 different ways. Considering the fact that this will be a round-trip, 12·12 = 144 different round-trips can be done.

Yanıt / The answer is E

7.

yüzler basamağı hundreds place	onlar basamağı tens place	birler basamağı ones place
9	10	5

{1, 3, 5, 7, 9}

⇒ 9·10·5 = 450

Yanıt / The Answer Is E

8. 6 kişiden 4 kişi $\binom{6}{4}$ yolla seçilebilir. 4 kişi yuvarlak bir masa etrafında (4 – 1)! = 3! farklı şekilde oturabileceğinden;

$$\binom{6}{4}\cdot 3! = \frac{6\cdot 5\cdot 4!}{2!\cdot 4!}\cdot 6 = 90$$

4 people from among 7 people can be chosen in $\binom{6}{4}$ ways. Since 4 people be seated around a table in (4 – 1)! = 3! different ways, then the answer is

$$\binom{6}{4}\cdot 3! = \frac{6\cdot 5\cdot 4!}{2!\cdot 4!}\cdot 6 = 90$$

Yanıt / The answer is E

9. Kırmızı kalem / *The number of red pencils* = x tane

Siyah kalem / *The number of black pencils* = x + 4 tane

Toplam kalem / *The sum of the pencils* = 2x + 4 tane

Rastgele çekilen kalemin siyah olma olasılığı

The probability of picking a black pen is

$$\frac{x+4}{2x+4} = \frac{4}{7}$$

$$7x + 28 = 8x + 16$$

$$12 = x \Rightarrow 2\cdot 12 + 4 = 28$$

Yanıt / The answer is A

10. 5 kız, 6 erkekten oluşan bir grupta toplam 11 öğrenci vardır.

There are 11 students in a group that consists of 5 girls and 6 boys.

$$\frac{5}{11}\cdot\frac{4}{10}\cdot\frac{3}{9} = \frac{2}{33}$$

Yanıt / The answer is C

11.

H	A	T	A	Y
$\frac{1}{5}$	$\frac{2}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1

$$\frac{1}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot 1 = \frac{1}{60}$$

Yanıt / The answer is E

12. İki topun da mavi olma olasılığı

The probability of picking two blue balls is

$$\frac{3}{12}\cdot\frac{6}{18} = \frac{1}{12}$$

İki topun da yeşil olma olasılığı

The probability of picking two green balls is

$$\frac{9}{12}\cdot\frac{12}{18} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{12} + \frac{1}{2} = \frac{7}{12}$$

Yanıt / The answer is A

1. $P(7, 3) = ?$

Yanıt / The Answer is: **210**

2. $P(6, 4) + C(4, 2) = ?$

Yanıt / The Answer is: **366**

3. $C(5, 3) + P(5, 5) = ?$

Yanıt / The Answer is: **130**

4. $C(n, 2) + C(n, 3) = 35 \Rightarrow n = ?$

Yanıt / The Answer is: **6**

5. $C(9, 3) - P(5, 2) = ?$

Yanıt / The Answer is: **64**

6. $5 \cdot C(n, 3) = C(n, 4) \Rightarrow n = ?$

Yanıt / The Answer is: **23**

7. **Anne, baba ve 4 çocuktan oluşan bir aile, çocukların bir arada olması koşuluyla, yan yana kaç farklı şekilde sıralanabilir?**
In how many different ways a family that consists of a mother, a father and 4 children may appear side by side if the children must remain together?

Yanıt / The Answer is: **144**

8. **6 mühendis ve 7 öğretmen arasından, bir mühendis ve bir doktor kaç farklı şekilde seçilebilir?**
In how many different ways an engineer and a doctor can be chosen from among 6 engineers and 7 teachers?

Yanıt / The Answer is: **42**

9. **A = {1, 2, 3, 4, 5, 6} kümesinin elemanlarıyla rakamları birbirinden farklı, üç basamaklı kaç farklı sayı yazılabilir?**

How many three-digit numbers such that its digits are different can be written by using the members of the set A = {1, 2, 3, 4, 5, 6} ?

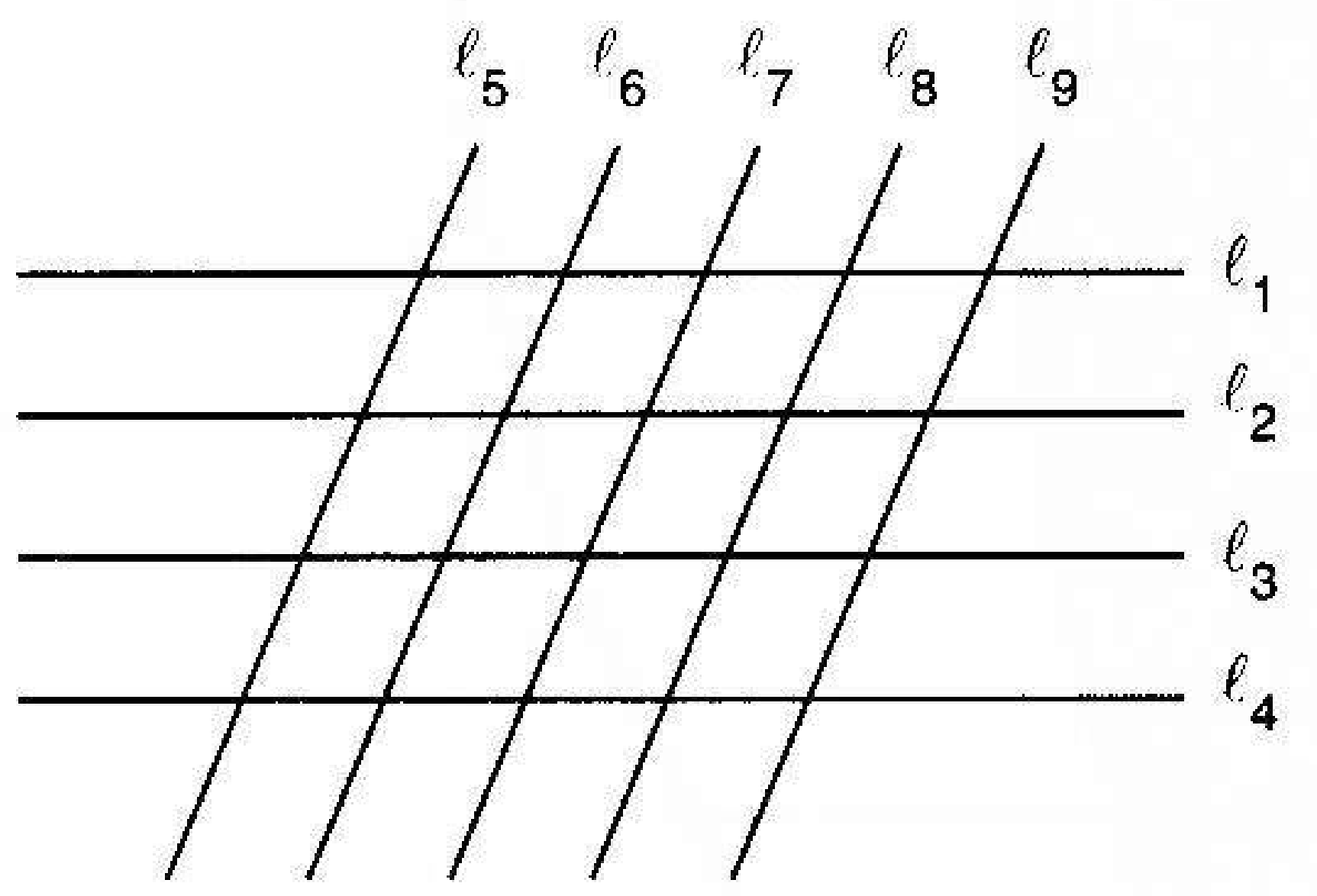
Yanıt / The Answer is: **120**

10. **5 Fransız, 4 İngiliz yuvarlık bir masa etrafında Fransızlar yan yana oturmak koşuluyla kaç farklı şekilde oturabilir?**

In how many different ways can 5 French, and 4 British be seated around a table in condition of French people apprearing side by side?

Yanıt / The Answer is: **$4! \cdot 5!$**

11.

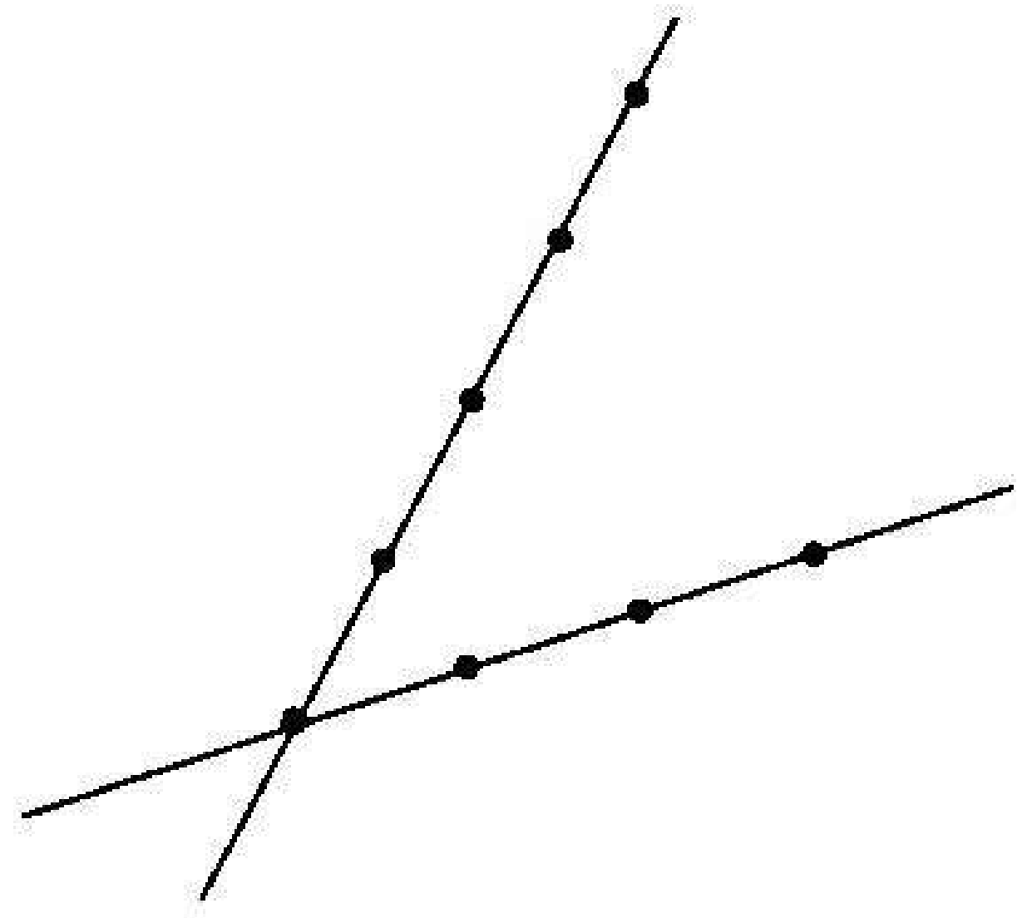


$\ell_1 // \ell_2 // \ell_3 // \ell_4$ ve $\ell_5 // \ell_6 // \ell_7 // \ell_8 // \ell_9$ olduğuna göre, kaç farklı paralelkenar vardır?

If $\ell_1 // \ell_2 // \ell_3 // \ell_4$ and $\ell_5 // \ell_6 // \ell_7 // \ell_8 // \ell_9$, how many different parallelogram in the figure?

Yanıt / The Answer is: **60**

12.



Şekilde 8 noktanın üçünü köşe kabul eden kaç farklı üçgen çizilebilir?

How many different triangles can be drawn by using 3 of the 8 points in the figure as the edges of the triangle?

Yanıt / The Answer is: **42**

13. **K = {1, 2, 3, 4, 5, 6, 7, 8} kümesinin elemanlarından biri rastgele seçiliyor. Bu elemanın çift olma olasılığı kaçtır?**

What is the probability of picking an even number when a member from the set K = {1, 2, 3, 4, 5, 6, 7, 8} is picked randomly?

Yanıt / The Answer is: **1/2**

14. **1'den 100'e kadar numaralandırılmış yüz top bir torbaya yerleştiriliyor. Torbadan rastgele çekilen bir topun çift olduğu bilindiğine göre, 50'den küçük olma olasılığı kaçtır?**

100 balls that are enumerated from 1 to 100 are put in a pouch. If it is known that the number on the ball is even, what is the probability of being smaller than 50?

Yanıt / The Answer is: **12/25**

15. **Ali'nin bir matematik sorusunu çözme olasılığı 1/10 iken, Ahmet'in çözememe olasılığı 1/5'tir. Buna göre aynı soruyu Ahmet'in çözüp, Ali'nin çözememe olasılığı kaçtır?**

The probability of Ali solving a Mathematics question is 1/10 and Ahmet not solving is 1/5. According to that, what is the probability of that question is being solved by Ahmet and not by Ali?

Yanıt / The Answer is: **18/25**

16. **Yüzleri 1'den 6'ya kadar numaralandırılan farklı iki zar düz bir zemine atılıyor. Üste gelen sayılar toplamının, 6 olduğu bilindiğine göre, bu iki sayının eşit olma olasılığı kaçtır?**

Two dice are rolled at the same time. If it is known that the sum of the numbers that appears is 6, what is the probability of being equal for those numbers?

Yanıt / The Answer is: **1/5**

1. **$C(8, 2) + P(3, 2) = ?$**

A) 30

B) 32

C) 34

D) 36

E) 38

2. $P(n, 2) + C(n, 2) = 108 \Rightarrow$ **$n = ?$**

A) 5

B) 6

C) 7

D) 8

E) 9

3. $P(n, 2) + 3 \cdot C(n, 3) = 50 \Rightarrow$ **$n = ?$**

A) 3

B) 4

C) 5

D) 6

E) 7

4. $y \neq 5$,

$P(x, y) = 6! \Rightarrow$ **$x + y = ?$**

A) 8

B) 10

C) 12

D) 14

E) 16

5. $C(n, 0) + C(n, 1) + 2 \cdot C(n, 2) = 37 \Rightarrow$ **$n = ?$**

A) 5

B) 6

C) 7

D) 8

E) 9

6. $\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^8 \Rightarrow$ **$n = ?$**

A) 6

B) 7

C) 8

D) 9

E) 10

7. **5 tabanında üç basamaklı kaç farklı sayı yazılabilir?**
How many different three-digit numbers can be written with the base 5?

A) 5 B) 25 C) 50
D) 75 E) 100

8. **1, 2, 3, 4 ve 5 rakamları kullanılarak rakamları birbirinden farklı iki basamaklı kaç sayı yazılabilir?**
How many different two-digit numbers such that its digits are different can be written by using the digits 1, 2, 3, 4 and 5?

A) 5
B) 10
C) 12
D) 20
E) 25

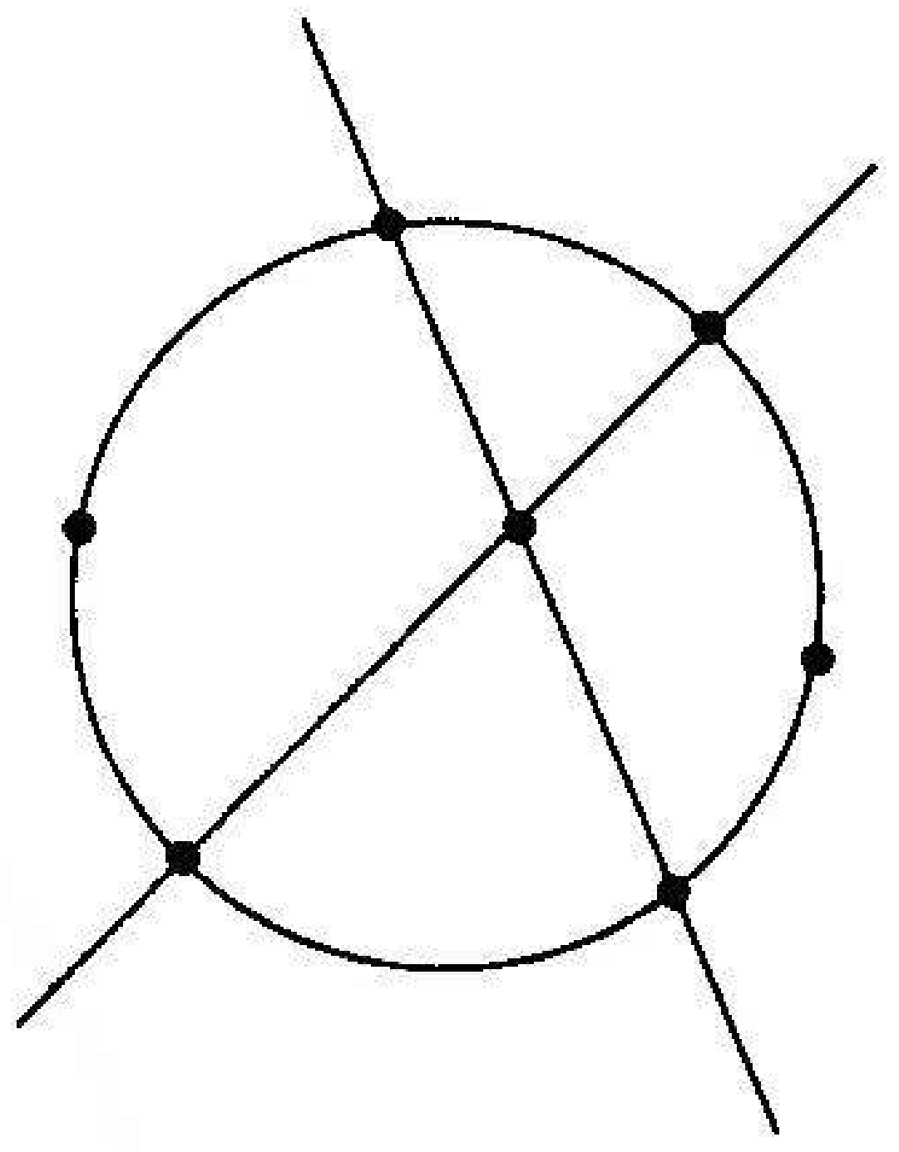
9. **METROPOL sözcüğündeki harflerin yerleri değiştirilerek anlamlı ya da anlamsız kaç sözcük yazılabilir?**
How many different meaningful or meaningless words can be written by using the letters of the word METROPOL?

A)120 B) 6! C) $2 \cdot 6!$
D) $\frac{8!}{2}$ E) $\frac{8!}{2! \cdot 2!}$

10. **6 öğrenci içerisinden 4 kişilik bir grup ve bu grup içerisinden de bir başkan kaç farklı şekilde seçilebilir?**
In how many different ways a group of 4 people from among 6 students and a head within the group can be chosen?

A) 10 B) 24 C) 30
D) 45 E) 60

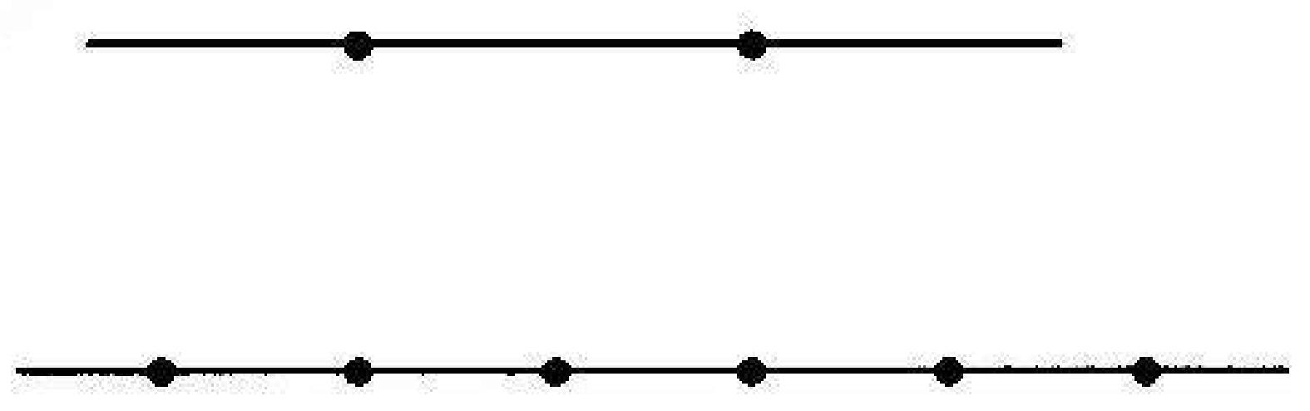
11.



Köşeleri şekildeki 7 noktadan herhangi üçü olan kaç farklı üçgen çizilebilir?
How many different triangles can be drawn by using the points in the figure as the edges of the triangles?

A) 30 B) 33 C) 35
D) 49 E) 52

12.



Şekildeki paralel iki doğru üzerinde 8 nokta verilmiştir. Bu 8 noktadan üçü köşesi olan kaç farklı üçgen çizilebilir?
8 points located on two parallel lines given above. How many different triangles can be drawn by using those points as the edges on triangles?

A) 64 B) 36 C) 24
D) 16 E) 8

YANITLAR / ANSWERS

1. B	2. E	3. C	4. C	5. B	6. C	7. E	8. D
9. D	10. E	11. B	12. B				

1. **Düzgün bir zar atılıyor. Üste gelen yüzünün 5'ten küçük olma olasılığı kaçtır?**

What is the probability of getting a number smaller than 5 when a dice is rolled?

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) $\frac{1}{2}$

2. **Bir torbada 2 kırmızı, 5 beyaz, 7 mavi bilye vardır. Torbadan rastgele çekilen bilyenin kırmızı olma olasılığı kaçtır?**

There are 2 red, 5 white, and 7 marbles in a pouch. What is the probability of getting a red marble when a marble is picked randomly from the pouch?

A) $\frac{1}{14}$ B) $\frac{1}{7}$ C) $\frac{3}{14}$ D) $\frac{5}{14}$ E) $\frac{2}{7}$

3. **4 çift, 5 tek sayıdan rastgele seçilen iki sayının toplamının tek olma olasılığı kaçtır?**

What is the probability of picking two numbers such that their sum is an odd number, when two numbers are chosen from among 4 even and 5 odd numbers?

A) $\frac{1}{4}$ B) $\frac{1}{5}$ C) $\frac{1}{10}$ D) $\frac{1}{20}$ E) $\frac{5}{9}$

4. **Üç madeni para atılıyor. Bunlardan ikisinin yazı, birinin tura gelme olasılığı kaçtır?**

What is the probability of getting two tails and one heads when 3 coins are tossed?

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{8}$ E) $\frac{5}{8}$

5. **Bir sınıfta 15 kız, 10 erkek öğrenci vardır. Buna göre, sınıftan rastgele seçilen bir öğrencinin kız olma olasılığı kaçtır?**

There are 15 girls and 10 boys in a classroom. What is the probability of choosing a girl when a student is chosen randomly from among the classroom?

A) $\frac{1}{25}$ B) $\frac{1}{20}$ C) $\frac{1}{15}$ D) $\frac{1}{10}$ E) $\frac{3}{5}$

6. **1'den 6'ya kadar numaralandırılan bir zar düz bir zemine atılıyor. Buna göre, üste gelen yüzün asal sayı olma olasılığı kaçtır?**

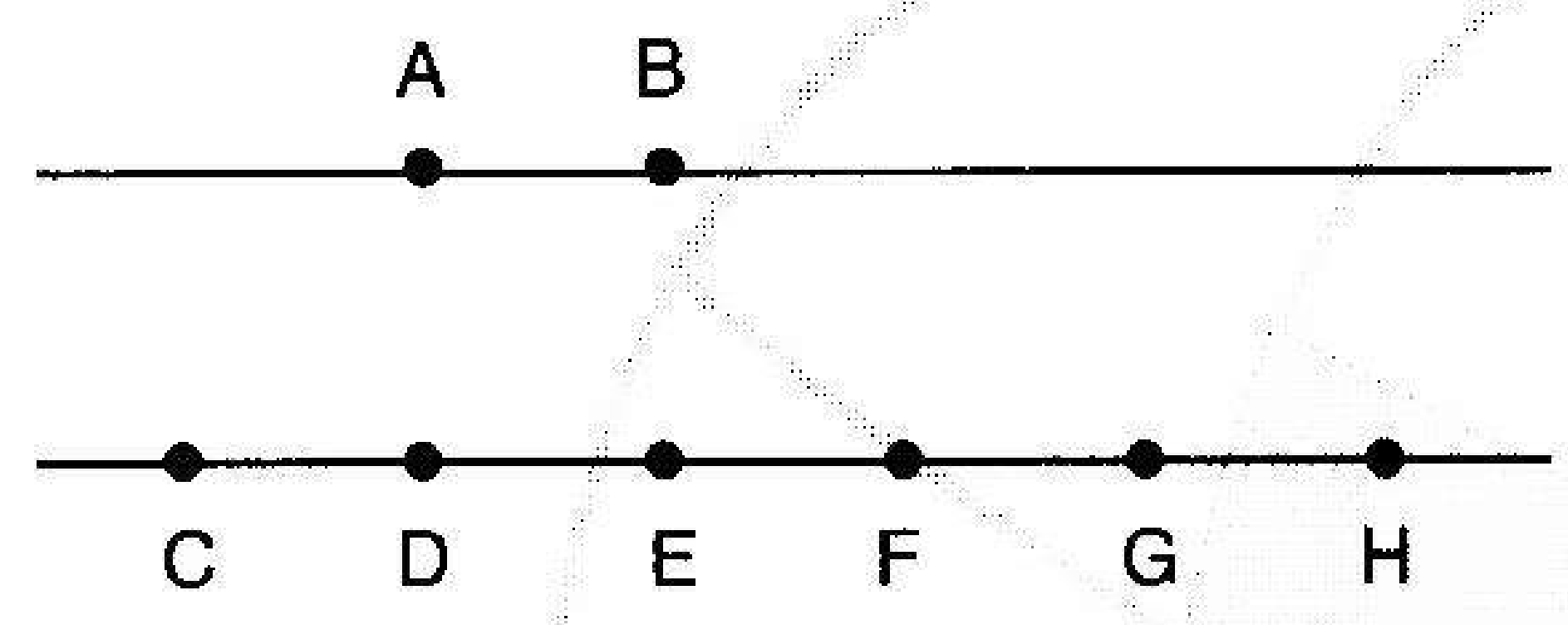
What is the probability of getting a prime number when a dice is rolled?

A) $\frac{1}{6}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

7. $P(A)=\frac{3}{5} \Rightarrow P(A')=?$

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) $\frac{1}{2}$

8.



Şekildeki paralel doğrular üzerinde bulunan 8 tane noktadan rastgele seçilen 3 tanesinin bir üçgen oluşturma olasılığı kaçtır?

3 points are chosen from among the points that located on the parallel lines above. What is the probability of choosing 3 points that constitutes a triangle?

A) $\frac{1}{8}$ B) $\frac{1}{64}$ C) $\frac{9}{14}$ D) $\frac{9}{64}$ E) $\frac{2}{9}$

9. **6 kadın 4 erkekten oluşan bir grup içinden rastgele 3 kişi seçiliyor. Buna göre, kişilerin hepsinin erkek olma olasılığı kaçtır?**

3 persons are chosen randomly from among a group of 6 women and 4 men. What is the probability of choosing 3 men?

A) $\frac{2}{3}$ B) $\frac{4}{7}$ C) $\frac{1}{10}$ D) $\frac{1}{20}$ E) $\frac{1}{30}$

10. **Samet ve Selçuk'un bir hedefi vurma olasılığı sırasıyla 2/3 ve 3/4'tür. Buna göre, Samet'in hedefi vurup, Selçuk'un vuramama olasılığı kaçtır?**

The probability of hitting a target by Samet and Selçuk is 2/3 and 3/4, respectively. What is the probability of hitting the target by Samet and missing the target by Selçuk when they shoot at the same time?

A) $\frac{1}{8}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

11. 1 3 6 7 9 10

Şekildeki gibi üzerine sayıların yazıldığı kartlardan rastgele seçilen ikisinin çarpımının çift sayı olma olasılığı kaçtır?

What is the probability of choosing 2 numbers such that their product is an even number, from among the cards given in the figure?

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) $\frac{5}{6}$

12. **Bir gruptaki kadınların sayısı erkeklerin sayısından 3 fazladır. Grupta rastgele seçilen birinin kadın olma olasılığı 8/13 ise, bu grupta kaç erkek vardır?**

In a group, the number of women exceeds the number of men by 3. How many men are there in this group if the probability of choosing a woman when a person is chosen randomly from among the group is 8/13?

A) 5 B) 6 C) 7 D) 8 E) 9

YANITLAR / ANSWERS							
1. D	**2. B**	**3. E**	**4. D**	**5. E**	**6. E**	**7. B**	**8. C**
9. E	**10. B**	**11. C**	**12. A**				

1. **Bir torbada 1 den 6 ya kadar numaralandırılmış 6 bilye vardır. Bu torbadan çekileni tekrar torbaya koymama koşulu ile sırayla üç bilye çekiliyor. Birinci çekilişte 2 numaralı, ikinci çekilişte 3 numaralı, üçüncü çekilişte 5 numaralı bilyenin çekilmesi olasılığı kaçtır?**
There are 6 marbles which enumerated from 1 to 6 in a pouch. 3 marbles are picked successively from the pouch in condition of not putting back. What is the probability of picking the marble number 2 at the first picking, picking the marble number 3 at the second picking and picking the marble number 5 at the third picking?

A) $\frac{9}{20}$ B) $\frac{37}{60}$ C) $\frac{1}{60}$ D) $\frac{1}{9}$ E) $\frac{1}{120}$

2. **Bir köprüden geçiş ücreti olarak her bir kamyondan 5 TL, dolmuştan 3 TL ve taksiden 2 TL alınmaktadır. Köprüden 2 dolmuş, 2 taksi ve 3 kamyon geçiyor. Köprüden geçen bu 7 araçtan herhangi 5 inden alınan toplam paranın 20 TL olma olasılığı kaçtır?**
The toll for a bridge is 5 tl for a truck, 3 tl for a minibus and 2 tl for a car. If 2 minibus, 2 cars and 3 trucks pass this bridge, what is the probability of getting 20 tl from any of the 5 vehicles from among them?

A) $\frac{1}{7}$ B) $\frac{3}{7}$ C) $\frac{5}{7}$ D) $\frac{4}{21}$ E) $\frac{5}{21}$

3. **Bir sınıfta 8 erkek öğrenci ve 6 kız öğrenci vardır. Bu öğrencilerden seçilen 6 kişilik kurulda, 4 erkek öğrenci ve 2 kız öğrencinin bulunması olasılığı kaçtır?**
There are 8 male, 6 female students in a class. What is the probability of forming a committee that contains 4 male and 2 female students from among the studens in the class?

A) $\frac{25}{78}$ B) $\frac{26}{35}$ C) $\frac{15}{76}$ D) $\frac{5}{34}$ E) $\frac{50}{143}$

4. **Bir sınıfın 18 erkek, 16 kız öğrencisi pikniğe gitmişler ve pikniğe giden kızların ve erkeklerin yarısı güneşten yanıyor. Bu 34 öğrenci arasından rastgele seçilen bir öğrencinin kız ya da güneşten yanmış olma olasılığı kaçtır?**
18 male, 16 female students of a classroom went to picnic and half of the female students and half of the male students had sunburn. What is the probability of choosing 1 male and 1 female students who had sunburn from among these 34 students?

A) $\frac{1}{2}$ B) $\frac{8}{17}$ C) $\frac{4}{17}$ D) $\frac{6}{17}$ E) $\frac{25}{34}$

5. **Üç madeni para birlikte atılıyor. Bu 3 paradan herhangi ikisinin tura, birinin yazı gelme olasılığı kaçtır?**
3 coins are tossed simultaneously. What is the probability of getting 2 heads and 1 tails?

A) $\frac{1}{2}$ B) $\frac{5}{8}$ C) $\frac{3}{8}$ D) $\frac{2}{3}$ E) $\frac{1}{4}$

6. **Bir öğrencinin, matematik sınavından geçme olasılığı 5/6, fizik sınavından geçme olasılığı 1/4 tür. Bu öğrencinin iki sınavdan da geçme olasılığı 1/2 olduğuna göre, bu öğrencinin matematik veya fizik dersinden başarılı olma olasılığı kaçtır?**
For a student; the probability of passing the mathematics test is 5/6, passing the physics test is 1/4. If the probability of passing both lessons is 1/2, what is the probability of passing the mathematics test or the physics test?

A) $\frac{19}{24}$ B) $\frac{13}{24}$ C) $\frac{11}{12}$ D) $\frac{7}{12}$ E) $\frac{3}{4}$

7. A, B, C isimli üç at yarışacaktır. A ve C nin kendi aralarında yarışı kazanma olasılıkları aynı olduğu halde her birinin B ye göre kazanma olasılığı, B nin kazanma olasılığının üç katıdır. Buna göre, C veya B atının yarışı kazanma olasılığı kaçtır?

Three horses, named as A,B, and C, participated in a race. If the probability of winning the race of A and C is equal and trebled the probability of winning the race of B. According to that, what is the probability of winning the race of B or C?

A) $\frac{3}{49}$ B) $\frac{4}{49}$ C) $\frac{3}{14}$ D) $\frac{8}{7}$ E) $\frac{4}{7}$

8. A ve B gibi iki atıcıdan, A nın hedefi vurma olasılığı 1/4 ve B'nin hedefi vurma olasılığı ise 3/5'tir. İki atıcı birlikte hedefe ateş ediyorlar ve hedef vuruluyor. Buna göre, hedefi yalnız birinin vurmuş olması olasılığı kaçtır?

For two shooters named as A and B, the probability of hitting the target of A is 1/4 and the probability of hitting the target of B is 3/5. Both of the shooters shoot simultaneously and the target is hit. According to that, what is the probability of being hit for the target by only one of the shooters?

A) $\frac{1}{10}$ B) $\frac{7}{20}$ C) $\frac{11}{20}$ D) $\frac{9}{20}$ E) $\frac{1}{2}$

9. Ali'nin bir sınavdaki sorulardan her birini çözme olasılığı çözememe olasılığının 3 katıdır. 4 soruluk bir sınavda Ali'nin soruların hepsini çözmüş olma olasılığı nedir?

The probability of Ali to solve each of the questions in an exam is trebled the probability of him not to. In a test that contains 4 questions, what is the probability of Ali to solve all of the questions?

A) $\frac{9}{16}$ B) $\frac{27}{64}$ C) $\frac{9}{32}$ D) $\frac{3}{8}$ E) $\frac{81}{256}$

10. Hilesiz bir para 6 kez atılıyor. 3 kez tura gelme olasılığı kaçtır?

What is the probability of getting 3 heads when a coin is tossed 6 times?

A) $\frac{1}{16}$ B) $\frac{3}{16}$ C) $\frac{5}{16}$ D) $\frac{7}{16}$ E) $\frac{9}{16}$

11. Bir zar peşpeşe 3 kez atılıyor. Bu atışlardan 2 sinde 3, birinde 3 den farklı bir sayı gelme olasılığı nedir?

What is the probability of getting 3 two times and getting another number for the rest when a dice is rolled 6 times?

A) $\frac{1}{72}$ B) $\frac{5}{72}$ C) $\frac{7}{72}$ D) $\frac{11}{72}$ E) $\frac{13}{72}$

12. Bir zarın bir yüzü sarı, iki yüzü mavi, üç yüzü de kırmızıdır. Bu zar iki kez atılırsa, iki atış sonunda bir kez mavi bir kez de kırmızı yüzün üzerine düşme olasılığı kaçtır?

What is the probability of falling down one time on the blue face and one time on the red face when a dice which has one yellow, two blue, and three red faces rolled two times?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

YANITLAR / ANSWERS

1. E 2. D 3. E 4. C 5. C 6. D 7. E 8. C
9. E 10. C 11. B 12. E

KÜMELER
SETS

BÖLÜM 2
CHAPTER

Tanım: Nesnelerin iyi tanımlanmış bir listesidir. Kümeyi oluşturan nesnelere *kümenin elemanları* denir. Bir A kümesinin eleman sayısı n(A) veya s(A) ile biçiminde gösterilir.

$\in$ kümenin elemanı

$\notin$ kümenin elemanı değil diye okunur.

$A = \{1, 3, a, b\}$ kümesi için $3 \in A$, fakat $4 \notin A$ dır.

***Definition:** A set is a well defined collection of objects. Every item within the brackets is called an element(or member) of the set. The number of elements of a set A is represented by n(A) or s(A).*

$\in$ reads is an element of

$\notin$ reads is not an element of

For the set $A = \{1, 3, a, b\}$ we can say $3 \in A$, but $4 \notin A$.

NOT / NOTE

{ } veya $\varnothing$ **boş küme** olarak adlandırılır ve hiç bir eleman içermez.

*{ } or $\varnothing$ is called the **empty set** and contains no elements.*

$A = \{\ \} \Rightarrow n(A) = 0$

$A = \{\varnothing\} \Rightarrow n(A) = 1$

ALT KÜME / SUBSET

Bir A kümesinin her elemanı, bir B kümesinin de elemanı ise A'ya B'nin *alt kümesi* denir.

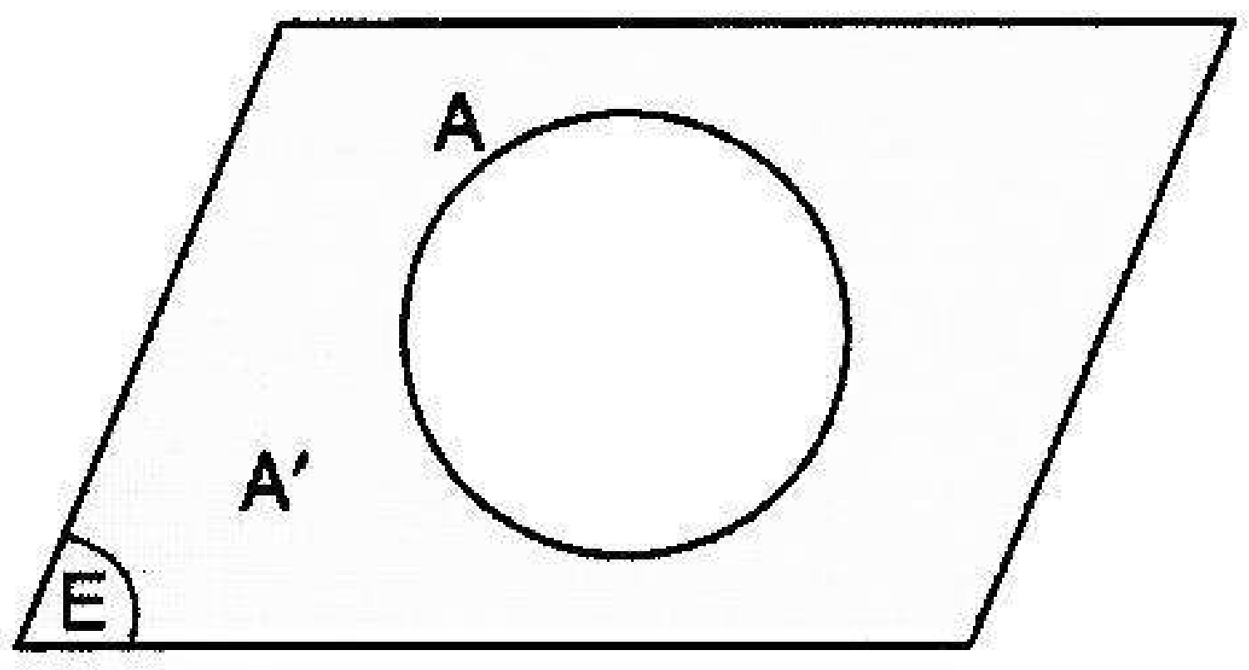
$A \subset B$ biçiminde gösterilir.

If every element in A is also an element of B, then we say that A is a subset of the set B. It is represented by $A \subset B$.

Alt Kümenin Özellikleri / Properties of a Subset

1. Her küme, kendisinin alt kümesidir ($A \subset A$).
   Every set is subset of itself ($A \subset A$).
2. Boş küme, her kümenin alt kümesidir ($\varnothing \subset A$).
   Empty set is subset of every set ($\varnothing \subset A$).
3. $A \subset B$ ve *(and)* $B \subset C \Rightarrow A \subset C$
4. $A \subset B$ ve *(and)* $B \subset A \Rightarrow A = B$
5. n elemanlı bir kümenin alt kümelerinin sayısı 2^n dir.
   The number of subsets of a set with n elements is equal to 2^n.

TÜMLEME / COMPLEMENTARY SET



Üzerinde işlem yapılan tüm kümeleri kapsayan kümeye *evrensel küme* denir. Evrensel küme **E** veya **U** ile gösterilir.

E kümesinin A kümesinde olmayan elemanlarının oluşturduğu kümeye *A kümesinin tümleyeni* denir. A kümesinin tümleyeni A′ ile gösterilir.

A set comprising all sets, on which operations are done, is called the universal set and represented by E or U.

The complement of A is the set of all elements of E which are not in A. The complement of A is denoted by A′.

Tümleyenin Özellikleri / Properties of Complementary Sets

1. $E' = \varnothing$
2. $\varnothing' = E$
3. $(A')' = A$
4. $A \subset B \Leftrightarrow B' \subset A'$

ÖRNEK / EXAMPLE 1

$n(A) + n(B') = 17$

$n(A') + n(B) = 13 \Rightarrow$ **n(E) = ?**

A) 14 B) 15 C) 16 D) 17 E) 18

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

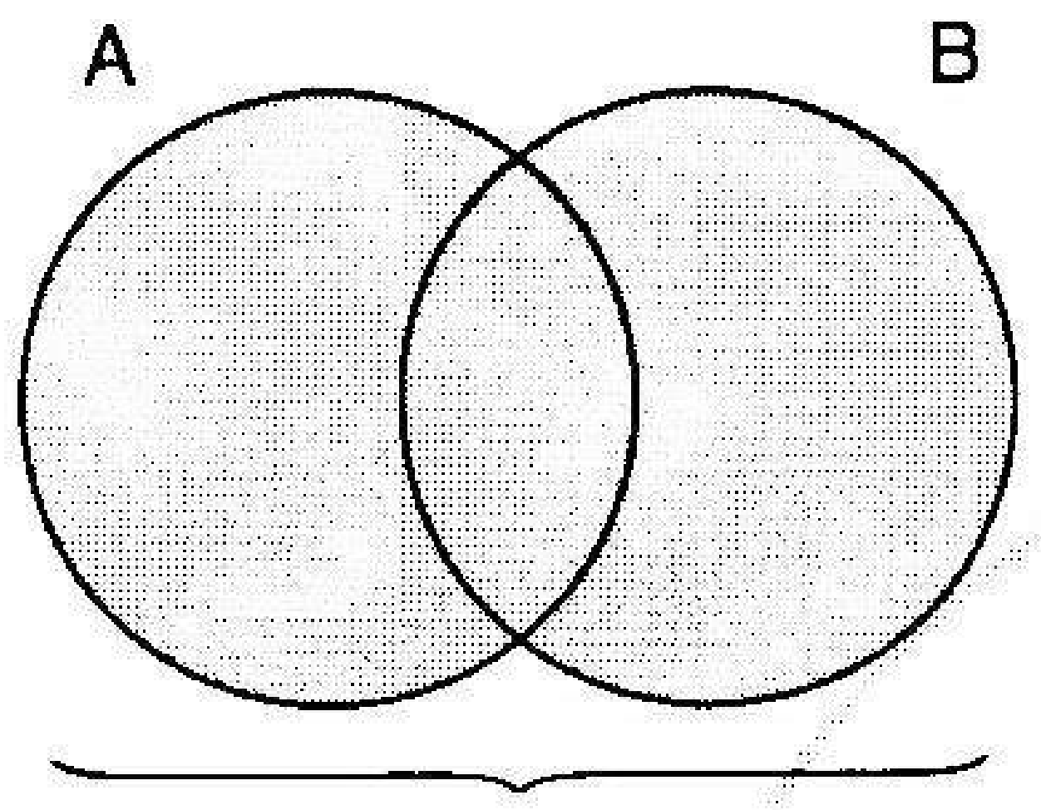
$$\begin{array}{rr} & n(A) + n(B') = 17 \\ + & n(A') + n(B) = 13 \\ \hline & n(E) + n(E) = 30 \\ & 2 \cdot n(E) = 30 \\ & n(E) = 15 \end{array}$$

BİRLEŞİM / UNION

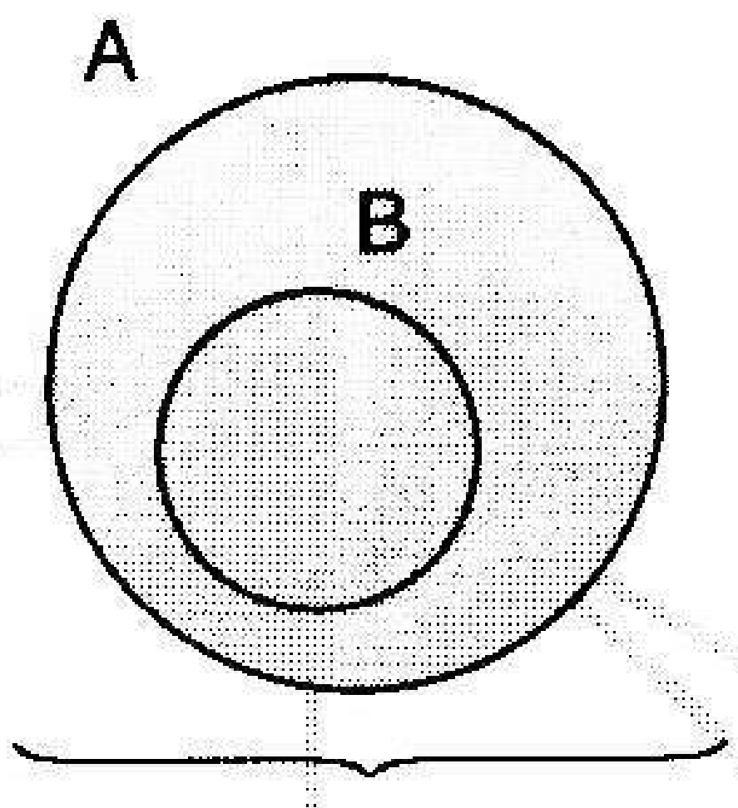
A ve B kümeleri için ortak elemanları bir kez kullanmak üzere A nın ve B nin elemanlarından oluşturduğumuz yeni kümeye A ile B nin *birleşim kümesi* denir. $A \cup B$ ile gösterilir.

$A \cup B$ is the union of sets A and B and is the set made up of all elements which are in A or B.

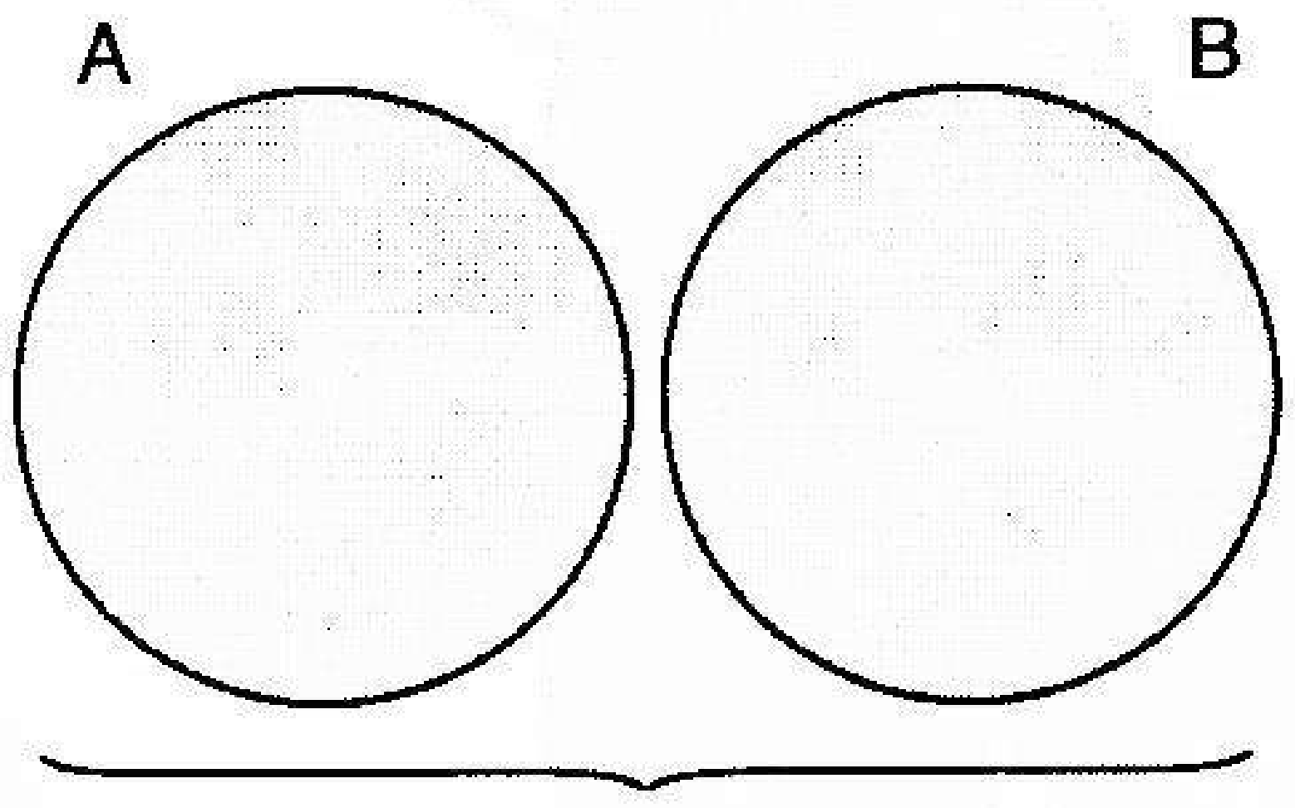
$$\mathbf{A \cup B = \{x \mid x \in A \text{ veya } \textit{(or)}\ x \in B\}}$$



$A \cup B$



$B \subset A \Rightarrow A \cup B = A$



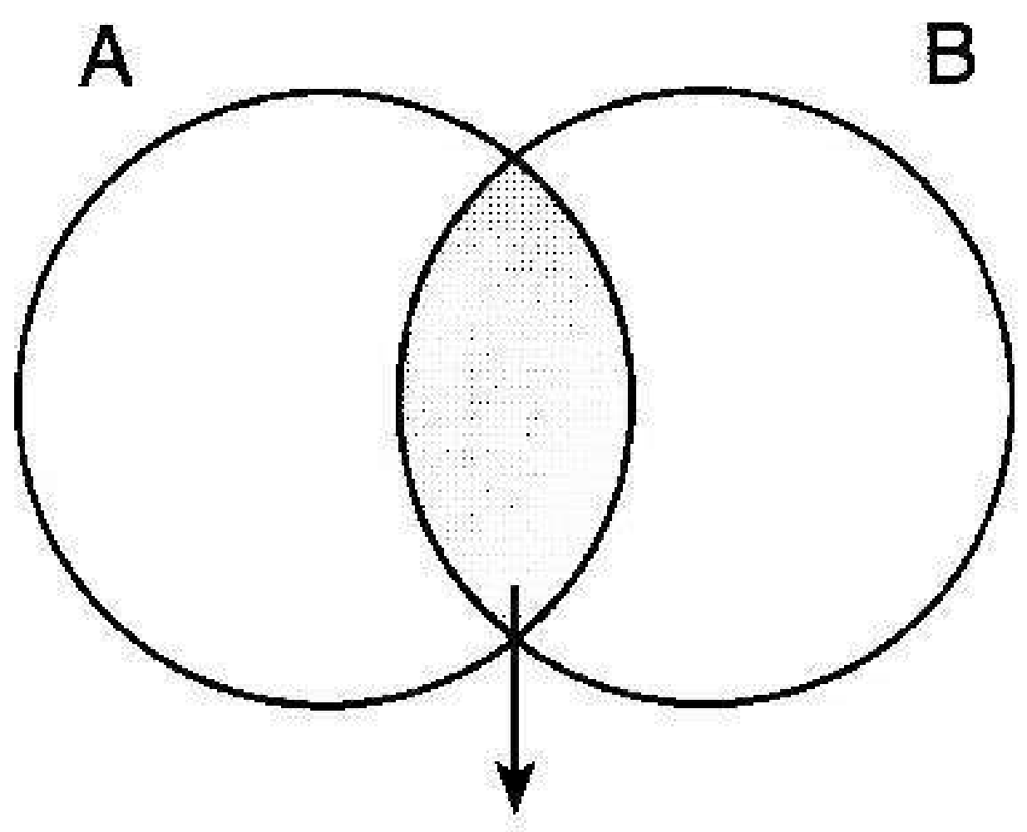
$A \cup B$

KESİŞİM / INTERSECTION

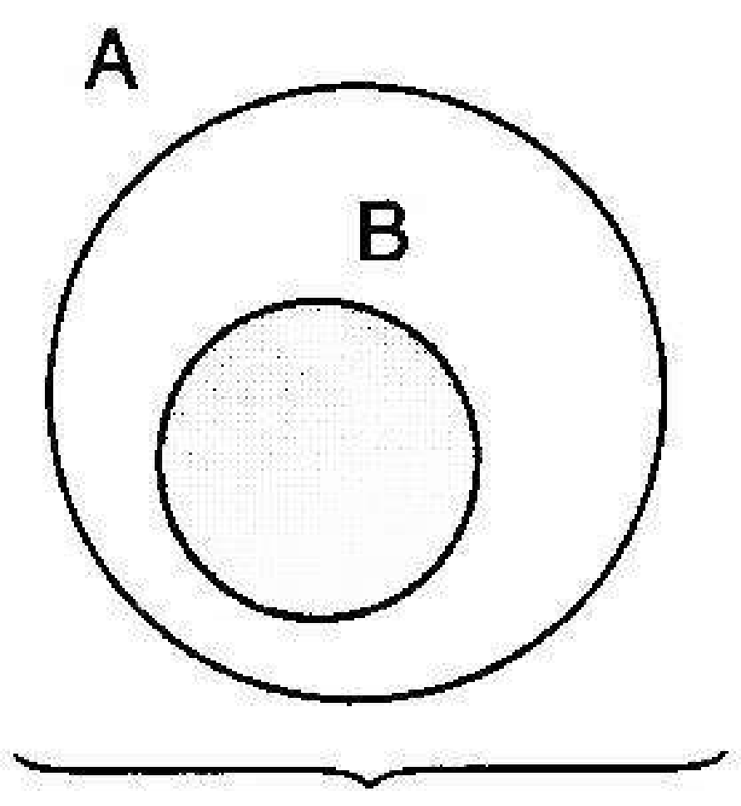
A ve B kümeleri için bu iki kümenin ortak olan elemanlarının oluşturduğu kümeye A ile B nin *kesişim kümesi* denir. $A \cap B$ ile gösterilir.

$A \cap B$ is the intersection of sets A and B and is the set made up of all elements which are in both A and B.

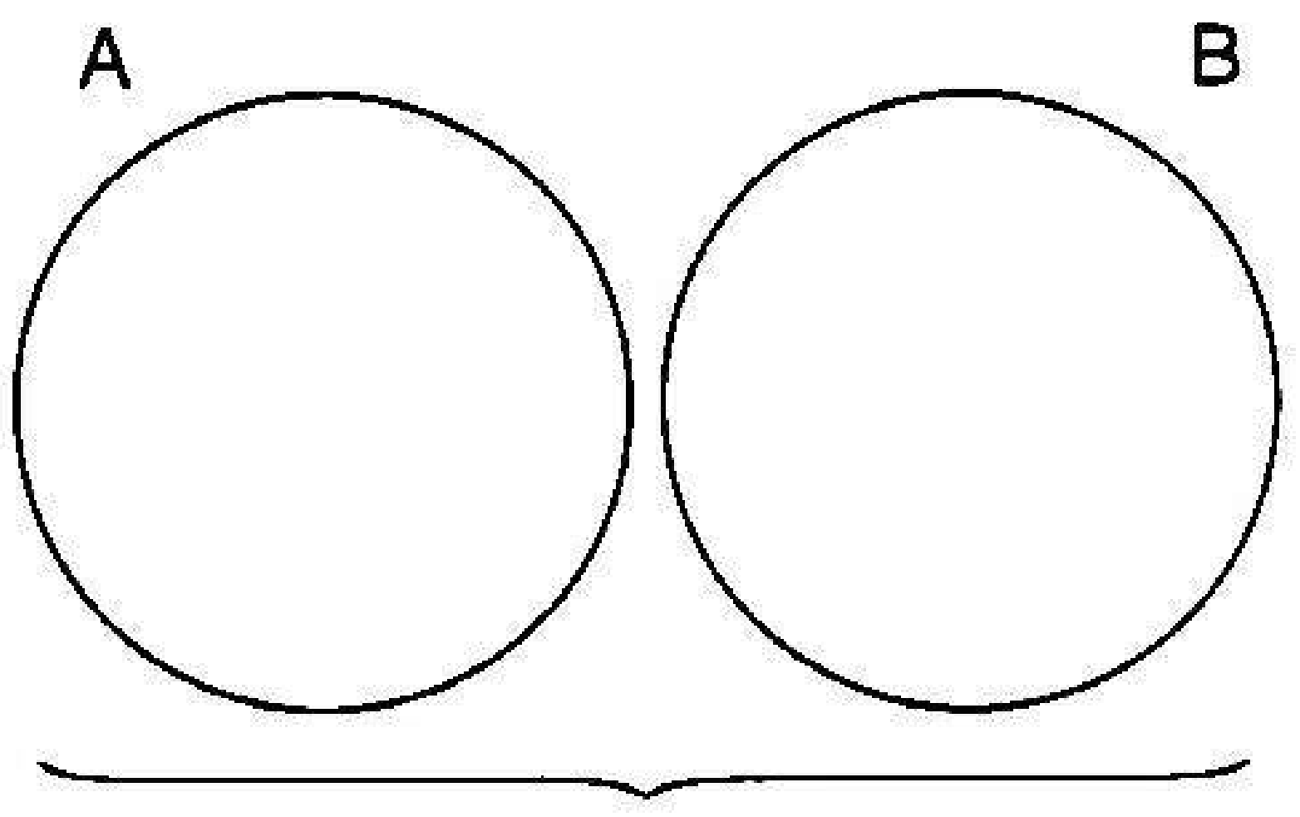
$$\mathbf{A \cap B = \{x \mid x \in A \text{ ve } \textit{(and)}\ x \in B\}}$$



$A \cap B$



$B \subset A \Rightarrow A \cap B = B$



$A \cap B = \varnothing$

Birleşim ve Kesişimin Özellikleri / Properties of Union and Intersection

1. $A \cup B = B \cup A$

$A \cap B = B \cap A$

2. $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

3. $A \cup A = A$ ve (*and*) $A \cap A = A$

4. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. $(A \cup B)' = A' \cap B'$ ve (*and*) $(A \cap B)' = A' \cup B'$ [De Morgan kuralları / *De Morgan rules*]

6. $A \cup \emptyset = A$ ve (*and*) $A \cap \emptyset = \emptyset$

7. $A \cup E = E$ ve (*and*) $A \cap E = A$

8. $A \cup A' = E$ ve (*and*) $A \cap A' = \emptyset$

9. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

10. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

11. $n(A) + n(A') = n(E)$

ÖRNEK / EXAMPLE 2

$n(A) = 2x + 5$, $n(B) = 7 + x$

$n(A \cap B) = 3x + 1 \Rightarrow$ **$n(A \cup B) = ?$**

A) 8 B) 11 C) 12 D) 16 E) 19

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A) = 2x + 5$,

$n(B) = 7 + x$,

$$n(A \cap B) = 3x + 1 \Rightarrow \begin{aligned} n(A \cup B) &= 2x + 5 + 7 + x - (3x + 1) \\ &= 3x + 12 - 3x - 1 \\ n(A \cup B) &= 11 \end{aligned}$$

PEKİŞTİRME / REINFORCE

$n(A) = 7$,

$n(B) = 8$,

$n(A \cup B) = 9 \Rightarrow n(A \cap B) = ?$

YANIT / ANSWER : 6

ÖRNEK / EXAMPLE 3

$A \not\subset B$,

$n(A) = 5$,

$n(B) = 8$

$\Rightarrow$ $\max(n(A \cup B)) + \min(n(A \cup B)) = ?$

A) 21 B) 22 C) 23 D) 24 E) 25

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\max(n(A \cup B)) = n(A) + n(B) - (n(A \cap B))$

$= 5 + 8 - 0 = 13$

$\min(n(A \cup B)) = n(A) + n(B) - \min(n(A \cap B))$

$= 5 + 8 - 4 = 9$

$\Rightarrow \max\ (n(A \cup B)) + \min\ (n(A \cap B)) = 13 + 9 = 22$

ÖRNEK / EXAMPLE 4

$[(A \cap B) \cup (A \cap B')]' \cap A = ?$

A) $\varnothing$ B) A C) A′ D) B E) E

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

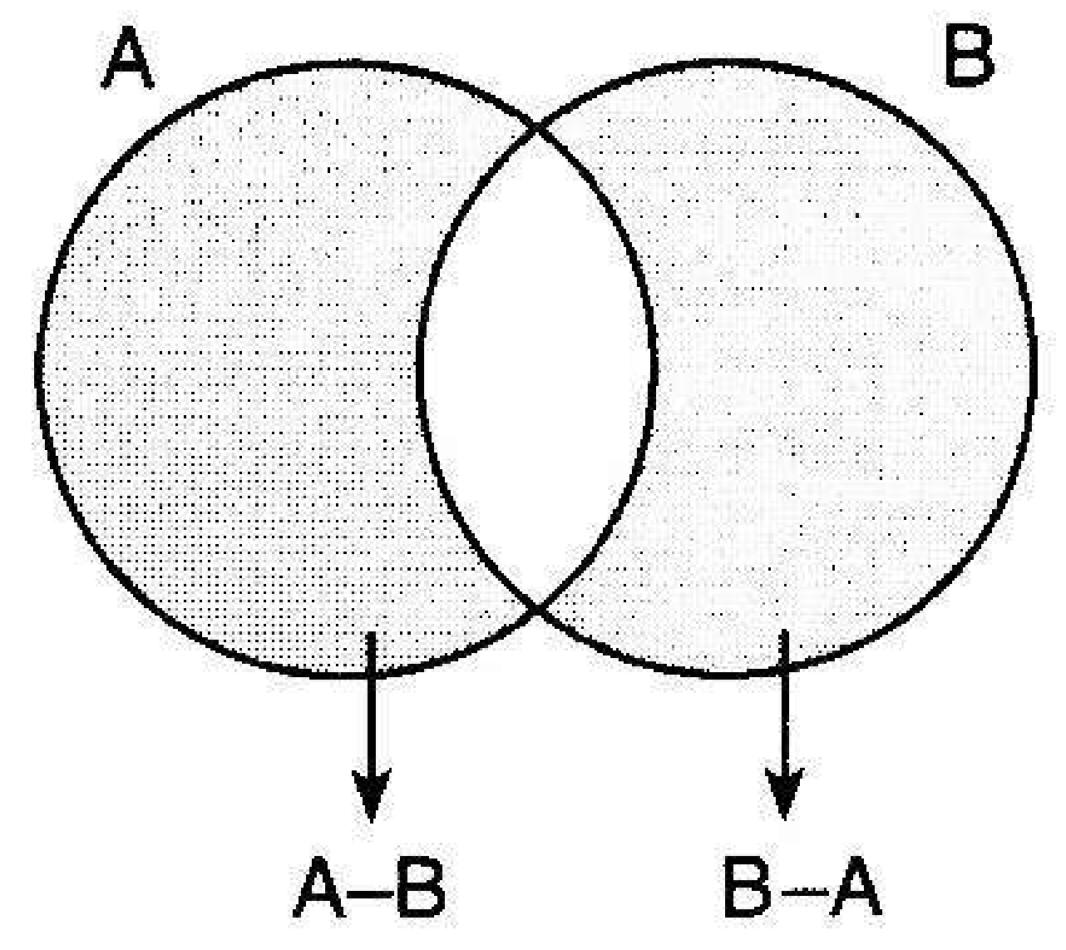
$$[(A \cap B) \cup (A \cap B')]' \cap A = [A \cup (B \cap B')]' \cap A$$

$$= [A \cup \varnothing]' \cap A$$

$$= A' \cap A$$

$$= \varnothing$$

FARK İŞLEMİ / SUBTRACTION



A ve B kümeleri için A'nın elemanı olup B'nin elemanı olmayan elemanların oluşturduğu kümeye *A fark B* denir. A–B ya da A\B biçiminde gösterilir.

For the sets A and B, a set composed of elements that exist in A but, do not exist in B is called A minus B. It is represented by A–B or A \ B.

$$\mathbf{A - B = A \setminus B = \{x \mid x \in A \text{ ve } \textit{(and)}\ x \notin B\}}$$

Fark İşleminin Özellikleri / Properties of Subtraction

1. $A - B = A \cap B'$
2. $A \subset B \Rightarrow A - B = \varnothing$
3. $A - B = A - (A \cap B)$
4. $E - A = A'$
5. $A - \varnothing = A$
6. $(A - B)' = (A \cap B')' = A' \cup B$
7. $(A - B) \cup B = A \cup B$

ÖRNEK / EXAMPLE 5

$n(A \cup B) = 21$, $n(A \cap B) = 3$,

$n(A) = 3 \cdot n(B) \Rightarrow$ **$n(B \setminus A) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cap B) = n(A) + n(B) - 3 = 21$

$n(A) = 3 \cdot n(B) \Rightarrow 3 \cdot n(B) + n(B) = 24$

$n(B) = 6$

$n(B \setminus A) = n(B) - n(A \cap B) = 6 - 3 = 3$

PEKİŞTİRME / REINFORCE

$n(A - B) = 3$,

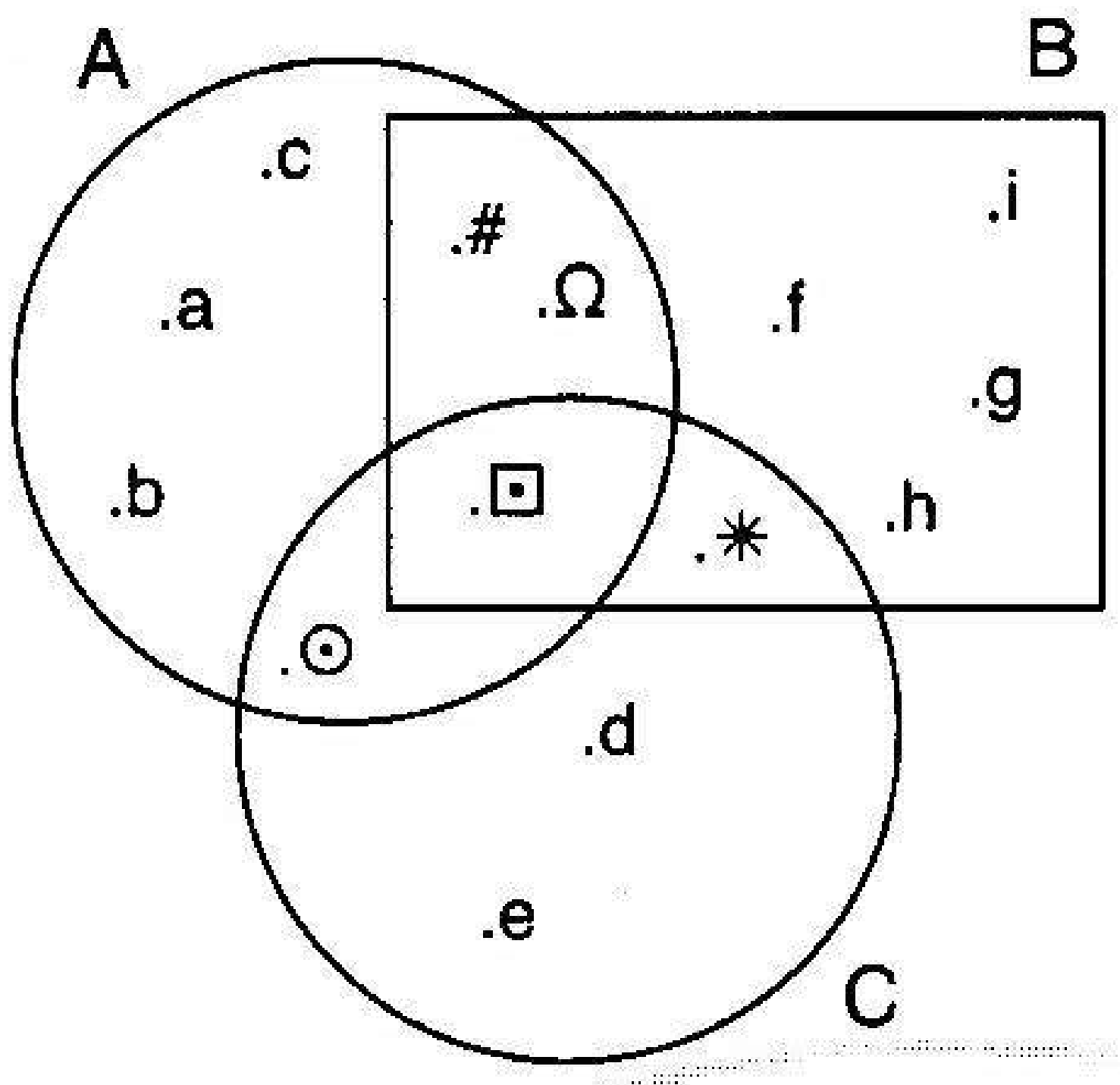
$n(A \cap B) = 2$,

$n(B) = 2 \cdot n(A) \Rightarrow n(A \cup B) = ?$

YANIT / ANSWER : 13

ÖRNEK / EXAMPLE 6

İSTANBUL ÜNİ. YÖS 2013

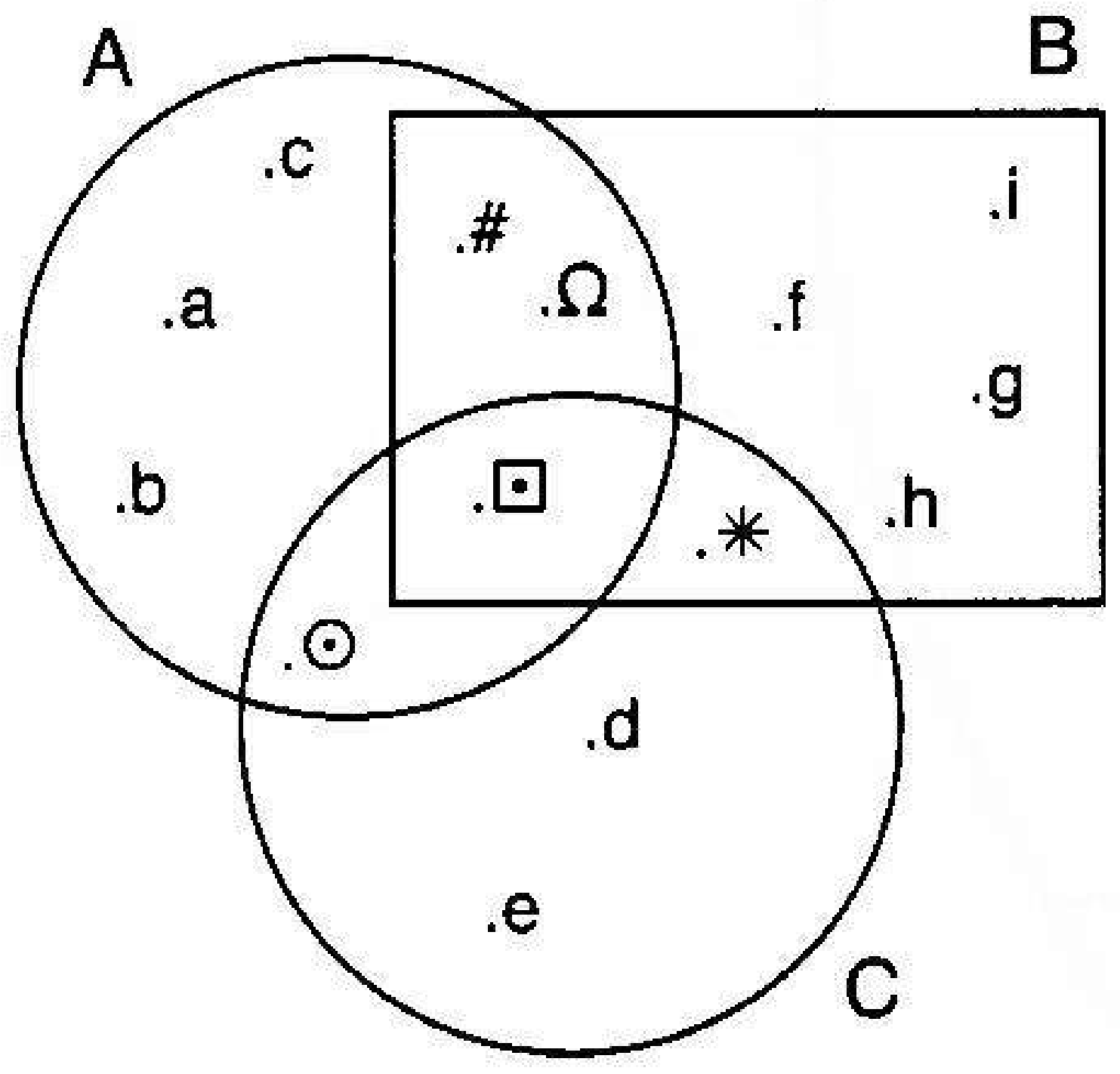


[(A ∪ B) \ C′] = ?

A) e B) {a, b, c} C) {#, Ω} D) {⊙, ⊡, ✳} E) {f, g, h, i}

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**



[(A ∪ B) \ C′] = [(A ∪ B) ∩ C] = {⊙, ⊡, ✳}

ÖRNEK / EXAMPLE 7

ANKARA ÜNİ. YÖS 2013

A = { x : –2 < x < 3, x ∈ Z}

B = { y : –3 < y < 2, y ∈ Z}

⇒ **n(A∩B) = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

PEKİŞTİRME / REINFORCE

A∩B = {1, a, b, 3},

B∩C = {a, c, 2}

⇒ n(B∩(A∪C)) = ?

YANIT / ANSWER : 6

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

A = { –1, 0, 1, 2}

B = { –2, –1, 0, 1}

A∩B = {–1, 0, 1} ⇒ n(A∩B) = 3

ÖRNEK / EXAMPLE 8

ANKARA ÜNİ. YÖS 2015

$n(A) = 4,\ n(B) = 8,$

$n(A) \cdot n(A \cup B) = 36$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$n(A) \cdot n(A \cup B) = 4 \cdot n(A \cup B) = 36$

$n(A \cup B) = 9$

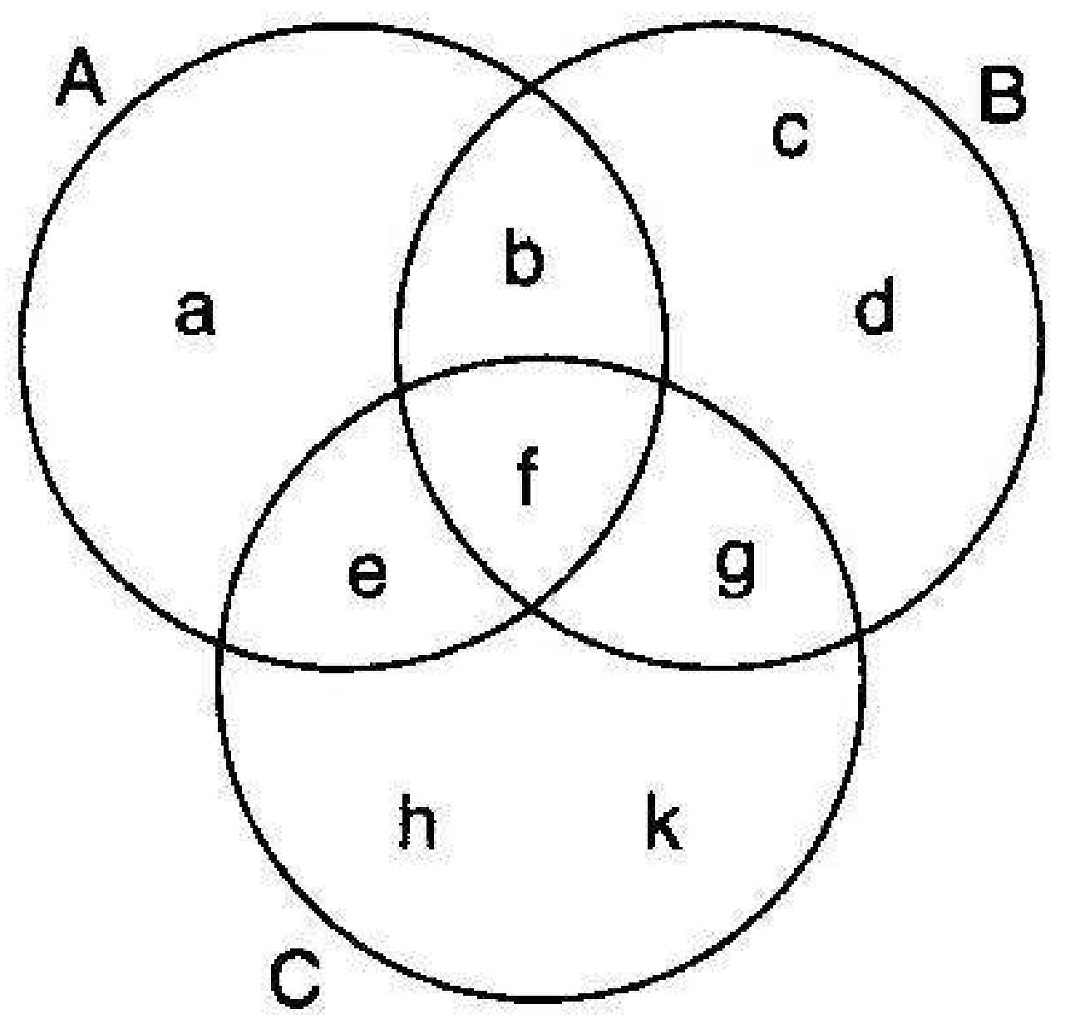
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$9 = 4 + 8 - n(A \cap B)$

$n(A \cap B) = 3$

ÖRNEK / EXAMPLE 9

YILDIZ TEKNİK ÜNİV. YÖS 2016



$A / B = \{x \mid x \in A \wedge x \notin B\}$

$B / C = \{x \mid x \in B \wedge x \notin C\}$

$(A / C) \cap (A / B) = ?$

A) f B) b C) e D) a E) g

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$A \setminus C = \{a, b\}$

$A \setminus B = \{a, e\}$

$(A \setminus C) \cap (A \setminus B) = \{a\}$

1. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

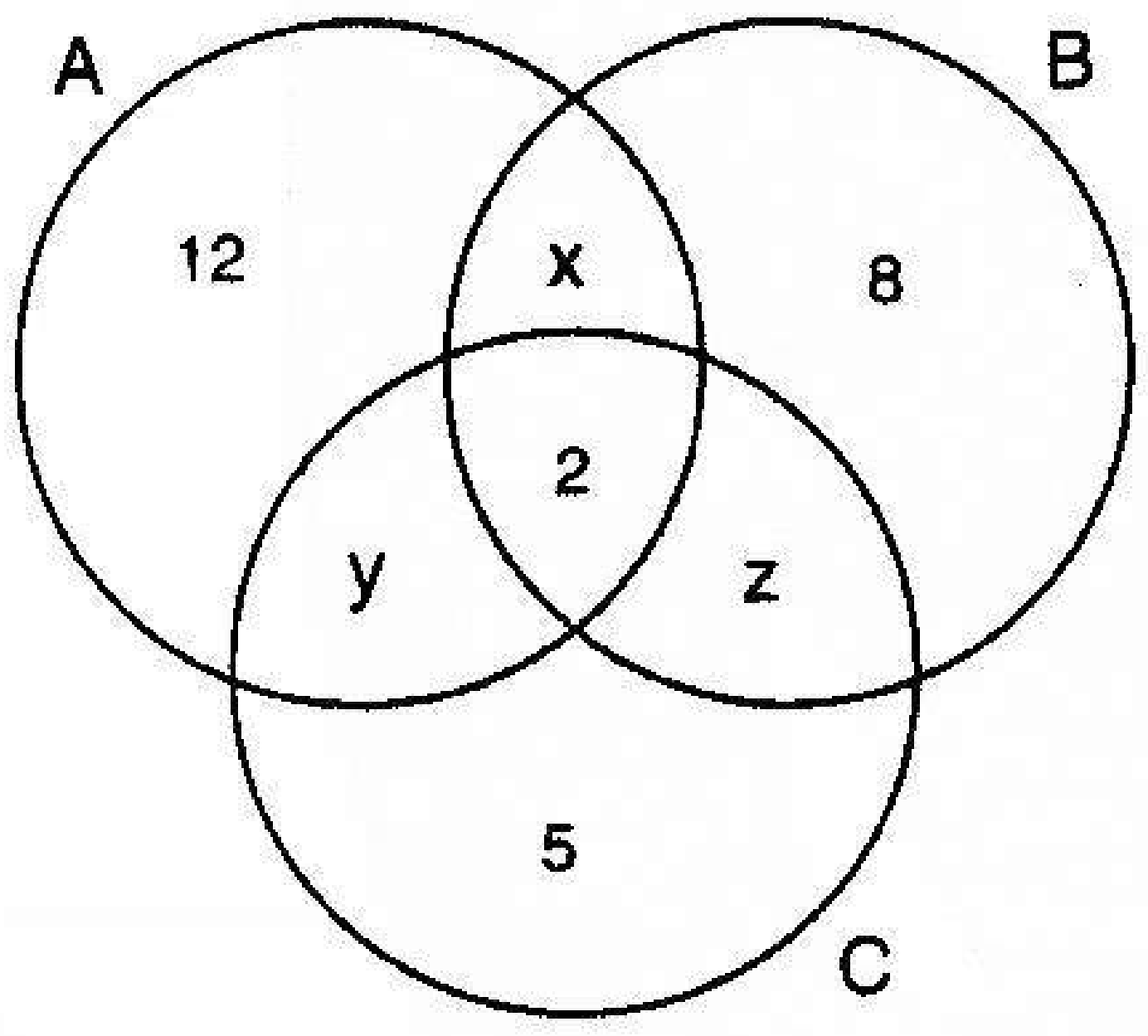
$A \cap B = \{3, 4, 5, 8\}$

$B \setminus A = \{2, 6, 7\} \Rightarrow$ **$A \setminus B = ?$**

A) {2, 4}
B) {1, 9}
C) {1, 7}
D) {3, 8}
E) {2, 9}

2. $n(A) + n(B) + n(C) = 41$

$\Rightarrow$ **$x + y + z = ?$**



A) 2
B) 4
C) 5
D) 8
E) 10

3. $n(A \cup B) = 21$,

$n(A) = 2 \cdot n(B) - 6$,

$n(A \cap B) = 6 \Rightarrow$ **$n(A \setminus B) = ?$**

A) 7
B) 8
C) 9
D) 10
E) 11

4. $n(A \setminus B) = 3n(B \setminus A) = 2n(A \cap B)$,

$n(A \cup B) = 44 \Rightarrow$ **$n(A) = ?$**

A) 28
B) 30
C) 32
D) 36
E) 40

5. $n(A) = 4n(B \setminus A)$, $n(A \cap B) = 3$

$n(A \setminus B) + n(B \setminus A) = 12 \Rightarrow$ **$n(A \cup B) = ?$**

A) 9
B) 12
C) 15
D) 18
E) 21

6. $A = \{ x ; -3 \le x < 4, x \in Z \}$

$B = \{ x ; | x - 3 | < 3, x \in Z \}$

$\Rightarrow$ **$n(A \cap B) = ?$**

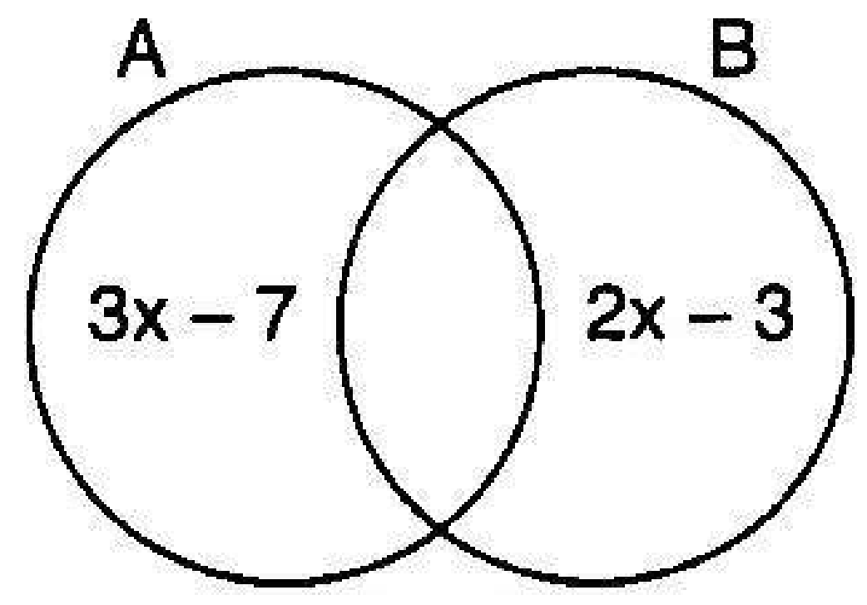
A) 3
B) 4
C) 5
D) 6
E) 7

7. $n(A-B) = 3x-7$,

$n(B-A) = 2x-3$

$n(A-B) = n(B-A)$

$n(A \cup B) = 14 \Rightarrow$ **$n(A \cap B) = ?$**



A) 2
B) 3
C) 4
D) 5
E) 6

8. **$[-2, 6] \cap [0, 3) = ?$**

A) $[-2, 6]$
B) $[0,3)$
C) $[-2, 0]$
D) $[3, 6]$
E) $(3, 6)$

9. $n(A \cup B) = 20$,

$n(A-B) = 8$,

$n(B-A) = 10 \Rightarrow$ **$n(A \cap B) = ?$**

A) 1
B) 2
C) 3
D) 7
E) 15

10. $n(E) = 42$

$n(A') = 18$

$n(B') = 20$

$n(A' \cap B') = 6 \Rightarrow$ **$n(A \cap B) = ?$**

A) 6
B) 7
C) 8
D) 10
E) 12

11. $A = \{x \mid 150 < x < 500,\ x = 4 \cdot k,\ k \in Z\}$

$B = \{y \mid 48 \leq y < 450,\ y = 6 \cdot m,\ m \in Z\}$

$n(A \cap B) = ?$

A) 23
B) 24
C) 25
D) 26
E) 27

12. $A = \{x \mid x \in Z,\ x^2 \leq 4\}$

$B = \{x \mid x \in Z,\ |x| \leq 1\} \quad \Rightarrow \quad$ **$B - A = ?$**

A) A
B) B
C) $\{-1, 1, 2\}$
D) $\varnothing$
E) $\{-1, 0, 1\}$

YANITLAR / ANSWERS							
1. B	**2. C**	**3. D**	**4. D**	**5. C**	**6. A**	**7. C**	**8. B**
9. B	**10. D**	**11. C**	**12. D**				

1.

$$\left.\begin{array}{l} A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ A \cap B = \{3, 4, 5, 8\} \end{array}\right\} \Rightarrow B = \{2, 3, 4, 5, 6, 7, 8\}$$

$$\left.\begin{array}{r} A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ B = \{2, 3, 4, 5, 6, 7, 8\} \end{array}\right\} \Rightarrow A \setminus B = \{1, 9\}$$

Yanıt / The Answer is B

2.

$$\begin{array}{rl} & n(A) = 12 + 2 + x + y \\ & n(B) = 8 + 2 + x + z \\ + & n(C) = 5 + 2 + y + z \\ \hline \end{array}$$

$$\begin{aligned} n(A) + n(B) + n(C) &= 31 + 2(x + y + z) \\ 41 &= 31 + 2(x + y + z) \\ 10 &= 2(x + y + z) \\ 5 &= x + y + z \end{aligned}$$

Yanıt / The Answer is C

---

3. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\begin{aligned} n(A \cup B) &= 2 \cdot n(B) - 6 + n(B) - n(A \cap B) \\ 21 &= 3 \cdot n(B) - 6 - 6 \\ 33 &= 3 \cdot n(B) \\ 11 &= n(B) \end{aligned}$$

$n(B) = 11 \Rightarrow n(A) = 16$

$n(A \setminus B) = n(A) - n(A \cap B)$

$n(A \setminus B) = 16 - 6 = 10$

Yanıt / The Answer is D

---

4. $n(A \setminus B) = 3 \cdot n(B \setminus A) = 2 \cdot n(A \cap B),$

$$\begin{aligned} \Rightarrow\ & n(A \setminus B) = 6k \\ & n(B \setminus A) = 2k \\ & n(A \cap B) = 3k \end{aligned}$$

$n(A) = 9k, \quad n(B) = 5k$

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 44 &= 9k + 5k - 3k \\ 44 &= 11k \\ 4 &= k \end{aligned}$$

$\Rightarrow n(A) = 9k = 9 \cdot 4 = 36$

Yanıt / The Answer is D

---

5. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\begin{aligned} \Rightarrow n(A \cup B) &= n(A \setminus B) + n(B \setminus A) + n(A \cap B) \\ &= 12 + 3 \\ &= 15 \end{aligned}$$

Yanıt / The Answer is C

---

6. $A = \{-3, -2, -1, 0, 1, 2, 3\}$

$|x - 3| < 3 \Rightarrow -3 < x - 3 < 3$

$0 < x < 6$

$B = \{1, 2, 3, 4, 5\}$

$A \cap B = \{1, 2, 3\} \Rightarrow n(A \cap B) = 3$

Yanıt / The Answer is A

7. $n(A-B) = n(B-A) \Rightarrow 3x-7 = 2x-3$

$x = 4$

$n(A-B) = 5,\ n(B-A) = 5$

$n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$

$14 = 5 + 5 + n(A \cap B)$

$n(A \cap B) = 4$

Yanıt / The Answer is C

---

8. $[-2, 6] = \{x \mid -2 \le x \le 6,\ x \in R\}$

$[0, 3) = \{x \mid 0 \le x < 3,\ x \in R\}$

$[-2, 6] \cap [0, 3) = \{x \mid 0 \le x < 3,\ x \in R\}$

$= [0, 3)$

Yanıt / The Answer is B

---

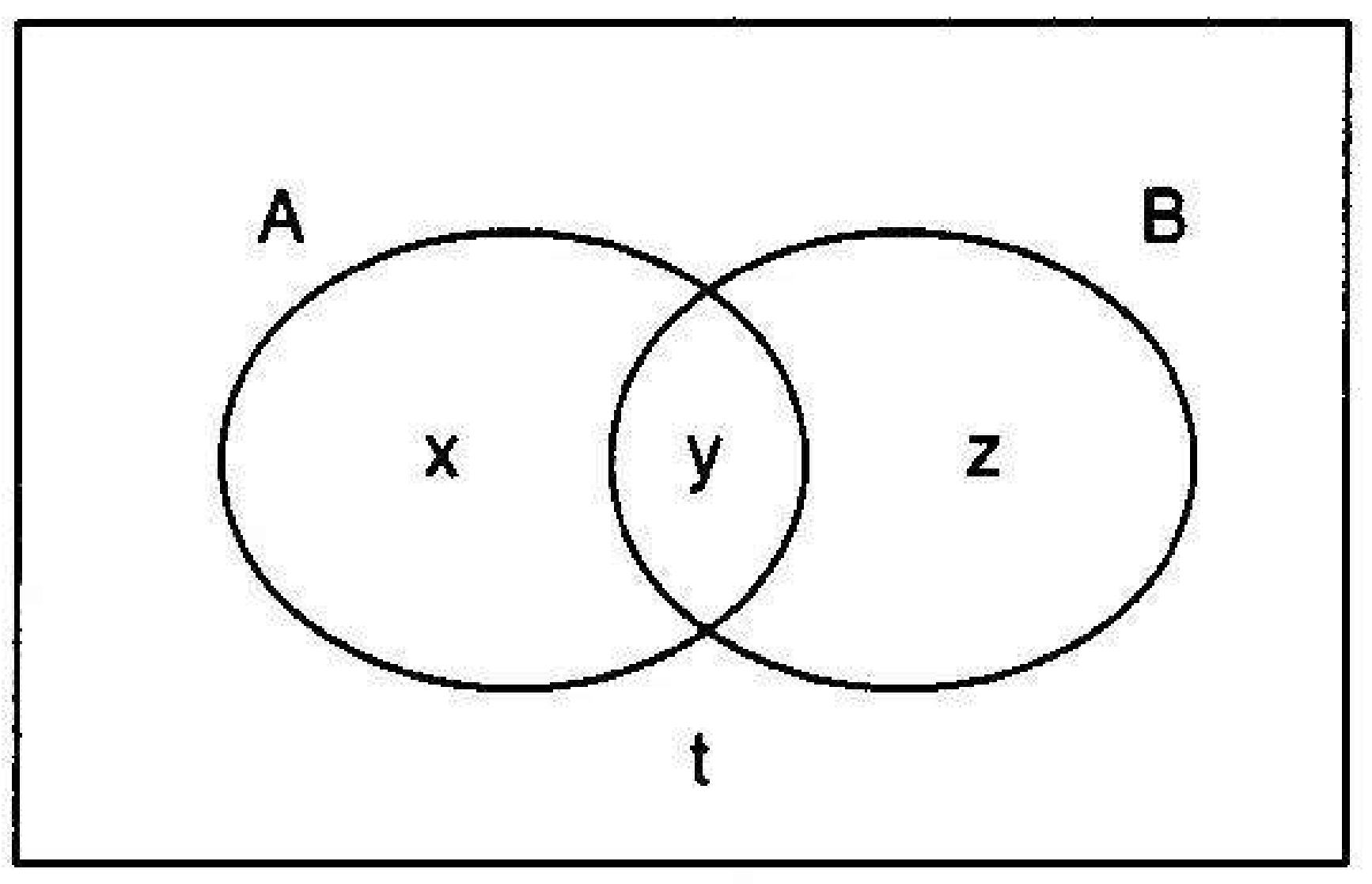
9. $n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$

$20 = 8 + 10 + n(A \cap B)$

$n(A \cap B) = 2$

Yanıt / The Answer is B

10.



$n(A' \cap B') = n(A \cup B)' = t = 6$

$n(A') = z + t = 18 \Rightarrow z = 12$

$n(B') = x + t = 20 \Rightarrow x = 14$

$n(E) = x + y + z + t = 42 \Rightarrow n(A \cap B) = y = 10$

Yanıt / The Answer is D

---

11. $A = \{x \mid 150 < x < 500,\quad x = 4 \cdot k,\ k \in Z\}$

$B = \{y \mid 48 \le y < 450,\quad y = 6 \cdot m,\ m \in Z\}$

$A \cap B = \{z \mid 150 < z < 450,\ z = 12 \cdot n,\ n \in Z\}$

$\Rightarrow n(A \cap B) = \dfrac{450-150}{12} = 25$

Yanıt / The Answer is C

---

12. $A = \{-2, -1, 0, 1, 2\}$

$B = \{-1, 0, 1\} \Rightarrow B - A = \varnothing$

Yanıt / The Answer is D

1. $[(A \cap B) \cup (A \cap C) \cup (B \cap C)]' \cap (A \cap B \cap C) = ?$

Yanıt / The Answer is: $\varnothing$

---

2. $A = \{ x \mid |x - 2| < 8;\ x \in \mathbb{Z} \}$

$B = \{ x \mid |2 - x| < 4;\ x \in \mathbb{Z} \}$

$\Rightarrow n(A \setminus B) = ?$

Yanıt / The Answer is: **8**

---

3. $n(A) = 8,$

$n(B) = 6,$

$n(A \cap B) \neq 0$

$\Rightarrow \max(n(A - B)) + \min(n(A - B)) = ?$

Yanıt / The Answer is: **9**

---

4. $A, B \neq \varnothing \quad A, B \subset E$

$n(A \cap B)' = 55,\ n(A \cup B)' = 12$

$n(A) + n(B) = 45 \quad \Rightarrow \quad n(E) = ?$

Yanıt / The Answer is: **56**

---

5. $\frac{1}{3} \cdot n(A \setminus B) = 3 \cdot n(B \setminus A) = \frac{1}{5} \cdot n(A \cap B)$

$n(A \cup B) = 50 \quad \Rightarrow n(A \cap B) = ?$

Yanıt / The Answer is: **30**

---

6. $n(A \cup B) = 34$

$n(A \cap B') = 2 \cdot n(B \cap A')$ ve (and)

$n(A \cap B) = 4 \Rightarrow n(A) = ?$

Yanıt / The Answer is: **24**

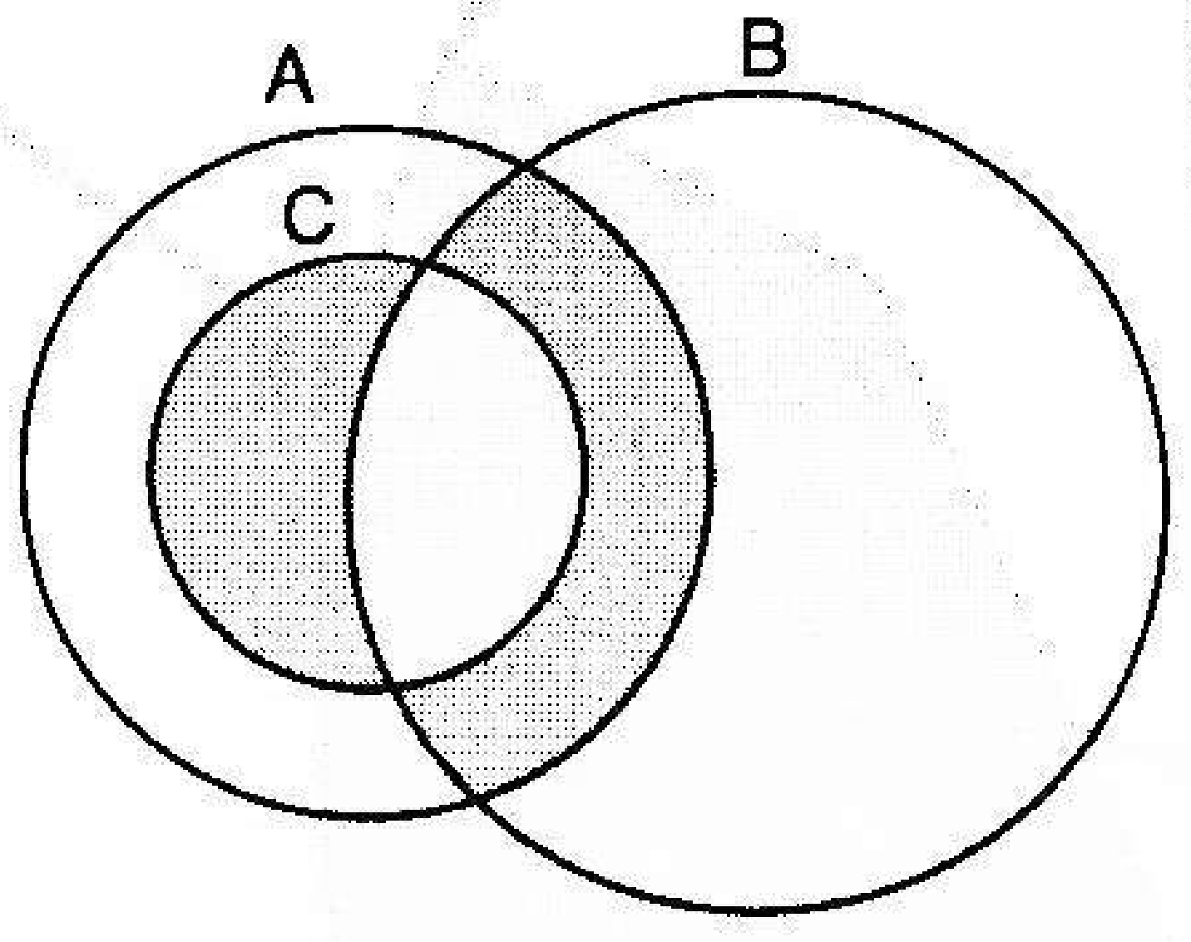
7. $n(A \cap B') + n(B \cap A') = 6 \cdot n(A \cap B)$

$n(A) = 3 \cdot n(B)$ ve (and) $n(A \cup B) = 42 \Rightarrow$ **$n(A) = ?$**

Yanıt / The Answer is: **36**

8. **Taralı alan = ?**

Shaded area



Yanıt / The Answer is: **$[(A \cap B) \setminus C) \cup (C \setminus B)]$**

9. $n(A) = 3 \cdot n(B)$

$4 \cdot n(A \cap B) = n(B \setminus A)$

$n(A \cup B) = 38 \Rightarrow$ **$n(A \setminus B) = ?$**

Yanıt / The Answer is: **28**

10. $n(A \setminus B) = 9, \quad n(B \setminus A) = 7,$

$n(A) + n(B) = 36 \Rightarrow$ **$n(A \cup B) = ?$**

Yanıt / The Answer is: **26**

11. $A = \{x \mid 3 \leq x \leq 1600,\ x = 6k,\ k \in N\}$

$B = \{y \mid 8 < y < 1200,\ y = 9n,\ n \in N\}$

$n(A \cap B) = ?$

Yanıt / The Answer is: **66**

12. $A = \{x \mid 100 < x \leq 400,\ x = 8 \cdot k,\ k \in Z\}$

$B = \{y \mid 75 \leq y < 390,\ y = 6 \cdot m,\ m \in Z\}$

$n(A \cup B) = ?$

Yanıt / The Answer is: **78**

1. $A = \{x \mid |x - 1| \le 5\}$,
$B = \{x \mid |x + 1| < 4\}$
$\Rightarrow \mathbf{A \cap B = ?}$

A) $(-5, 6)$
B) $[-4, 5)$
C) $[-4, 3)$
D) $(-2, 6]$
E) $[0, 4]$

2. $n(A \cap B) = 6$, $n(A) = 22$
$n(B \cap C) = 5$, $n(B) = 17$
$n(A \cap C) = 8$, $n(C) = 24$
$n(A \cap B \cap C) = 4$
$\Rightarrow \mathbf{n(A \cup B \cup C) = ?}$

A) 38
B) 40
C) 42
D) 45
E) 48

3. $A \subset B \subset C \Rightarrow \mathbf{(A' \cap C') \cup (A' \cap B') = ?}$

A) A
B) B
C) C
D) A′
E) B′

4. $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$A' = \{3, 5, 8, 9, 10\}$,
$B = \{2, 3, 4, 6, 7, 8\}$
$\Rightarrow \mathbf{A \cap B' = ?}$

A) {1}
B) {2}
C) {3}
D) {4, 6}
E) {5, 8}

5. $n(A) = 10$,
$n(B) = 7$,
$n(A \setminus B) = 5 \Rightarrow \mathbf{n(A \cup B) = ?}$

A) 8
B) 9
C) 10
D) 11
E) 12

6. $A \cup B = E$,
$A \cap B \neq \varnothing$,
$n(A) = 12$,
$n(B) = 15 \Rightarrow \mathbf{max\,[\,n(A \cap B)'\,] = ?}$

A) 23
B) 24
C) 25
D) 26
E) 27

7. $n(A \cup B) = 28$,

$n(A) = 2 \cdot n(B) = 3 \cdot n(A \cap B)$

$\Rightarrow$ $n(B \setminus A) = ?$

A) 4 B) 7 C) 8

D) 16 E) 20

8. $n(E) = 35$,

$n(A') = 12$,

$n(B') = 21$,

$n(A \cup B) = 24$

$\Rightarrow$ $n(A \cap B) = ?$

A) 11 B) 12 C) 13

D) 14 E) 15

9. $n(E) = n(A \cup B \cup C) = 30$,

$A \subset B$,

$A \cap C = \varnothing$

$n(B \cap C) = 6$

$n(A \setminus C) = 8$

$n(C') = 17$

$\Rightarrow$ $n(C \setminus B) = ?$

A) 5 B) 6 C) 7

D) 8 E) 9

10. **Taralı Alan**

The shaded Area = ?

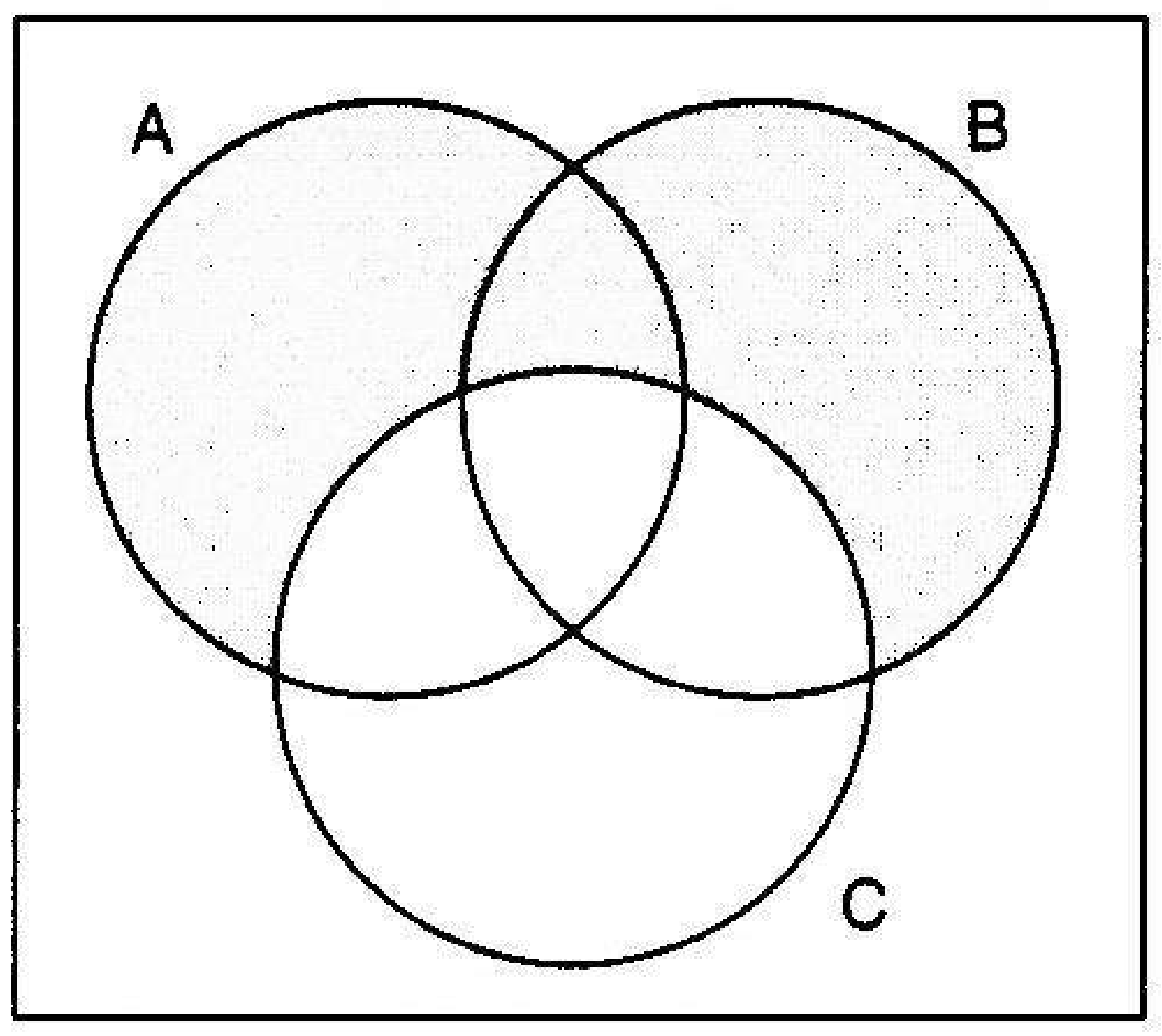
A) $(A \cup B) \setminus C$

B) $(A \setminus B) \cup C$

C) $C \setminus (A \cap B)$

D) $(A \cup B) \cap C$

E) $(A \setminus B) \cup (C \setminus B)$



11. $A = (2, 6]$,

$B = [-1, 5]$

$\Rightarrow$ $A \cap B = ?$

A) (2, 3] B) [0, 5] C) (2, 5]

D) (2, 4) E) [3, 5]

12. $A = (-1, 7]$ $B = [3, 10)$ **$\Rightarrow$ $A \setminus B = ?$**

A) (–1, 3) B) (–1, 3] C) (–2, 7)

D) [–2, 7] E) (3, 8)

YANITLAR / ANSWERS							
1. C	2. E	3. E	4. A	5. E	6. C	7. A	8. C
9. C	10. A	11. C	12. A				

1. $n(A \cup B) = 11$,

$3 \cdot n(A) = n(B)$,

$n(A \cap B) = 1$

$\Rightarrow$ **$n(B \setminus A) = ?$**

A) 5 B) 6 C) 7

D) 8 E) 9

2. $n(A) = 8$,

$n(B) = 3$,

$n(A \cap B) \neq \varnothing$,

$B \not\subset A$

$\Rightarrow$ **$\max(n(A \cup B)) + \min(n(A \cup B)) = ?$**

A) 17 B) 18 C) 19

D) 20 E) 21

3. $n(A \cap B') = 2n(A \cap B) = 3n(B \setminus A)$,

$n(A \cup B) = 44$

$\Rightarrow$ **$n(B) = ?$**

A) 16 B) 18 C) 20

D) 22 E) 24

4. $n[(A \setminus B) \cup (B \setminus A)] = 12$,

$n(A' \cup B') = 16$

$\Rightarrow$ **$n(A \cup B)' = ?$**

A) 1 B) 4 C) 6

D) 8 E) 12

5. **$[A \cup (A \cup B')'] \cap (A \cup B') = ?$**

A) A B) B C) A′

D) B′ E) ∅

6. $(B \cap C) \cup (A \cap C) = \varnothing$,

$n(A \cup B)' = 10$,

$n(A \cup B \cup C)' = 4$

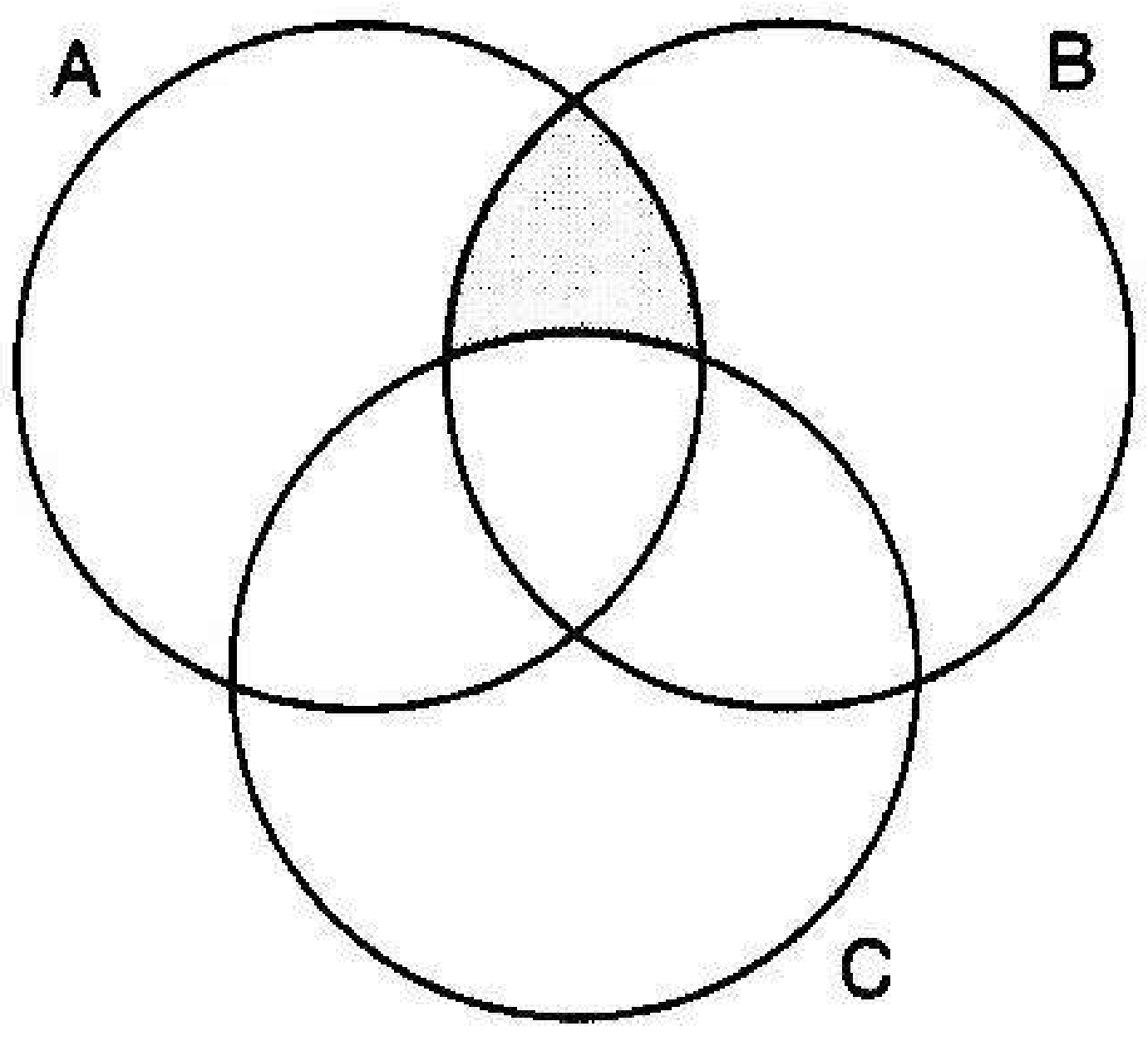
$\Rightarrow$ **$n(C) = ?$**

A) 3 B) 4 C) 5

D) 6 E) 7

7. **Taralı Alan**

The shaded Area = ?



A) $(A \cup B) \setminus C$

B) $C \setminus (A \cup B)$

C) $(A \cap C) \setminus B$

D) $(B \cap C) \setminus A$

E) $(A \cap B) \setminus C$

---

8. $A = \{x \mid 100 < x \leq 400,\ x = 3k,\ k \in N\}$

$B = \{y \mid 150 \leq y < 450,\ y = 4k,\ k \in N\}$

$\Rightarrow$ **$n(A \cap B)$ = ?**

A) 19 B) 20 C) 21

D) 22 E) 23

---

9. $A = (-3, 5]$,

$B = [1, 8)$

$\Rightarrow$ **$A \cap B$ = ?**

A) $[1, 5]$ B) $[2, 4]$ C) $[-2, 6)$

D) $(1, 7)$ E) $[0, 6)$

---

10. $n(A \setminus B') = 3$,

$n(A' \cap B) = 5$,

$n(A \cup B) = 16$

$\Rightarrow$ **$n(A \cap B')$= ?**

A) 4 B) 5 C) 6

D) 7 E) 8

---

11. $n(A \setminus B) = 4$,

$n(B') = 7$,

$n(A') = 8$,

$n(A \cup B) = 21$

$\Rightarrow$ **$n(A \cap B)$ = ?**

A) 6 B) 8 C) 9

D) 10 E) 12

---

12. $C \subset B \subset A$,

$n(C') = 9$,

$n(B') = 2$,

$n(C) = 1$

$\Rightarrow$ **$n(B)$ = ?**

A) 4 B) 5 C) 6

D) 7 E) 8

YANITLAR / ANSWERS

1. D 2. C 3. C 4. B 5. A 6. D 7. E 8. C
9. A 10. E 11. E 12. E

1. $A \not\subset B$,

$A \cap B \neq \varnothing$,

$n(A) = 20$,

$n(B) = 12$

$\Rightarrow$ **$\max(n(A \cup B)) + \min(n(A \cup B)) = ?$**

A) 49

B) 50

C) 51

D) 52

E) 53

2. $n(A \cup B) = 2 \cdot n(A' \cap B) = 3 \cdot n(A \cap B)$,

$n(A) = 24$

$\Rightarrow$ **$n(B) = ?$**

A) 8

B) 16

C) 24

D) 32

E) 40

3. $n(A) + n(B') = 12$,

$n(A') + n(B) = 8$,

$n(C') = 4$

$\Rightarrow$ **$n(C) = ?$**

A) 4

B) 5

C) 6

D) 7

E) 8

4. $A = \{x \mid 50 < x \leq 120,\ x = 3k,\ k \in N\}$

$B = \{y \mid 70 < y \leq 150,\ y = 2k,\ k \in N\}$

$\Rightarrow$ **$n(A \setminus B) = ?$**

A) 14

B) 15

C) 16

D) 17

E) 18

5. $E = \{1, 2, 3, 4, 5, 6\}$,

$(A \cup B)' = \{3\}$,

$A \setminus B = \{1\}$,

$A' \cup B' = \{1, 3, 5\}$

$\Rightarrow$ **$B = ?$**

A) $\{2, 4, 5, 6\}$

B) $\{1, 2, 3\}$

C) $\{2, 3, 4\}$

D) $\{3, 5, 6\}$

E) $\{1, 4, 5, 6\}$

6. $n(E) = 42$,

$n(A' \cap B') = 8$,

$n(B) = 12$,

$n(A') = 10$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 5

B) 6

C) 8

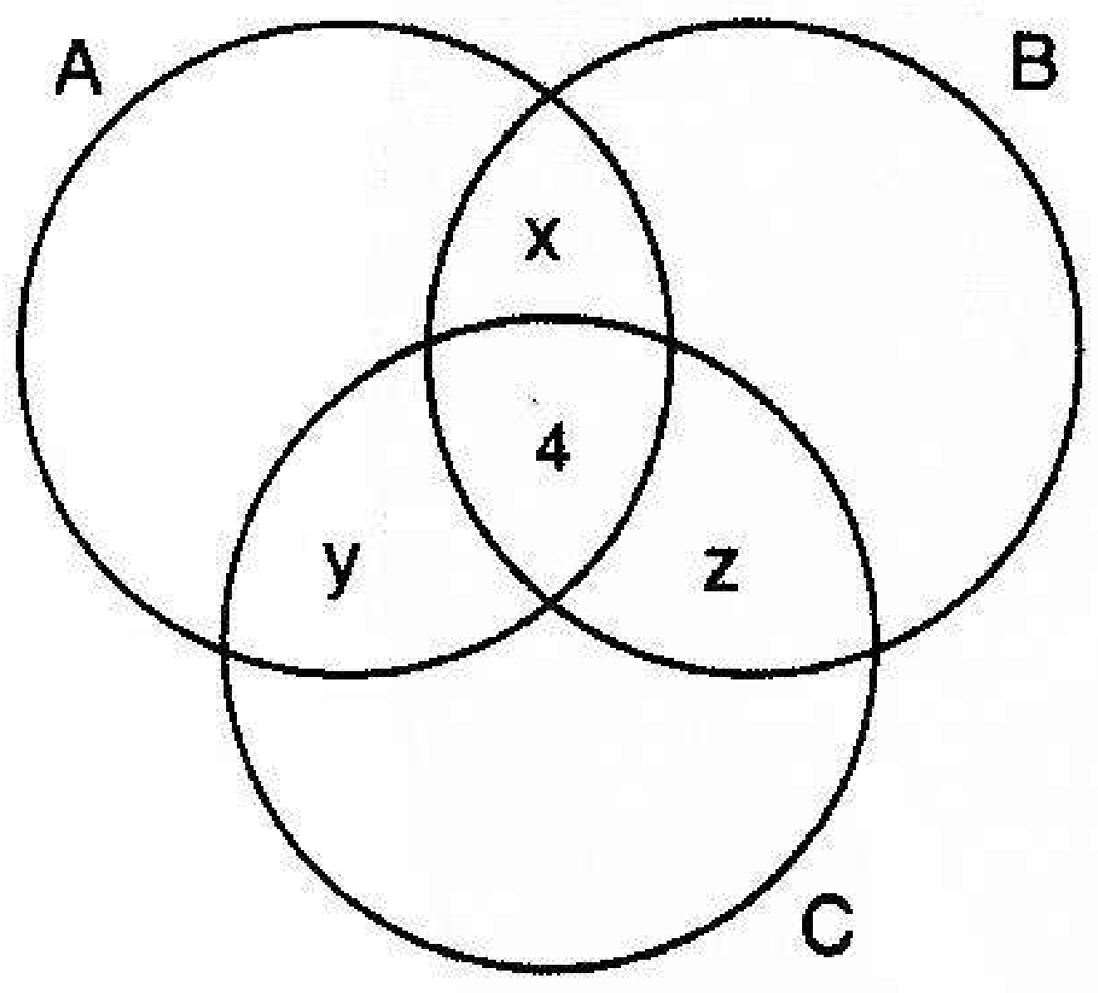
D) 9

E) 10

7. $n(A \cup B) = 20$,
$n(A \cap B) = 4$,
$2 \cdot n(A) = n(B)$
$\Rightarrow$ **$n(A) = ?$**

A) 7
B) 8
C) 9
D) 10
E) 11

8. $n(A \cup B \cup C) = 30$,
$n(A) = 15$,
$n(B) = 14$,
$n(C) = 16$
$\Rightarrow$ **$x + y + z = ?$**



A) 5 B) 6 C) 7
D) 8 E) 9

9. $n(A' \cap B') = 2$,
$n(A) = 12$,
$n(E) = 20$
$\Rightarrow$ **$n(B \setminus A) = ?$**

A) 6
B) 7
C) 8
D) 9
E) 10

10. $A = \{x \mid |x| > 3, x \in R\}$,
$B = \{x \mid |x - 1| \leq 4, x \in R\}$
$\Rightarrow$ **$A \cap B = ?$**

A) (1, 3]
B) (2, 4]
C) (3, 5]
D) (4, 6]
E) (5, 7]

11. $n(A) + n(B) = n(A \cup B)$,
$n(A) = 8$,
$n(B') = 10$
$\Rightarrow$ **$n(A' \cap B') = ?$**

A) 1 B) 2 C) 3
D) 4 E) 5

12. $A = \{x \mid 50 < x \leq 100, x = 2k, k \in N\}$,
$B = \{y \mid 60 \leq x < 120, x = 3k, k \in N\}$
$\Rightarrow$ **$n(A \cup B) = ?$**

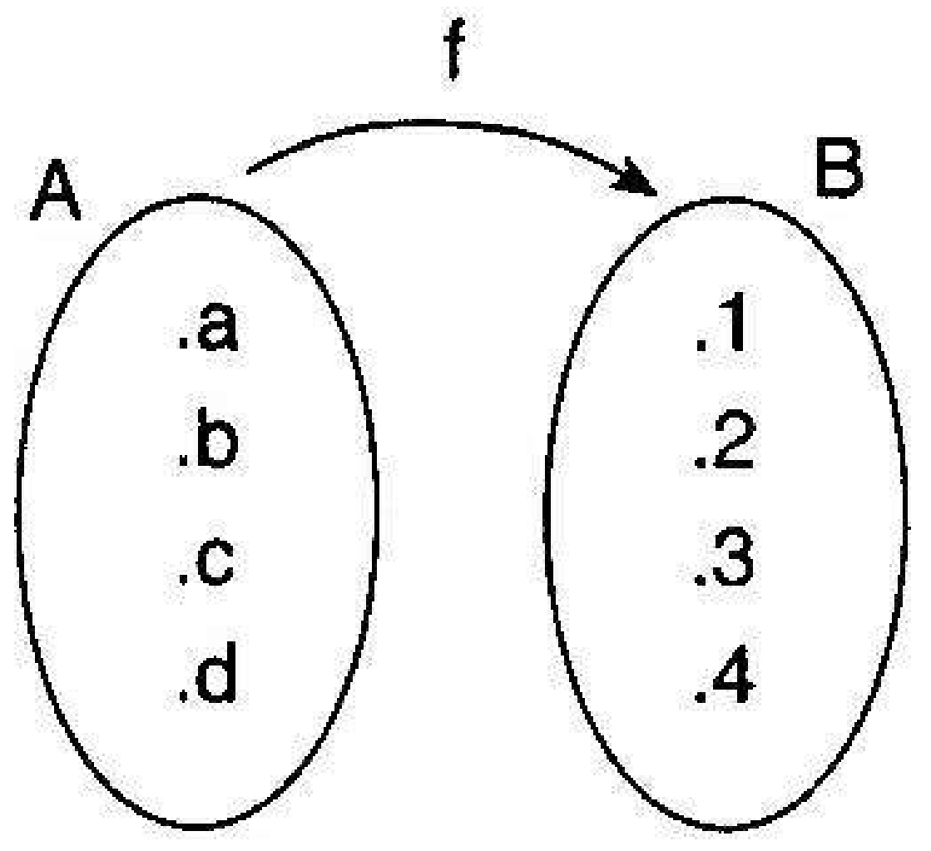
A) 34 B) 35 C) 36
D) 37 E) 38

YANITLAR / ANSWERS

1. D	**2. E**	**3. C**	**4. B**	**5. A**	**6. E**	**7. B**	**8. C**
9. A	**10. C**	**11. B**	**12. E**				

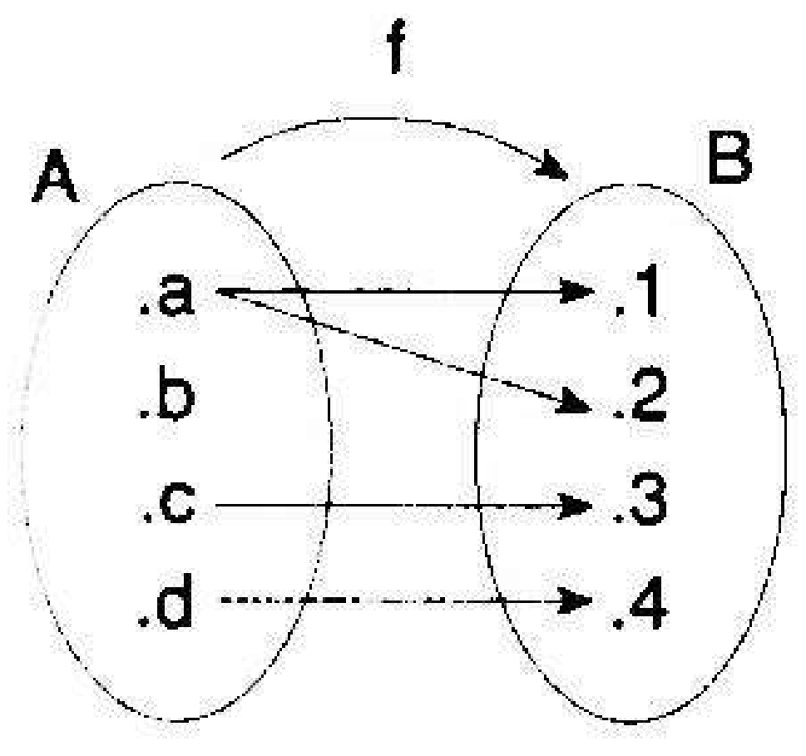
FONKSİYONLAR
FUNCTIONS

BÖLÜM 3
CHAPTER 3

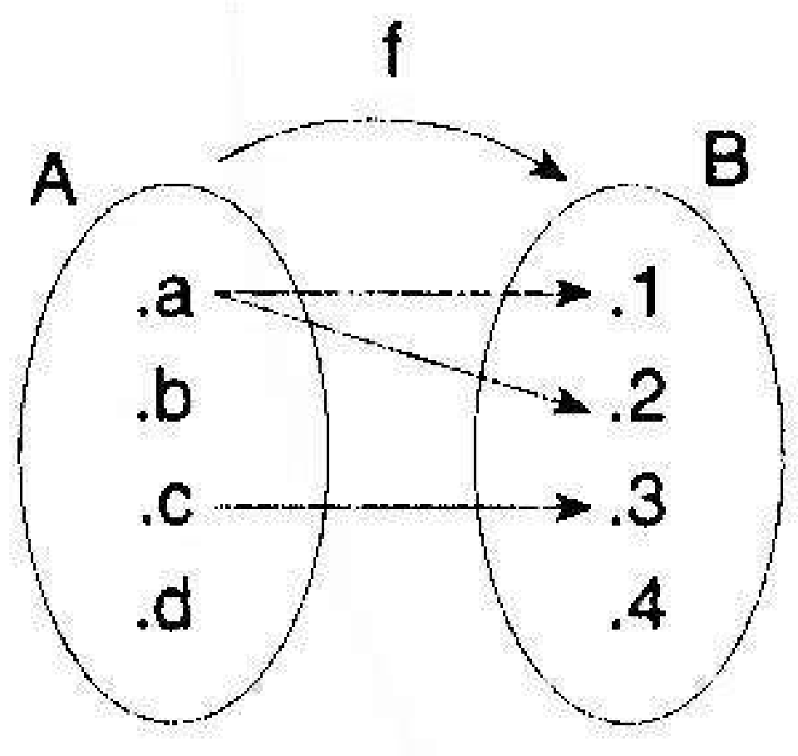


Tanım: A' dan B' ye bir f bağıntısı A' nın her elemanını B' de bir ve yalnız bir elemanla eşliyorsa ve A'da eşlenmeyen eleman kalmıyorsa f, A' dan B' ye bir fonksiyondur denir ve f: A → B ile gösterilir. A kümesine *tanım kümesi*, B kümesine *değer kümesi* denir.

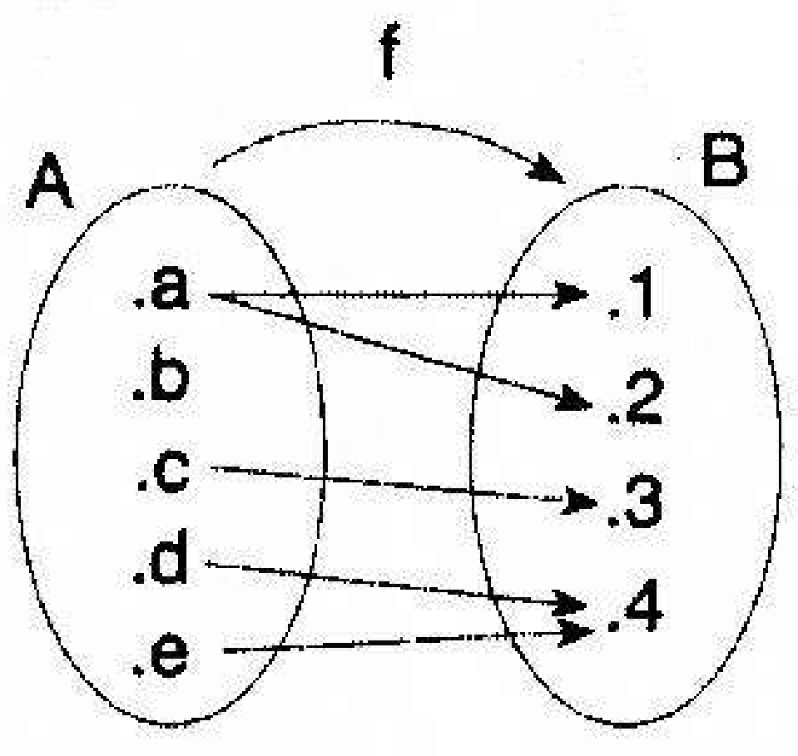
Definition: *If a relation f from the set A to set B makes an assignment to each element in the set A and assigns one element of A to only one element in the set B then f is called a function. Set A is called the domain and set B is called the range of the function.*



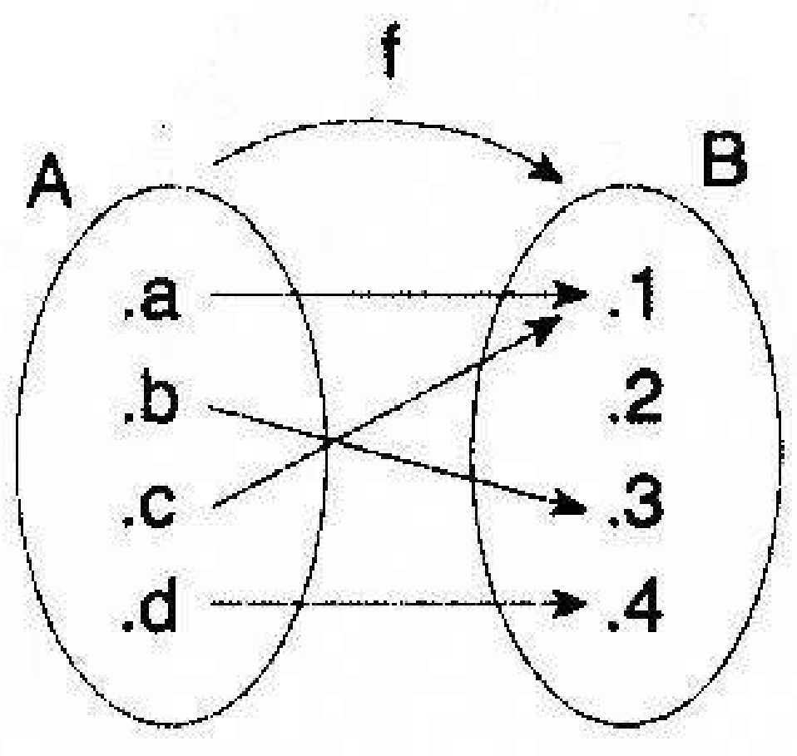
Fonksiyon değil
Not a function



Fonksiyon değil
Not a function



Fonksiyon değil
Not a function



Fonksiyon
A function

ÖRNEK / EXAMPLE 1

f : A → B,

f(x) = 3x + 1, A = {–2, 0, 1, 3} ⇒ **B = ?**

A) {–3, 0, 4, 7}
B) {–1, 2, 5, 7}
C) {–5, 1, 4, 10}
D) {–4, 1, 5,12}
E) {–2, 3, 7, 13}

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

A = {–2, 0, 1, 3} , f(x) = 3x + 1
x = –2 ⇒ f(–2) = 3·(–2) + 1 = –5
x = 0 ⇒ f(0) = 3·(0) + 1 = 1
x = 1 ⇒ f(1) = 3·1 + 1 = 4
x = 3 ⇒ f(3) = 3·3 + 1 = 10
⇒ B = {–5, 1, 4, 10}

PEKİŞTİRME / REINFORCE

f : A → B,
f(x) = x – 5,
B = {–1, 0, 1} ⇒ A = ?

YANIT / ANSWER : {4, 5, 6}

Düşünün / Think about it

$A > 1$,

$x_1, x_2, x_3, \ldots x_k$ asal sayılar
prime numbers

$A = x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot x_3^{\alpha_3} \cdot \ldots \cdot x_k^{\alpha_k}$

$f(A) = \alpha_1 + \alpha_2 + \ldots + \alpha_k \Rightarrow$ **$f(164) = ?$**

Yanıt / Answer

3

Toplama ve Çıkarma İşlemi / Addition and Subtraction

$(f \mp g)(x) = f(x) \mp g(x)$

ÖRNEK / EXAMPLE 2

$f(x) = 3x - 7$,

$g(x) = 4x + 6 \Rightarrow$ **$(f + g)(-1) = ?$**

A) –17 B) –14 C) –11 D) –8 E) –5

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\left.\begin{array}{l} f(x) = 3x - 7 \\ g(x) = 4x + 6 \end{array}\right\} \Rightarrow (f+g)(x) = 7x - 1$$

$$(f+g)(-1) = 7 \cdot (-1) - 1 = -8$$

Çarpma ve Bölme İşlemi / Multiplication and Division

$(f \cdot g)(x) = f(x) \cdot g(x)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

FONKSİYON ÇEŞİTLERİ / TYPES of FUNCTION

1. Birim (Etkisiz) Fonksiyon / *Identity Function*
2. Sabit Fonksiyon / *Constant Function*
3. Ters Fonksiyon / *Inverse Function*
4. Bileşke Fonksiyon / *Composite Function*
5. Permütasyon Fonksiyonu / *Permutation Function*
6. Doğrusal Fonksiyon / *Linear Function*

BİRİM (ETKİSİZ) FONKSİYON / IDENTITY FUNCTION

$f(x) = x \Rightarrow f$ birim fonksiyondur. $f(x) = x$ ile gösterilir.

$f(x) = x \Rightarrow f$ is the identity function. It is denoted by $I(x) = x$.

ÖRNEK / EXAMPLE 3

f(x) = (8 – a)x + 4b + 12 fonksiyonu birim fonksiyon olduğuna göre, **a·b kaçtır?**

If f(x) = (8 – a)x + 4b + 12 is an identity function, what is the value of a·b ?

A) –21 B) –20 C) –18 D) –16 E) –14

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

f(x) birim fonksiyon olduğundan f(x) = x tir.

Since f(x) is an identity function, f(x) = x.

$f(x) = (8 - a)x + 4b + 12 = x$

$8 - a = 1 \quad , \quad 4b + 12 = 0$

$a = 7 \qquad\qquad b = -3$

$a \cdot b = 7 \cdot (-3) = -21$

SABİT FONKSİYON / CONSTANT FUNCTION

$f(x) = c,\ c \in R \Rightarrow$ f sabit fonksiyondur.

$f(x) = c,\ c \in R \Rightarrow$ f is a constant function.

ÖRNEK / EXAMPLE 4

f(x) bir sabit fonksiyondur. / *f(x) is a constant function.*

$$f(x) = \frac{12x^2 - 6x + b}{ax^2 + 18x - 9} \Rightarrow \mathbf{f(7) = ?}$$

A) $-\frac{1}{4}$ B) $-\frac{1}{3}$ C) $-\frac{1}{2}$ D) 0 E) $\frac{1}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$f(x) = \frac{12x^2 - 6x + b}{ax^2 + 18x - 9}$$ [sabit fonksiyon / *constant function*]

$$\frac{12}{a} = \frac{-6}{18} = \frac{b}{-9}$$

$$\frac{12}{a} = \frac{-6}{18} \Rightarrow a = -36$$

$$-\frac{6}{18} = \frac{b}{-9} \Rightarrow b = 3$$

$$f(x) = \frac{12x^2 - 6x + 3}{-36x^2 + 18x - 9} = -\frac{1}{3}$$

f(x) bir sabit fonksiyon ise / *If f(x) is a constant function then*

$$f(7) = -\frac{1}{3}.$$

NOT / NOTE

Bir fonksiyonun tersini bulmak için,
To the find the inverse of a function,

- ✓ *x yalnız bırakılır. /* isolate to x.
- ✓ *y yerine x, x yerine* f^{-1} *yazılır.*
  write x instead of y and f^{-1} *instead of x.*

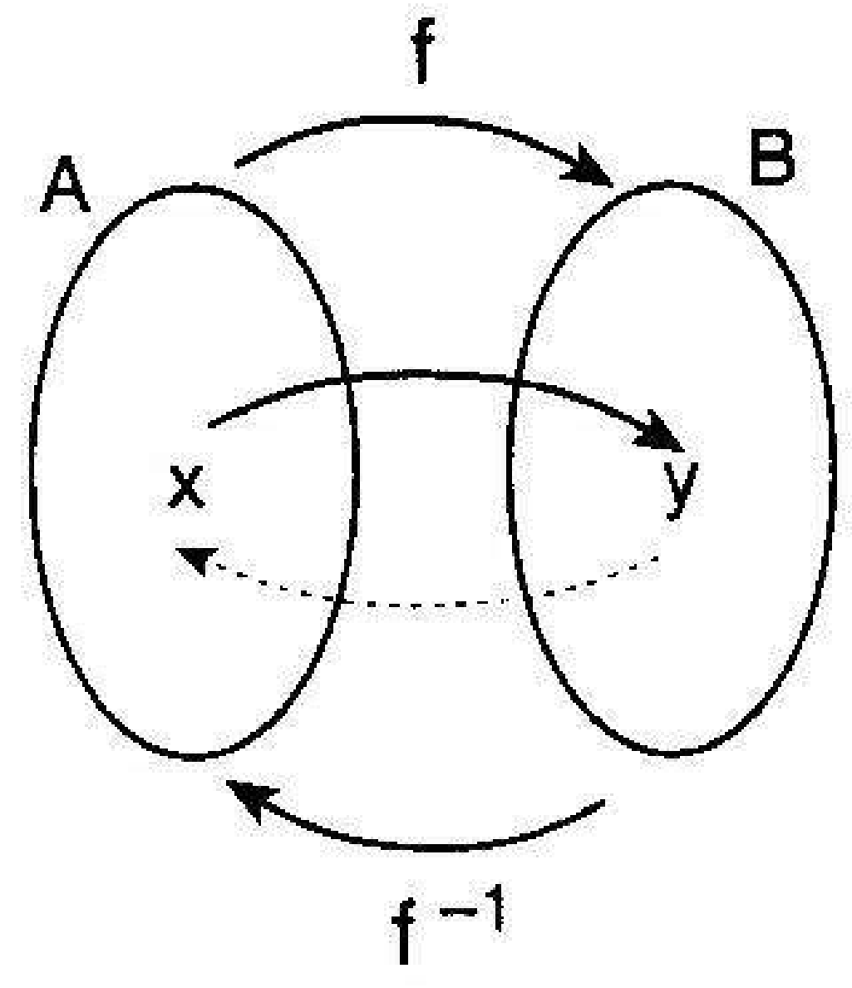
NOT / NOTE

$f(x) = y \Rightarrow f^{-1}(y) = x$

TERS FONKSİYON / INVERSE FUNCTION

$f : A \rightarrow B$;

$f^{-1} : B \rightarrow A$



$f(x) = y \quad \Rightarrow \quad f^{-1}(y) = x$

Bir Fonksiyonun Tersini Bulmada Pratik Formüller
Practice Formulas to the Inverse of a Function

a. $f(x) = ax + b \quad \Rightarrow \quad f^{-1}(x) = \dfrac{x-b}{a}$

b. $f(x) = \dfrac{ax+b}{cx+d} \quad \Rightarrow \quad f^{-1}(x) = \dfrac{-dx+b}{cx-a}$

ÖRNEK / EXAMPLE 5

$f(x) = ax + b$, $f(2) = 6$,
$f(3) = 8 \Rightarrow$ $\mathbf{f^{-1}(x) = ?}$

A) $\dfrac{x+2}{2}$ B) $\dfrac{2x-1}{2}$ C) $\dfrac{2-x}{2}$

D) $\dfrac{2x-2}{3}$ E) $\dfrac{x-2}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$f(x) = ax + b$

$x = 2 \Rightarrow f(2) = 2a + b = 6$

$x = 3 \Rightarrow f(3) = 3a + b = 8$

$a = 2$, $b = 2 \Rightarrow$

$f(x) = 2x + 2 \Rightarrow f^{-1}(x) = \dfrac{x-2}{2}$

ÖRNEK / EXAMPLE 6

$f : R - \{a\} \rightarrow R - \{b\}$,

$f(x) = \dfrac{x+3}{x-2} \Rightarrow$ **a + b = ?**

A) –2 B) –1 C) 1 D) 2 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$f : R - \{a\} \rightarrow R - \{b\}$,

$f(x) = \dfrac{x+3}{x-2} \Rightarrow a - 2 = 0$

$a = 2$

$f^{-1} : R - \{b\} \rightarrow R - \{a\}$,

$f^{-1}(x) = \dfrac{2x+3}{x-1} \Rightarrow b - 1 = 0$

$b = 1$

$a + b = 2 + 1 = 3$

PEKİŞTİRME / REINFORCE

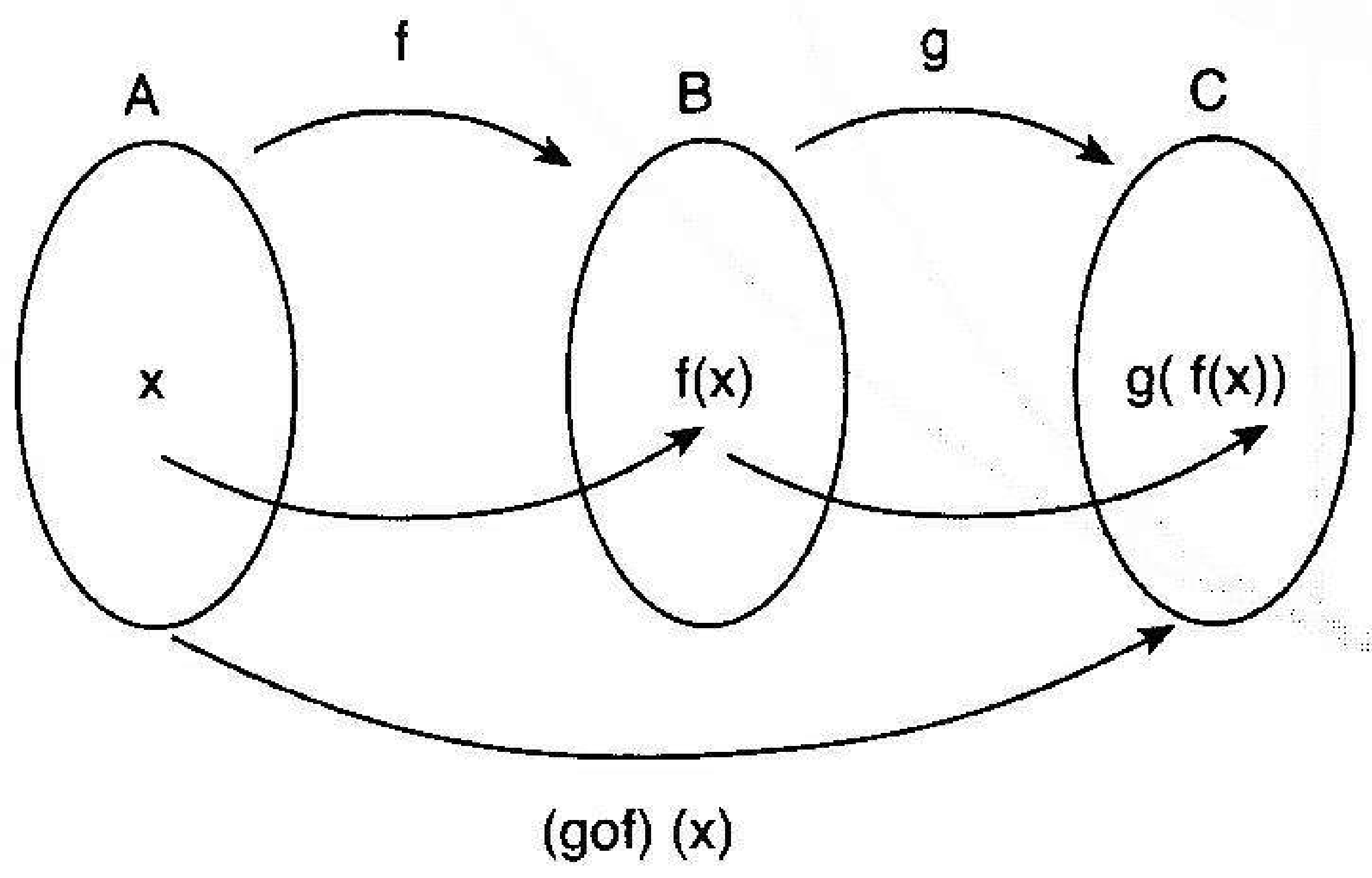
$g(x) = 3x - 4$,

$f(x) = 2x - 1$

$\Rightarrow gof(1) = ?$

YANIT / ANSWER : –1

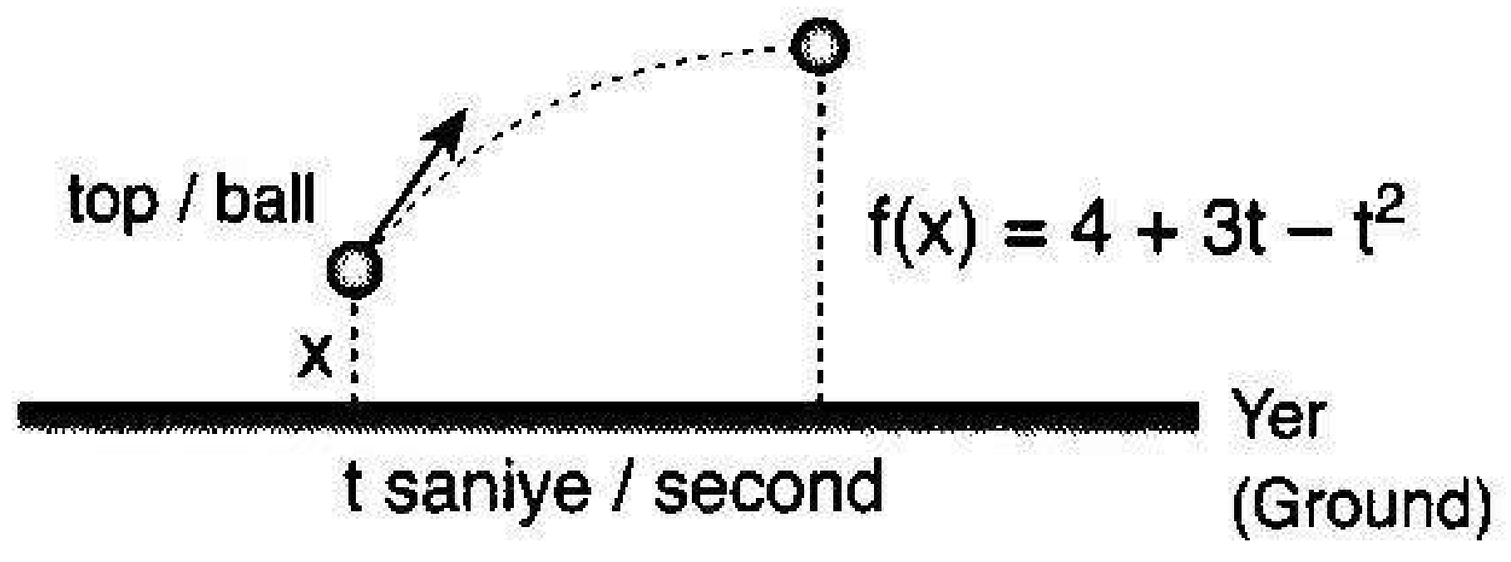
BİLEŞKE FONKSİYON / COMPOSITE FUNCTION



Bileşke Fonksiyonunun Özellikleri / Properties of Composite Functions

1. $fog \neq gof$
2. $(fog)oh = fo(goh)$
3. $(fog)^{-1}(x) = (g^{-1}of^{-1})(x)$
4. $(f^{-1}of)(x) = (fof^{-1})(x) = I(x)$
5. $(foI)(x) = (Iof)(x) = f(x)$
6. $(fog)(x) = h(x) \Rightarrow g(x) = f^{-1}(h(x))$
7. $(fog)(x) = h(x) \Rightarrow f(x) = h(g^{-1}(x))$

Düşünün / Think about it



$\Rightarrow$ **x = ?**

ÖRNEK / EXAMPLE 7

$$\left.\begin{array}{l} f(x) = 2x+4 \\ g(x) = x+1 \end{array}\right\} \Rightarrow \mathbf{(fog)(2) = ?}$$

A) 4 B) 6 C) 8 D) 10 E) 12

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left.\begin{array}{l} f(x) = 2x+4 \\ g(x) = x+1 \end{array}\right\} \Rightarrow (fog)(x) = 2\cdot(x+1)+4 = 2x+6$$

$(fog)(2) = 2\cdot 2+6 = 10$

ÖRNEK / EXAMPLE 8

$a \in R^+$,

$f(x) = ax + b$, $fof(x) = 9x + 4 \Rightarrow$ **f(2) = ?**

A) 1 B) 5 C) 7 D) 10 E) 15

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$f(x) = ax + b \Rightarrow (fof)(x) = a(ax + b) + b$

$(fof)(x) = a^2x + ab + b = 9x + 4$

$a^2 = 9$, $a > 0 \Rightarrow a = 3$

$ab + b = 3b + b = 4$

$b = 1$

$f(x) = 3x + 1$

$f(2) = 3\cdot 2 + 1 = 7$

Yanıt / Answer

4

ÖRNEK / EXAMPLE 9

$fog^{-1}(x) = 3x + 2,$

$gof(x) = 2x - 1 \Rightarrow$ **$fof(3) = ?$**

A) 17 B) 19 C) 21 D) 23 E) 25

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$fog^{-1}(x) \text{ o } gof(x) = fof(x)$

$\Rightarrow fof(x) = 3 \cdot (2x - 1) + 2$

$fof(x) = 6x - 1$

$fof(3) = 6 \cdot 3 - 1 = 17$

PERMÜTASYON FONKSİYONU / PERMUTATION FUNCTION

Tanım ve değer kümeleri aynı olan fonksiyondur.

The permutation function is a function whose domain and range sets are same.

$$f = \begin{pmatrix} a & b & c & d \\ d & c & b & a \end{pmatrix} \Rightarrow \begin{cases} f(a) = d, & f^{-1}(d) = a \\ f(b) = c, & f^{-1}(c) = b \\ f(c) = b, & f^{-1}(b) = c \\ f(d) = a, & f^{-1}(a) = d \end{cases}$$

ÖRNEK / EXAMPLE 10

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, \; fog = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} \Rightarrow \mathbf{g = ?}$$

A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$

D) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$

E) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

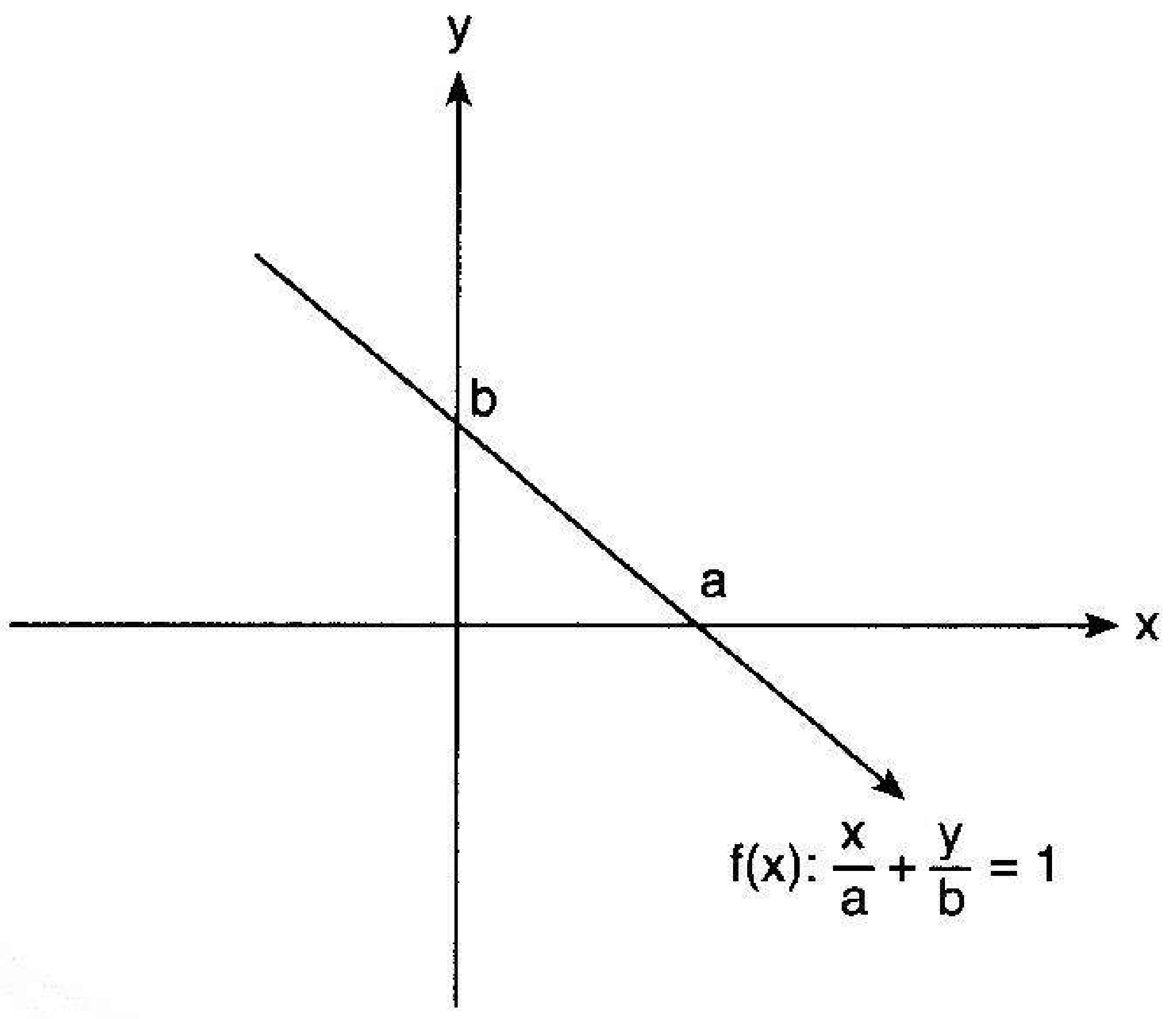
ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$f^{-1} o \, (fog) = g \Rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} o \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

DOĞRUSAL FONKSİYON / LINEAR FUNCTION



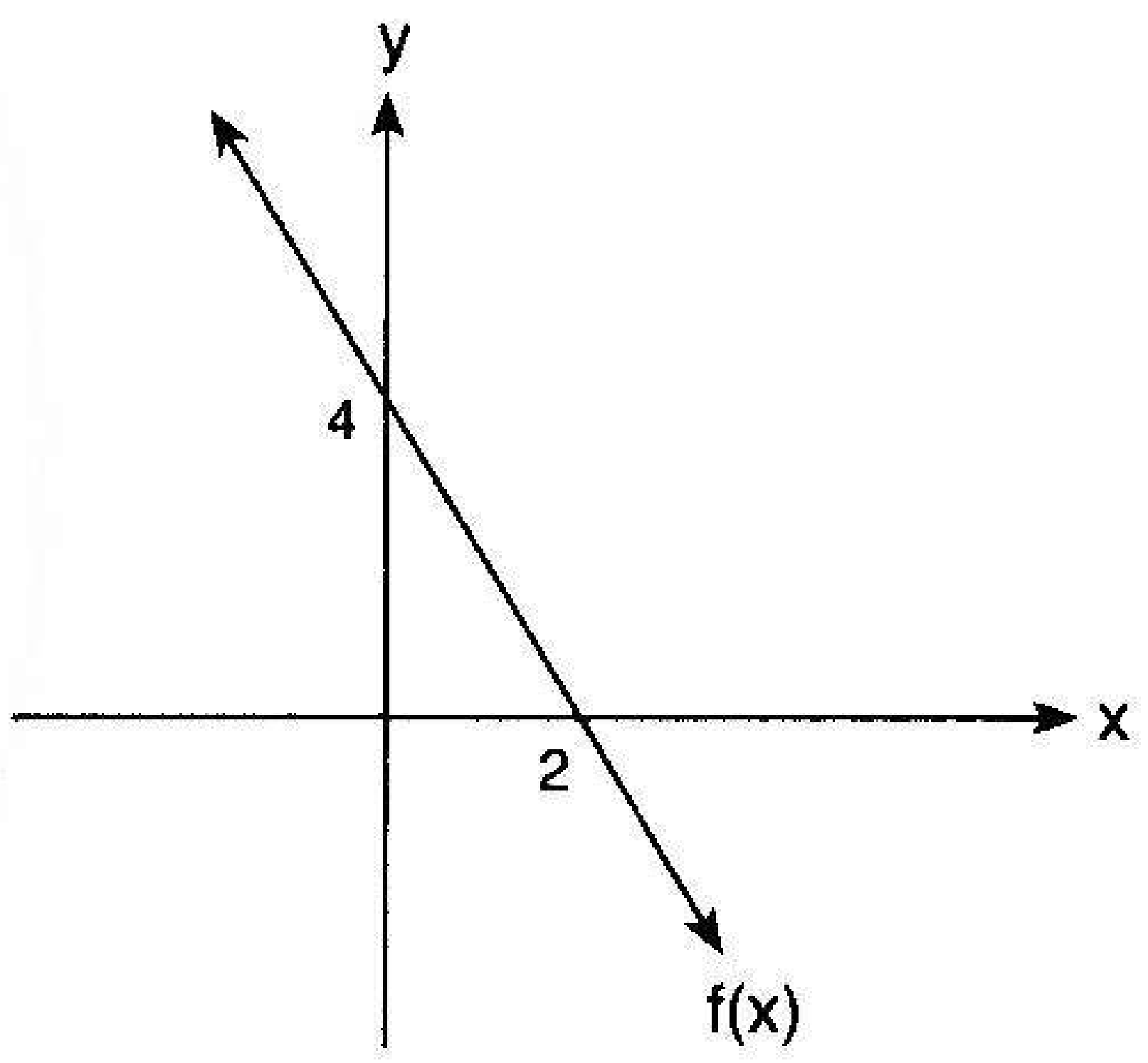
a ve b sabit sayılar olmak üzere, y = ax + b şeklindeki fonksiyonlardır.

a and b are constants. y = ax + b is a linear function.

ÖRNEK / EXAMPLE 11

$f(x) = ax + b$

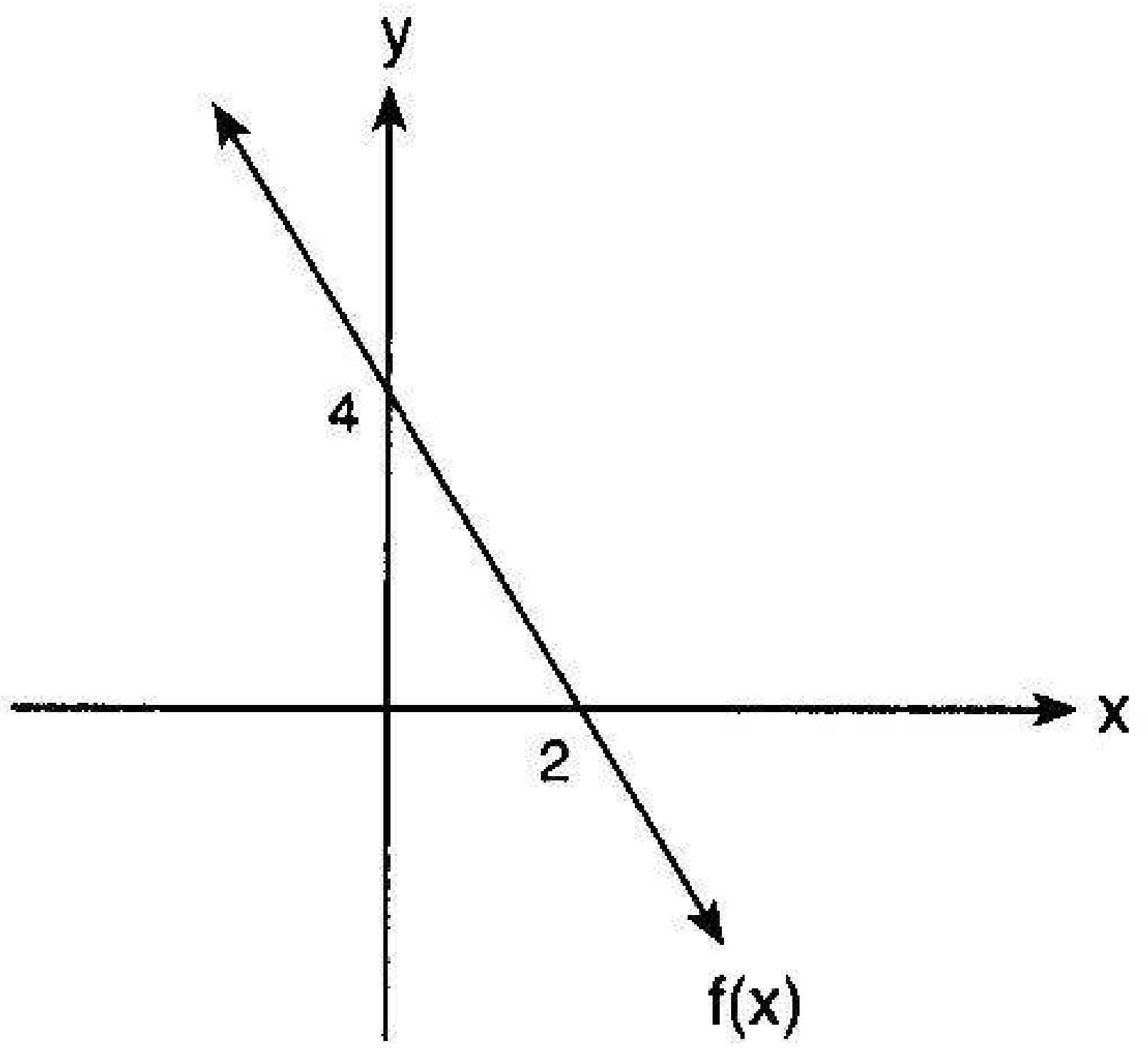
$\Rightarrow$ **f(1) = ?**



A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**



$\frac{x}{2}+\frac{y}{4}=1$

$f(x) = y = 4 - 2x$

$f(1) = 4 - 2 = 2$

ÖRNEK / EXAMPLE 12

YILDIZ TEKNİK ÜNİ. YÖS 2013

$f, g: R \to R$

$g(x) = x - 1$

$f(x + 1) = 2g(x - 1)$

$\Rightarrow$ **$(gof^{-1})(x) = ?$**

A) $\frac{x+3}{2}$ B) $\frac{x-2}{x-1}$ C) $\frac{1}{3x}$ D) $\frac{x+4}{2}$ E) $\frac{5}{x+1}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$g(x-1) = x-2$$

$$f(x+1) = 2g(x-1) = 2x-4$$

$$f(x-1+1) = 2(x-1)-4$$

$$f(x) = 2x-6$$

$$f^{-1}(x) = \frac{x+6}{2}$$

$$g\left(\frac{x+6}{2}\right) = \frac{x+6}{2} - 1 = \frac{x+4}{2}$$

PEKİŞTİRME / REINFORCE

$f(x) = x + 1,$

$g^{-1}(x) = 3x - 4$

$\Rightarrow fog(2) = ?$

YANIT / ANSWER : 3

ÖRNEK / EXAMPLE 13

İSTANBUL ÜNİ. YÖS 2013

$f(x) = 3^{2x+1},$

$f(2x) = m \cdot (f(x))^2 \Rightarrow$ **$m = ?$**

A) 3 B) 2 C) 1 D) $\frac{1}{3}$ E) $\frac{1}{9}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$3^{2(2x)+1} = m(3^{2x+1})^2$$

$$3 \cdot 3^{4x} = m \cdot 3^{4x} \cdot 3^2$$

$$m = \frac{3}{9}$$

$$m = \frac{1}{3}$$

PEKİŞTİRME / REINFORCE

$f(x) = 3^x \Rightarrow f(2x) = ?$

YANIT / ANSWER : $f^2(x)$

ÖRNEK / EXAMPLE 14

ANKARA ÜNİV. YÖS 2015

$f(2x + 1) = 3x + 4,$

$f^{-1}(a) = 1 \Rightarrow \mathbf{a = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$f^{-1}(a) = 1 \Rightarrow f(1) = a$

$x = 0 \Rightarrow f(1) = 4 \Rightarrow a = 4$

ÖRNEK / EXAMPLE 15

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$f : R \to R$

$\mathbf{f(x) = 2^{x+3}} \quad \Rightarrow \mathbf{f(m + n - 2) = ?}$

A) $\dfrac{f(m+n)}{2}$

B) $\dfrac{f(m)\cdot f(n)}{32}$

C) $f(m) \cdot f(n)$

D) $2^{f(m) - f(n)}$

E) $[f(m)]^{f(n) - 1}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$f(m + n - 2) = 2^{m+n-2+3} = 2^{m+n+1}$

$$\left.\begin{array}{l} f(m) = 2^{m+3} \\ \\ f(n) = 2^{n+3} \end{array}\right\} \frac{f(m)\cdot f(n)}{32} = \frac{2^{m+3}\cdot 2^{n+3}}{2^5} = \frac{2^{m+n+6}}{2^5} = 2^{m+n+1}$$

1. $f(x) = 2x + 1 \Rightarrow$ **$f([-1, 4)) = ?$**

A) $[-1, 9]$

B) $[-1, 9)$

C) $(-1, 9]$

D) $(-1, 9)$

E) $\{-1, 9\}$

2. $f(2x + 1) = 6x + 4 \Rightarrow$ **$f(1) = ?$**

A) 3

B) 4

C) 5

D) 6

E) 10

3. $f(x) = \begin{cases} 2x - 5, & x < 1 \\ \dfrac{x}{2}, & x \geq 3 \end{cases} \Rightarrow$ **$f(4) + 2 \cdot f(0) = ?$**

A) 3

B) 2

C) -8

D) -9

E) -10

4. $f, g : R \to R$,

$f(x) = x + 1$,

$g(x) = x^3 \Rightarrow$ **$(gof^{-1})(-1) = ?$**

A) -6

B) -8

C) -10

D) -12

E) -27

5. $\left.\begin{array}{l} f(x) = 2x + 15 \\ g(x) = 8x - 1 \end{array}\right\} \Rightarrow$ **$(g^{-1}of)(0) = ?$**

A) -2

B) -1

C) 0

D) 1

E) 2

6. $a > 0$,

$f(x) = \dfrac{-3x}{x + a}$, $f(x) = f^{-1}(x) \Rightarrow$ **$a = ?$**

A) 1

B) 2

C) 3

D) 4

E) 6

7. $f\left(\dfrac{2x-1}{3x+1}\right)=\dfrac{x-1}{x-2} \Rightarrow \mathbf{f^{-1}(2)=?}$

A) $\dfrac{1}{2}$　　B) $-\dfrac{1}{2}$　　C) $\dfrac{1}{4}$

D) $-\dfrac{1}{4}$　　E) -1

8. $f(x) = x + 2$,

$(fog)(x)=\dfrac{x+1}{x-2} \Rightarrow \mathbf{g(x)=?}$

A) $\dfrac{x-1}{x-2}$　　B) $\dfrac{-x+1}{x-2}$　　C) $\dfrac{-x+3}{x-2}$

D) $\dfrac{-x+5}{x-2}$　　E) $\dfrac{x+5}{x-2}$

9. $f : R \to R$,

$2 \cdot f(x-2) + 3 \cdot f(2+x) = x + 5 \Rightarrow \mathbf{f(1)=?}$

A) -3　　B) $-\dfrac{7}{5}$　　C) -1

D) $\dfrac{2}{5}$　　E) $\dfrac{28}{25}$

10. $f^{-1}\left(\dfrac{4x+m}{x+1}\right)=3x-4$, $f(5)=2 \Rightarrow \mathbf{m=?}$

A) –4　　B) –3　　C) 5

D) 6　　E) 7

11. $f(x) = ax + b$,

$g(x) = 3x - 2$,

$(gof)(x) = g^{-1}(x) \Rightarrow \mathbf{a+b=?}$

A) 1　　B) 2　　C) 3

D) 4　　E) 5

12. $$\left.\begin{array}{l} f(x)=\dfrac{x^2-2}{x-1} \\ g(x)=x+1 \end{array}\right\} \Rightarrow \mathbf{(fog^{-1})(3)=?}$$

A) 2　　B) 3　　C) 4

D) 5　　E) 6

YANITLAR / ANSWERS

1. B	2. B	3. C	4. B	5. E	6. C	7. A	8. D
9. E	10. A	11. A	12. A				

1. $f(x) = 2x + 1$

$x = -1 \Rightarrow f(-1) = 2 \cdot (-1) + 1$

$= -1$

$x = 4 \Rightarrow f(4) = 2 \cdot 4 + 1 = 9$

$\Rightarrow f([-1, 4)) = [-1, 9)$

Yanıt / The Answer is B

---

2. $2x + 1 = 1$

$2x = 0$

$x = 0 \Rightarrow f(1) = 6 \cdot 0 + 4 = 4$

Yanıt / The Answer is B

---

3. $4 > 3 \Rightarrow f(x) = \frac{x}{2}$

$f(4) = \frac{4}{2} = 2$

$0 < 1 \Rightarrow f(x) = 2x - 5$

$f(0) = 2 \cdot 0 - 5 = -5$

$f(4) + 2 \cdot f(0) = 2 + 2 \cdot (-5)$

$= 2 - 10 = -8$

Yanıt / The Answer is C

---

4. $f(x) = x + 1 \Rightarrow f^{-1}(x) = x - 1$

$g(x) = x^3$

$(gof^{-1})(x) = g[f^{-1}(x)]$

$= (x - 1)^3$

$(gof^{-1})(-1) = (-1 - 1)^3 = -8$

Yanıt / The Answer is B

---

5. $f(x) = 2x + 15$

$g(x) = 8x - 1 \Rightarrow g^{-1}(x) = \frac{x+1}{8}$

$(g^{-1}of)(0) = g^{-1}(2 \cdot 0 + 15)$

$= g^{-1}(15)$

$= \frac{15+1}{8}$

$= 2$

Yanıt / The Answer is E

---

6. $f(x) = \frac{-3x}{x+a} \Rightarrow f^{-1}(x) = \frac{-ax}{x+3}$

$\frac{-3x}{x+a} = \frac{-ax}{x+3}$

$3x + 9 = ax + a^2$

$a^2 = 9$

$a = 3$

Yanıt / The Answer is C

7. $f\left(\frac{2x-1}{3x+1}\right)=\frac{x-1}{x-2} \Rightarrow f^{-1}\left(\frac{x-1}{x-2}\right)=\frac{2x-1}{3x+1}$

$$\frac{x-1}{x-2}=2 \Rightarrow 2x-4=x-1$$
$$x=3$$

$$f^{-1}(2)=\frac{2\cdot 3-1}{3\cdot 3+1}=\frac{5}{10}=\frac{1}{2}$$

Yanıt / The Answer is A

---

8. $f(x)=x+2 \Rightarrow f^{-1}(x)=x-2$

$$(f^{-1}\ o\ (fog)(x))=g(x)$$

$$\frac{x+1}{x-2}-2=g(x)$$

$$g(x)=\frac{x+1-2x+4}{x-2}=\frac{-x+5}{x-2}$$

Yanıt / The Answer is D

---

9. $f(x)=ax+b \Rightarrow f(x-2)=ax-2a+b$

$f(x)=ax+b \Rightarrow f(2+x)=ax+2a+b$

$$2\cdot f(x-2)+3\cdot f(2+x)=x+5$$
$$2ax-4a+2b+3ax+6a+3b=x+5$$
$$5ax+2a+5b=x+5$$

$$5a=1 \Rightarrow a=\frac{1}{5}$$

$$2\cdot\frac{1}{5}+5b=5 \Rightarrow b=\frac{23}{25}$$

$$f(x)=\frac{x}{5}+\frac{23}{25}$$

$$f(1)=\frac{1}{5}+\frac{23}{25}=\frac{28}{25}$$

Yanıt / The Answer is E

---

10. $f(5)=2 \Rightarrow f^{-1}(2)=5$

$$f^{-1}\left(\frac{4x+m}{x+1}\right)=3x-4 \Rightarrow 3x-4=5$$
$$x=3$$

$$\Rightarrow \frac{4\cdot x+m}{x+1}=2$$
$$\frac{4\cdot 3+m}{3+1}=2$$
$$m=-4$$

Yanıt / The Answer is A

---

11. $\left.\begin{array}{l} f(x)=ax+b \\ g(x)=3x-2 \end{array}\right\} \Rightarrow (gof)(x)=3\cdot(ax+b)-2$

$$(gof)(x)=g^{-1}(x)$$

$$3ax+3b-2=\frac{x+2}{3}$$

$$9ax+9b-6=x+2$$

$$\Rightarrow a=\frac{1}{9}\ ,\ b=\frac{8}{9}$$

$$a+b=\frac{1}{9}+\frac{8}{9}=\frac{9}{9}=1$$

Yanıt / The Answer is A

---

12. $g(x)=x+1 \Rightarrow g^{-1}(x)=x-1$

$$(fog^{-1})(3)=f(3-1)$$
$$=f(2)$$
$$=\frac{2^2-2}{2-1}=2$$

Yanıt / The Answer is A

1. $f(x \cdot y) = f(x) + f(y)$, $f(2) = 5 \Rightarrow$ **$f(8) = ?$**

A) 15
B) 25
C) 45
D) 75
E) 125

2. $g(x) = 2^x + 1 \Rightarrow$ **$g^{-1}(9) = ?$**

A) 2
B) 3
C) 4
D) 5
E) 6

3. $f(4x-2) = \dfrac{3x-1}{x-2} \Rightarrow$ **$f^{-1}(8) = ?$**

A) –3
B) 5
C) 10
D) 13
E) 17

4. $f(x) = \begin{cases} 4x+3 & x<1 \\ x^2+1 & x \geq 1 \end{cases}$

$g(x) = 2x + 5 \Rightarrow$ **$fog(-1) = ?$**

A) 7
B) 8
C) 9
D) 10
E) 11

5. $f(2x^2 - x) = 4x^2 - 2x + 5 \Rightarrow$ **$f(3) = ?$**

A) 7
B) 8
C) 9
D) 10
E) 11

6. $f(x) = 4x - 9$,

$(g^{-1}of)(-2) = 5 \Rightarrow$ **$g(5) = ?$**

A) –17
B) –19
C) –21
D) –23
E) –25

7. $f(x+1) = 3x - 2$

$g(x-1) = 4x + 5 \Rightarrow \mathbf{gof^{-1}(4) = ?}$

A) 21

B) 22

C) 23

D) 24

E) 25

8. $f(x) = 3x^2 + 4,\ g(x) = 3x - 2$

$[f^{-1} og]^{-1}(x) = x^2 + a \Rightarrow \mathbf{a = ?}$

A) 2

B) 3

C) 4

D) 5

E) 6

9. $f(x, y) = \max(x^2 - 2, xy + 3)$

$g(x,y) = \min(3x + y, 2x - y)$

$\Rightarrow \mathbf{f(-1, 4) + g(-2, 5) = ?}$

A) −15

B) −13

C) −10

D) −6

E) −3

10. $f(x) = ax + b,\ f^{-1}(7) = 3,$

$f^{-1}(-5) = -1 \Rightarrow \mathbf{f^{-1}(10) = ?}$

A) 1

B) 2

C) 3

D) 4

E) 5

11. $\dfrac{\mathbf{fof^{-1}(2) + f(1)}}{\mathbf{f^{-1}(-2) + f(0)}} = \mathbf{?}$

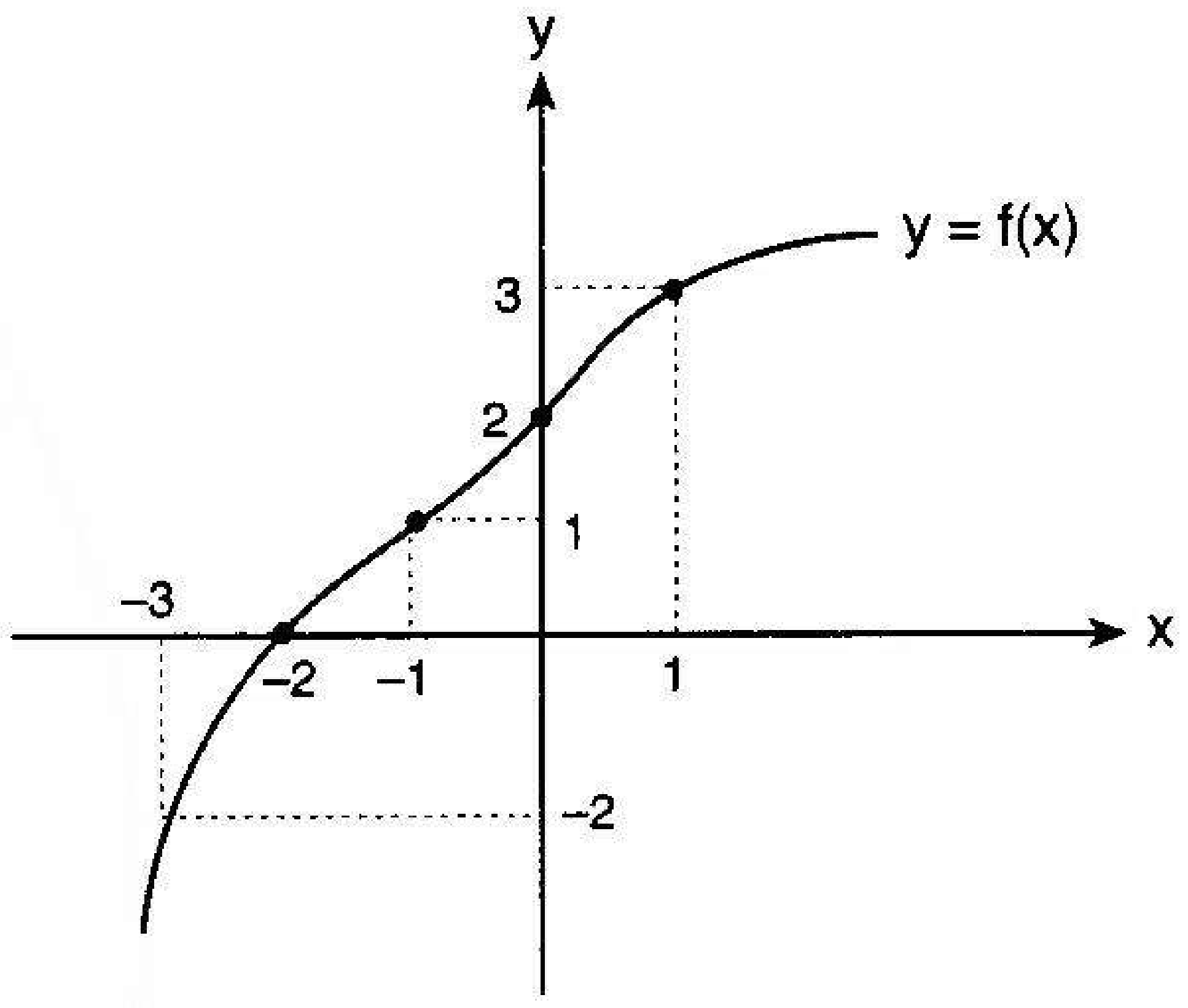
A) −5

B) −4

C) −3

D) −2

E) −1



12. $\dfrac{\mathbf{f(f(1))}}{\mathbf{f^{-1}(-3) + f(3)}} = \mathbf{?}$

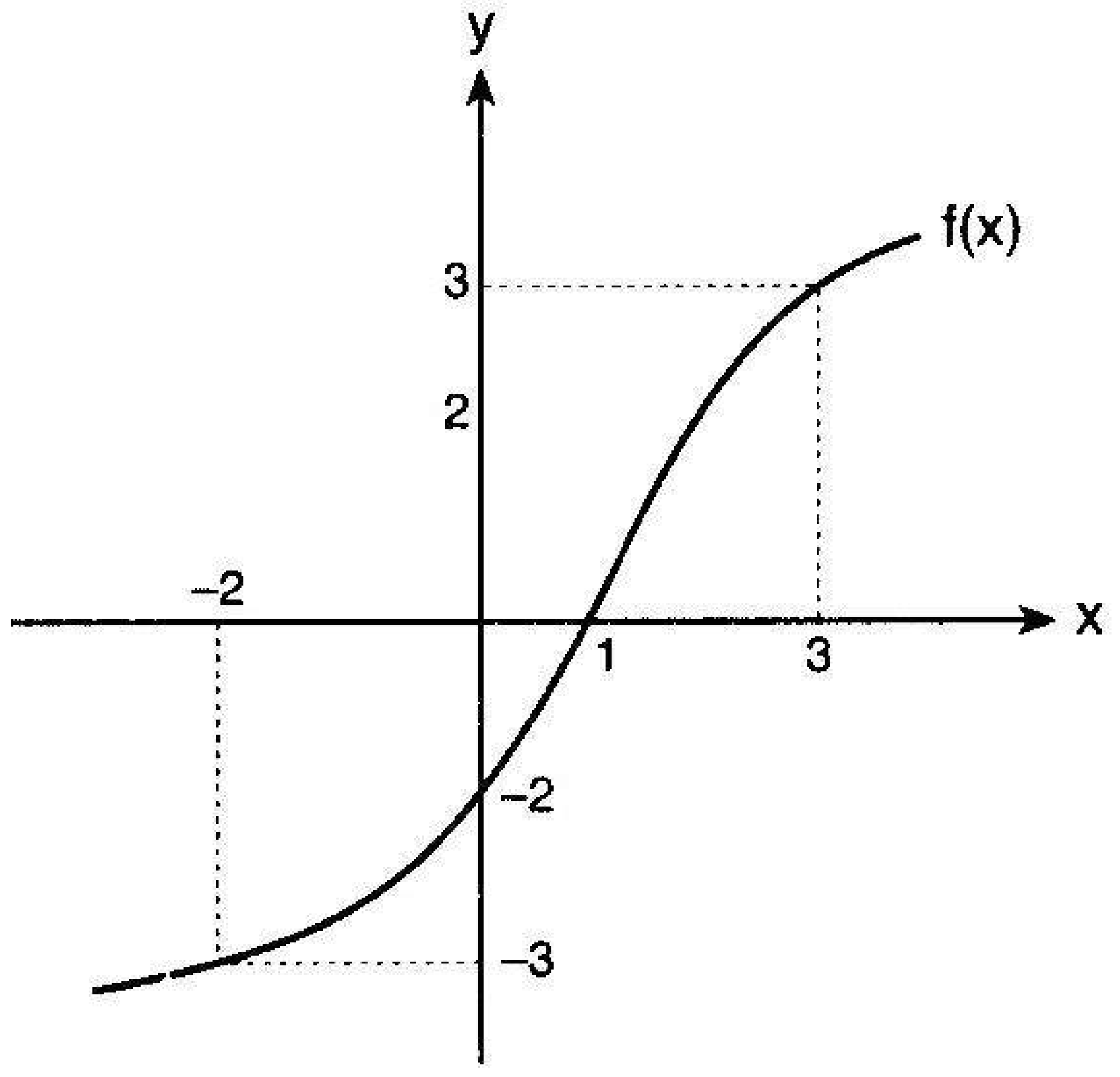
A) −2

B) −1

C) 0

D) 1

E) 2



YANITLAR / ANSWERS

1. A	**2. B**	**3. C**	**4. D**	**5. E**	**6. A**	**7. A**	**8. A**
9. C	**10. D**	**11. A**	**12. A**				

1. $f(8) = f(2 \cdot 4)$

$f(8) = f(2) + f(4)$

$f(8) = f(2) + f(2 \cdot 2)$

$f(8) = f(2) + f(2) + f(2)$

$f(2) = 5 \Rightarrow f(8) = 3 \cdot 5 = 15$

Yanıt / The Answer is A

2. $g(x) = 2^x + 1 \Rightarrow g^{-1}(2^x + 1) = x$

$2^x + 1 = 9 \Rightarrow x = 3$

$g^{-1}(9) = 3$

Yanıt / The Answer is B

3. $f(4x-2) = \dfrac{3x-1}{x-2}$

$f^{-1}\left(\dfrac{3x-1}{x-2}\right) = 4x-2$

$\dfrac{3x-1}{x-2} = 8 \Rightarrow 3x-1 = 8x-16$

$15 = 5x$

$x = 3$

$f^{-1}(8) = 4 \cdot 3 - 2 = 10$

Yanıt / The Answer is C

4. $fog(-1) = f(g(-1))$

$g(x) = 2x + 5 \Rightarrow g(-1) = 2 \cdot (-1) + 5$

$g(-1) = 3$

$f(g(-1)) = f(3)$

$3 \geq 1 \Rightarrow f(x) = x^2 + 1$

$f(3) = 3^2 + 1$

$f(3) = 10$

Yanıt / The Answer is D

5. $f(2x^2 - x) = 4x^2 - 2x + 5$

$f(2x^2 - x) = 2 \cdot (2x^2 - x) + 5$

$\Rightarrow f(3) = 2 \cdot 3 + 5 = 11$

Yanıt / The Answer is E

6. $f(x) = 4x - 9,$

$(g^{-1}of)(-2) = 5$

$g^{-1}(f(-2)) = 5$

$g(5) = f(-2)$

$= 4 \cdot (-2) - 9 = -17$

Yanıt / The Answer is A

7. $f(x+1) = 3x - 2 \Rightarrow f(x) = 3x - 5$

$g(x-1) = 4x + 5 \Rightarrow g(x) = 4x + 9$

$gof^{-1}(4) = g(f^{-1}(4))$

$$f^{-1}(x) = \frac{x+5}{3} \Rightarrow f^{-1}(4) = 3$$

$$g(f^{-1}(4)) = g(3) = 4 \cdot 3 + 9 = 21$$

Yanıt / The Answer is A

---

8. $\left[f^{-1}og\right]^{-1}(x) = (g^{-1}of)(x) = x^2 + a$

$$g^{-1}(x) = \frac{x+2}{3} \Rightarrow (g^{-1}of)(x) = \frac{3x^2+4+2}{3}$$

$$(g^{-1}of)(x) = x^2 + 2 = x^2 + a \Rightarrow a = 2$$

Yanıt / The Answer is A

---

9. $f(x, y) = \max(x^2 - 2, xy + 3) \Rightarrow f(-1, 4) = -1$

$g(x,y) = \min(3x + y, 2x - y) \Rightarrow g(-2, 4) = -9$

$\Rightarrow f(-1, 4) + g(-2, 5) = (-1) + (-9) = -10$

Yanıt / The Answer is C

---

10. $f^{-1}(7) = 3 \Rightarrow f(3) = 7$

$f^{-1}(-5) = -1 \Rightarrow f(-1) = -5$

$f(x) = ax + b \Rightarrow 3a + b = 7$

$-a + b = -5$

$\Rightarrow a = 3, \ b = -2$

$$f(x) = 3x - 2$$

$$f^{-1}(x) = \frac{x+2}{3}$$

$$f^{-1}(10) = \frac{10+2}{3}$$

$$f^{-1}(10) = \frac{12}{3} = 4$$

Yanıt / The Answer is D

---

11. $(fof^{-1})(2) = I(2) = 2$

$f(1) = 3,$

$f(0) = 2,$

$f^{-1}(-2) = -3$

$$\Rightarrow \frac{fof^{-1}(2) + f(1)}{f^{-1}(-2) + f(0)} = \frac{2+3}{-3+2} = -5$$

Yanıt / The Answer is A

---

12. $f(1) = 0 \Rightarrow f(f(1)) = -2$

$f^{-1}(3) = 3$

$f^{-1}(-3) = -2$

$$\Rightarrow \frac{f(f(1))}{f^{-1}(-3) + f(3)} = \frac{-2}{3-2} = -2$$

Yanıt / The Answer is A

1. $f\left(\frac{x+1}{2}\right)=3x-4 \Rightarrow$ **$f(3)=?$**

Yanıt / The Answer is: **11**

2. $f(3x-2)=4x+7,$

$f(4)=2n+3 \Rightarrow$ **$n=?$**

Yanıt / The Answer is: **6**

3. $f(x, y)=3x+y \Rightarrow$ **$f(-1, 3)=?$**

Yanıt / The Answer is: **0**

4. $f(4x+2)=\frac{2x-1}{x+3} \Rightarrow$ **$f^{-1}(1)=?$**

Yanıt / The Answer is: **18**

5. $f(x)=\begin{cases} \frac{3x+b}{4} & x\leq 1 \\ x\cdot(x-a) & x>1 \end{cases}$

$f(3)=6,\ f(-1)=2 \Rightarrow$ **$a+b=?$**

Yanıt / The Answer is: **12**

6. $f(x)=\frac{x-5}{2x+3} \Rightarrow$ **$f(3x)=?$**

Yanıt / The Answer is: $\mathbf{\frac{3x-5}{6x+3}}$

7. $f(x) = 3^{2x-3} + 1 \Rightarrow \mathbf{f^{-1}(28) = ?}$

Yanıt / The Answer is: **3**

8. $f(x) = 2f(x-1) + 4,$

$f(5) = 64 \Rightarrow \mathbf{f(2) = ?}$

Yanıt / The Answer is: $\frac{9}{2}$

9. $f(x) = \begin{cases} 3x^2 + 1 & x \le 1 \\ 4x - 7 & x > 1 \end{cases} \Rightarrow \mathbf{f(0) + f(2) = ?}$

Yanıt / The Answer is: **2**

10. $f(x) = \dfrac{3x+a}{x-3}, \quad g(x) = x + 5,$

$fog^{-1}(3) = 2 \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **–4**

11. $f(x) = \begin{cases} 3x - 7 & x \le 0 \\ x^2 + 2 & x > 0 \end{cases}$

$g(x) = 4x - 3 \Rightarrow \mathbf{fog^{-1}(5) = ?}$

Yanıt / The Answer is: **6**

12. $f : R \to R, \ f : x \to y = f(x)$

$f(x-3) = \dfrac{ax}{3x+1}, \ f(x) = f^{-1}(x) \Rightarrow \mathbf{f(2) = ?}$

Yanıt / The Answer is: $-\frac{25}{8}$

13. $f(2x+1) = 4x - 3,$

$2 \cdot f(x) - f(x+1) = 5 \quad \Rightarrow \quad \mathbf{x = ?}$

Yanıt / The Answer is: **6**

14. $f(x-1) = \frac{1}{3}x^2 + x - m\ ,$

$f(2) = 4 \Rightarrow \mathbf{f(1-m) = ?}$

Yanıt / The Answer is: **–2**

15. $f(x) = \frac{2x+1}{x-1}\ ,$

$g(x) = \frac{x+a}{2x-5}$

$(g^{-1}of)(2) = 3 \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **2**

16. $f(x) = 3x^2 - 5,$

$g(x) = x + 1$

$\Rightarrow \mathbf{fog^{-1}(-1) = ?}$

Yanıt / The Answer is: **7**

17. $f(x) = ax + 3\ ,$

$g(x) = \frac{1}{2}x - a,$

$(fog)^{-1}(-3) = 2 \Rightarrow \mathbf{a_1 \cdot a_2 = ?}$

Yanıt / The Answer is: **–6**

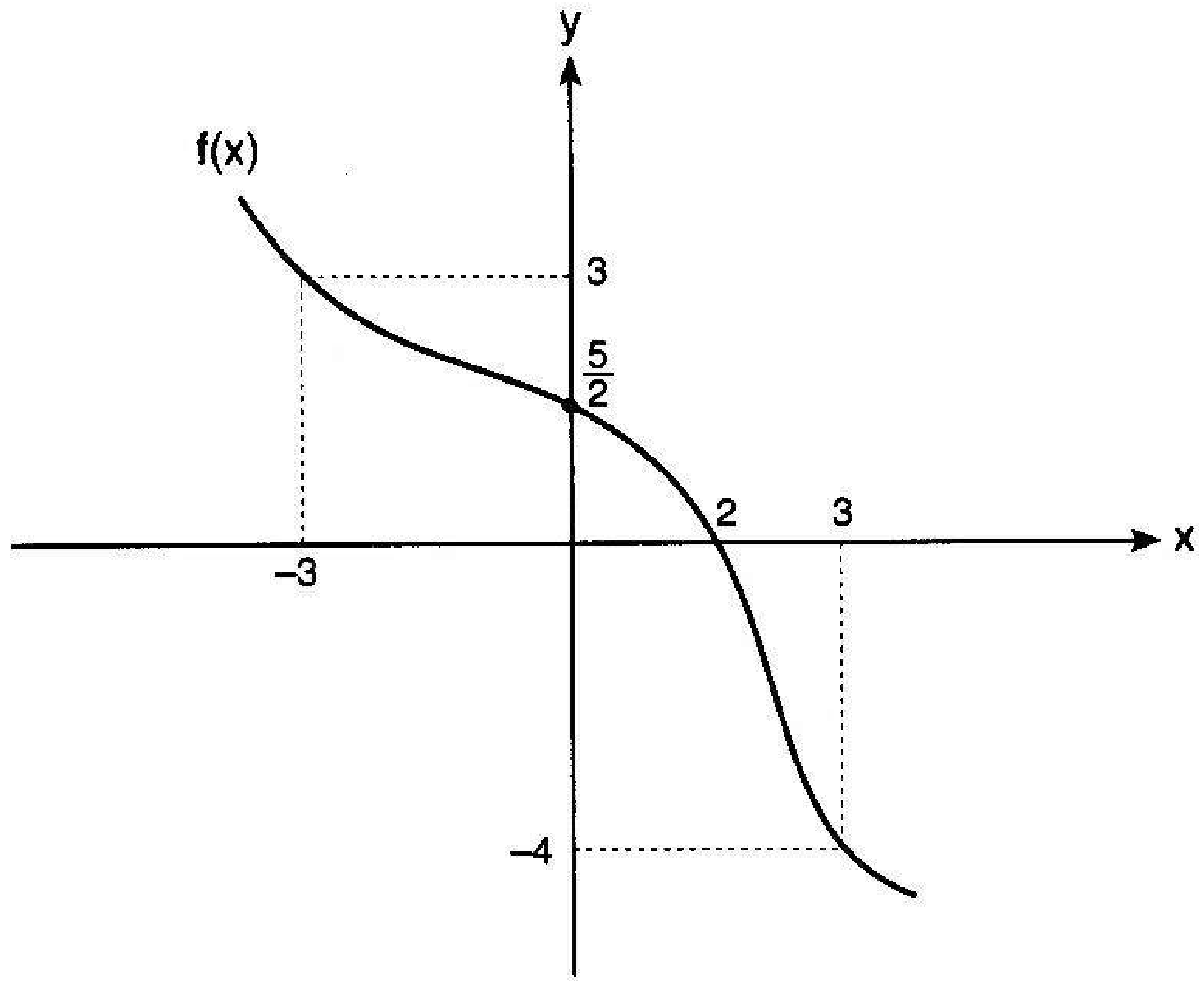
18. $f(x) = \frac{3x-a}{2x+1}\ ,$

$g(x) = \frac{x-1}{x+1}\ ,$

$(f^{-1}og)(3) = \frac{1}{4} \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **0**

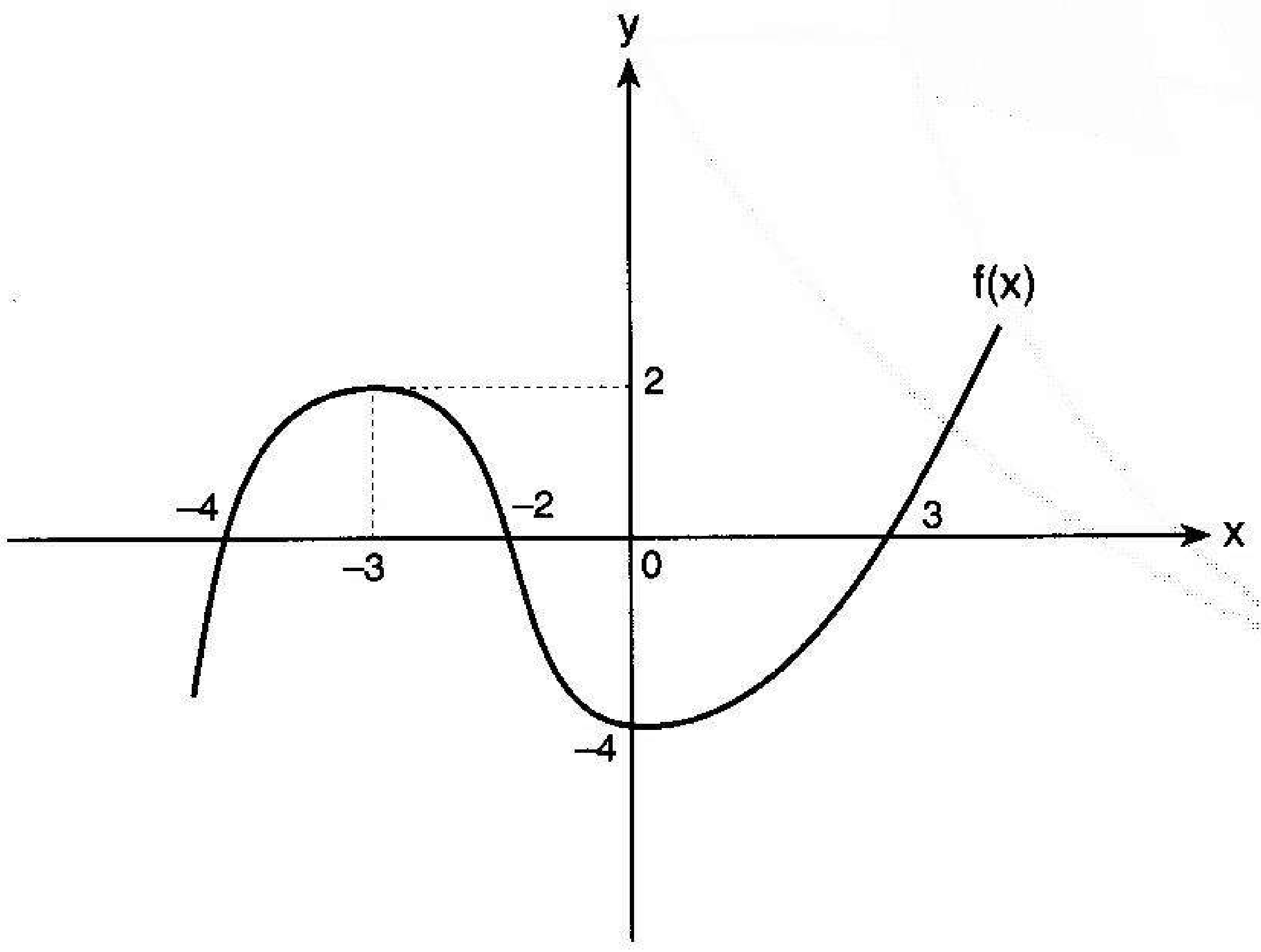
19.



$$\frac{f(-3)+f(2)}{f(3)+f(0)}=?$$

Yanıt / The Answer is: **–2**

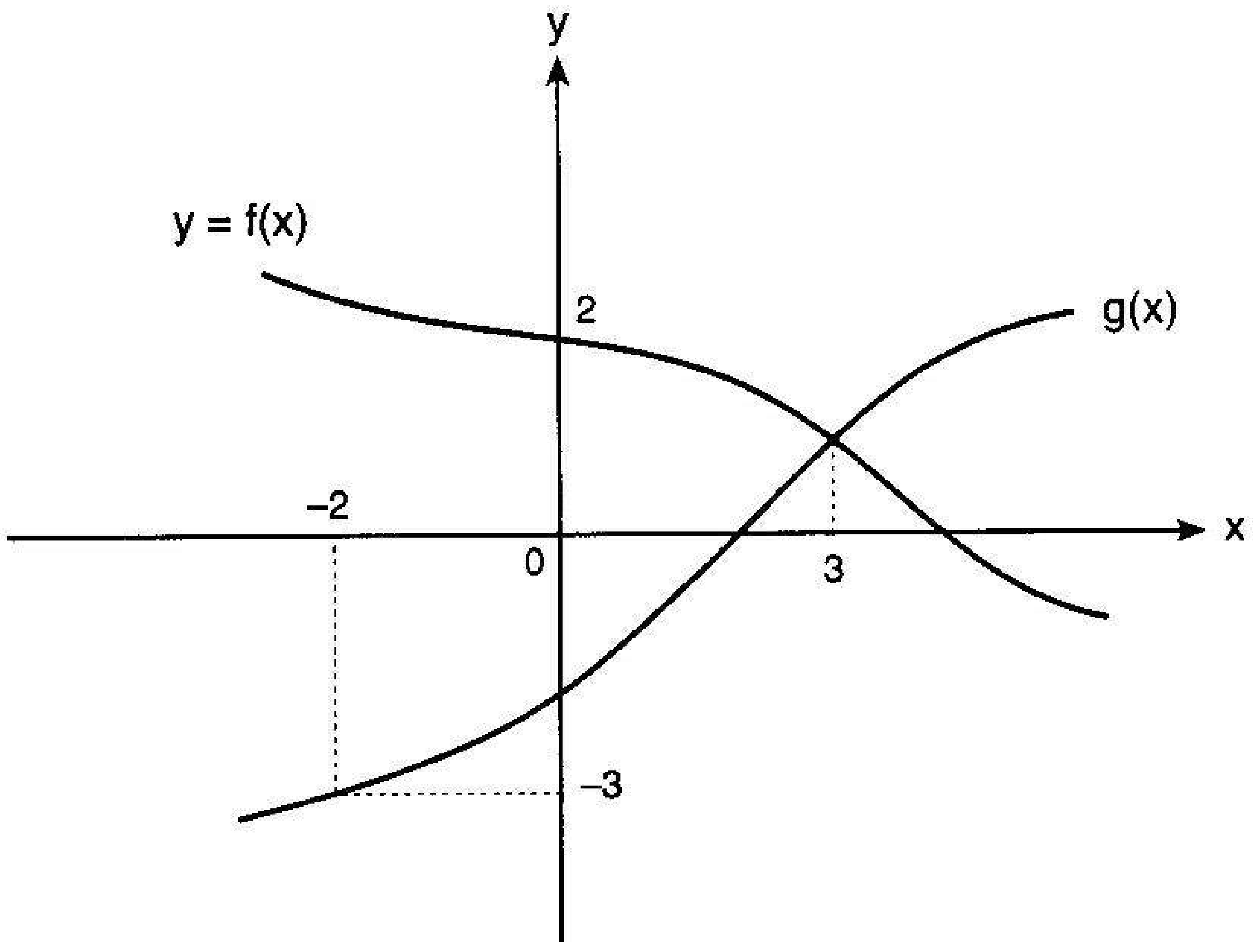
20.



$$\frac{f(3)+f(-3)}{f^{-1}(-4)+f^{-1}(2)}=?$$

Yanıt / The Answer is: $\frac{-2}{3}$

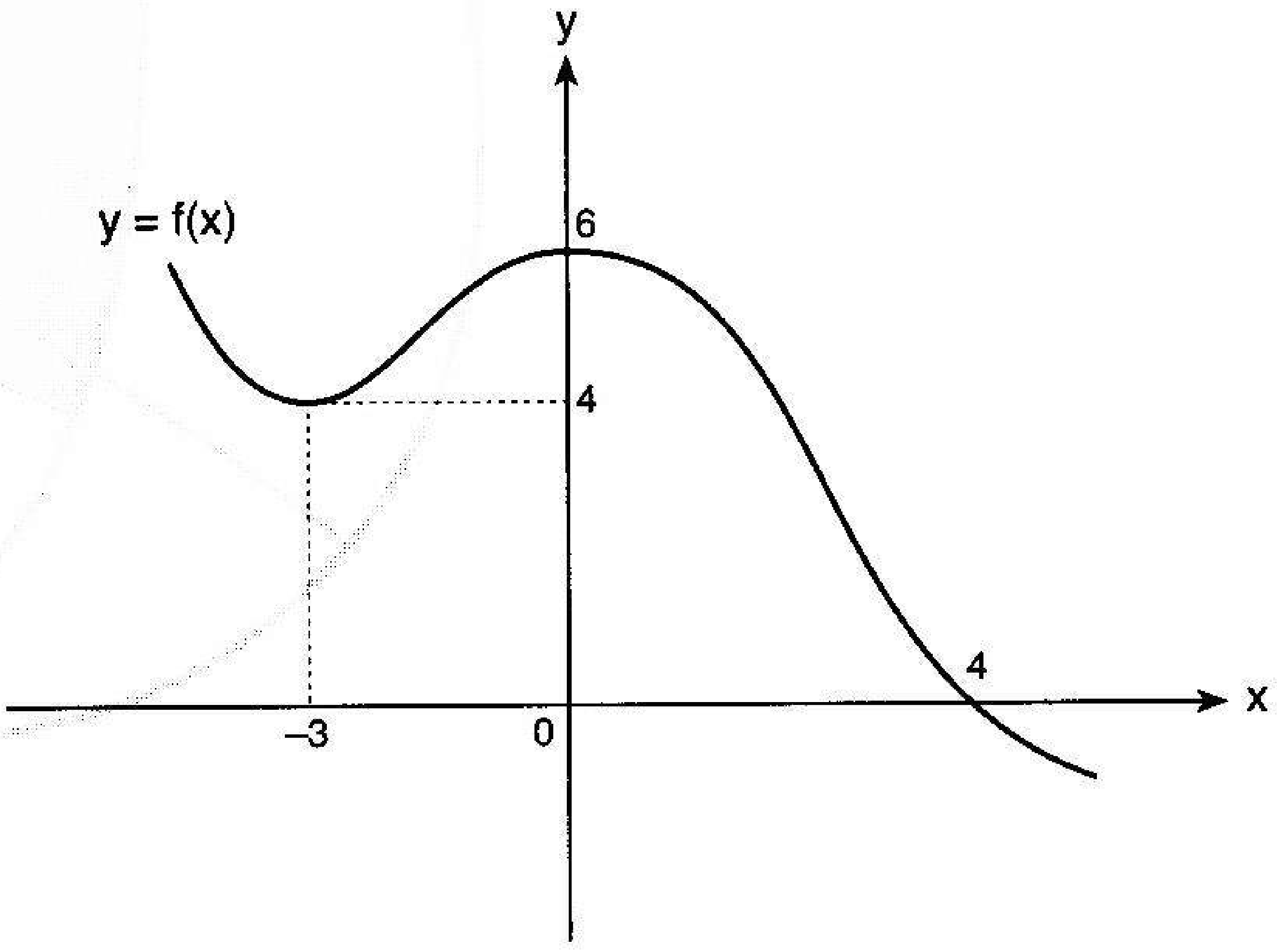
21.



$(g^{-1}of)(3)+f^{-1}(a)=-g(-2) \Rightarrow$ **a = ?**

Yanıt / The Answer is: **2**

22.



$(fofof)(k)=6 \Rightarrow$ **k = ?**

Yanıt / The Answer is: **–3**

1. $f : A \to B$,

$f(x) = 4x - 3$,

$B = \{1, 9, 17\} \Rightarrow$ **A = ?**

A) $\{-2, 0, 1\}$

B) $\{3, 7, 8\}$

C) $\{2, 5, 6\}$

D) $\{3, 6, 7\}$

E) $\{1, 3, 5\}$

2. $f(x) = \begin{cases} 3x - 2 & x \le 3 \\ x + 1 & 3 < x \end{cases}$

$\Rightarrow$ **f(−2) + f(4) = ?**

A) −6

B) −3

C) 1

D) 2

E) 5

3. $f(x + y) = f(x) \cdot f(y)$

$f(3) = 2 \Rightarrow$ **f(9) = ?**

A) 2

B) 4

C) 8

D) 16

E) 32

4. $f(x - 3) = 2x + 1 \Rightarrow$ **$f^{-1}(5)$ = ?**

A) −1

B) 3

C) 6

D) 8

E) 10

5. $f(4x - 3) = 2x - 5$,

$f^{-1}(1) = a + 2 \Rightarrow$ **a = ?**

A) 6

B) 7

C) 8

D) 9

E) 10

6. $g(x) = ax^2 + b$,

$g(2) = 11$,

$g(-1) = 2 \Rightarrow$ **g(4) = ?**

A) 45

B) 46

C) 47

D) 48

E) 49

7. $f(x) = 4x - 2$

$g(x) = 3x + 5 \Rightarrow$ **$fog(-1) = ?$**

A) 1

B) 3

C) 5

D) 6

E) 9

8. $f(x) = \frac{x+1}{x-1}$

$g(x) = 4x - 1 \Rightarrow$ **$(f^{-1} \text{ o } g)^{-1}(2) = ?$**

A) –12

B) –9

C) –7

D) –3

E) 1

9. $f(x + 1) = (x - 2) \cdot f(x)$

$f(3) = 1 \Rightarrow$ **$f(50) = ?$**

A) 47!

B) 48!

C) 49!

D) 50!

E) 51!

10. $f(x) = 2x + a,$

$f(x + 2) + f(x - 1) = 4x + 8 \Rightarrow$ **$a = ?$**

A) –2

B) –1

C) 2

D) 3

E) 4

11. $f(x + 1) = 2f(x) + 1,$

$f(1) = -2 \Rightarrow$ **$f(4) = ?$**

A) –1

B) –3

C) –5

D) –7

E) –9

12. $f(x) + 2f(-x) = x^2 - 3x \Rightarrow$ **$f(1) = ?$**

A) 2

B) $\frac{8}{3}$

C) 3

D) $\frac{10}{3}$

E) $\frac{14}{3}$

YANITLAR / ANSWERS							
1. E	**2. B**	**3. C**	**4. A**	**5. B**	**6. C**	**7. D**	**8. E**
9. B	**10. D**	**11. E**	**12. D**				

1. $f(x) = 4x + 5$

$g(x) = \begin{cases} x-3 & x \geq 1 \\ x^2+1 & x<1 \end{cases} \Rightarrow$ **gof(−1) = ?**

A) −5 B) −4 C) −3 D) −2 E) −1

2. $f(x+1) = 2x - 3 \Rightarrow$ **f(x) = ?**

A) $3x + 1$ B) $2x - 5$ C) $2x + 1$ D) $4x - 3$ E) $4x - 2$

3. $f(x) = \dfrac{x+1}{2x-1} \Rightarrow$ **f(2x) = ?**

A) $\dfrac{4f(x)+1}{2f(x)+5}$ B) $\dfrac{3f(x)+1}{2f(x)-3}$ C) $\dfrac{2f(x)-1}{4f(x)+2}$ D) $\dfrac{3f(x)+4}{2f(x)-3}$ E) $\dfrac{4f(x)+3}{2f(x)-4}$

4. $f(x) = ax + b$

$f(1) = 1,\ f^{-1}(7) = 3 \Rightarrow$ **a − b = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

5. $f(x) = 3x + a,\ (g^{-1}of)^{-1}(x) = \dfrac{x+2}{2x-1}$

$g(1) = 8 \Rightarrow$ **a = ?**

A) −8 B) −6 C) −4 D) −2 E) −1

6. $f(x) = 3x - 2,$

$g(x) = 4x + 1$

$h(x) = x^2 - 2 \Rightarrow$ **fogoh(−1) = ?**

A) −15 B) −13 C) −11 D) −9 E) −7

7. $f(x) = x - 1$, $gof(x) = x^2 - 2 \Rightarrow$ **$g(x) = ?$**

A) $x^2 + x + 3$

B) $x^2 + 3x - 1$

C) $x^2 - 2x + 3$

D) $x^2 + 2x - 1$

E) $x^2 + x - 5$

8. $f(x) = \dfrac{x-1}{2x+3}$

$(g^{-1}of)^{-1}(x) = x^2 + 2 \Rightarrow$ **$g(1) = ?$**

A) $\dfrac{1}{9}$

B) $\dfrac{2}{9}$

C) $\dfrac{4}{9}$

D) $\dfrac{5}{9}$

E) $\dfrac{7}{9}$

9. $g(x) = \dfrac{3x-2}{x+1}$ $fog(x) = x \Rightarrow$ **$f(-2) = ?$**

A) -1

B) 0

C) 1

D) 2

E) 3

10. $a > 0$,

$f(x) = 2x^2 + 1$, $g(x) = 4 - x$,

$gof(a) = -15 \Rightarrow$ **$a = ?$**

A) 3

B) 4

C) 5

D) 6

E) 7

11. $f(x) = 2^{x+1} \Rightarrow$ **$f(2x) = ?$**

A) $2f^2(x)$

B) $f^2(x)$

C) $\dfrac{f^2(x)}{2}$

D) $\dfrac{f^2(x)}{4}$

E) $\dfrac{f^2(x)}{8}$

12. $f: R - \{a\} \to R - \{b\}$,

$f(x) = \dfrac{x+4}{3x+6} \Rightarrow$ **$a + b = ?$**

A) $-\dfrac{5}{3}$

B) $-\dfrac{4}{3}$

C) -1

D) $-\dfrac{2}{3}$

E) $-\dfrac{1}{3}$

YANITLAR / ANSWERS

1. D 2. B 3. A 4. E 5. E 6. C 7. D 8. B

9. B 10. A 11. C 12. A

1. $f(x+1) = 2 \cdot f(x) + 3x$,

$f(1) = -2 \Rightarrow \mathbf{f(5) = ?}$

A) 38 B) 41 C) 44 D) 46 E) 48

2. $f(2^{3x+2}) = 9x^2 - 3x + 5 \Rightarrow \mathbf{f(8) = ?}$

A) 4 B) 5 C) 8 D) 12 E) 16

3. $[f(x)]^2 - 4x \cdot f(x) + 3 = 0$

$\Rightarrow \mathbf{f^{-1}(3) = ?}$

A) 1

B) 2

C) 3

D) 4

E) 5

4. $f(x) = x^3 - 2x^2 + ax + 4$

$f^{-1}(14) = 2 \Rightarrow \mathbf{a = ?}$

A) 5 B) 8 C) 9 D) 12 E) 14

5. $fof^{-1}(4x+7) = 15 \Rightarrow \mathbf{x = ?}$

A) −2 B) −1 C) 0 D) 1 E) 2

6. $f(2x+1) = \dfrac{x-3}{x+4} \Rightarrow \mathbf{f(x) = ?}$

A) $\dfrac{x+3}{x-2}$ B) $\dfrac{x-7}{x+7}$ C) $\dfrac{x-5}{x+4}$ D) $\dfrac{x-6}{x+1}$ E) $\dfrac{x+8}{x-2}$

7. $f(x^2 - 4x) = 5x^2 - 20x + 3 \Rightarrow \mathbf{f^{-1}(13) = ?}$

A) –8 B) –6 C) –4 D) –1 E) 2

8. $2^{f(x)} = x + 7 \Rightarrow \mathbf{f(1) + f(25) = ?}$

A) 2 B) 4 C) 6 D) 8 E) 10

9. $f(x + 1) = f(x) + 4$

$f(1) = 2 \Rightarrow \mathbf{f(20) = ?}$

A) 76 B) 77 C) 78 D) 79 E) 80

10. $(fog^{-1})^{-1}(x) = \dfrac{2x+1}{3x-2}$

$f(x) = 4x + 5 \Rightarrow \mathbf{g^{-1}(3) = ?}$

A) –3 B) –2 C) –1 D) 0 E) 1

11. $f(x) = ax + b,$

$f(3) = 5$

$f^{-1}(9) = 5 \Rightarrow \mathbf{f(2) = ?}$

A) –8 B) –6 C) –5 D) –1 E) 3

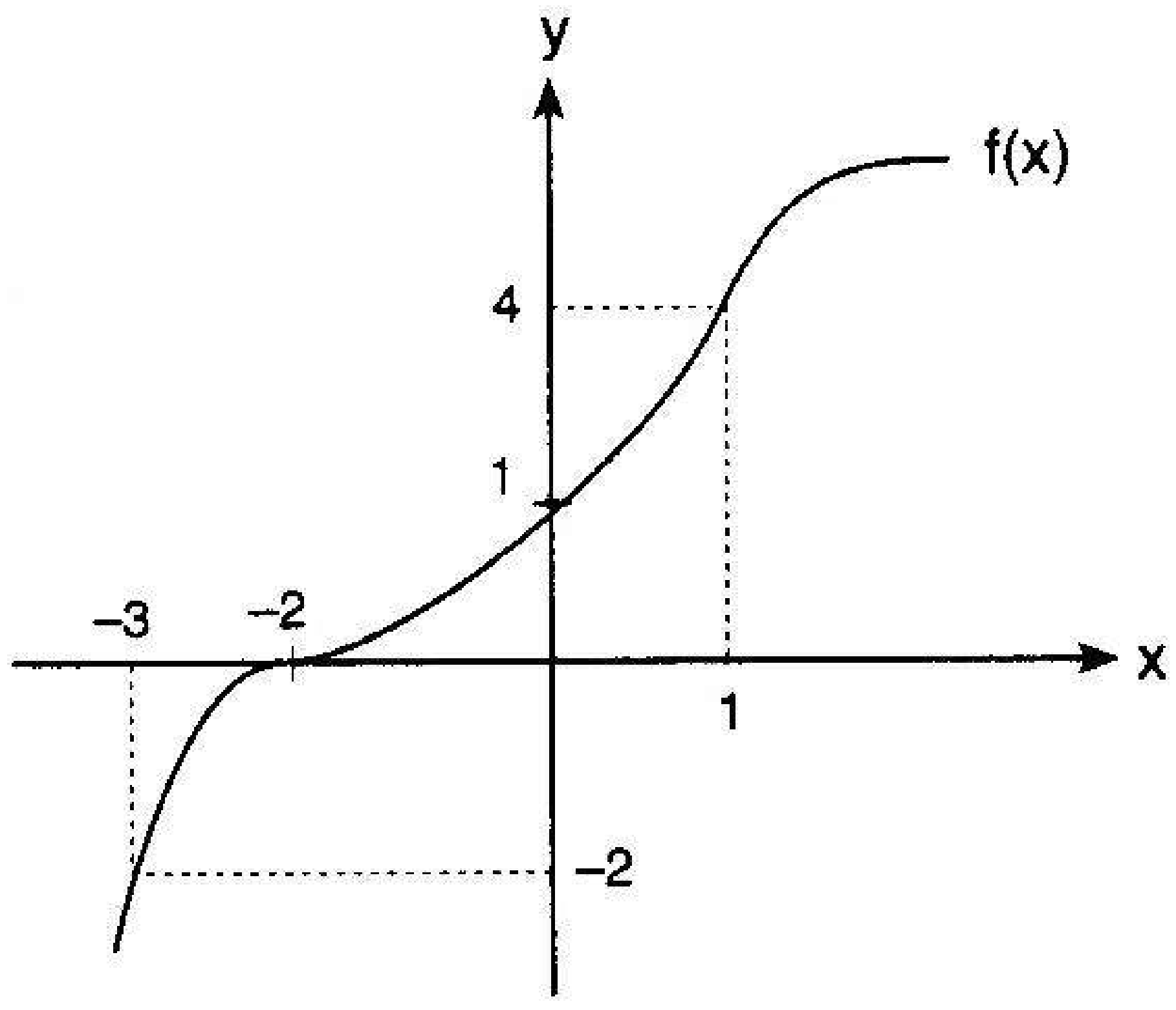
12. $f\left(\dfrac{6}{x}\right) + 2 \cdot f\left(\dfrac{x}{6}\right) = 3x \Rightarrow \mathbf{f(3) = ?}$

A) 24 B) 27 C) 30 D) 34 E) 36

YANITLAR / ANSWERS

1. D 2. B 3. A 4. A 5. E 6. B 7. E 8. D
9. C 10. C 11. E 12. D

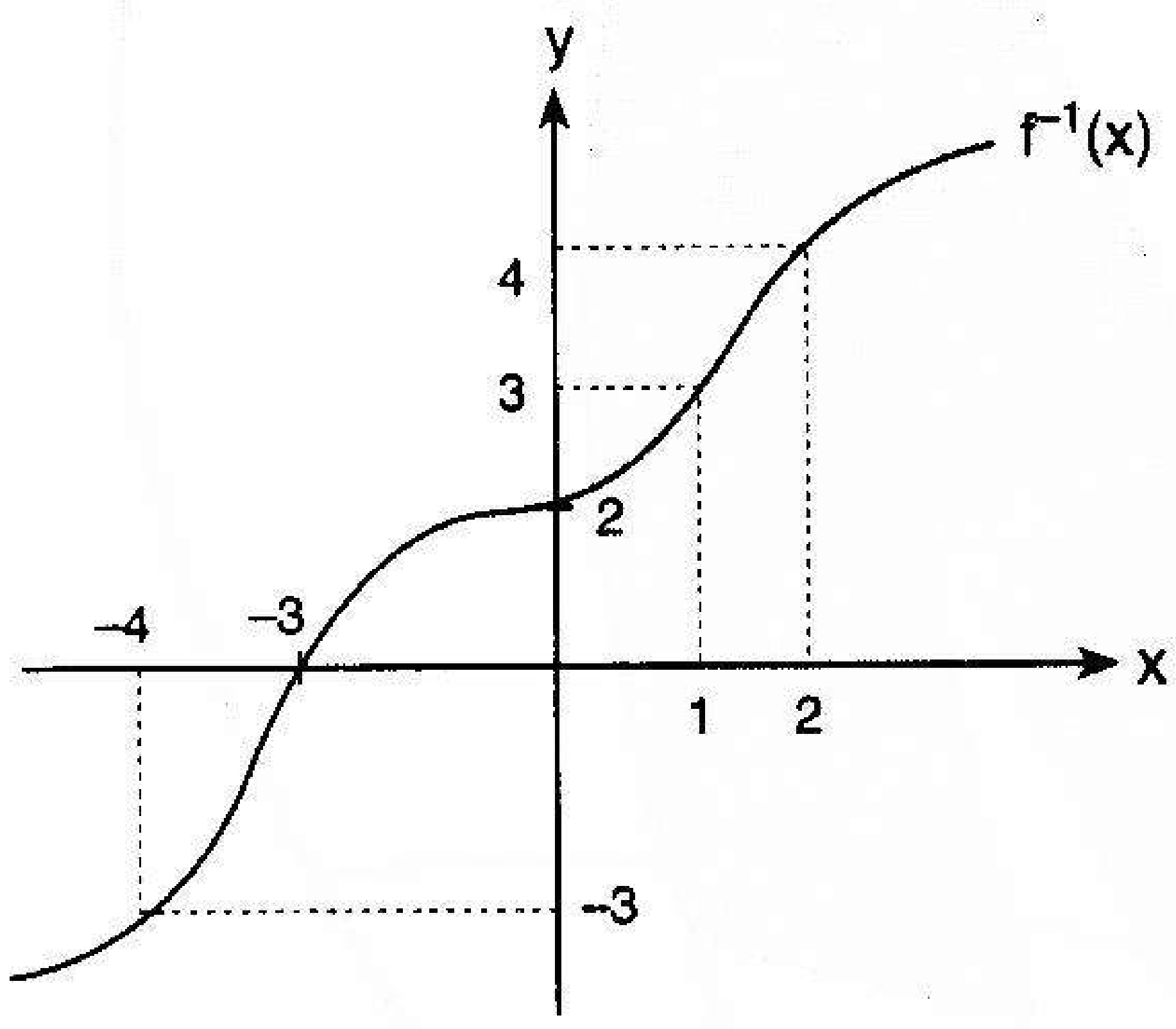
1. $\dfrac{f(0)+f^{-1}(-2)}{f(1)+f^{-1}(0)}=?$



A) –1 B) 0 C) 1

D) 2 E) 3

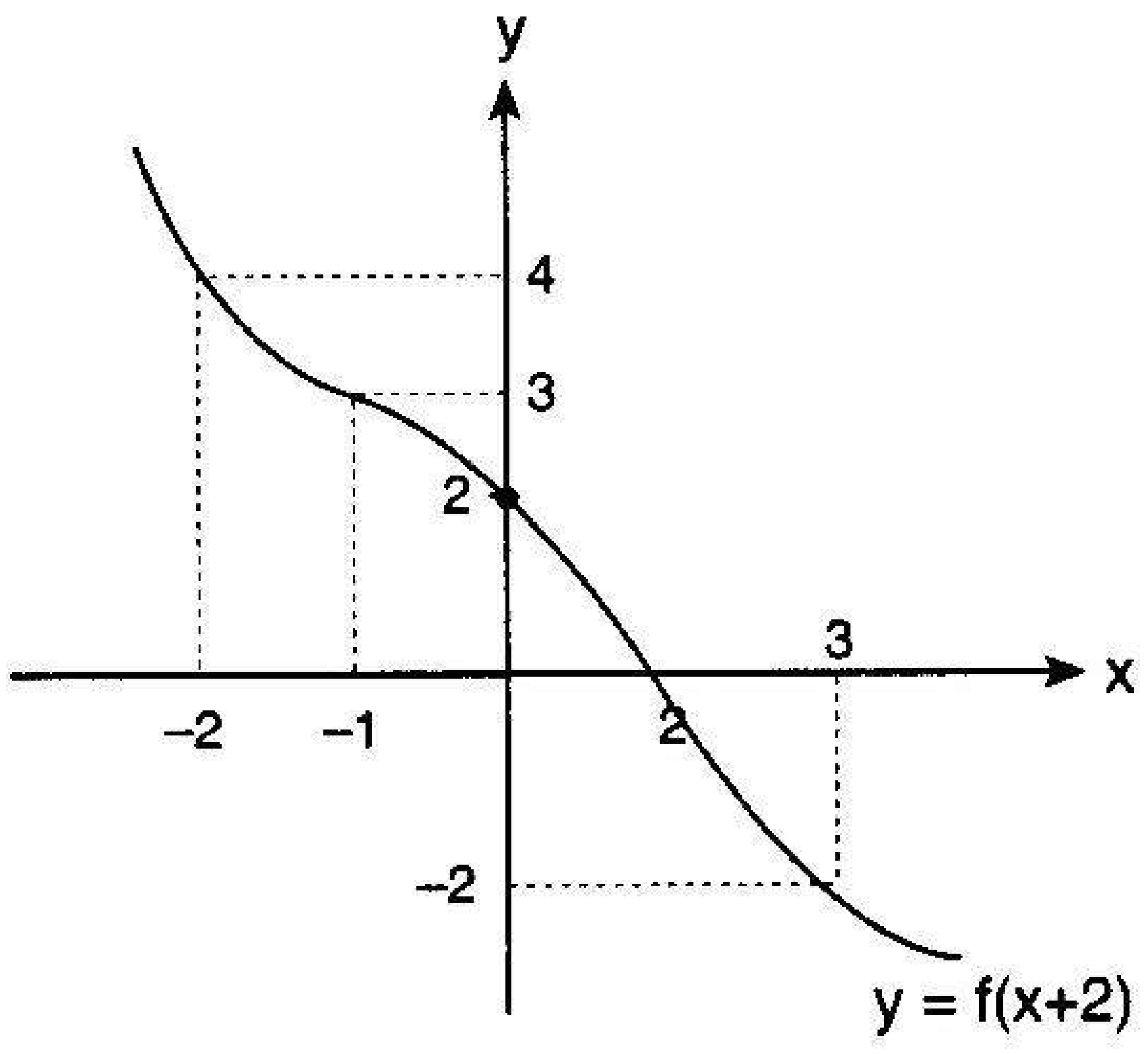
2. fof(2) = ?



A) –4 B) –3 C) 0

D) 1 E) 2

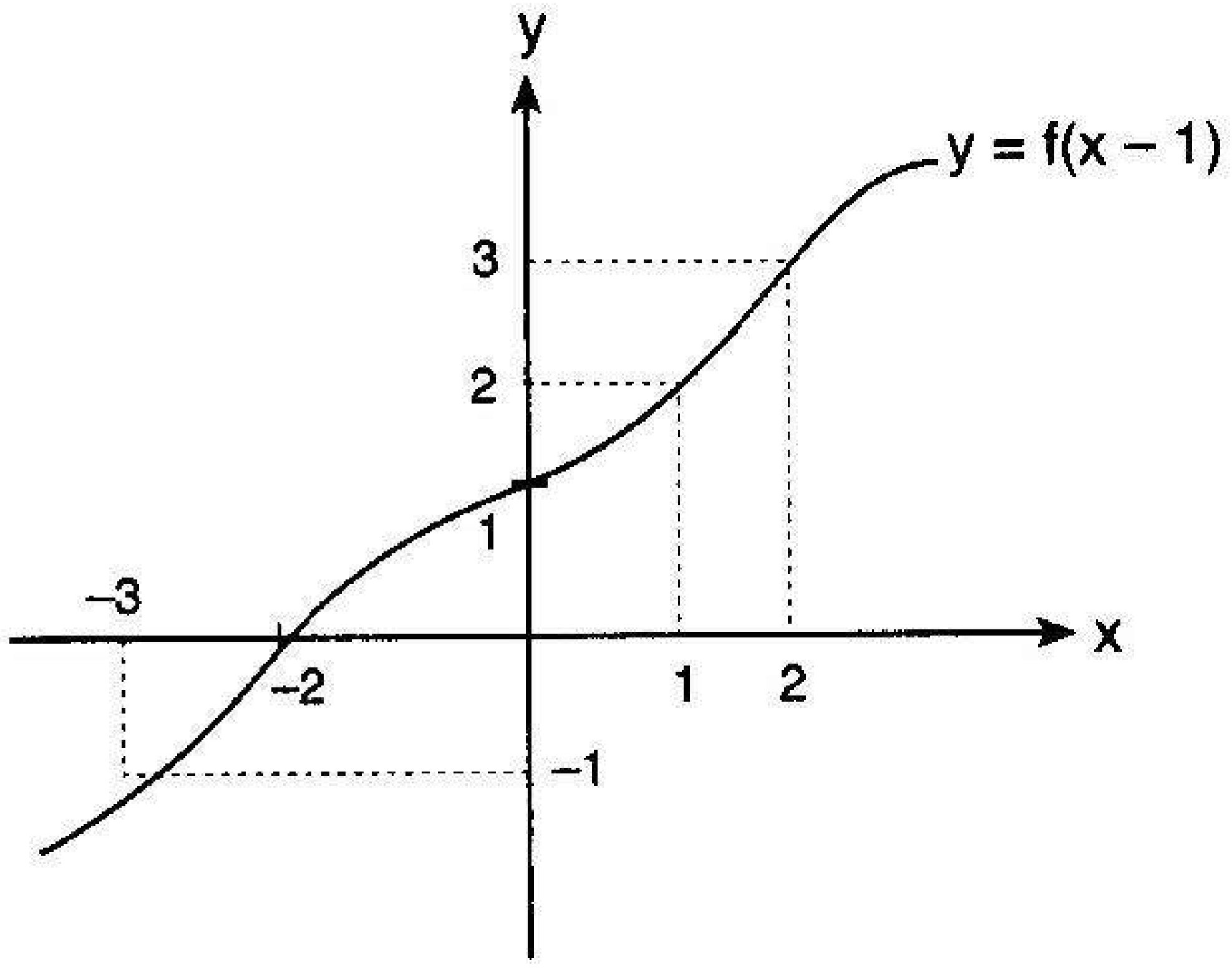
3. $\dfrac{f(1)+f(5)}{f^{-1}(2)+f^{-1}(4)}=?$



A) $-\frac{1}{2}$ B) 0 C) $\frac{1}{2}$

D) 1 E) $\frac{3}{2}$

4. $\dfrac{f(-4)+f^{-1}(3)}{f(0)+f^{-1}(2)}=?$

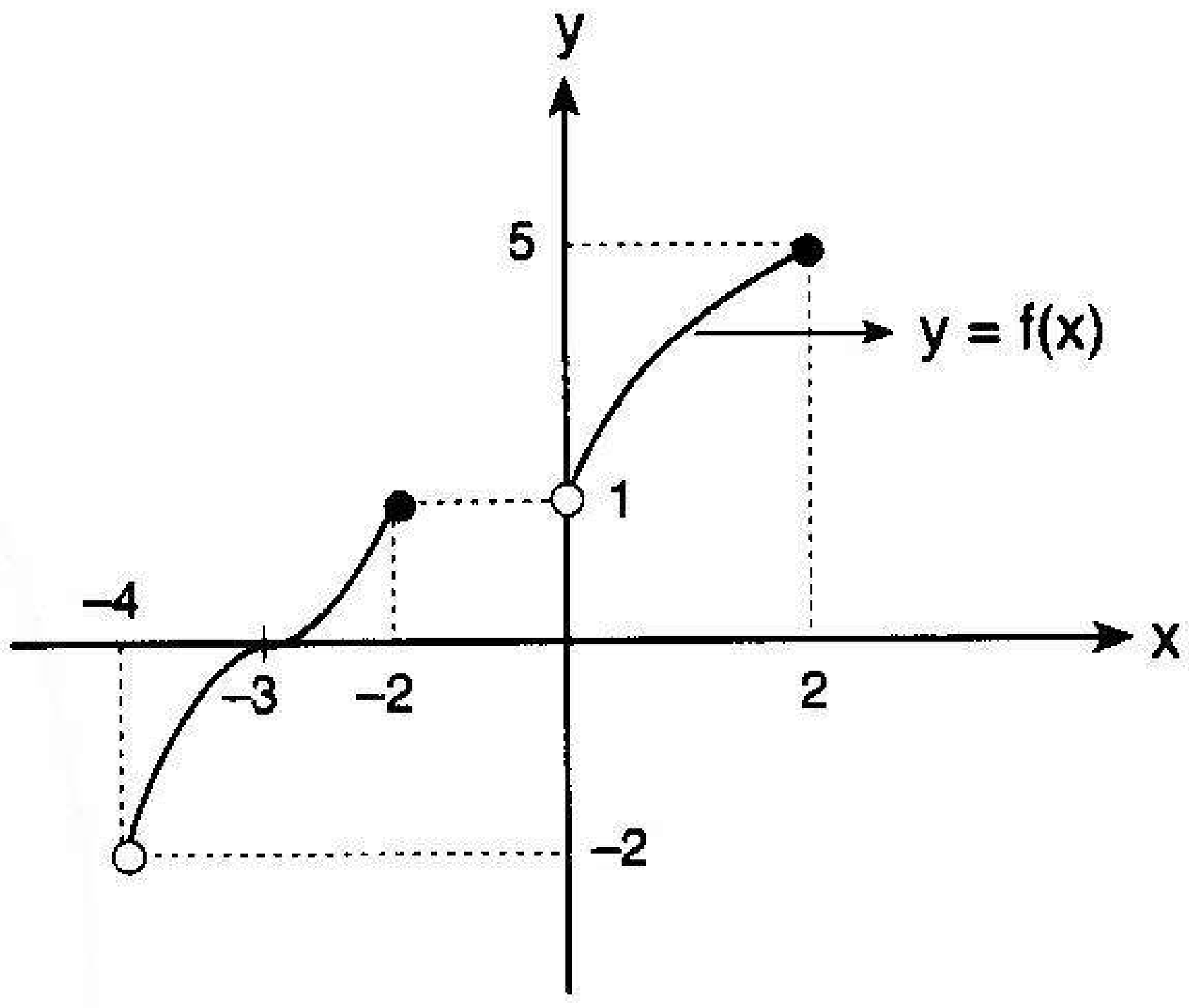


A) –2 B) –1 C) 0

D) 1 E) 2

5. f : A → B

⇒ **B = ?**



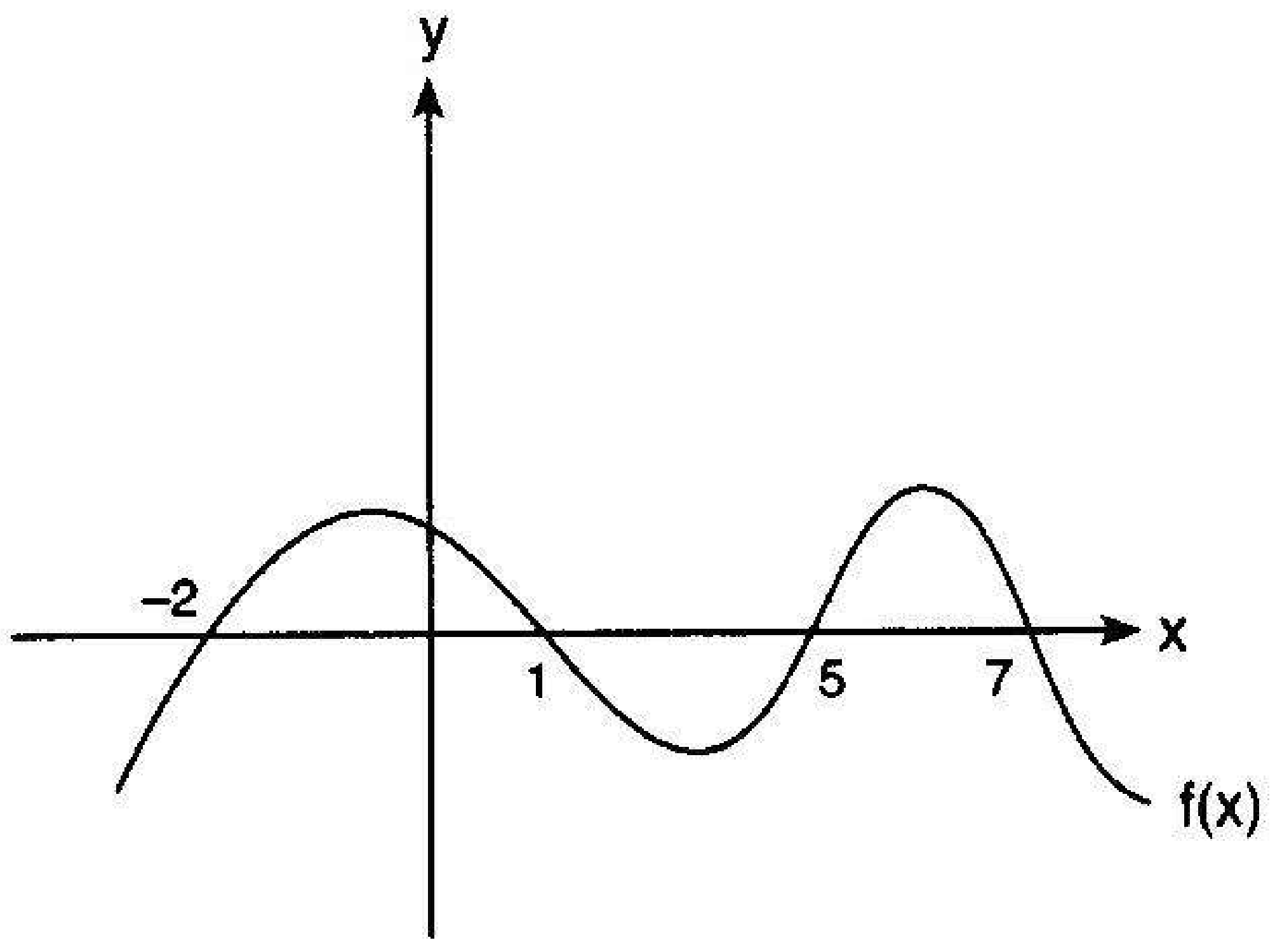
A) (–4, 5] B) (–2, 5] C) [–4, 2]

D) [–3, 1) E) (–2, 2)

6. $x \in Z$,

$f(x) > 0$

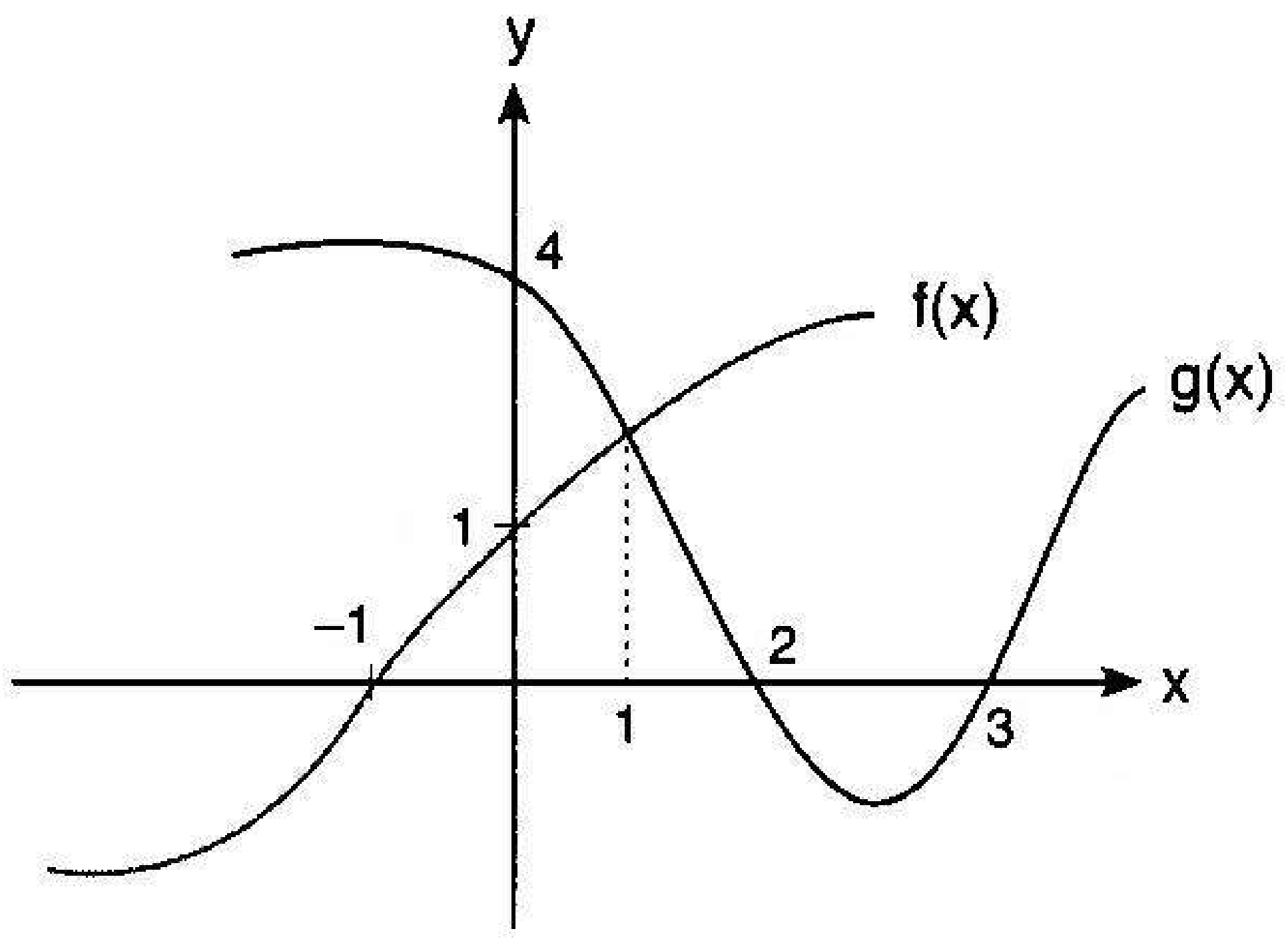
$\Rightarrow \sum x = ?$



A) 3 B) 5 C) 7

D) 8 E) 9

7.

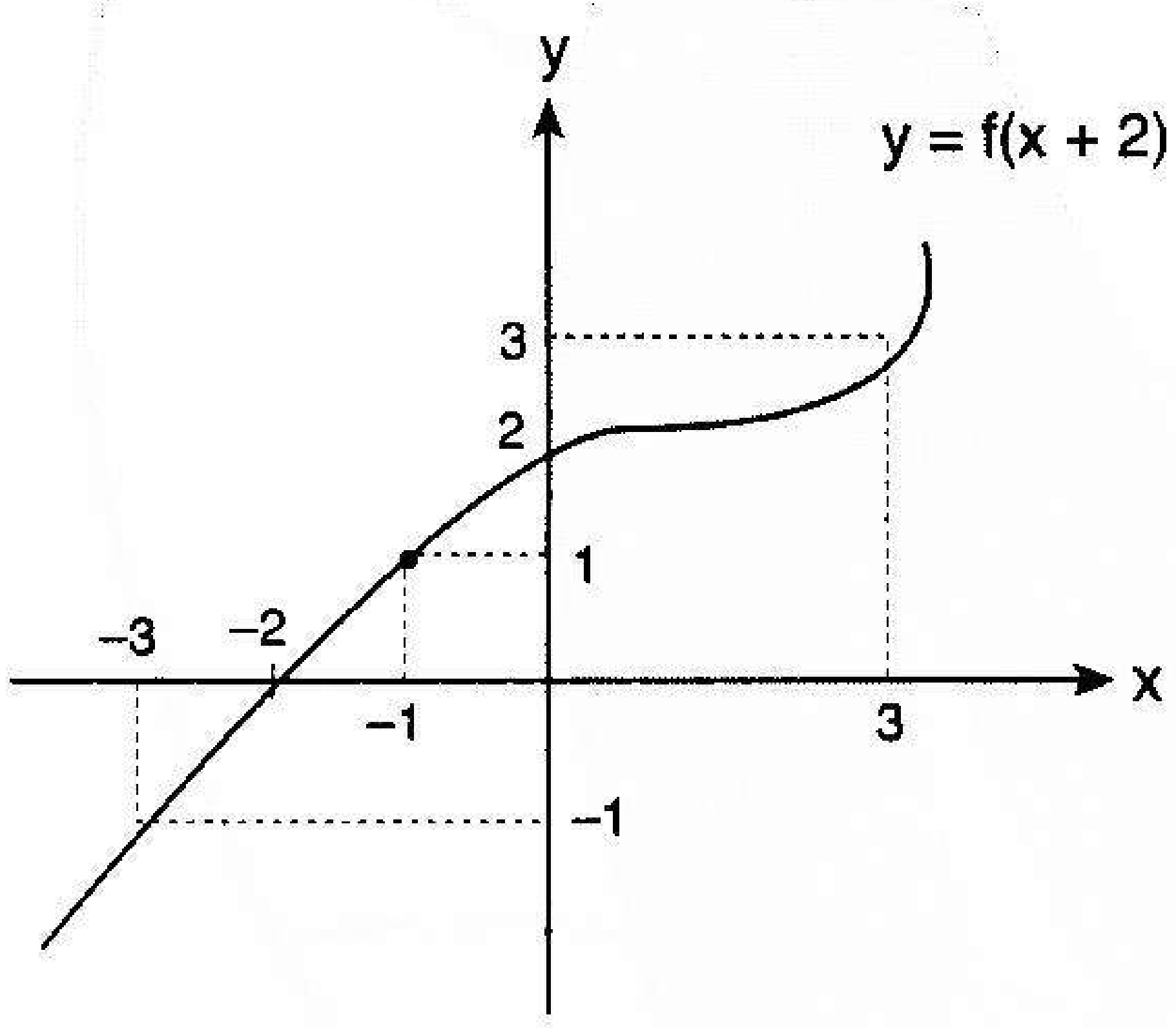


$f^{-1}og(1) + fog(2) = ?$

A) 2 B) 3 C) 5

D) 7 E) 8

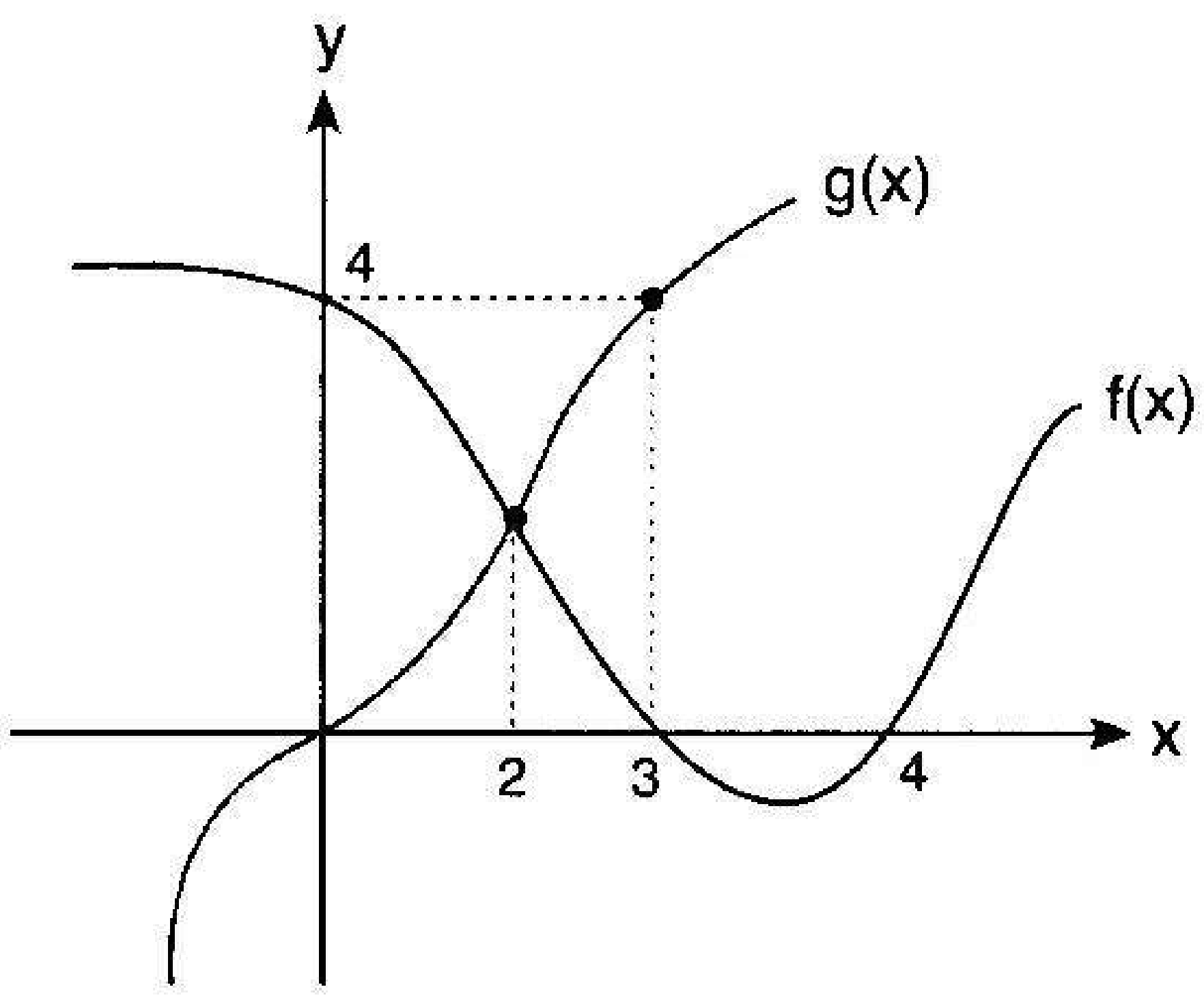
8. **$fof(0) = ?$**



A) –1 B) 0 C) 1

D) 2 E) 3

9.



$g^{-1}of(2) + fog(3) = ?$

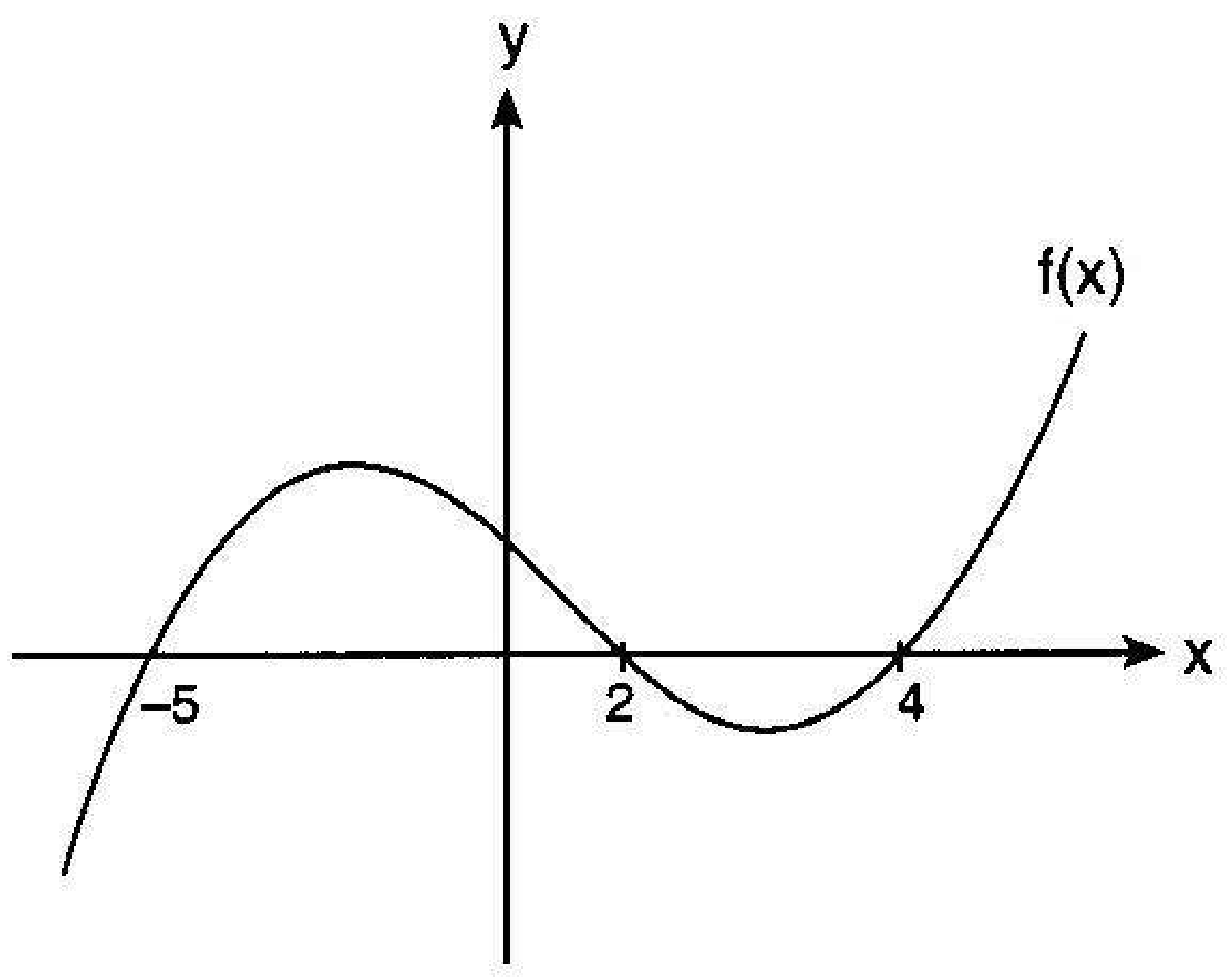
A) –1 B) 1 C) 2

D) 4 E) 5

10. $x \in Z$,

$x \cdot f(x) < 0$

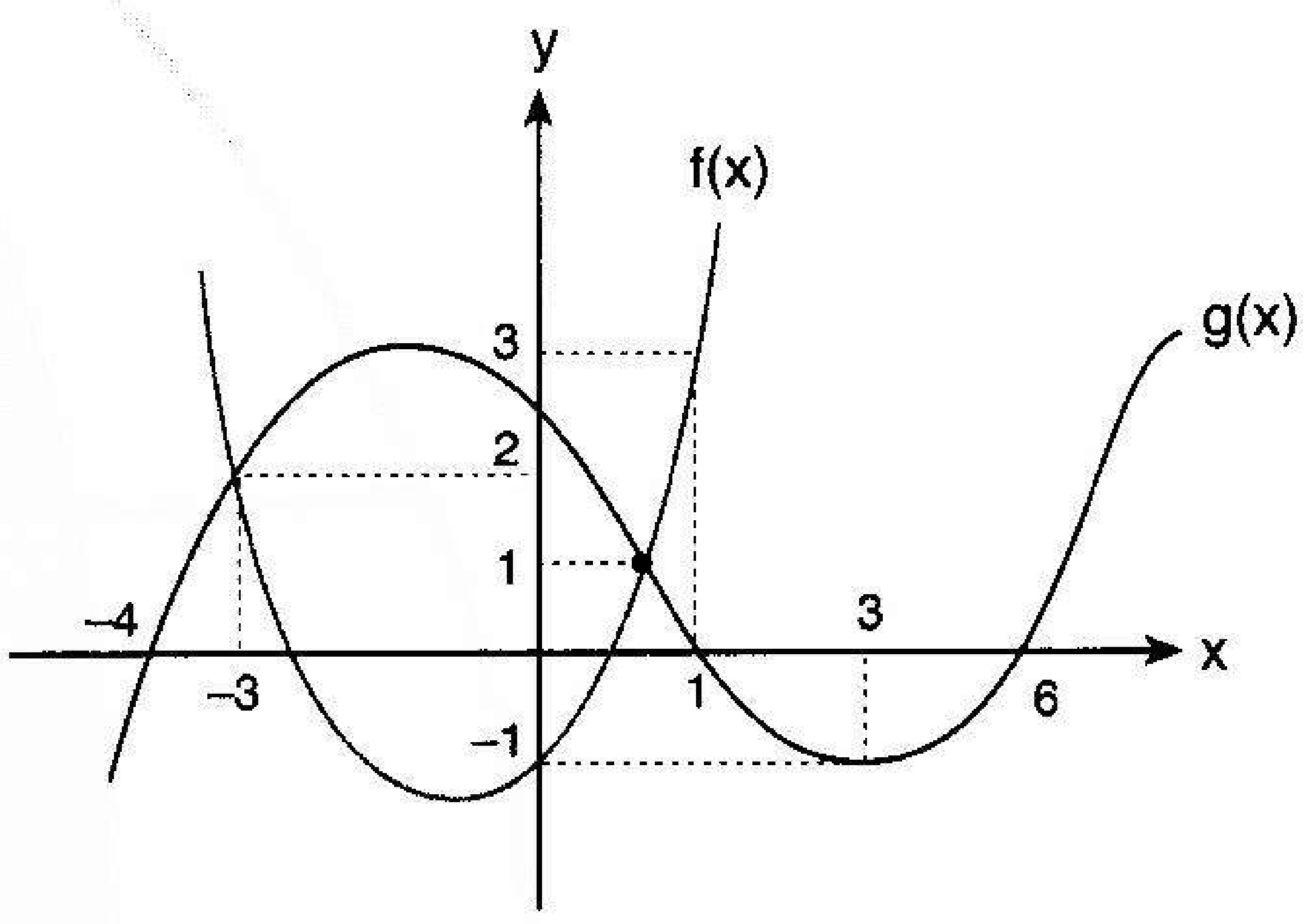
$\Rightarrow \sum x = ?$



A) –1 B) –2 C) –3

D) –5 E) –7

11.



$gof(1) + fog^{-1}(1) = ?$

A) –3 B) –2 C) –1

D) 0 E) 1

12. $x \in Z$,

$x \cdot f(x) > 0$

$\Rightarrow \sum x = ?$

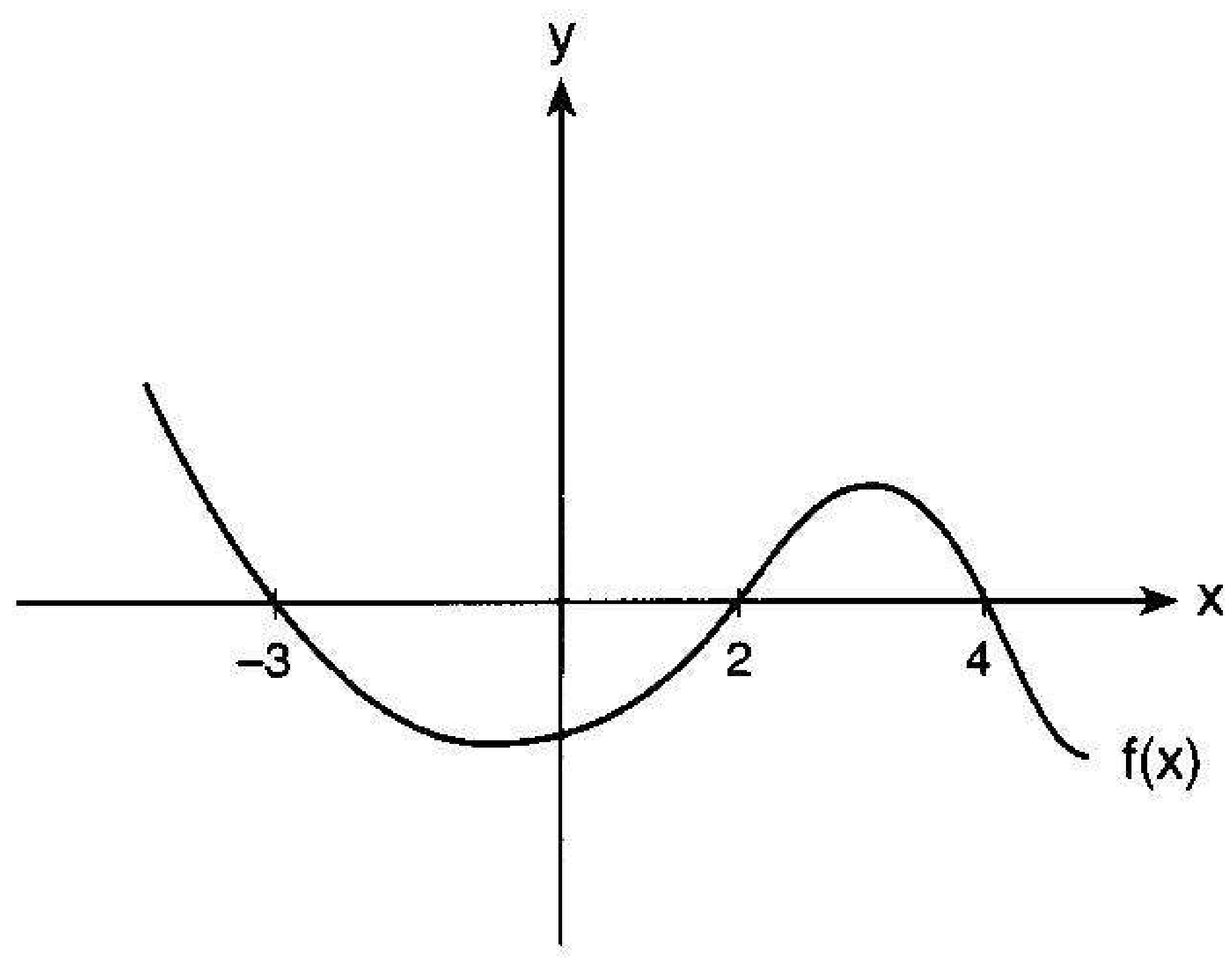
A) –5

B) –3

C) 0

D) 3

E) 5



YANITLAR / ANSWERS							
1. A	**2. B**	**3. C**	**4. C**	**5. B**	**6. B**	**7. A**	**8. B**
9. C	**10. E**	**11. D**	**12. C**				

İŞLEM ve MODULER ARİTMETİK SİSTEMLERİ
OPERATION and MODULAR ARITHMETIC SYSTEMS

BÖLÜM
CHAPTER 4

İŞLEM / OPERATION

Tanım: $A \neq \varnothing$ olmak üzere $A \times A$ kümesinden A'ya tanımlanan her fonksiyona, A'da *ikili işlem* ya da kısaca *işlem* denir.

İşlemler $\oplus$, ●, $\ast$, $\square$, Δ gibi simgelerle gösterilir.

Definition: *Let $A \neq \varnothing$, each function defined from $A \times A$ to A is called as a binary operation or operation.*

Operations are shown by symbols like $\oplus$, ●, $\ast$, $\square$, Δ.

ÖRNEK / EXAMPLE 1

$x \square y = 3x - 2y + 4$

$(2 \square a) \square 3 = 34$

$\Rightarrow$ **a = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$x \square y = 3x - 2y + 4$

$2 \square a = 3 \cdot 2 - 2 \cdot a + 4 = 10 - 2a$

$\Rightarrow (2 \square a) \square 3 = (10 - 2a) \square 3$

$= 3(10 - 2a) - 2 \cdot 3 + 4$

$28 - 6a = 34$

$6a = -6$

$a = -1$

PEKİŞTİRME / REINFORCE

$a \Delta b = 2a - b$

$\Rightarrow (3 \Delta 1) \Delta 2 = ?$

YANIT / ANSWER : 8

Düşünün / Think about it

$$\star x \star = \begin{cases} x & , \quad x \in Z \\ \text{x'ten küçük en büyük tamsayı} \\ \textit{maximum integer less than x} & , \quad x \notin Z \end{cases}$$

$\Rightarrow \star\pi\star - \star 2 \star = ?$

Yanıt / Answer

1

ÖRNEK / EXAMPLE 2

$$m \otimes n = \begin{cases} 3m - n & , \quad m < n \\ m^n & , \quad m \geq n \end{cases}$$

$(3 \otimes 5) \otimes 4 = ?$

A) 16 B) 56 C) 256 D) 276 E) 512

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$3 < 5 \Rightarrow 3 \otimes 5 = 3 \cdot 3 - 5 = 4$

$4 \geq 4 \Rightarrow 4 \otimes 4 = 4^4 = 256$

ÖRNEK / EXAMPLE 3

YILDIZ TEKNİK ÜNİV. YÖS 2011

$$3^{2a} \nabla \frac{2^5}{b} = 6ab - 7$$

$9 \nabla 8 = ?$

A) 5 B) 17 C) 23 D) 31 E) 47

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$3^{2a} = 9 = 3^2 \Rightarrow 2a = 2$

$a = 1$

$\frac{2^5}{b} = 8 \Rightarrow 32 = 8b$

$b = 4$

$9 \nabla 8 = 6 \cdot 1 \cdot 4 - 7 = 24 - 7 = 17$

MODÜLER ARİTMETİK / MODULAR SYSTEMS

$$\begin{array}{r|l} x & m \\ \cline{2-2} - \quad & k \\ \cline{1-1} y & \end{array}$$

$0 \leq y < m$, $x = km + y$

Örneğin; Z nin 5 ile bölümünden kalanlar 0, 1, 2, 3, 4 sayılarından biri olur.

Bundan dolayı, $Z_{/5} = \{0, 1, 2, 3, 4\}$ biçiminde gösterilir.

$0 \leq y < m$, $x = km + y$

For example; if Z is divided by 5, the remainders are 0,1, 2, 3, or 4.

Thus, $Z_{/5} = \{0, 1, 2, 3, 4\}$.

BİR SAYININ KUVVETİNİN BİRLER BASAMAĞI
ONES PLACE of an EXPONENTIALS

Bir sayının birler basamağını bulma işlemi (mod10)'a göre kalanın bulunması ile eş anlamlıdır.

To find ones place of an exponentials is same as to find the remainder when divided by 10 (mod 10).

Mod'un Özellikleri / Properties of Mod

$x \equiv y \pmod{m}$

$u \equiv v \pmod{m}$,

1. $x + u = (y + v) \pmod{m}$
2. $x - u = (y - v) \pmod{m}$
3. $x \cdot u = y \cdot v \pmod{m}$
4. $x^n = y^n \pmod{m}$
5. $ax = ay \pmod{m} \quad a \neq 0$

Düşünün / Think about it

x asal sayı / *prime number*

$y \in Z^+$,

$y^x - y \equiv 0 \pmod{x}$,

$4\cdot(4^4 - 1) \equiv k \pmod{5} \Rightarrow$ **k = ?**

Yanıt / Answer

0

ÖRNEK / EXAMPLE 4

$$\left.\begin{array}{l} 13 \equiv x \pmod{5} \\ -17 \equiv y \pmod{6} \end{array}\right\} \Rightarrow x + y = ?$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{array}{r|l} 13 & 5 \\ \hline -10 & 2 \\ \hline ❸ \longrightarrow x & \end{array} \qquad \begin{array}{r|l} 17 & 6 \\ \hline -12 & 2 \\ \hline ❺ & \end{array}$$

$\Rightarrow y = -5 + 6 = 1$

$x + y = 3 + 1 = 4$

ÖRNEK / EXAMPLE 5

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$16^8 + 16^7$

sayısının 5 ile bölümünden kalan kaçtır?

What is the remainder when the number $16^8 + 16^7$ divided by 5?

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$16^8 + 16^7 \equiv x \text{ (mod5)}$

$16 \equiv 1 \text{ (mod5)} \Rightarrow 16^8 \equiv 1\text{(mod5)}$

$16 \equiv 1 \text{ (mod5)} \Rightarrow 16^7 \equiv 1\text{(mod5)}$

$\Rightarrow x = 2$

1. $\frac{3}{x\Delta y}=\frac{1}{x}+\frac{2}{y} \Rightarrow \mathbf{2\,\Delta\,3=?}$

A) $\frac{10}{7}$

B) $\frac{12}{7}$

C) 2

D) $\frac{16}{7}$

E) $\frac{18}{7}$

2. $(2x+y)\,\square\,(x-y)=4x-y \Rightarrow \mathbf{4\,\square\,(-1)=?}$

A) –1

B) 0

C) 1

D) 2

E) 3

3. $x\,\Delta\,y=3x-y+2(y\,\Delta\,x) \Rightarrow \mathbf{2\,\Delta\,1=?}$

A) $-\frac{5}{3}$

B) –2

C) $-\frac{7}{3}$

D) $-\frac{8}{3}$

E) –3

4. $x \bigstar y=\max(3x+y,\ 2x-y)$,

$x\,\square\,y=\min(x^{y},\ 3x+y)$

$\Rightarrow \mathbf{(2 \bigstar 1)\,\square\,3=?}$

A) 15

B) 17

C) 20

D) 24

E) 26

5. $a\,\square\,b=(3a-b,\ 2a+b)$,

$(a,b)\,\Delta\,(c,d)=a\cdot b+cd$

$\Rightarrow \mathbf{(3\,\square\,-1)\,\Delta\,(-1\,\square\,2)=?}$

A) 47

B) 48

C) 49

D) 50

E) 51

6. $\mathbf{(2\,\Delta\,1)\,\Delta\,(4\,\Delta\,5)=?}$

Δ	1	2	3	4	5
1	4	5	1	2	3
2	5	1	2	3	4
3	1	2	3	4	5
4	2	3	4	5	1
5	3	4	5	1	2

A) 1

B) 2

C) 3

D) 4

E) 5

7.

$$\begin{array}{r|l} A & 5 \\ \cline{2-2} - & B \\ \hline 2 & \end{array} \qquad \begin{array}{r|l} B & 2 \\ \cline{2-2} - & \\ \hline 1 & \end{array}$$

$A \equiv x \pmod{10} \Rightarrow$ **x = ?**

A) 3
B) 4
C) 5
D) 6
E) 7

8. $1! + 2! + \ldots + 55! \equiv x \pmod{5} \Rightarrow$ **x = ?**

A) 0
B) 1
C) 2
D) 3
E) 4

9. $12! - 8! \equiv x \pmod{7} \Rightarrow$ **x = ?**

A) 0
B) 1
C) 2
D) 3
E) 4

10. $3^{18} \equiv x \pmod{8} \Rightarrow$ **x = ?**

A) 1
B) 2
C) 4
D) 5
E) 7

11. $\left.\begin{array}{l} -17 \equiv x \pmod{4} \\ 71 \equiv y \pmod{6} \end{array}\right\} \Rightarrow$ **x + y = ?**

A) 3
B) 4
C) 6
D) 8
E) 9

12. $-\dfrac{2}{3} \equiv x \pmod{5} \Rightarrow$ **x = ?**

A) 0
B) 1
C) 2
D) 3
E) 4

YANITLAR / ANSWERS

1. E	**2. D**	**3. C**	**4. D**	**5. D**	**6. C**	**7. E**	**8. D**
9. A	**10. A**	**11. D**	**12. B**				

1. $x = 2,\ y = 3 \Rightarrow \dfrac{3}{2\Delta 3} = \dfrac{1}{\underset{(3)}{2}} + \dfrac{2}{\underset{(2)}{3}}$

$$\frac{3}{2\Delta 3} = \frac{7}{6}$$

$$18 = 7(2\Delta 3)$$

$$2\Delta 3 = \frac{18}{7}$$

Yanıt / The answer is E

2.

$$\begin{array}{r} 2x + y = 4 \\ +\ \ x - y = -1 \\ \hline 3x = 3 \end{array}$$

$x = 1 \Rightarrow y = 2$

$\Rightarrow 4 \square (-1) = 4 \cdot 1 - 2 = 2$

Yanıt / The answer is D

3. $x \Delta y = 3x - y + 2(y \Delta x)$

$2 \Delta 1 = 6 - 1 + 2(1 \Delta 2)$

$1 \Delta 2 = 3 - 2 + 2(2 \Delta 1)$

$2 \Delta 1 = 5 + 2\,(1 \Delta 2)$

$\quad = 5 + 2\,(1 + 2(2 \Delta 1))$

$2 \Delta 1 = 5 + 2\ + 4(2 \Delta 1)$

$-3(2 \Delta 1) = 7 \Rightarrow\ 2 \Delta 1 = -\dfrac{7}{3}$

Yanıt / The answer is C

4. $2 \bigstar 1 = \max(3 \cdot 2 + 1,\ 2 \cdot 2 - 1),$

$\quad = \max(7, 3) = 7$

$7 \square 3 = \min(7^3, 24) = 24$

Yanıt / The answer is D

5. $3 \square -1 = (3 \cdot 3 + 1,\ 2 \cdot 3 - 1) = (10, 5)$

$-1 \square 2 = (-3 - 2,\ -2 + 2) = (-5, 0)$

$\Rightarrow\ (10, 5)\ \Delta\ (-5, 0) = 10 \cdot 5 + 0 = 50$

Yanıt / The answer is D

6. $2 \Delta 1 = 5$

$4 \Delta 5 = 1$

$\Rightarrow\ (2 \Delta 1) \Delta (4 \Delta 5) = 5 \Delta 1 = 3$

Yanıt / The answer is C

7. $\begin{array}{r|l} A & 5 \\ \cline{2-2} - & B \\ \hline 2 & \end{array}$ $\qquad$ $\begin{array}{r|l} B & 2 \\ \cline{2-2} - & C \\ \hline 1 & \end{array}$

$\Rightarrow A = 5B + 2,\ B = 2C + 1$

$A = 5(2C + 1) + 2 = 10C + 7$

$\Rightarrow A \equiv 7 \pmod{10}$

Yanıt / The answer is E

8. $1! = 1$

$2! = 1 \cdot 2 = 2$

$3! = 1 \cdot 2 \cdot 3 = 6$

$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120 \equiv 0 \pmod 5$

$6! = 6 \cdot 5! \equiv 0 \pmod 5$

$\vdots$

$\Rightarrow 1 + 2 + 6 + 24 + 0 + 0 + \ldots + 0 \equiv x \pmod 5$

$33 \equiv x \pmod 5$

$\begin{array}{r|l} 33 & 5 \\ \cline{2-2} - & 6 \\ \hline ❸ & \end{array} \longrightarrow x$

Yanıt / The answer is D

9. $8! = 8 \cdot 7! \equiv 0 \pmod 7$

$\Rightarrow x = 0$

Yanıt / The answer is A

10. $3^1 \equiv 3 \pmod 8$

$3^2 \equiv 1 \pmod 8$

$\begin{array}{r|l} 18 & 2 \\ \cline{2-2} -\ 18 & 9 \\ \hline ⓿ & \end{array} \longrightarrow 3^0 = 1 = x$

Yanıt / The answer is A

11. $\begin{array}{r|l} 17 & 4 \\ \cline{2-2} -\ 16 & 4 \\ \hline 1 & \end{array} \Rightarrow x = -1 + 4 = 3$

$\begin{array}{r|l} 71 & 6 \\ \cline{2-2} -\ 66 & 11 \\ \hline ❺ & \end{array} \longrightarrow y$

$x + y = 3 + 5 = 8$

Yanıt / The answer is D

12. $Z_{/5}$,

$$-\frac{2}{3} \equiv \frac{-2+5}{3} = \frac{3}{3} = 1$$

$$\Rightarrow -\frac{2}{3} \equiv 1 \pmod 5$$

Yanıt / The answer is B

1. $\dfrac{3}{a \bigcirc b} = \dfrac{2}{a} + \dfrac{5}{b} \Rightarrow \mathbf{3 \bigcirc 2 = ?}$

Yanıt / The Answer is: $\mathbf{\dfrac{18}{19}}$

---

2. $x \bigstar y = x^y + y \Rightarrow \mathbf{(2 \bigstar (-1)) \bigstar 2 = ?}$

Yanıt / The Answer is: $\mathbf{\dfrac{9}{4}}$

---

3. $a \text{ ✿ } b = \begin{cases} 2a - b & a \cdot b > 0 \\ a + 3b & a \cdot b \leq 0 \end{cases}$

$\Rightarrow \mathbf{(1 \text{ ✿ } 0) \text{ ✿ } (2 \text{ ✿ } -1) = ?}$

Yanıt / The Answer is: **–2**

---

4. $\left.\begin{array}{l} a \bigstar b = 4a + b - 3 \\ a \square b = 2a - b + 1 \end{array}\right\} \Rightarrow \mathbf{(3 \bigstar 2) \square (-2 \bigstar 1) = ?}$

Yanıt / The Answer is: **33**

---

5. $a \square b = 3a - b + 2$

$a \bigstar b = a \cdot b - 4$

$(2 \square -1) \bigstar x = 41 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **5**

---

6. $a \Delta b = \max(a - 2b, a + b)$

$a \bigstar b = \min(4 - a + b, a^b) \Rightarrow \mathbf{(2 \Delta -1) \bigstar (3 \Delta 0) = ?}$

Yanıt / The Answer is: **3**

7. $\left.\begin{aligned} 213 &\equiv x \pmod 4 \\ -35 &\equiv y \pmod 8 \end{aligned}\right\} \Rightarrow \mathbf{x + y = ?}$

Yanıt / The Answer is: **6**

8. $4x - 3 \equiv x + 3 \pmod 6 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **2**

9. $3x - 2 \equiv x + 5 \pmod 7 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **0**

10. $x < 100,$

$x \equiv 3 \pmod 5 \Rightarrow \mathbf{max(x) = ?}$

Yanıt / The Answer is: **98**

11. $50 < x,$

$12 \equiv x \pmod 7 \Rightarrow \mathbf{min(x) = ?}$

Yanıt / The Answer is: **54**

12. $2^{25} \equiv x \pmod 7 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **2**

1. $x \Delta y = 2x + 3y - 2$

$(2 \Delta m) \Delta 1 = -13 \Rightarrow$ **$m = ?$**

A) -9

B) -7

C) -6

D) -5

E) -3

2. $(x + y) \Delta (x - y) = 2x + y - 2 \Rightarrow$ **$3 \Delta 5 = ?$**

A) 1

B) 2

C) 3

D) 4

E) 5

3. $a \Delta b = 3a + 2b + 2(b \Delta a) \Rightarrow$ **$3 \Delta 1 = ?$**

A) $-\frac{28}{3}$

B) $-\frac{29}{3}$

C) -10

D) $-\frac{31}{3}$

E) $-\frac{32}{3}$

4. $a \Delta b = 3a - 2b$, $(3 \Delta 2) \Delta x = 19 \Rightarrow$ **$x = ?$**

A) -8

B) -7

C) -4

D) -2

E) -1

5. $a \bigstar b = 2a - b + 3$

$\Rightarrow$ **$(1 \square 2) \bigstar (4 \square 3) = ?$**

A) -2

B) 1

C) 3

D) 6

E) 8

□	1	2	3	4	5
1	1	2	3	4	5
2	2	3	4	5	1
3	3	4	5	1	2
4	4	5	1	2	3
5	5	1	2	3	4

6. $(1 \Delta x) \Delta (b \Delta 2) = a$

$\Rightarrow$ **$x = ?$**

A) 1

B) 2

C) a

D) b

E) c

Δ	1	2	a	b	c
1	b	c	1	2	a
2	c	1	2	a	b
a	1	2	a	b	c
b	2	a	b	c	1
c	a	b	c	1	2

7. $\left.\begin{array}{l} x + y \equiv 3 \pmod{6} \\ x - y \equiv 5 \pmod{6} \end{array}\right\} \Rightarrow$ **$x \cdot y \equiv ?\ (\text{mod } 6)$**

A) 1
B) 2
C) 3
D) 4
E) 5

8. $2! + 4! + \ldots + 30! \equiv x \pmod{7} \Rightarrow$ **$x = ?$**

A) 2
B) 3
C) 4
D) 5
E) 6

9. $1! + 3! + \ldots + 81! \equiv x \pmod{15} \Rightarrow$ **$x = ?$**

A) 3
B) 4
C) 5
D) 6
E) 7

10. $4x + 3 = x - 2 \pmod{7} \Rightarrow$ **$x = ?$**

A) 1
B) 2
C) 3
D) 4
E) 5

11. $x < 80$,
$21 \equiv x \pmod{4} \Rightarrow$ **$\max(x) = ?$**

A) 75
B) 76
C) 77
D) 78
E) 79

12. $3^{52} \equiv x \pmod{6} \Rightarrow$ **$x = ?$**

A) 1
B) 2
C) 3
D) 4
E) 5

YANITLAR / ANSWERS

1. E	2. E	3. B	4. D	5. D	6. E	7. B	8. C
9. E	10. C	11. C	12. C				

ÜNİTE 6

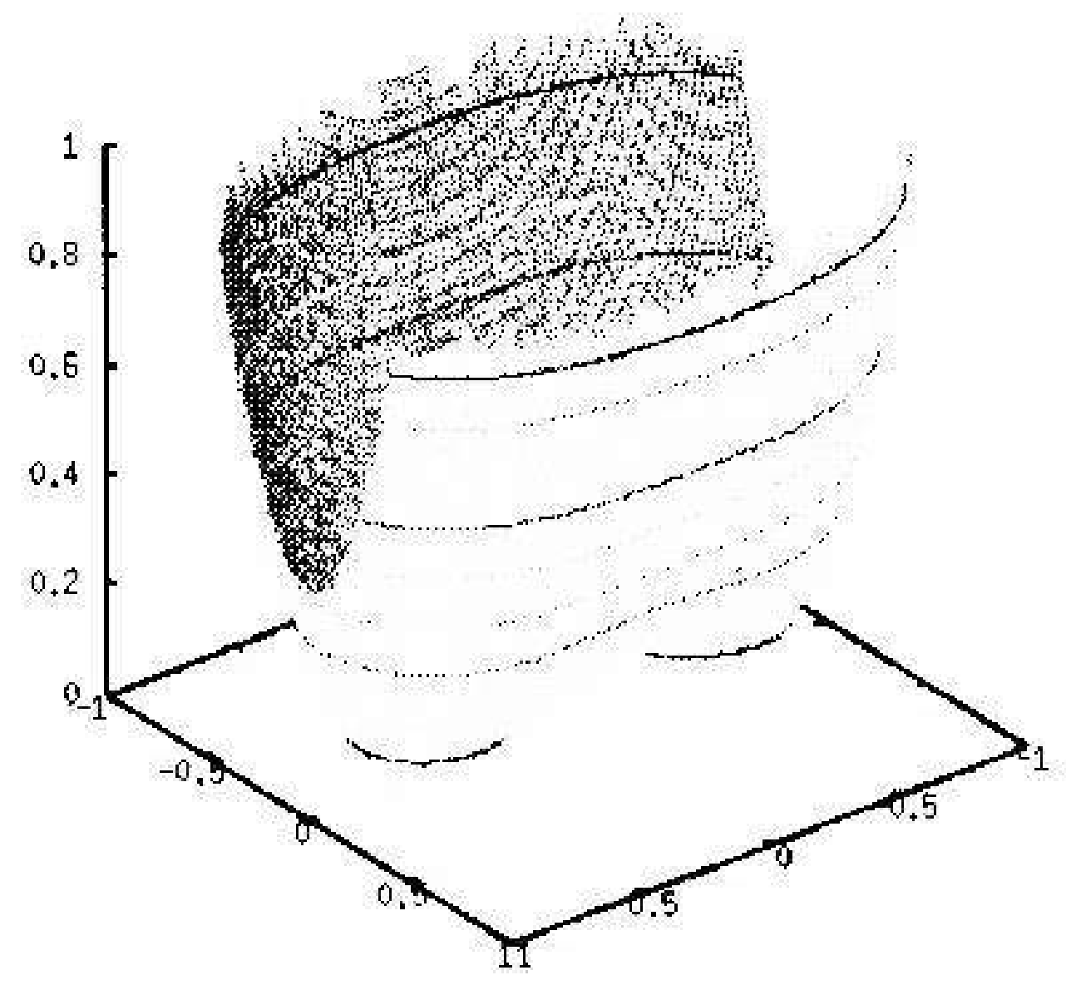
UNIT

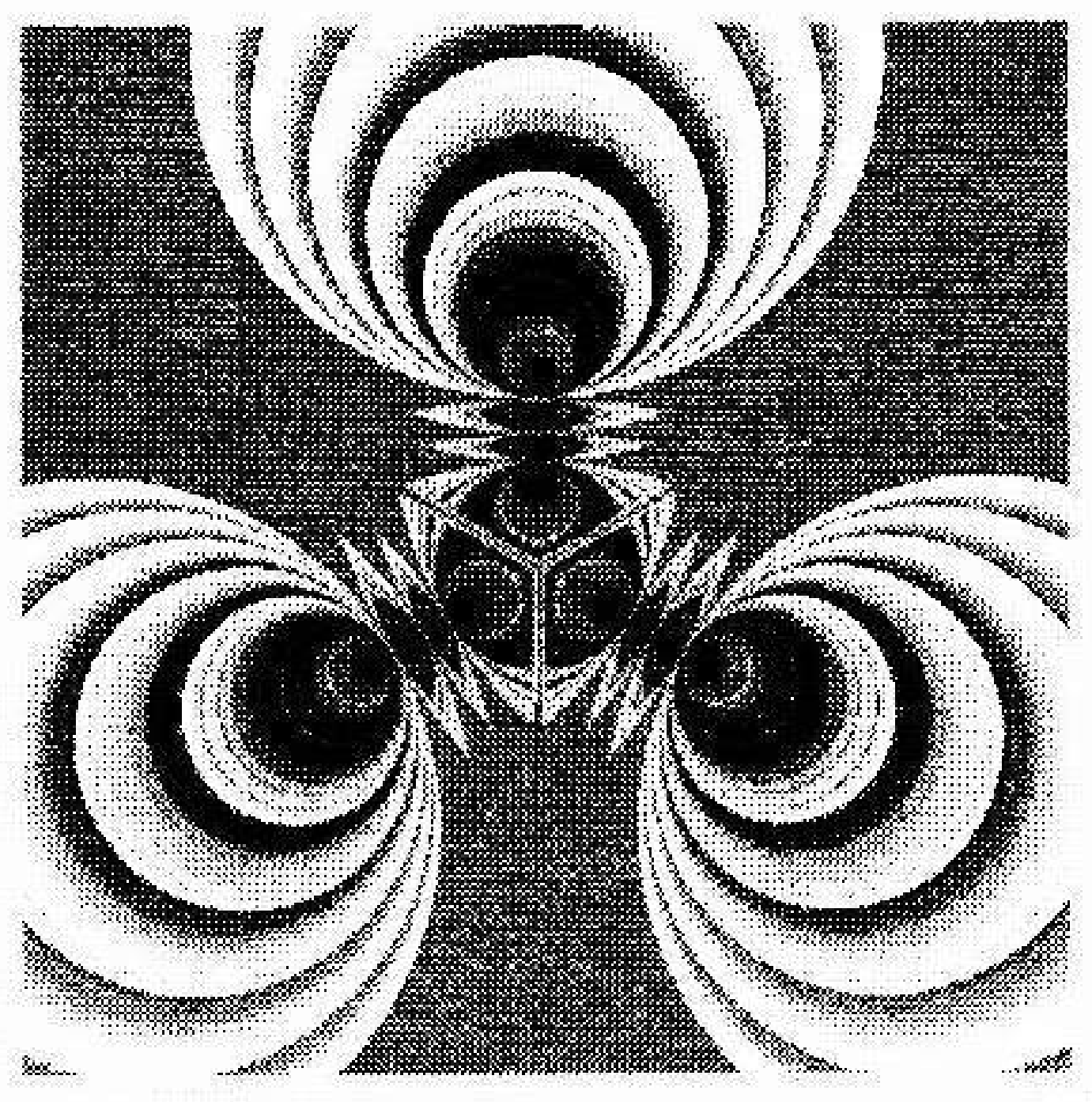
Polinomlar, ikinci dereceden denklemler, eşitsizlikler ve fonksiyonlar bu ünitede ayrıntılı anlatılmıştır. Polinom, fonksiyonun özel durumudur. Bu nedenle, fonksiyonları iyi öğrendiyseniz, polinomlarda zorlanmazsınız. Benzer şekilde, ikinci dereceden denklemler, eşitsizlikler ve fonksiyonlar, birinci dereceden denklem ve eşitsizliklerin genelleştirilmiş halidir. Ayrıca çarpanlara ayırma konusuna, ikinci dereceden denklemler konusunda kısaca değinilmiştir.

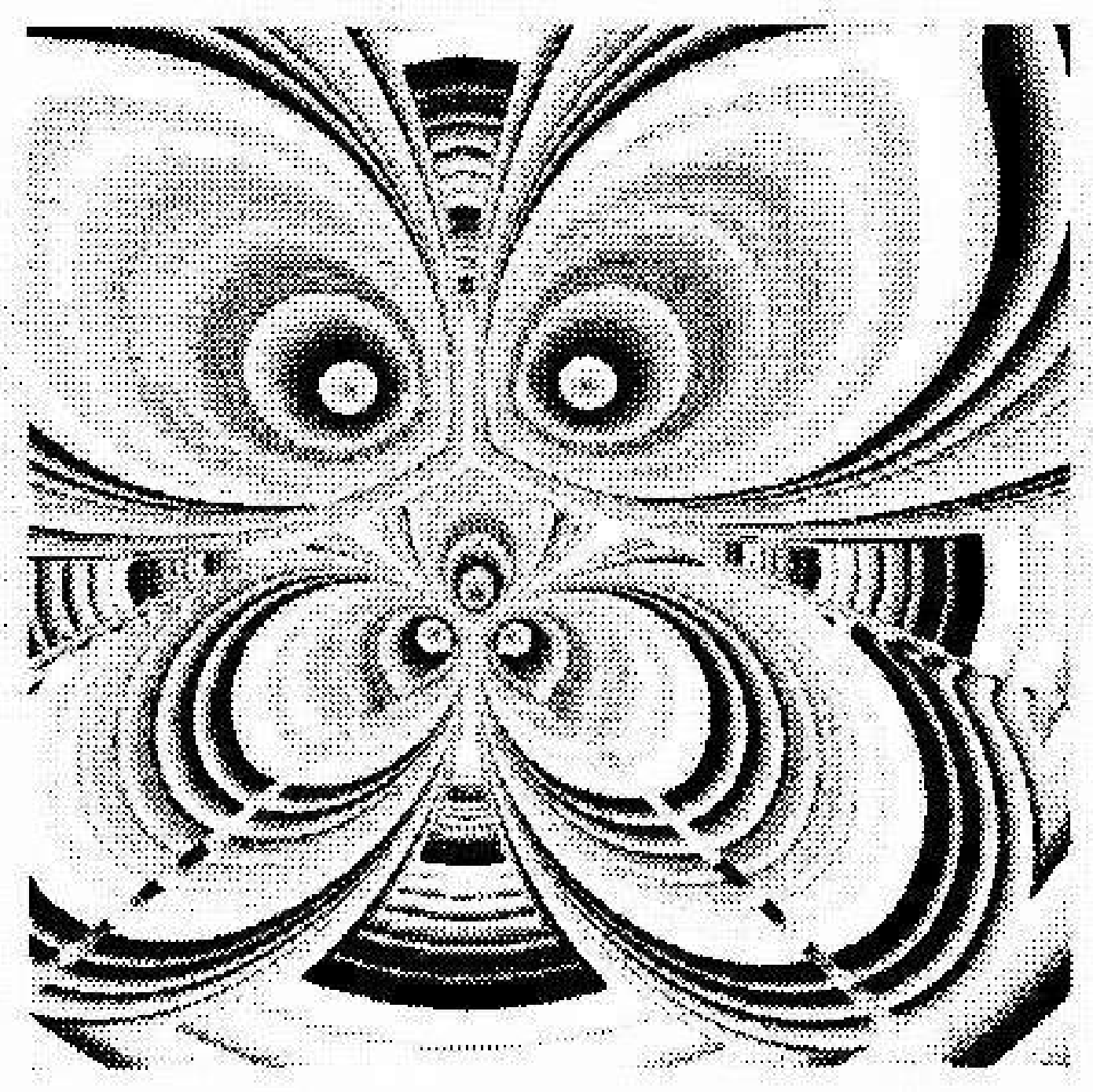
Bu ünite; 111 çözümlü örnek, 48 alıştırma, ve 108 test sorusundan oluşmaktadır.

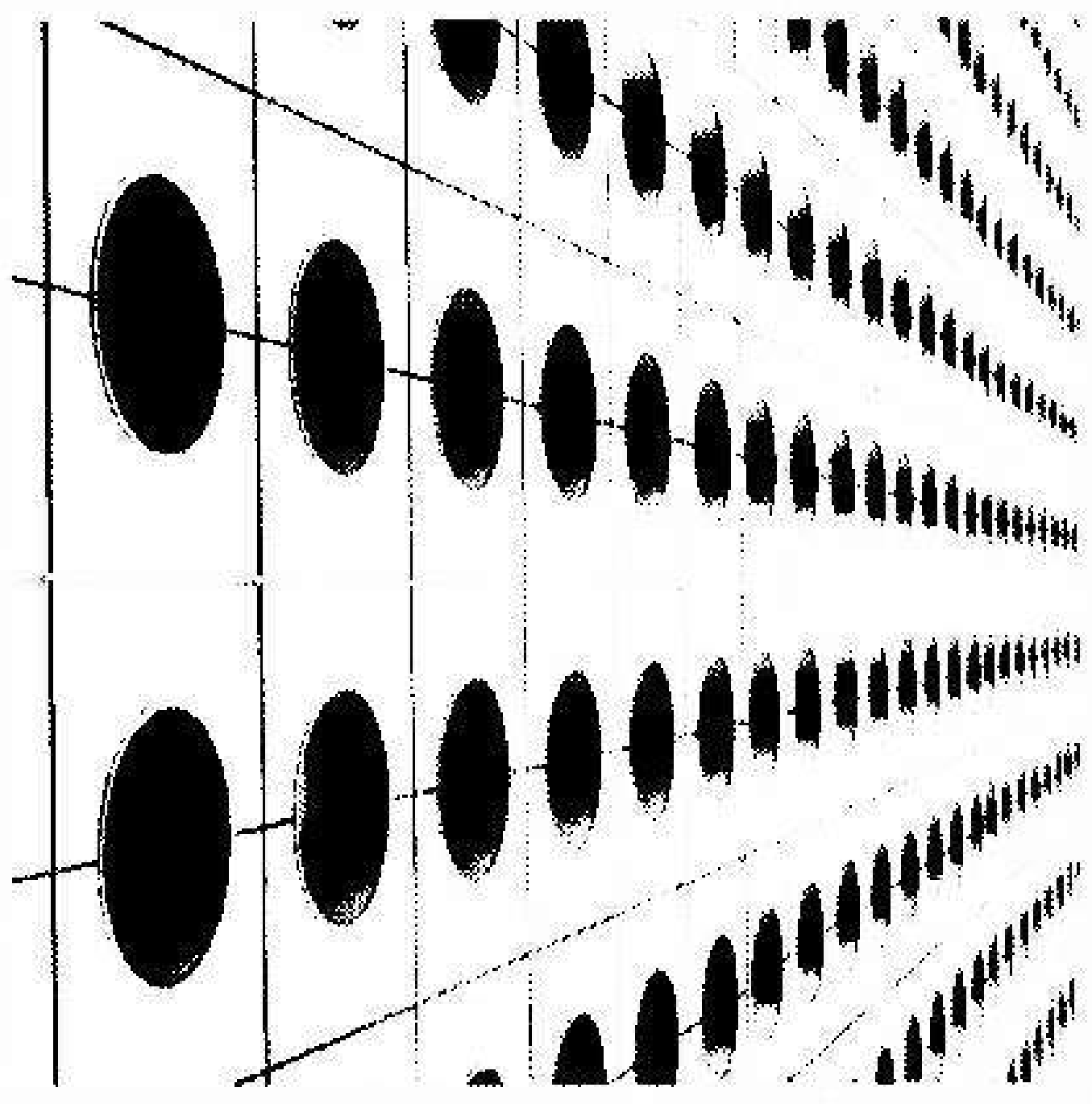
In this unit, polynomials, quadratic equations, inequalities and functions are explained in detailed. Polynomial is special situation of function. Moreover, if you learned functions, polynomials are very easy. Similarly, quadratic equations, inequalities and functions are generalized of the first order equations, inequalities. Also factorization is explanation shortly in quadratic equation.

This unit includes; 111 examples, 48 exercises, and 108 test questions.









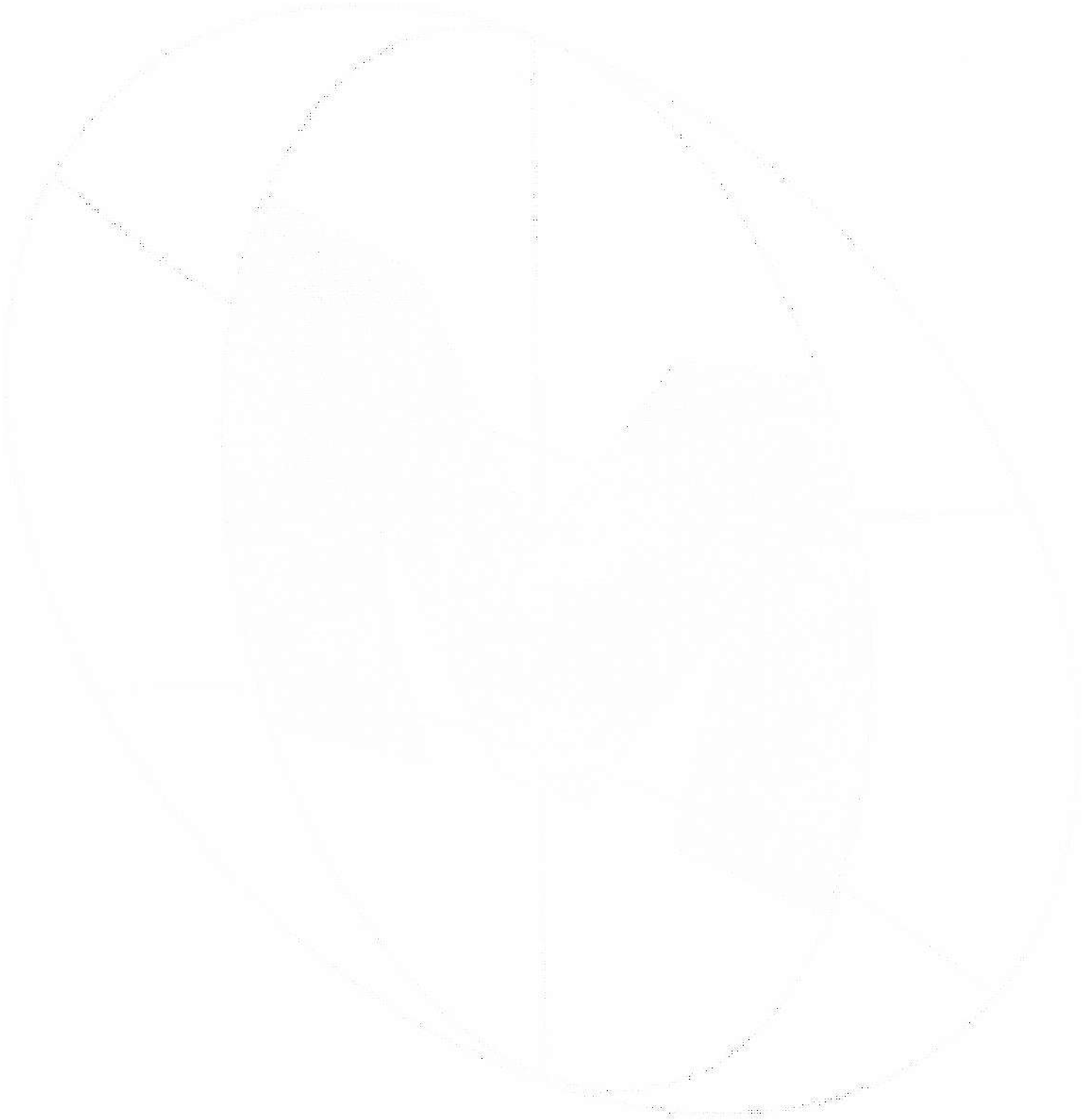
◆ **POLİNOMLAR / *POLYNOMIALS*** . . . . 333 – 354

◆ **İKİNCİ DERECEDEN DENKLEMLER**
QUADRATIC EQUATIONS . . . . 355 – 376

◆ **İKİNCİ DERECEDEN EŞİTSİZLİKLER**
QUADRATIC INEQUALITIES . . . . 377 – 392

◆ **İKİNCİ DERECEDEN FONKSİYONLAR**
QUADRATIC FUNCTIONS . . . . 393 – 414

BU SAYFADA KONU MATERYALİ YOKTUR.



NO SUBJECT MATERIAL ON THIS PAGE.

POLİNOMLAR
POLYNOMIALS

BÖLÜM 1
CHAPTER

Tanım : a_0 , a_1, a_2.......$a_n \in R$, $n \in N$ olmak üzere,
$P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ biçiminde tanımlanan ifadelere *polinom* denir.
a_0, a_1, a_2 … a_n polinomun katsayılarıdır.

Definition: *A polynomial is any function f of the form* $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ *where* a_0, a_1 , a_2 ....$a_n \in R$ *and* $n \in N$.
a_0, a_1, … a_n *are the coefficients of the polynomial.*

ÖRNEK / EXAMPLE 1

P(x) bir polinomdur. / *P(x) is a polynomial.*

$P(x) = (a + 3)x^{-2} + (a + b - 2)x^{-1} + 3x^2 - 5 \Rightarrow$ **$a \cdot b = ?$**

A) –17 B) –15 C) 0 D) 15 E) 17

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$P(x) = (a + 3)x^{-2} + (a + b - 2)x^{-1} + 3x^2 - 5$

$a + 3 = 0 \Rightarrow a = -3$

$a + b - 2 = 0 \Rightarrow b = 5$

$a \cdot b = (-3) \cdot 5 = -15$

ÖRNEK / EXAMPLE 2

P(x) bir polinomdur. / *P(x) is a polynomial.*

$P(x) = x^{\frac{9+n}{n}} - 2x + 3 \Rightarrow$ **$\sum n = ?$**

A) 4 B) 7 C) 13 D) 15 E) 18

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$P(x) = x^{\frac{9+n}{n}} - 2x + 3 \Rightarrow \frac{9+n}{n} \in N$

$\frac{9}{n} + 1 \in N \Rightarrow n = 9, 3, 1, -9$

$\Rightarrow \sum n = 9 + 3 + 1 - 9 = 4$

PEKİŞTİRME / REINFORCE

$P(3x + 1) = 2x^2 + x - 1$

$\Rightarrow P(1) = ?$

YANIT / ANSWER : -1

PEKİŞTİRME / REINFORCE

$P(x^3) = x^6 + x^9 - x^3$

$\Rightarrow P(x) = ?$

YANIT / ANSWER : $x^3 + x^2 - x$

ÖRNEK / EXAMPLE 3

$P(x - 2) = 3x^2 - 6x + 2$

$\Rightarrow$ **P(−1) = ?**

A) −5 B) −1 C) 4 D) 11 E) 15

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$P(x - 2) = 3x^2 - 6x + 2$

$x - 2 = -1 \Rightarrow x = 1$

$P(-1) = 3 \cdot 1^2 - 6 \cdot 1 + 2$

$= 3 - 6 + 2$

$= -1$

ÖRNEK / EXAMPLE 4

$P(x^2) = x^6 - 4x^4 + x^2 - 3$

$\Rightarrow$ **P(−2) = ?**

A) −36 B) −34 C) −32 D) −30 E) −29

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$P(x^2) = x^6 - 4x^4 + x^2 - 3$

$P(x^2) = (x^2)^3 - 4(x^2)^2 + x^2 - 3$

$P(-2) = (-2)^3 - 4 \cdot (-2)^2 + (-2) - 3$

$= -29$

POLİNOMLARIN EŞİTLİĞİ / EQUALITY of POLYNOMIALS

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

$Q(x) = b_n x^n + b_{n-1} x^{n-1} + \ldots + b_1 x + b_0$

$P(x) = Q(x) \Rightarrow a_n = b_n\ ,\ a_{n-1} = b_{n-1}\ \ldots,\ a_1 = b_1\ ,\ a_0 = b_0$

ÖRNEK / EXAMPLE 5

$P(x) = (x^2 - 2x + a)\cdot(x + b)$,

$Q(x) = x^3 - 5x^2 + 8x - 6$,

$P(x) = Q(x) \Rightarrow$ **$a + b = ?$**

A) −4 B) −2 C) −1 D) 3 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$P(x) = (x^2 - 2x + a)\cdot(x + b)$

$= x^3 + (b - 2)x^2 + (a - 2b)x + ab$

$P(x) = Q(x) \Rightarrow x^3 + (b - 2)x^2 + (a - 2b)x + ab = x^3 - 5x^2 + 8x - 6$

$b - 2 = -5 \Rightarrow b = -3$

$a - 2b = 8 \Rightarrow a = 2$

$a + b = 2 + (-3) = -1$

PEKİŞTİRME / REINFORCE

$P(x) = 2x^2 - 3x + 5$,

$Q(x) = (a - 1)x^2 + (b + 1)x + c$,

$P(x) = Q(x) \Rightarrow a + b + c = ?$

YANIT / ANSWER : 4

POLİNOMUN DERECESİ / DEGREE of a POLYNOMIAL

Bir P(x) polinomunda, derecesi en büyük olan terimin derecesine *polinomun derecesi* denir. Polinomun derecesi der [P(x)] ile gösterilir.

The degree of term with the maximum degree is called degree of polynomial in the polynomial P(x). Degree of the polynomial is denoted by der [P(x)].

ÖRNEK / EXAMPLE 6

$\text{der}[P(x)\cdot Q(x)] = 6$,

$\text{der}\left[\frac{P(x)}{Q(x)}\right] = 2 \Rightarrow$ **$\text{der}[P(Q(x))] = ?$**

A) 2 B) 4 C) 8 D) 12 E) 16

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\text{der}[P(x)] = m$

$\text{der}[Q(x)] = n$

$\Rightarrow \text{der}[P(x)\cdot Q(x)] = m + n = 6$

$\text{der}\left[\frac{P(x)}{Q(x)}\right] = m - n = 2$

$\Rightarrow m = 4,\ n = 2$

$\text{der}[P(Q(x))] = 4\cdot 2 = 8$

Düşünün / Think about it

$(1 + x + \dots\dots\dots + x^{12})^3 = \dots\dots + Ax^{12}$

$\Rightarrow A = ?$

POLİNOMLARDA TOPLAMA ve ÇIKARMA
ADDITION and SUBTRACTION in POLYNOMIAL

Toplama / Addition

İki polinom toplanırken dereceleri aynı olan terimlerin katsayıları toplanır ve toplam polinomu elde edilir.

To find the sum of two polynomials, we add the coeffients of the terms of the same degree.

ÖRNEK / EXAMPLE 7

$P(x) = 2x^4 - ax^3 + x^2 - (3 + b)x + 1,$

$Q(x) = (c + 1)x^4 - 2x^2 + 2x + 3,$

$P(x) + Q(x) = -x^2 + 4$

$\Rightarrow$ **a + b + c = ?**

A) – 7 B) –6 C) –5 D) – 4 E) – 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$P(x) + Q(x) = (c + 1 + 2)x^4 - ax^3 - x^2 + (2 - 3 - b)x + 4$

$= (c + 3)x^4 - ax^3 - x^2 + (-1 - b)x + 4$

$= -x^2 + 4$

$c + 3 = 0 \quad -a = 0 \quad -1 - b = 0$

$c = -3, \quad a = 0 \quad b = -1$

$a + b + c = 0 + (-1) + (-3) = -4$

Çıkarma / Subtraction

İki polinom çıkarılırken dereceleri aynı olan terimlerin katsayıları çıkarılır ve fark polinomu elde edilir.

To find the difference of two polynomials, we subtract the coefficients of the terms of the same degree.

Yanıt / Answer

91

ÖRNEK / EXAMPLE 8

$P(x) = x^4 - x^3 + 2x^2 + 3x + 2,$

$Q(x) = -x^4 + 3x^2 - x + 5$

$\Rightarrow$ **$P(x) - Q(x) = ?$**

A) $x^4 - x^3 - x^2 + 4x - 3$

B) $x^4 - x^3 + x^2 + 4x - 3$

C) $2x^4 - x^3 - x^2 + 4x - 3$

D) $2x^4 - x^3 - x^2 + 4x + 3$

E) $2x^4 + x^3 + x^2 + 4x - 3$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$P(x) = x^4 - x^3 + 2x^2 + 3x + 2$

$Q(x) = -x^4 + 3x^2 - x + 5$

$\Rightarrow P(x) - Q(x) = 2x^4 - x^3 - x^2 + 4x - 3$

POLİNOMLARDA ÇARPMA / MULTIPLICATION in POLYNOMIAL

İki polinomun çarpımı polinomlardan birinin her teriminin diğer polinomun her bir terimi ile ayrı ayrı çarpımlarından elde edilen terimlerin toplamına eşittir.

The product of two polynomials equals the sum of the terms that multiplying each terms of one polynomial with each terms of the other polynomial.

ÖRNEK / EXAMPLE 9

$ax^3 + 2x^2 + bx + c = (x + 1)\cdot(x - 2)\cdot(x + 3)$

$\Rightarrow$ **$a + b + c = ?$**

A) –9 B) –10 C) 13 D) 14 E) 21

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$ax^3 + 2x^2 + bx + c = (x + 1)\cdot(x - 2)\cdot(x + 3)$

$ax^3 + 2x^2 + bx + c = (x^2 - x - 2)\cdot(x + 3)$

$ax^3 + 2x^2 + bx + c = x^3 + 2x^2 - 5x - 6$

$\Rightarrow a = 1,\ b = -5,\ c = -6$

$a + b + c = 1 + (-5) + (-6) = -10$

POLİNOMLARDA BÖLME / DIVISION in POLYNOMIAL

Bölme Özdeşliği / *Identity of Division*

$$\begin{array}{r|l} P(x) & Q(x) \\ \cline{2-2} - & T(x) \\ \hline K(x) & \end{array} \Rightarrow$$

P(x) = Bölünen / *Dividend*
Q(x) = Bölen / *Divisor*
T(x) = Bölüm / *Quotient*
K(x) = Kalan / *Remainder*

1. der $[P(x)] \geq$ der $[Q(x)]$
2. der $[K(x)] <$ der $[Q(x)]$
3. $P(x) = Q(x) \cdot T(x) + K(x)$

ÖRNEK / EXAMPLE 10

$$\begin{array}{r|l} P(x) & x-3 \\ \cline{2-2} - & x^2-2x+1 \\ \hline 4 & \end{array} \Rightarrow \mathbf{P(1) = ?}$$

A) 1 B) 2 C) 4 D) 6 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$P(x) = (x-3) \cdot (x^2-2x+1) + 4$

$P(x) = x^3 - 5x^2 + 7x + 1$

$P(1) = 1 - 5 + 7 + 1 = 4$

ÖRNEK / EXAMPLE 11

$P(x) = x^3 - 2x^2 + x - 2$

$$\begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - & Q(x) \\ \hline 0 & \end{array} \Rightarrow \mathbf{Q(1) = ?}$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

Çözüm / *Solution* 1:

$$\begin{array}{r|l} x^3-2x^2+x-2 & x-2 \\ \cline{2-2} -\quad -/\ x^3-2x^2 & x^2+1 \\ \cline{1-1} 0+0+x-2 & \\ +\quad -/\ x-2 & \\ \cline{1-1} 0 & \end{array}$$

$Q(x) = x^2 + 1$

$Q(1) = 1^2 + 1 = 2$

Çözüm / *Solution* 2:

$P(x) = (x-2) \cdot Q(x) + 0$

$\Rightarrow Q(x) = \dfrac{P(x)}{x-2}$

$Q(1) = \dfrac{P(1)}{1-2} = \dfrac{1^3 - 2 \cdot 1^2 + 1 - 2}{-1}$

$= 2$

P(x) Polinomunun ax + b ile Bölümünden Elde Edilen Kalan
The Remainder of the Polynomial P(x) Divided by ax + b

Bir P(x) polinomunun ax + b ile bölümünden kalanı bulmak için $ax + b = 0 \Rightarrow x = -\frac{b}{a}$ belirlenir. P(x) polinomunda, x'in yerine $-\frac{b}{a}$ yazılırsa işlemin sonucu kalanı verir.

ax + b birinci dereceden bir polinom olduğu için kalan sıfırıncı dereceden bir polinom, yani; reel bir sayı olmak zorundadır.

To find the remainder of P(x) divided by (ax + b) the following is determined:

$ax + b = 0 \Rightarrow x = -\frac{b}{a}$

If we write $-\frac{b}{a}$ *instead of x in P(x), then the result gives the remainder. Since ax + b is a first order polynomial, the remainder must always be equal to a real number, which is a zero polynomial.*

$$\begin{array}{r|l} P(x) & ax+b \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow K(x) = P\left(\frac{-b}{a}\right)$$

ÖRNEK / EXAMPLE 12

$P(x) = x^3 - 2x^2 + x - 1$

$$\begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) –1 B) 0 C) 1 D) 2 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x - 2 = 0 \Rightarrow x = 2$

$$\begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow K(x) = P(2)$$

$P(x) = x^3 - 2x^2 + x - 1$

$P(2) = 2^3 - 2 \cdot 2^2 + 2 - 1$

$= 1$

PEKİŞTİRME / REINFORCE

$$\begin{array}{r|l} 3x^3 - 2x^2 + x - 1 & x+2 \\ \cline{2-2} - \quad & \\ \hline ? & \end{array}$$

YANIT / ANSWER : –35

ÖRNEK / EXAMPLE 13

$P(x) = x^3 + 2x^2 + 12$

$$\begin{array}{r|l} P(x) & 2x+4 \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) 4 B) 8 C) 12 D) 16 E) 20

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$2x + 4 = 0 \Rightarrow x = -2$

$$\begin{array}{r|l} P(x) & 2x+4 \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow K(x) = P(-2)$$

$P(x) = x^3 + 2x^2 + 12$

$P(-2) = (-2)^3 + 2\cdot(-2)^2 + 12$

$= -8 + 8 + 12 = 12$

P(x) Polinomunun $x^n \mp a$ ile Bölümünden Elde Edilen Kalan
The Remainder of P(x) Divided by $x^n \mp a$

Bir P(x) polinomunun $x^n \pm a$ ile bölümünden kalanını bulmak için $x^n \pm a = 0$'dan $x^n = \pm a$ bulunur. Polinomda x^n yerine ± a değeri yerine yazılarak kalan bulunur.

To find the remainder of P(x) divided by ($x^n \pm a$), insert ($x^n = \pm a$) in the polynomial P(x), which writing ± a instead of x^n in the polynomial.

ÖRNEK / EXAMPLE 14

$P(x) = x^6 - 2x^4 - 3x^2 + 8$

$$\begin{array}{r|l} P(x) & x^2+\sqrt{3} \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x^2 + \sqrt{3} = 0 \Rightarrow x^2 = -\sqrt{3}$

$P(x) = x^6 - 2x^4 - 3x^2 + 8$

$= (x^2)^3 - 2(x^2)^2 + 3x^2 + 8$

$\Rightarrow K(x) = (-\sqrt{3})^3 - 2\cdot(-\sqrt{3})^2 - 3\cdot(-\sqrt{3}) + 8$

$= -3\sqrt{3} - 6 + 3\sqrt{3} + 8 = 2$

P(x) Polinomunun (x – a) · (x – b) · (x – c)... Çarpımı ile Bölünebilmesi
Divided by (x – a) · (x – b) · (x – c)... of the Polynomial P(x)

P(x) polinomu (x – a)·(x – b)·(x – c)... çarpımı ile tam bölünebiliyorsa (x – a), (x – b), (x – c) ... çarpanlarıyla da ayrı ayrı tam bölünebilir.

If the polynomial P(x) is divided by (x – a)·(x – b)·(x – c)· ..., it is divided by the factor (x – a)·(x – b)·(x – c)· ...

ÖRNEK / EXAMPLE 15

$$\begin{array}{r|l} P(x) & x^2+2x-15 \\ \cline{2-2} - & \\ \hline 2x+1 & \end{array} \quad \Rightarrow \mathbf{P(-5) = ?}$$

A) –7 B) –8 C) –9 D) –10 E) –11

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{array}{r|l} P(x) & x^2+2x-15 \\ \cline{2-2} & Q(x) \\ - & \\ \hline 2x+1 & \end{array} \quad \Rightarrow P(x) = (x^2+2x-15)\cdot Q(x) + 2x + 1$$

$x^2 - 2x - 15 = (x + 5)\cdot(x - 3)$

$\Rightarrow \quad P(x) = (x + 5)\cdot(x - 3) \cdot Q(x) + 2x + 1$

$P(-5) = (-5 + 5)\cdot(-5 - 3)\cdot Q(-5) + 2\cdot(-5) + 1$

$P(-5) = -9$

ÖRNEK / EXAMPLE 16

$$\begin{array}{r|l} P(x) & x^2-5x+6 \\ \cline{2-2} - & Q(x) \\ \hline x+3 & \end{array} \qquad \begin{array}{r|l} P(x) & x-3 \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \quad \Rightarrow \mathbf{K(x) = ?}$$

A) –4 B) –1 C) 3 D) 4 E) 6

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$P(x) = (x^2 - 5x + 6) \cdot Q(x) + x + 3$

$\quad = (x - 3)\cdot(x - 2) \cdot Q(x) + x + 3$

$$\begin{array}{r|l} P(x) & x-3 \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \quad \Rightarrow K(x) = P(3)$$

$P(3) = 0 + 6 = 6$

P(x) Polinomunun $(ax + b)^n$ ile Bölünebilmesi
Divided by $(ax + b)^n$ of a Polynomial P(x)

P(x) polinomu $(ax + b)^n$ polinomu ile tam bölünebiliyorsa, $P'(x)$, $P''(x)$, $P'''(x)$ … $P^{(n-1)}(x)$ polinomları da ax + b ile tam bölünebilir.

Burada $P'(x)$, $P''(x)$, … , $P^{(n-1)}(x)$ polinomları P(x) polinomunun 1, 2, 3. ve (n – 1). mertebeden türevleridir.

If the polynomial P(x) is divided by $(ax + b)^n$, the polynomial $P'(x)$, $P''(x)$, $P'''(x)$ … $P^{(n-1)}(x)$ is divided by ax + b where $P'(x)$, $P''(x)$, $P'''(x)$ … $P^{(n-1)}(x)$ are 1, 2, … (n – 1). order derivatives of the polynomial P(x).

ÖRNEK / EXAMPLE 17

$P(x) = x^3 + ax + b - 4$,

$$\begin{array}{r|l} P(x) & (x-1)^2 \\ \cline{2-2} - \quad & \\ \hline 0 & \end{array} \Rightarrow a \cdot b = ?$$

A) –18 B) –16 C) –14 D) –12 E) –10

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

Çözüm / *Solution* 1:

$x^2 - 2x + 1 = 0 \Rightarrow x^2 = 2x - 1$

$P(x) = x^2 \cdot x + ax + b - 4$

$= (2x - 1) \cdot x + ax + b - 4$

$= 2x^2 - x + ax + b - 4$

$= 4x - 2 + ax - x + b - 4$

$(a + 3)x + b - 6 = 0$

$a + 3 = 0 \qquad b - 6 = 0$

$a = -3 \qquad b = 6$

$\Rightarrow a \cdot b = (-3) \cdot 6 = -18$

Çözüm / *Solution* 2:

$x - 1 = 0 \Rightarrow x = 1$

$x = 1 \Rightarrow P(1) = 0,\ P'(1) = 0$

$P(x) = x^3 + ax + b - 4$

$P(1) = 1 + a + b - 4 = 0 \Rightarrow a + b = 3$

$P'(x) = 3x^2 + a$

$P'(1) = 3 + a = 0 \Rightarrow a = -3$

$a = -3 \Rightarrow b = 6$

$a \cdot b = (-3) \cdot 6 = -18$

ÖRNEK / EXAMPLE 18

YILDIZ TEKNİK ÜNİ. YÖS 2011

$$\frac{P(x-2)}{Q(x+2)} = x^2 - x + 4$$

$Q(5) = 1$

$P(1) = ?$

A) 4 B) 10 C) 16 D) 24 E) 34

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$x - 2 = 1 \Rightarrow x = 3$

$$\frac{P(1)}{Q(5)} = 3^2 - 3 + 4 \Rightarrow P(1) = 10 \cdot Q(5) = 10$$

ÖRNEK / EXAMPLE 19

ANKARA ÜNİ. YÖS 2013

$P(x) = (x^2 - 1) \cdot Q(x) + 4x + 5$,

$P(x) = (x - 1) \cdot K(x) + R$

$\Rightarrow$ **$R = ?$**

A) 7 B) 8 C) 9 D) 10 E) 11

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x = 1 \Rightarrow P(1) = 0 \cdot Q(1) + 4 + 5 = 9$

$x = 1 \Rightarrow P(1) = (1 - 1) \cdot K(1) + R$

$9 = 0 + R$

$9 = R$

ÖRNEK / EXAMPLE 20

İSTANBUL ÜNİ. YÖS 2013

$$\begin{array}{r|l} P(x) & x+1 \\ \cline{2-2} - & A(x) \\ \hline -6 & \end{array} \qquad \begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - & B(x) \\ \hline 6 & \end{array} \qquad \begin{array}{r|l} P(x) & (x+1)(x-2) \\ \cline{2-2} - & C(x) \\ \hline ax+b & \end{array}$$

ax + b = ?

A) 4x + 2

B) 4x – 2

C) 2x – 2

D) x – 4

E) 2x + 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{array}{r|l} P(x) & x+1 \\ \cline{2-2} - & A(x) \\ \hline -6 & \end{array} \Rightarrow P(-1) = -6 \qquad \begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - & B(x) \\ \hline 6 & \end{array} \Rightarrow P(2) = 6$$

$$\begin{array}{r|l} P(x) & (x+1)(x-2) \\ \cline{2-2} - & C(x) \\ \hline ax+b & \end{array} \Rightarrow P(x) = (x+1)(x-2) \cdot C(x) + ax + b$$

$x = -1 \Rightarrow -a + b = -6$

$x = 2 \Rightarrow 2a + b = 6$

$a = 4, \quad b = -2$

$\Rightarrow ax + b = 4x - 2$

ÖRNEK / EXAMPLE 21

MARMARA ÜNİV. YÖS 2016

$$\begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - & \\ \hline 4 & \end{array} \qquad \begin{array}{r|l} Q(x) & x+2 \\ \cline{2-2} - & \\ \hline 3 & \end{array}$$

$R(x) = 3\,P(2x) - 4Q(-2x) \Rightarrow$ **R(1) = ?**

A) 0 B) 3 C) 7 D) 10 E) 14

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - & \\ \hline 4 & \end{array} \Rightarrow P(2) = 4$$

$$\begin{array}{r|l} Q(x) & x+2 \\ \cline{2-2} - & \\ \hline 3 & \end{array} \Rightarrow Q(-2) = 3$$

$R(x) = 3P(2x) - 4Q(-2x)$

$R(1) = 3 \cdot P(2) - 4 \cdot Q(-2) = 3 \cdot 4 - 4 \cdot 3 = 12 - 12 = 0$

1. $(x+3)\cdot P(x) = 2x^2 - x + a \Rightarrow$ **$P(-2) = ?$**

A) -20 B) -15 C) -12 D) -11 E) -8

2. $P(x-2) = 3x^2 - 2x + 3,$

$$\begin{array}{r|l} P(x) & x+1 \\ \cline{2-2} \underline{-\qquad} & \\ K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) -2 B) 0 C) 1 D) 3 E) 4

3. $P(x) = 2x^3 + mx - 2,$

$$\begin{array}{r|l} P(x) & x^2+x \\ \cline{2-2} \underline{-\qquad} & \\ 6x-2 & \end{array} \Rightarrow \mathbf{m = ?}$$

A) 2 B) 4 C) 7 D) 8 E) 10

4.

$$\begin{array}{r|l} P(x) & x^2-3x-10 \\ \cline{2-2} \underline{-\qquad} & \\ 4x-2 & \end{array} \qquad \begin{array}{r|l} P(x) & x+2 \\ \cline{2-2} \underline{-\qquad} & \\ K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) -10 B) -7 C) -5 D) -3 E) -2

5. $P(x) + 2P(-x) = 3x^2 - 4x + 5 \Rightarrow$ **$P(1) = ?$**

A) $\frac{17}{3}$ B) $\frac{19}{3}$ C) $\frac{20}{3}$ D) $\frac{23}{3}$ E) $\frac{25}{3}$

6.

$$\begin{array}{r|l} P(x) & x+3 \\ \cline{2-2} \underline{-\qquad} & \\ 2 & \end{array} \qquad \begin{array}{r|l} Q(x) & x+3 \\ \cline{2-2} \underline{-\qquad} & \\ -1 & \end{array}$$

$$\begin{array}{r|l} 2\cdot P(x)\cdot x + Q(x)\cdot x^2 - 3x + a & x+3 \\ \cline{2-2} \underline{-\qquad\qquad\qquad\qquad} & \\ 0 & \end{array} \Rightarrow \mathbf{a = ?}$$

A) 4 B) 8 C) 9 D) 12 E) 16

7. $\dfrac{P(x-2)}{Q(x+3)} = x^2 + x - 3,\ Q(0) = 2 \Rightarrow$ **$P(-5) = ?$**

A) 0 B) 2 C) 4 D) 6 E) 8

8.
$$\begin{array}{r|l} x^4 - 4x^3 + x^2 + ax + 1 & x^3 - 1 \\ \cline{2-2} - \qquad\qquad\qquad & \\ \hline x^2 - x - 3 & \end{array} \Rightarrow \mathbf{a = ?}$$

A) –9 B) –7 C) –5 D) –4 E) –2

9.
$$\begin{array}{r|l} 2x^{20} - 3x^{15} + x^{10} - x^5 + 3 & x^5 - 2 \\ \cline{2-2} - \qquad\qquad\qquad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) 12 B) 13 C) 14 D) 15 E) 16

10. $P(x) = -3 \cdot x^{40} + 12 \cdot x^{20} - 12 \Rightarrow$ **$P\left(\sqrt[5]{2}\right) = ?$**

A) –768 B) – 640 C) –612 D) – 588 E) – 542

11. $(x^2 + x + 1) \cdot P(x + 5) = x^3 - 1 \Rightarrow$ **$P(x) = ?$**

A) x + 6 B) x – 5 C) x – 6 D) x + 5 E) x – 1

12. $P(x) = x^3 - 3mx^2 + 9x + n,$

$$\begin{array}{r|l} P(x) & (x-1)^2 \\ \cline{2-2} - \quad & \\ \hline 0 & \end{array} \Rightarrow \mathbf{n = ?}$$

A) –4 B) –2 C) 0 D) 2 E) 4

YANITLAR / ANSWERS							
1. D	**2. E**	**3. B**	**4. A**	**5. C**	**6. D**	**7. D**	**8. E**
9. B	**10. D**	**11. C**	**12. A**				

1. $(x+3)\cdot P(x) = 2x^2 - x + a$

$x = -3 \Rightarrow (-3+3)\cdot P(-3) = 2\cdot(-3)^2 - (-3) + a$

$0 = 18 + 3 + a$

$a = -21$

$x = -2 \Rightarrow (-2+3)\cdot P(-2) = 2\cdot(-2)^2 - (-2) - 21$

$P(-2) = 8 + 2 - 21$

$= -11$

Yanıt / The Answer is D

2.

$$\begin{array}{l|l} P(x) & x+1 \\ \hline K(x) & \end{array} \Rightarrow K(x) = P(-1)$$

$P(x-2) = 3x^2 - 2x + 3$

$x - 2 = -1 \Rightarrow x = 1$

$P(-1) = 3\cdot 1^2 - 2\cdot 1 + 3$

$= 4$

Yanıt / The Answer is E

---

3. $x^2 + x = 0 \Rightarrow x^2 = -x$

$P(x) = 2x^3 + mx - 2$

$= 2x^2 \cdot x + mx - 2$

$x^2 = -x \Rightarrow P(x) = 2\cdot(-x)\cdot x + mx - 2$

$= -2x^2 + mx - 2$

$= 2x + mx - 2$

$(m+2)x - 2 = 6x - 2$

$m = 4$

Yanıt / The Answer is B

---

4. $x^2 - 3x - 10 = (x-5)\cdot(x+2)$

$P(x) = (x^2 - 3x - 10)\cdot Q(x) + 4x - 2$

$x + 2 = 0 \Rightarrow x = -2$

$P(-2) = 0 + 4\cdot(-2) - 2$

$= -10$

Yanıt / The Answer is A

---

5. $P(x) = ax^2 + bx + c$

$P(-x) = ax^2 - bx + c$

$\Rightarrow P(x) + 2\cdot P(-x) = 3ax^2 - bx + 3c$

$3x^2 - 4x + 5 = 3ax^2 - bx + 3c$

$a = 1 \;,\; b = 4 \;,\; c = \frac{5}{3}$

$P(x) = x^2 + 4x + \frac{5}{3}$

$P(1) = 1 + 4 + \frac{5}{3} = \frac{20}{3}$

Yanıt / The Answer is C

---

6. $P(-3) = 2 \;,\; Q(-3) = -1$

$2\cdot P(x)\cdot x + Q(x)\cdot x^2 - 3x + a = 2\cdot P(-3)\cdot(-3) + Q(-3)\cdot(-3)^2 - 3\cdot(-3) + a$

$0 = 2\cdot(2)\cdot(-3) + (-1)\cdot 9 + 9 + a$

$0 = -12 + a \Rightarrow a = 12$

Yanıt / The Answer is D

7. $\dfrac{P(x-2)}{Q(x+3)} = x^2 + x - 3$

$P(x-2) = (x^2 + x - 3) \cdot Q(x+3)$

$x + 3 = 0 \Rightarrow x = -3$

$P(-3-2) = ((-3)^2 - 3 - 3) \cdot Q(0)$

$P(-5) = 3 \cdot 2 = 6$

Yanıt / The Answer is D

8. $x^3 - 1 = 0 \Rightarrow x^3 = 1$

$x^4 - 4x^3 + x^2 + ax + 1 = (x^3 - 1) \cdot Q(x) + x^2 - x - 3$

$x^3 \cdot x - 4x^3 + x^2 + ax + 1 = (x^3 - 1) \cdot Q(x) + x^2 - x - 3$

$x^3 = 1 \Rightarrow x - 4 + x^2 + ax + 1 = x^2 - x - 3$

$x^2 + (a+1)x - 3 = x^2 - x - 3$

$\Rightarrow a = -2$

Yanıt / The Answer is E

9. $2x^{20} - 3x^{15} + x^{10} - x^5 + 3 = (x^5 - 2) \cdot Q(x) + K(x)$

$x^5 - 2 = 0 \Rightarrow x^5 = 2$

$2 \cdot (x^5)^4 - 3 \cdot (x^5)^3 + (x^5)^2 - x^5 + 3 = (x^5 - 2) \cdot Q(x) + K(x)$

$2 \cdot 2^4 - 3 \cdot 2^3 + 2^2 - 2 + 3 = K(x)$

$32 - 24 + 4 + 1 = K(x)$

$K(x) = 13$

Yanıt / The Answer is B

10. $P(x) = -3 \cdot (x^5)^8 + 12 \cdot (x^5)^4 - 12$

$$x = \sqrt[5]{2} \Rightarrow P(\sqrt[5]{2}) = -3 \cdot \left(\left(\sqrt[5]{2}\right)^5\right)^8 + 12\left(\left(\sqrt[5]{2}\right)^5\right)^4 - 12$$

$$= -3 \cdot 2^8 + 12 \cdot 2^4 - 12 = -588$$

Yanıt / The Answer is D

11. $(x^2 + x + 1) \cdot P(x+5) = x^3 - 1$

$$P(x+5) = \frac{x^3 - 1}{x^2 + x + 1}$$

$$= \frac{(x-1) \cdot (x^2 + x + 1)}{x^2 + x + 1}$$

$$= x - 1$$

$P(x - 5 + 5) = x - 5 - 1$

$P(x) = x - 6$

Yanıt / The Answer is C

12. $P(x) = x^3 - 3mx^2 + 9x + n$

$P'(x) = 3x^2 - 6mx + 9$

$x = 1 \Rightarrow P(1) = 1 - 3m + 9 + n = 0 \Rightarrow 3m - n = 10$

$P'(1) = 3 - 6m + 9 = 0 \Rightarrow m = 2$

$3m - n = 10$

$3 \cdot 2 - n = 10$

$6 - n = 10$

$n = -4$

Yanıt / The Answer is A

1. $P(x-1) + P(x+1) = 4x + 6 \Rightarrow \mathbf{P(x) = ?}$

Yanıt / The Answer is: **2x + 3**

---

2. $(x-2) \cdot P(x) = 3x^2 + ax + 4 \Rightarrow \mathbf{P(1) = ?}$

Yanıt / The Answer is: **1**

---

3. $P(x) = ax^3 - 2x^2 + 2x - 1,$

$$\begin{array}{l|l} P(x) & \underline{x+2} \\ \underline{-} & \\ -29 & \end{array} \Rightarrow \mathbf{a = ?}$$

Yanıt / The Answer is: **2**

4. $P(x) = (2x-1) \cdot Q(x+3) + 3x - 2,$

$Q(1) = 3 \Rightarrow \mathbf{P(-2) = ?}$

Yanıt / The Answer is: **–23**

---

5. $x \cdot P(x) = x^3 - 2x^2 + x - a \Rightarrow \mathbf{P(-1) = ?}$

Yanıt / The Answer is: **4**

---

6. $P(x) = x^2 - ax + 3,$

$$\begin{array}{l|l} P(x+1) & \underline{x-2} \\ \underline{-} & \\ 6 & \end{array} \Rightarrow \mathbf{a = ?}$$

Yanıt / The Answer is: **2**

7. $P(x) = x^3 - x^2 + mx - 2,$

$$\begin{array}{r|l} P(x) & x^2-x+1 \\ \cline{2-2} - & \\ \hline 6x-2 & \end{array} \Rightarrow \mathbf{m = ?}$$

Yanıt / The Answer is: **7**

8.

$$\begin{array}{r|l} x^3-3x^2+mx-n & x^2-2x-8 \\ \cline{2-2} - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{m + n = ?}$$

Yanıt / The Answer is: **–14**

9. $\text{der}\,[P(x) \cdot Q(x^2)] = 11,$

$$\text{der}\left[\frac{P^2(x)}{Q(x)}\right] = 2 \Rightarrow \mathbf{der\ [P(x) + Q(x)] = ?}$$

Yanıt / The Answer is: **4**

10.

$$\begin{array}{r|l} 3x^3-x^2+ax-5 & x^2-x \\ \cline{2-2} - & \\ \hline 4x-5 & \end{array} \Rightarrow \mathbf{a = ?}$$

Yanıt / The Answer is: **2**

11.

$$\begin{array}{r|l} x^3-2x^2+3x-1 & x^2+3 \\ \cline{2-2} - & Q(x) \\ \hline 5 & \end{array} \Rightarrow \mathbf{Q(3) = ?}$$

Yanıt / The Answer is: **1**

12. $P(x) = x^3 + 3x^2 + 3x + 4 \Rightarrow \mathbf{P\left(\sqrt[3]{2} - 1\right) = ?}$

Yanıt / The Answer is: **5**

1. P(x) bir polinomdur. / P(x) is a polynomial.

$P(x) = x^{\frac{12}{n}} + 2x^{n-3} - 3x + 5 \Rightarrow \sum n = ?$

A) 10
B) 13
C) 18
D) 25
E) 28

2. $P(x) = 4x^3 - 2x^2 + 3x - 4 \Rightarrow P(-1) = ?$

A) –13
B) –9
C) –3
D) 4
E) 7

3. $P(x + 3) = 2x^3 + ax + 5$, $P(4) = 10 \Rightarrow a = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

4. $P(2x - 1) = 2x^2 + (m + 1)x - 6$,
$P(3) = 12 \Rightarrow m = ?$

A) 2
B) 4
C) 5
D) 6
E) 8

5. $P(-5x) = -125x^3 + 25x^2 - 10x + 4 \Rightarrow P(3) = ?$

A) 40
B) 42
C) 44
D) 45
E) 46

6. $P(x + 2) = 3x^2 - 2x + 5 \Rightarrow P(x) = ?$

A) $3x^2 - 10x + 20$
B) $3x^2 - 14x + 21$
C) $3x^2 - 16x + 8$
D) $3x^2 - 8x + 10$
E) $3x^2 - 18x + 15$

7. $P(x-1) = x^2 + 3x - 5 \Rightarrow$ **$P(x) = ?$**

A) $x^2 + 5x - 1$

B) $x^2 + 3x + 8$

C) $x^2 - 4x + 6$

D) $x^2 + 3x + 3$

E) $x^2 - 2x + 8$

8. $P(x+4) = 3x^2 - x + 2 \Rightarrow$ **$P(x+1) = ?$**

A) $3x^2 - 19x + 32$

B) $3x^2 + 8x - 4$

C) $3x^2 - 15x + 25$

D) $3x^2 - 7x - 12$

E) $3x^2 + 12x - 8$

9. $(x^3 - 4x^2 + 8x + 5)\cdot(x^2 - 2x + 4) = \ldots + ax^2 \ldots$

$\Rightarrow$ **$a = ?$**

A) –20

B) –22

C) –25

D) –27

E) –30

10. $P(x) = 4x^2 - (a-4)x + 7$

$Q(x) = (b+2)x^3 - (c-b)x^2 + 3x - d + 1$

$P(x) = Q(x) \Rightarrow$ **$a + b + c + d = ?$**

A) –3

B) –5

C) –7

D) –9

E) –13

11. $\text{der}[P(x)] = 3$,

$\text{der}[Q(x)] = 2 \Rightarrow$ **$d[P(Q(x))] = ?$**

A) 2

B) 3

C) 4

D) 6

E) 9

12. $\text{der}[P^2(x)\cdot Q(x)] = 8$,

$$\text{der}\left[\frac{P(x)}{Q(x)}\right] = 1 \Rightarrow \mathbf{der}\left[Q(x)\cdot x^2\right] = ?$$

A) 4

B) 5

C) 6

D) 7

E) 8

YANITLAR / ANSWERS							
1. D	2. A	3. C	4. B	5. E	6. B	7. A	8. A
9. D	10. E	11. D	12. A				

1. $x^3 + ax + b = (x^2 - 1)\cdot Q(x) + 3x - 1 \Rightarrow$ **$a - 2b = ?$**

A) 1 B) 2 C) 3

D) 4 E) 5

2. $P(x) = 3x^2 - 6x + 7,$

$$\begin{array}{r|l} P(x) & x-1 \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) 2 B) 4 C) 5

D) 6 E) 8

3. $P(x) = x^3 + ax^2 - 3x + 1,$

$$\begin{array}{r|l} P(x) & 2x-4 \\ \cline{2-2} - \quad & \\ \hline 15 & \end{array} \Rightarrow \mathbf{a = ?}$$

A) 1 B) 2 C) 3

D) 5 E) 7

4. $P(x + 3) = 2x^2 - 3x + 1$

$$\begin{array}{r|l} P(x) & x+1 \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) 26 B) 30 C) 35

D) 42 E) 45

5. $P(x + 1) = -x^2 - 3x + 1,$

$$\begin{array}{r|l} P(x-2) & x+3 \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) −17 B) −20 C) −23

D) −28 E) −32

6. $P(x - 2) = 2x^2 + 3x - 1,$

$$\begin{array}{r|l} P(x+1) & x \\ \cline{2-2} - \quad & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) 18 B) 20 C) 24

D) 26 E) 28

7. $P(x) = x^6 + 8x^3 - 6x^2 + (n+2)x - 12$

$$\begin{array}{r|l} P(x) & x-1 \\ \hline - & \\ \hline -2 & \end{array} \Rightarrow \begin{array}{r|l} \mathbf{P(x)} & \mathbf{x+1} \\ \hline - & \\ \hline \mathbf{?} & \end{array}$$

A) 32 B) 24 C) −12

D) −24 E) −32

8.

$$\left.\begin{array}{l} P(x) = ax^4 + 3bx^2 - 9x + 12 \\ \begin{array}{r|l} P(x) & x^2 - 2x + 1 \\ \hline - & \\ \hline 0 & \end{array} \end{array}\right\} \Rightarrow \mathbf{a + b = ?}$$

A) 4 B) 8 C) 9

D) 12 E) 15

9. $P(x) = 12x^{20} - 3x^{12} + x^6 - x + 4,$

$$\begin{array}{r|l} P(x) & x^5 - 1 \\ \hline - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) $-2x^2 + 5$

B) $-3x^2 + 16$

C) $x^2 - x + 3$

D) $4x^2 - 7$

E) $5x^2 + 1$

10. $P(x) = 2x^3 - 3x^2 + x - 1,$

$$\begin{array}{r|l} P(x) & x^2 - x - 1 \\ \hline - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) $3x + 1$

B) $2x + 3$

C) $2x - 2$

D) $2x + 7$

E) $4x - 1$

11. $P(x) = x^4 - 3x^3 + x^2 + x - 2$

$$\begin{array}{r|l} P(x) & x^2 + x + 2 \\ \hline - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) $4x - 2$ B) $5x + 7$ C) $4x - 3$

D) $6x - 3$ E) $6x - 8$

12.

$$\begin{array}{r|l} P(x) & x-2 \\ \hline - & \\ \hline 5 & \end{array} \qquad \begin{array}{r|l} P(x) & x+1 \\ \hline - & \\ \hline -4 & \end{array}$$

$$\begin{array}{r|l} P(x) & x^2 - x - 2 \\ \hline - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) $2x + 3$ B) $3x + 7$ C) $x - 4$

D) $2x + 1$ E) $3x - 1$

YANITLAR / ANSWERS

1. D	**2. B**	**3. C**	**4. E**	**5. A**	**6. D**	**7. E**	**8. A**
9. B	**10. C**	**11. E**	**12. E**				

İKİNCİ DERECEDEN DENKLEMLER
QUADRATIC EQUATIONS

BÖLÜM
CHAPTER **2**

Tanım : a, b, c ∈ R, a ≠ 0 olmak üzere, $ax^2 + bx + c = 0$ biçimindeki eşitliklere *ikinci dereceden denklem* denir.
a, b, c sayılarına *denklemin katsayıları* denir. Denklemi sağlayan sayılara *denklemin kökleri*, köklerin kümesine de *çözüm kümesi* denir.

Definition: *An equation in the form $ax^2 + bx + c = 0$ is called an quadratic equation. Where a, b, c ∈ R, a ≠ 0.*
The numbers of a,b and c are coefficients in the equation. Numbers satisfying the equation are called roots of the equation, the set containing the roots are called solution set.

İKİNCİ DERECEDEN DENKLEMLERİN ÇÖZÜM KÜMESİNİN BULUNMASI
FINDING the SOLUTION SET of the QUADRATIC EQUATIONS

1. ÇARPANLARA AYIRMA YÖNTEMİ / FACTORIZATION METHOD

a, b, c ∈ R, a ≠ 0 olmak üzere, $ax^2 + bx + c = 0$ denklemi çarpanlarına ayrılıyorsa denklemin kökleri çarpanlara ayırarak bulunur.

If the equation $ax^2 + bx + c = 0$ is factorization, the roots of equation is found by the factorization method.

ÖRNEK / EXAMPLE 1

$3x^2 + 4x + 1 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{2, -3\}$

B) $\left\{\frac{1}{5}, 2\right\}$

C) $\left\{-1, -\frac{1}{3}\right\}$

D) $\left\{-\frac{1}{2}, \frac{3}{2}\right\}$

E) ∅

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$3x^2 + 4x + 1 = 0$

$\downarrow \qquad \downarrow$

$3x \qquad 1$

$x \qquad 1$

$3x^2 + 4x + 1 = (3x + 1)\cdot(x + 1) = 0 \Rightarrow 3x + 1 = 0 \qquad x + 1 = 0$

$x_1 = -\frac{1}{3} \qquad x_2 = -1$

$ÇK(SS) = \left\{-1, -\frac{1}{3}\right\}$

PEKİŞTİRME / REINFORCE

$x^2 - x - 6 = 0 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : {–2, 3}

ÖRNEK / EXAMPLE 2

$2x^3 + 3x^2 - 18x - 27 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$ B) $\left\{-1, \frac{3}{2}, 2\right\}$ C) $\left\{1, \frac{5}{2}, 3\right\}$ D) $\left\{-3, -\frac{3}{2}, 3\right\}$ E) $\left\{-2, 1, \frac{5}{2}\right\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$2x^3 + 3x^2 - 18x - 27 = 0$$

$$x^2(2x + 3) - 9(2x + 3) = 0$$

$$(2x + 3) \cdot (x^2 - 9) = 0$$

$$\Rightarrow x_1 = -\frac{3}{2}, \quad x_2 = -3, \quad x_2 = 3$$

$$ÇK(SS) = \left\{-3, -\frac{3}{2}, 3\right\}$$

İRDELEME / STUDY TIP

Çarpanlara ayırma yöntemi, çarpanlara ayrılabilen ikinci dereceden denklemlerde etkilidir. Fakat birçok ikinci dereceden denklem, çarpanlara ayrılmayabilir. Bu tür denklemleri çözmek için, genel bir formül diskriminant yöntemidir.

The factorization method are efficient for special quadratic equations that are factorable. There are, however, many quadratic equations that cannot be solved efficiently by factorization. The general formula is the discriminant method that can be used to solve any quadratic equation.

2. DİSKRİMİNANT YÖNTEMİ / THE DISCRIMINANT METHOD

$ax^2 + bx + c = 0$ denklemi için;

$\Delta = b^2 - 4ac$ sayısına denklemin *diskriminantı* denir.

For the equation $ax^2 + bx + c$,

$\Delta = b^2 - 4ac$ is called the discriminant of the equation.

In equation $ax^2 + bx + c = 0$ denkleminde,

a. $\Delta > 0$ ise, denklemin birbirinden farklı iki reel çözümü vardır.

If $\Delta > 0$, the equation has two distinct real solutions.

Bu kökler aşağıdaki gibidir.

These roots are as follows:

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a},$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

b. $\Delta = 0$ ise, denklemin birbirine eşit iki çözümü vardır.

If $\Delta = 0$, there are two equal repeated real solutions.

Bu kökler aşağıdaki gibidir.

These roots are as follows:

$$x_1 = x_2 = -\frac{b}{2a}$$

c. $\Delta < 0$ ise denklemin reel kökü yoktur, karmaşık kök vardır. Çözüm kümesi boş kümedir.

If $\Delta < 0$, the equation has imaginary solutions. The solution set is an empty set.

ÖRNEK / EXAMPLE 3

$x^2 - 4x + 1 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{\sqrt{3}, 2\}$
B) $\{2 - \sqrt{3}, 2 + \sqrt{3}\}$
C) $\{3 - \sqrt{3}\}$
D) $\{3 + \sqrt{3}, 3 - \sqrt{3}\}$
E) $\{4\sqrt{3}, 2\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$x^2 - 4x + 1 = 0 \Rightarrow \Delta = b^2 - 4ac$$
$$= 16 - 4 \cdot 1 \cdot 1$$
$$= 12$$

$$\Delta = 12 \Rightarrow x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{4 + 2\sqrt{3}}{2}$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{4 - 2\sqrt{3}}{2}$$

$$\Rightarrow \text{ÇK(SS)} = \left\{2 - \sqrt{3}, 2 + \sqrt{3}\right\}$$

ÖRNEK / EXAMPLE 4

$2x^2 - 3x + 10 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$
B) $\{-1, 3\}$
C) $\{1, 2\}$
D) $\{-2, 1\}$
E) $\{0, 1\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$2x^2 - 3x + 10 = 0 \Rightarrow \Delta = b^2 - 4ac$$
$$= (-3)^2 - 4 \cdot 2 \cdot 10$$
$$= 9 - 80 = -72$$

$$\Delta = -72 < 0 \Rightarrow \text{ÇK(SS)} = \varnothing$$

PEKİŞTİRME / REINFORCE

$x^2 + x - 1 = 0 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : $(-1 \mp \sqrt{5}) / 2$

PEKİŞTİRME / REINFORCE

$x^2 - x + 1 = 0 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : $\varnothing$

3. İKİNCİ DERECEDEN DENKLEME DÖNÜŞTÜRÜLEBİLEN DENKLEMLER EQUATIONS THAT CAN BE CONVERTED INTO QUADRATIC EQUATIONS

A. $f(x) \cdot g(x) = 0 \Rightarrow f(x) = 0$ veya (or) $g(x) = 0$

ÖRNEK / EXAMPLE 5

$x^3 - 2x = 0 \Rightarrow$ ÇK(SS) = ?

A) $\{\sqrt{2}\}$

B) $\{0\}$

C) $\{-\sqrt{2}, 0, \sqrt{2}\}$

D) $\{-\sqrt{2}\}$

E) $\{-\sqrt{2}, \sqrt{2}\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x^3 - 2x = 0$

$x(x^2 - 2) = 0$

$x(x - \sqrt{2}) \cdot (x + \sqrt{2}) = 0$

$\Rightarrow$ ÇK(SS) = $\{-\sqrt{2}, 0, \sqrt{2}\}$

B. Değişken Değiştirerek Çözülebilen Denklemler Equations That Can Be Solved By Changing the Variable

ÖRNEK / EXAMPLE 6

$x^6 - 9x^3 + 8 = 0 \Rightarrow$ ÇK(SS) = ?

A) $\{-1, 1\}$

B) $\{-2, -1\}$

C) $\{1, 2\}$

D) $\{0, 3\}$

E) $\{-3, 1\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x^6 - 9x^3 + 8 = 0$

$x^3 = t \Rightarrow x^6 - 9x^3 + 8 = t^2 - 9t + 8$

$0 = (t - 8) \cdot (t - 1)$

$$\left.\begin{matrix} t = 8 \\ t = 1 \end{matrix}\right\} \Rightarrow \begin{matrix} x^3 = 8 \\ x^3 = 1 \end{matrix}$$

$\Rightarrow$ ÇK(SS) = $\{1, 2\}$

PEKİŞTİRME / REINFORCE

$x^{\frac{3}{2}} - x^{\frac{3}{4}} - 6 = 0$

YANIT / ANSWER : $3\sqrt[3]{3}$

ÖRNEK / EXAMPLE 7

$(x^2-5x)^2+10(x^2-5x)+24=0 \Rightarrow \sum x = ?$

A) 2 B) 5 C) 7 D) 8 E) 10

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$(x^2-5x)^2+10(x^2-5x)+24=0$

$x^2-5x=t \Rightarrow t^2+10t+24=0$

$(t+6)\cdot(t+4)=0$

$t_1=-6,\quad t_2=-4$

$x^2-5x=-6 \Rightarrow x^2-5x+6=0$

$x_1=3,\ x_2=2$

$x^2-5x=-4 \Rightarrow x^2-5x+4=0$

$x_3=1,\ x_4=4$

$\Rightarrow \sum x = 3+2+1+4=10$

C. Köklü Denklemler / Equations Involving Radicals

ÖRNEK / EXAMPLE 8

$x-4\sqrt{x+1}+4=0 \Rightarrow$ ÇK(SS) = ?

A) {1 , 4} B) {0 , 6} C) {8, 9}

D) {0 , 8} E) { – 2, 0}

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$x-4\sqrt{x+1}+4=0$

$x+4=4\sqrt{x+1}$

$x^2+8x+16=16(x+1)$

$x^2-8x=0$

$x(x-8)=0$

$\Rightarrow$ ÇK(SS) = {0, 8}

SONUÇLAR / CONCLUSIONS

1. $\frac{1}{x_1}+\frac{1}{x_2}=\frac{-b}{c}$
2. $x_1^2x_2+x_2^2x_1=x_1\cdot x_2(x_1+x_2)$

   $=\frac{c}{a}\cdot\frac{-b}{a}=\frac{-bc}{a^2}$
3. $x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2$

   $=\frac{b^2}{a^2}-\frac{2c}{a}$

PEKİŞTİRME / REINFORCE

$x^2-3x+t=0$,

$x_1\cdot x_2=5 \Rightarrow t=?$

YANIT / ANSWER : 5

KÖKLER İLE KATSAYILAR ARASINDAKİ BAĞINTILAR RELATIONSHIPS BETWEEN ROOTS and COEFFICIENTS

$ax^2+bx+c=0$ denkleminde $\Delta>0$ ise,

If $\Delta>0$ in the equation $ax^2+bx+c=0$,

$$x_1=\frac{-b+\sqrt{\Delta}}{2a}\ ,\ x_2=\frac{-b-\sqrt{\Delta}}{2a}$$

1. $x_1+x_2=\frac{-b}{a}$
2. $x_1\cdot x_2=\frac{c}{a}$
3. $|x_1-x_2|=\frac{\sqrt{\Delta}}{|a|}$

ÖRNEK / EXAMPLE 9

$2x^2-6x+k=0$,

$x_1=2x_2 \Rightarrow$ **k = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$2x^2-6x+k=0 \Rightarrow x_1+x_2=-\frac{b}{a}=\frac{6}{2}=3$$

$x_1+x_2=3$ [Verilen eşitlik / *Given equality*]

$2x_2+x_2=3$ [x_1 yerine $2x_2$ yaz. / *Substitute $2x_2$ for x_1.*]

$x_2=1$ [Sonuç / *Conclusion*]

$x_1=2$ [Sonuç / *Conclusion*]

$$x_1\cdot x_2=\frac{c}{a}=\frac{a}{2}$$

$$1\cdot 2=\frac{a}{2} \Rightarrow a=4$$

ÖRNEK / EXAMPLE 10

$3x^2 - 7x + 5 = 0 \Rightarrow \frac{1}{x_1} + \frac{1}{x_2} = ?$

A) $\frac{3}{5}$ B) $\frac{7}{5}$ C) 1 D) $\frac{5}{3}$ E) $\frac{7}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{1}{x_1} + \frac{1}{x_2} = -\frac{b}{c}$$

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{7}{5}$$

ÖRNEK / EXAMPLE 11

$x^2 + 3x - 2 = 0 \Rightarrow x_1 \cdot x_2^2 + x_1^2 \cdot x_2 = ?$

A) 2 B) 4 C) 5 D) 6 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$x_1 \cdot x_2^2 + x_1^2 \cdot x_2 = x_1 \cdot x_2(x_1 + x_2)$$

$$x_1 \cdot x_2^2 + x_1^2 \cdot x_2 = \frac{c}{a} \cdot \left(-\frac{b}{a}\right)$$

$$= -\frac{2}{1} \cdot \left(-\frac{3}{1}\right)$$

$$= 6$$

Düşünün / Think about it

$x^2 + (m + 4)x + (m - 1) = 0,$

$ÇK(SS) = \{x_1, x_2\},$

$x_1^2 + x_2^2 = \min \Rightarrow m = ?$

Yanıt / Answer

–3

NOT / NOTE

Kökleri x_1 ve x_2 olan ikinci dereceden denklem genel olarak aşağıdaki gibi yazılır.

In generally, we can write quadratic equations as follows that the roots x_1 and x_2.

$$(x - x_1)\cdot(x - x_2) = 0$$

KÖKLERİ VERİLEN İKİNCİ DERECEDEN DENKLEMİN YAZILMASI
WRITING QUADRATIC EQUATION THAT GIVEN ROOTS

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad [a \neq 0]$$

$$x_1 + x_2 = -\frac{b}{a}, \quad x_1 \cdot x_2 = \frac{c}{a}$$

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 - (x_1 + x_2)x + (x_1 \cdot x_2) = 0.$$

ÖRNEK / EXAMPLE 12

$x_1 = 1 - \sqrt{3}$, $x_2 = 1 + \sqrt{3}$ $\Rightarrow$ **$ax^2 + bx + c = ?$**

A) $x^2 - 2x - 2 = 0$

B) $x^2 - x - 2 = 0$

C) $x^2 - 2x - 1 = 0$

D) $x^2 - 2x = 0$

E) $x^2 + 2x - 2 = 0$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0$

$x_1 + x_2 = 1 - \sqrt{3} + 1 + \sqrt{3}$

$x_1 + x_2 = 2$

$x_1 \cdot x_2 = (1 - \sqrt{3}) \cdot (1 + \sqrt{3})$

$= 1 + \sqrt{3} - \sqrt{3} - 3$

$= -2$

$\Rightarrow x^2 - 2x - 2 = 0$

ÖRNEK / EXAMPLE 13

YÖS / 2008

$f(x) = 2x^2 + 3x - 1$

$f(x_1) = f(x_2) = 0$

$$\frac{1}{x_1} + \frac{1}{x_2} = ?$$

A) -3 B) $-\frac{3}{2}$ C) $-\frac{1}{2}$ D) 2 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 \cdot x_2}$$

$$= \frac{-b}{c}$$

$$= -\frac{3}{-1} = 3$$

ÖRNEK / EXAMPLE 14

YÖS / 2009

$f(x) = x^2 - ax + 4$

$f(x_1) = f(x_2) = 0$

$$\frac{2}{x_1} + x_2 = 3$$

a = ?

A) -2 B) -1 C) 2 D) 3 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{2}{x_1} + x_2 = \frac{2 + x_1 \cdot x_2}{x_1} = 3$$

$$= \frac{2+4}{x_1} = 3 \Rightarrow x_1 = 2$$

$f(2) = 2^2 - 2a + 4 = 0 \Rightarrow a = 4$

Düşünün / Think about it

$f(x) = x^2 + bx + c,$

$f(\sqrt{7 - 4\sqrt{3}}) = 0 \Rightarrow f(x) = ?$

Yanıt / Answer

$x^2 - 4x + 1$

Düşünün / Think about it

$3x^3 - 6x^2 + 9x + 6,$

$ÇK(SS) = \{x_1, x_2, x_3\}$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 = ?$

Yanıt / Answer

3

ÖRNEK / EXAMPLE 15

YILDIZ TEKNİK ÜNİ. YÖS 2013

$m > 0$

$m - \frac{6}{m} = 1$

$\Rightarrow$ **$m = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$m > 0$

$$m - \frac{6}{m} = 1 \Rightarrow m^2 - m - 6 = 0$$

$$(m-3)(m+2) = 0$$

$$m_1 = 3, \quad m_2 = -2$$

$m > 0 \Rightarrow m = 3$

ÖRNEK / EXAMPLE 16

İSTANBUL ÜNİ. YÖS 2013

$x^2 + (m^2 - 16)x - 2 + m = 0$

$x_1 + x_2 = 0 \Rightarrow$ **$m = ?$**

A) –4 B) –2 C) 0 D) 2 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$x^2 + (m^2 - 16)x - 2 + m = 0$

$x_1 + x_2 = 0$

$-\frac{b}{a} = 0$

$m^2 - 16 = 0$

$(m-4)(m+4) = 0$

$m_1 = 4 \quad m_2 = -4$

$m = 4 \Rightarrow x \notin R$

1. $x^2 - 5x + 8 = 0 \Rightarrow \mathbf{x_1^2 + x_2^2 = ?}$

A) 2

B) 5

C) 9

D) 10

E) 12

2. $2x^2 + mx - 2m + 1 = 0,$

$x_1 + x_2 = 3 \Rightarrow \mathbf{x_1 \cdot x_2 = ?}$

A) $\frac{13}{2}$

B) $\frac{15}{2}$

C) $\frac{17}{2}$

D) $\frac{19}{2}$

E) $\frac{21}{2}$

3. $m \in R^+,$

$x^2 - mx + 4 = 0, \quad \frac{1}{x_1} + \frac{1}{x_2} = \frac{1}{m} \Rightarrow \mathbf{m = ?}$

A) 1

B) 2

C) 4

D) 6

E) 8

4. $x^2 - (m + 5)x + 5m = 0,$

$x_1 = x_2 \Rightarrow \mathbf{m = ?}$

A) 2

B) 5

C) 7

D) 9

E) 10

5. $5^{2x} - 6 \cdot 5^{x+1} + 125 = 0 \Rightarrow \mathbf{\sum x = ?}$

A) –3

B) –1

C) 1

D) 3

E) 5

6. $x^2 + (2m + 1)x + 6 = 0$, ÇK(SS) $= \{x_1, x_2\}$

$x^2 + (5m - 1)x + 20 = 0$, ÇK(SS) $= \{x_1 + 2, x_2 + 2\}$

$\Rightarrow \mathbf{m = ?}$

A) $-\frac{1}{3}$

B) $-\frac{2}{3}$

C) -1

D) $-\frac{4}{3}$

E) $-\frac{5}{3}$

7. $x^2 + (2m-4)x + m - 9 = 0,\ x_1 = -5 \quad \Rightarrow \quad$ **$x_2 = ?$**

A) -2

B) -1

C) 0

D) 1

E) 2

8. $\frac{x-10}{x^2+x} - \frac{x}{x+1} + \frac{4}{x} = 0 \quad \Rightarrow$ **ÇK (SS) = ?**

A) $\{-1, 2\}$

B) $\{1, 2\}$

C) $\{1, 3\}$

D) $\{2, 3\}$

E) $\{-4, -3\}$

9. $\frac{x^2-7x+10}{x-5} = 0 \quad \Rightarrow$ **ÇK (SS) = ?**

A) $\{2, 5\}$

B) $\{5\}$

C) $\{2\}$

D) Ø

E) R

10. $x^2 - x + x^5 - x^4 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-1, 0, 1\}$

B) $\{-1, 0\}$

C) $\{0, 1\}$

D) $\{0\}$

E) $\{1\}$

11. $3^x + \frac{1}{3^x} - \frac{10}{3} = 0 \quad \Rightarrow \quad$ **$x_1 + x_2 = ?$**

A) -2

B) -1

C) 0

D) 1

E) 2

12. $x + 2 + \frac{m+2}{x-2} = 1,\ x_1 \cdot x_2 = 2 \Rightarrow$ **$m = ?$**

A) 2

B) 3

C) 4

D) 5

E) 6

YANITLAR / ANSWERS

1. C	**2. A**	**3. B**	**4. B**	**5. D**	**6. B**	**7. D**	**8. D**
9. C	**10. A**	**11. C**	**12. A**				

1. $x_1^2+x_2^2=(x_1+x_2)^2-2x_1\cdot x_2$

$$=\left(-\frac{b}{a}\right)^2-2\cdot\frac{c}{a}$$

$$=5^2-2\cdot 8$$

$$=9$$

Yanıt / The Answer is C

2. $x_1+x_2=3 \Rightarrow \dfrac{-m}{2}=3$

$$m=-6$$

$$x_1\cdot x_2=\frac{-2m+1}{2}=\frac{-2\cdot(-6)+1}{2}=\frac{13}{2}$$

Yanıt / The Answer is A

3. $\dfrac{1}{x_1}+\dfrac{1}{x_2}=\dfrac{x_1+x_2}{x_1\cdot x_2}=\dfrac{1}{m}$

$$\frac{-\frac{b}{a}}{\frac{c}{a}}=\frac{1}{m}$$

$$\frac{m}{4}=\frac{1}{m}$$

$$m^2=4$$

$m\in R^+ \Rightarrow m=2$

Yanıt / The Answer is B

4. $\left.\begin{array}{l} x^2-(m+5)x+5m=0 \\ x_1=x_2 \end{array}\right\} \Rightarrow \Delta=0$

$$\Delta=b^2-4ac$$

$$\Delta=(m+5)^2-4\cdot1\cdot5m=0$$

$$\Delta=m^2+10m+25-20m=0$$

$$\Delta=m^2-10m+25=0$$

$$(m-5)^2=0 \Rightarrow m=5$$

Yanıt / The Answer is B

5. $5^{2x}-6\cdot5^{x+1}+125=0$

$$5^x=t \Rightarrow t^2-30t+125=0$$

$$(t-5)\cdot(t-25)=0$$

$$t_1=5\ ,\ t_2=25$$

$$5^x=5 \Rightarrow x_1=1$$

$$5^x=25 \Rightarrow x_2=2$$

$$\sum x=1+2=3$$

Yanıt / The Answer is D

6. $x^2+(2m+1)x+6=0 \Rightarrow x_1+x_2=\dfrac{-2m-1}{1}$

$$x^2+(5m-1)x+20=0 \Rightarrow x_1+2+x_2+2=\frac{-5m+1}{1}$$

$$\frac{-2m-1}{1}+4=\frac{-5m+1}{1}$$

$$4=1-5m+2m+1$$

$$2=-3m$$

$$m=-\frac{2}{3}$$

Yanıt / The Answer is B

7. $x^2 + (2m-4) \cdot x + m - 9 = 0$

$x_1 = -5 \Rightarrow (-5)^2 + (2m-4) \cdot (-5) + m - 9 = 0$

$25 - 10m + 20 + m - 9 = 0$

$-9m + 36 = 0$

$-9m = -36$

$m = 4$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5$ veya (or) $x = 1$

Yanıt / The Answer is D

---

8. $$\frac{x-10}{x^2+x} - \underset{(x)}{\frac{x}{x+1}} + \underset{(x+1)}{\frac{4}{x}} = 0$$

$$\frac{x-10-x^2+4x+4}{x^2+x} = 0$$

$x - 10 - x^2 + 4x + 4 = 0$

$-x^2 + 5x - 6 = 0$

$x^2 - 5x + 6 = 0$

$(x-2)\cdot(x-3) = 0$

$x_1 = 2$, $x_1 = 3$

ÇK(SS) = {2 , 3}

Yanıt / The Answer is D

---

9. $$\frac{x^2-7x+10}{x-5} = 0$$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x - 5 = 0$ veya (or) $x - 2 = 0$

$x = 5$ $\qquad$ $x = 2$

$\Rightarrow$ ÇK(SS) = {2}

Yanıt / The Answer is C

---

10. $x^2 - x + x^5 - x^4 = 0$

$x(x-1) + x^4(x-1) = 0$

$(x-1)\cdot x(1 + x^3) = 0$

$x - 1 = 0$ $\qquad$ $x = 0$ $\qquad$ $1 + x^3 = 0$

$x = 1$ $\qquad$ $x^3 = -1$

$x = -1$

ÇK(SS) = {–1, 0, 1}

Yanıt / The Answer is A

---

11. $$3^x + \frac{1}{3^x} - \frac{10}{3} = 0$$

$$3^x = t \Rightarrow \underset{(3t)}{\frac{t}{1}} + \underset{(3)}{\frac{1}{t}} - \underset{(t)}{\frac{10}{3}} = 0$$

$$\frac{3t^2+3-10t}{3t} = 0$$

$3t^2 - 10t + 3 = 0$

$(3t-1)\cdot(t-3) = 0$

$3t - 1 = 0$ veya (or) $t - 3 = 0$

$t = \frac{1}{3}$ $\qquad$ $t = 3$

$3^x = \frac{1}{3} = 3^{-1}$ $\qquad$ $3^x = 3$

$x = -1$ $\qquad$ $x = 1$

$x_1 + x_2 = (-1) + 1 = 0$

Yanıt / The Answer is C

---

12. $$\underset{(x-2)}{\frac{x+2}{1}} + \frac{m+2}{x-2} = 1$$

$$\frac{x^2-4+m+2}{x-2} = 1$$

$$\frac{x^2+m-2}{x-2} = 1$$

$x^2 + m - 2 = x - 2$

$x^2 - x + m = 0$

$x_1 \cdot x_2 = \frac{c}{a}$

$m = 2$

Yanıt / The Answer is A

1. $x^2 - 6x + a = 0,$

$2x_1 - x_2 = 9 \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **5**

2. $\left(\frac{x-2}{2x-3}\right)^2 - \left(\frac{2x-3}{x-2}\right)^{-1} = 12 \Rightarrow \sum x = ?$

Yanıt / The Answer is: **3**

3. $x^2 + 2x - 2m + 1 = 0,$

$2x_1 + x_2 = 1 \Rightarrow \mathbf{m = ?}$

Yanıt / The Answer is: **8**

4. $\sqrt{x+2} = x - 4 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: **{2, 7}**

5. $x^2 - 3x - 1 = 0 \Rightarrow \frac{1}{x_1 - 2} + \frac{1}{x_2 - 2} = ?$

Yanıt / The Answer is: $\frac{1}{3}$

6. $x^2 - 3x - 2 = 0 \Rightarrow (x_1 - 3) \cdot (x_2 - 3) = ?$

Yanıt / The Answer is: **–2**

7. $x^2 - 3mx + (m + 2) = 0,$

$\frac{1}{x_1} + \frac{1}{x_2} = \frac{3}{2} \Rightarrow$ **m = ?**

Yanıt / The Answer is: **2**

8. $\left(x + \frac{1}{x}\right)^2 - 4\left(x + \frac{1}{x}\right) + 4 = 0 \Rightarrow$ **x = ?**

Yanıt / The Answer is: **1**

9. $x^2 + 3x + 5 = 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: ∅

10. $x^2 + 2x + m - 2 = 0,$

$2x_1 - x_2 = 5 \Rightarrow$ **m = ?**

Yanıt / The Answer is: **–1**

11. $x^2 - 16x + 2m + 6 = 0$

$x_1 = 3x_2 \Rightarrow$ **m = ?**

Yanıt / The Answer is: **21**

12. $(x^2 - 2x)^2 - 23(x^2 - 2x) + 120 = 0 \Rightarrow$ **$\sum$x = ?**

Yanıt / The Answer is: **4**

1. $\sqrt{x-5} + 3 = x - 4 \Rightarrow$ **ÇK(SS) = ?**

A) {6, 9} B) {6} C) {9}

D) ∅ E) R

2. $x^2 - 2mx + m^2 - 2m + 5 = 0$,

ÇK(SS) = ∅,

$m \in Z \Rightarrow$ **max(m) = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

3. $x^2 - mx + m + 4 = 0$,

$\frac{1}{x_1} + \frac{1}{x_2} = 3 \Rightarrow$ **m = ?**

A) 4 B) 2 C) –1

D) –4 E) –6

4. $2x^2 + (m + 1)x - 5 = 0$

$x_1(x_2 - 3) - x_2(3 - x_1) = 7 \Rightarrow$ **m = ?**

A) 10 B) 9 C) 8

D) 7 E) 6

5. $3x^2 - (3k - 6)x + 2k + 1 = 0$,

$x_1 < x_2$,

$|x_1| = x_2 \Rightarrow$ **k = ?**

A) 2 B) 3 C) 5

D) 6 E) 9

6. $2x^2 - mx + 6 = 0$,

$x_1 \cdot x_2^2 = 18 \Rightarrow$ **m = ?**

A) 5 B) 8 C) 10

D) 11 E) 13

7. $m \in R^+$,

$x^2 + mx - 5 = 0$,

$\frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{26}{25} \Rightarrow$ **$m = ?$**

A) 0 B) 2 C) 4 D) 5 E) 7

8. $mx^2 - 3mx + 9 = 0$,

$x_1 = x_2 \Rightarrow$ **$m = ?$**

A) –3 B) –1 C) 2 D) 4 E) 7

9. $(x^2 - x)^2 - 22(x^2 - x) + 40 = 0 \Rightarrow$ **$\sum x = ?$**

A) –10 B) –8 C) –5 D) –1 E) 2

10. $9^x - 4 \cdot 3^{x+1} + 27 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {–1, –2} B) {–1, 2} C) {1, 2} D) {1, 3} E) {2, 3}

11. $(m - 2)x^2 + 3mx - 4 = 0$, $x_1 = x_2 \Rightarrow$ **$\sum m = ?$**

A) -1 B) $-\frac{10}{9}$ C) $-\frac{14}{9}$ D) $-\frac{16}{9}$ E) $-\frac{20}{9}$

12. $x^2 - 7x + 5 = 0 \Rightarrow$ **$x_1^3 + x_2^3 = ?$**

A) 230 B) 238 C) 242 D) 248 E) 250

YANITLAR / ANSWERS

1. C 2. B 3. E 4. D 5. A 6. E 7. C 8. D
9. E 10. C 11. D 12. B

1. $2x^2 + x - 1 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{1}{2}, 1\right\}$ B) $\left\{-1, \frac{1}{2}\right\}$ C) $\{-2, -1\}$

D) $\{-1, 1\}$ E) $\{-1, 0\}$

2. $\frac{x-1}{x-2} - \frac{8}{3} = \frac{x-2}{x-1} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{5}{4}, \frac{5}{2}\right\}$ B) $\left\{\frac{1}{2}, \frac{2}{3}\right\}$ C) $\left\{-\frac{2}{3}, \frac{5}{2}\right\}$

D) $\left\{-\frac{1}{3}, 2\right\}$ E) $\{-1, 2\}$

3. $x^{-2} - 3 \cdot x^{-1} + 2 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{1}{3}, 1\right\}$ B) $\left\{-1, \frac{1}{2}\right\}$ C) $\left\{-2, \frac{1}{3}\right\}$

D) $\left\{\frac{1}{3}, \frac{1}{2}\right\}$ E) $\left\{\frac{1}{2}, 1\right\}$

4. $\sqrt{x+6} - x = 4 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-5\}$ B) $\{-2\}$ C) $\{-5, -2\}$

D) $\{-2, -3\}$ E) $\{-1, 0\}$

5. $\sqrt{x + \sqrt{x-2}} = 2 \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$ B) $\{2\}$ C) $\{3\}$

D) $\{3, 6\}$ E) $\{2, 3\}$

6. $4^{x^2+x-6} = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-2, 3\}$ B) $\{-1, 4\}$ C) $\{-2, 4\}$

D) $\{-3, 2\}$ E) $\{-3, 4\}$

7. $3^{x^2-x} = 729 \Rightarrow$ **ÇK(SS) = ?**

A) {–2, 3} B) {2, 3} C) {–1, 4}

D) {–3, 1} E) {2, 5}

8. $2 \cdot 3^{2x} - 3 \cdot 3^x - 9 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) ∅ B) {1} C) {3}

D) {4} E) {5}

9. $x^3 - 3x^2 - 4x = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {–1, 0, 3} B) {–1, 0, 4} C) {2, 4, 5}

D) {–2, 1, 2} E) {–3, 0, 2}

10. $x^2 + \frac{1}{x-5} = 4x - \frac{1}{5-x} \Rightarrow$ **$\sum x$ = ?**

A) 2 B) 3 C) 4

D) 5 E) 6

11. $\sqrt{2x+5} - 1 = x \Rightarrow$ **x = ?**

A) 2 B) 4 C) 6

D) 8 E) 10

12. $6x^2 - xy - y^2 = 50$,

$3x + y = 10 \Rightarrow$ **x = ?**

A) –1 B) 0 C) 1

D) 2 E) 3

YANITLAR / ANSWERS

1. B	2. A	3. E	4. B	5. C	6. D	7. A	8. B
9. B	10. C	11. A	12. E				

1. $m \in R^+$,

$mx^2 - 4mx + 8 = 0$,

$x_1 = x_2 \Rightarrow \mathbf{m = ?}$

A) 2

B) 3

C) 4

D) 5

E) 6

2. $x^2 + bx + c = 0$,

$x_1 = 3 + \sqrt{2} \Rightarrow \mathbf{b + c = ?}$

A) –1

B) 0

C) 1

D) 2

E) 3

3. $x^2 - (m + 1)x + 3 = 0$,

$x_1 = m \Rightarrow \mathbf{x_2 = ?}$

A) –3

B) –1

C) 1

D) 4

E) 6

4. $2x^2 - (m - 4)x - 3m = 0$,

$x_1 \cdot x_2^2 = -\frac{9}{2}m \Rightarrow \mathbf{m = ?}$

A) 1

B) 3

C) 4

D) 5

E) 8

5. $x^2 - 3x - 6 = 0 \Rightarrow \left(\frac{1}{x_1} + x_2\right) \cdot \left(\frac{1}{x_2} + x_1\right) = ?$

A) $-\frac{25}{6}$

B) $-\frac{19}{6}$

C) $-\frac{13}{6}$

D) $-\frac{7}{6}$

E) $-\frac{1}{6}$

6. $x^2 - (m - 3)x - 12 = 0$,

$x_1 = m \Rightarrow \mathbf{m = ?}$

A) –4

B) –2

C) 0

D) 2

E) 4

7. $x^2 - 5x + 2 = 0 \Rightarrow \mathbf{\dfrac{5}{x_1} + \dfrac{x_1 + x_2}{x_2} = ?}$

A) $\dfrac{23}{2}$

B) $\dfrac{25}{2}$

C) $\dfrac{27}{2}$

D) $\dfrac{29}{2}$

E) $\dfrac{31}{2}$

8. $x^2 + (3 - m)x + 2m + 1 = 0,$

$x_1 + x_2 = 5 \Rightarrow \mathbf{x_1 \cdot x_2^2 + x_1^2 \cdot x_2 = ?}$

A) 65

B) 70

C) 75

D) 80

E) 85

9. $4x^2 - 8x - 5m - 2 = 0,$

$2x_1 - x_2 = 7 \Rightarrow \mathbf{m = ?}$

A) 1

B) 2

C) 3

D) 4

E) 5

10. $x^2 + x_1 \cdot x - 4x_1 = 0,$

$ÇK(SS) = \{x_1, x_2\} \Rightarrow \mathbf{x_1 + x_2 = ?}$

A) 1

B) 2

C) –1

D) –2

E) –3

11. $x^2 - (m - 3)x + 2n - 1 = 0,\ ÇK(SS) = \{a, b\}$

$x^2 - (2m + 1)x + n - 5 = 0,\ ÇK(SS) = \{a, b\}$

$\Rightarrow \mathbf{m + n = ?}$

A) –4

B) –6

C) –8

D) –10

E) –12

12. $y = ax^2 + bx + c,$

$ÇK(SS) = \{2 - \sqrt{5}, 2 + \sqrt{5}\} \Rightarrow \mathbf{y = ?}$

A) $x^2 - 4x - 1$

B) $x^2 + 4x - 1$

C) $x^2 - x + 4$

D) $x^2 + 5x - 4$

E) $x^2 + x - 2$

YANITLAR / ANSWERS

1. A	**2. C**	**3. C**	**4. D**	**5. A**	**6. E**	**7. B**	**8. E**
9. B	**10. D**	**11. C**	**12. A**				

İKİNCİ DERECEDEN EŞİTSİZLİKLER
QUADRATIC INEQUALITIES

BÖLÜM 3
CHAPTER

Tanım: $f(x) \geq 0$, $f(x) \leq 0$, $f(x) > 0$, $f(x) < 0$ ifadelerine *eşitsizlik* denir.

Definition: *The expressions* $f(x) \geq 0$, $f(x) \leq 0$, $f(x) > 0$, $f(x) < 0$ *are called inequalities.*

1. $f(x) = ax + b$

x	$-\infty$		$-\frac{b}{a}$		$+\infty$
ax + b		a' nın işaretinin tersi *Opposite sign with a*	0	a' nın işareti ile aynı *Same sign with a*	

ÖRNEK / EXAMPLE 1

$2x - 6 > 0 \Rightarrow$ **ÇK(SS) = ?**

A) $x < -6$
B) $x < -3$
C) $x < 3$
D) $x > 3$
E) $x > -3$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$2x - 6 = 0 \Rightarrow x = 3$

x	$-\infty$		3		$+\infty$
2x − 6		−	○	+	

ÇK (SS) $= \{x \mid x > 3, x \in R\}$

ÖRNEK / EXAMPLE 2

$3x - 5 \leq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left(-\infty, \frac{5}{3}\right)$
B) $\left(-\infty, \frac{5}{3}\right]$
C) $\left(-\frac{5}{3}, \infty\right)$
D) $\left[+\frac{5}{3}, \infty\right)$
E) $\left[-\frac{5}{3}, \frac{5}{3}\right]$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$3x - 5 = 0 \Rightarrow x = \frac{5}{3}$

x	$-\infty$		$\frac{5}{3}$		$+\infty$
3x − 5		−	●	+	

$ÇK(SS) = \left(-\infty, \frac{5}{3}\right]$

AÇIKLAMA / EXPLANATION

$\{x \mid m \leq x \leq n;\ m, n \in R\} = [m, n]$

$\{x \mid m \leq x < n;\ m, n \in R\} = [m, n)$

$\{x \mid m < x \leq n;\ m, n \in R\} = (m, n]$

$\{x \mid m < x < n;\ m, n \in R\} = (m, n)$

PEKİŞTİRME / REINFORCE

$x^2 - x - 6 \leq 0 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : [−2, 3]

2. $f(x) = ax^2 + bx + c$

i. $\Delta > 0 \Rightarrow$

x	$-\infty$	x_1		x_2		$+\infty$
$ax^2 + bx+c$	a' nın işareti ile aynı *Same sign with a*	○	a' nın işaretinin tersi *Opposite sign with a*	○	a' nın işareti ile aynı *Same sign with a*	

ÖRNEK / EXAMPLE 3

$x^2 - 3x + 2 > 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -1) \cup (2, \infty)$
B) $(-\infty, 1) \cup (3, \infty)$
C) $(-\infty, 1) \cup (2, \infty)$
D) $(-\infty, 2) \cup (3, \infty)$
E) $(-\infty, -2) \cup (3, \infty)$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x - 2 = 0 \quad , \quad x - 1 = 0$

$x = 2 \qquad x = 1$

x	$-\infty$	1		2		$+\infty$
$x^2 - 3x + 2$	$+$	○	$-$	○	$+$	

ÇK(SS) = $(-\infty, 1) \cup (2, \infty)$

ii. $\Delta = 0 \Rightarrow$

x	$-\infty$	$-\frac{b}{2a}$		$+\infty$
$ax^2 + bx+c$	a' nın işareti ile aynı *Same sign with a*	○	a' nın işareti ile aynı *Same sign with a*	

ÖRNEK / EXAMPLE 4

$x^2 - 8x + 16 \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) R
B) $(-\infty, 0]$
C) $(-\infty, 4]$
D) $[4, 0)$
E) $(0, \infty)$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$x^2 - 8x + 16 = 0$

$(x - 4) \cdot (x - 4) = 0$

$\Rightarrow x_1 = x_2 = 4$

x		4	
$x^2 - 8x + 16$	$+$	●	$+$

$\Rightarrow$ ÇK(SS) = R

iii. $\Delta < 0 \Rightarrow$

x	$-\infty$		$+\infty$
$ax^2 + bx + c$		a' nın işareti ile aynı *Same sign with a*	

NOT / NOTE

$\forall x \in R,$

✓ $a > 0,\ \Delta < 0 \Rightarrow ax^2 + bx + c > 0$

✓ $a < 0,\ \Delta < 0 \Rightarrow ax^2 + bx + c < 0$

ÖRNEK / EXAMPLE 5

$\forall x \in R,\ f(x) < 0,$

$f(x) = ax^2 - (a+1)x + a + 1 < 0 \Rightarrow$ **$a \in$?**

A) $0 < a < \frac{1}{8}$ B) $0 < a < \frac{1}{3}$ C) $a < -1$

D) $a > -1$ E) $-1 < a < 2$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$f(x) = ax^2 - (a+1)x + a + 1 < 0$

$\forall x \in R,\ f(x) < 0 \Rightarrow a < 0,\ \Delta < 0$

$f(x) = ax^2 - (a+1)x + a + 1 \Rightarrow \Delta = (a+1)^2 - 4 \cdot a(a+1) < 0$

$= a^2 + 2a + 1 - 4a^2 - 4a < 0 = -3a^2 - 2a + 1 < 0$

$-3a^2 - 2a + 1 = 0$

$(3a - 1) \cdot (a + 1) = 0$

$a = \frac{1}{3}\ ,\quad a = -1$

x	$-\infty$		-1		$\frac{1}{3}$		$+\infty$
$-3a^2 - 2a + 1$		$-$	○	$+$	○	$-$	

$\Rightarrow a \in (-\infty, -1)$

ÖRNEK / EXAMPLE 6

$\forall x \in R,\ m \in Z,$

$x^2 + (m-2)x + 9 > 0 \Rightarrow$ **min(m) = ?**

A) 2 B) 1 C) -1 D) -2 E) -3

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x^2 + (m-2)x + 9 > 0 \Rightarrow \Delta < 0$

$\Delta = b^2 - 4ac = (m-2)^2 - 4 \cdot 1 \cdot 9 < 0$

$m^2 - 4m + 4 - 36 < 0$

$m^2 - 4m - 32 < 0$

$m^2 - 4m - 32 = 0$

$(m - 8) \cdot (m + 4) = 0$

$m = 8\ ,\quad m = -4$

m	$-\infty$		-4		8		$+\infty$
$m^2 - 4m - 32$		$+$	○	$-$	○	$+$	

$\Rightarrow \min(m) = -3$

ÖRNEK / EXAMPLE 7

$\frac{x^2-3x-10}{x^2+2x+4}<0 \Rightarrow ?<x<?$

A) $2<x<4$ B) $0<x<6$ C) $-1<x<5$

D) $-2<x<5$ E) $-3<x<4$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\frac{x^2-3x-10}{x^2+2x+4}<0$

$x^2-3x-10=0$

$(x-5)(x+2)=0 \Rightarrow x_1=5,\ x_2=-2$

$x^2+2x+4=0$

$\Delta=2^2-4\cdot1\cdot4$

$\Delta=-12 \Rightarrow$ ÇK(SS) $=\varnothing$

x	$-\infty$	-2		5	$+\infty$
$x^2-3x-10$	+	○	−	○	+
x^2+2x+4	+		+		+
$\frac{x^2+3x-10}{x^2+2x+4}$	+		−		+

ÖRNEK / EXAMPLE 8

$-x^2+x-5>0 \Rightarrow$ **ÇK (SS) = ?**

A) $-\infty<x<0$ B) R C) $\varnothing$

D) $0<x<\infty$ E) $-\infty<x<-1$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$-x^2+x-5>0$

$-x^2+x-5=0$

$\Delta=1^2-4\cdot(-1)\cdot(-5)$

$=1-20$

$=-19 \Rightarrow$ ÇK (SS) $=\varnothing$

x	$-\infty$		$+\infty$
$-x^2+x-5$	−	−	−

ÖRNEK / EXAMPLE 9

$a < x < b$, $x^2 - x < 6 \Rightarrow$ **(a , b) = ?**

A) $(-3, 4)$ B) $(-2, 1)$ C) $(-2, -1)$

D) $(-2, 3)$ E) $(3, 5)$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$x^2 - x < 6$

$x^2 - x - 6 < 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0$ veya (or) $x + 2 = 0$

$x = 3$ $x = -2$

x	$-\infty$		-2		3		$+\infty$
x^2-x-6		+	○	−	○	+	

ÇK(SS) $= -2 < x < 3 \Rightarrow (a, b) = (-2, 3)$.

ÖRNEK / EXAMPLE 10

$a \le x \le b$, $8 \ge x^2 + 2x \Rightarrow$ **[a , b] = ?**

A) $[1, 4]$ B) $[1, 2]$ C) $[-4, -1]$

D) $[-1, 2]$ E) $[-4, 2]$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$8 \ge x^2 + 2x$

$0 \ge x^2 + 2x - 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0$ veya (or) $x - 2 = 0$

$x = -4$ $x = 2$

x	$-\infty$		-4		2		$+\infty$
x^2+2x-8		+	●	−	●	+	
				$-4 \le x \le 2$			

$[a,b] = [-4, 2]$

EŞİTSİZLİK SİSTEMİ / SYSTEM of INEQUALITY

Birden fazla eşitsizliğin oluşturduğu sisteme eşitsizlik sistemi denir. Sistemi oluşturan eşitsizliklerin çözüm kümelerinin kesişimine ise, eşitsizlik sisteminin çözüm kümesi denir.
The system of more than one inequalities is called a system of inequality. The intersection of the solution sets of the inequalities is called a solution set of inequality system.

ÖRNEK / EXAMPLE 11

$$\left.\begin{matrix} x-3<0 \\ x^2-4>0 \end{matrix}\right\} \Rightarrow ÇK(SS)=?$$

A) $(-\infty, -2] \cup (2, 3)$ B) $(-\infty, -2) \cup (2, 3]$ C) $(-\infty, -2) \cup (2, 3)$
D) $(-\infty, -2) \cup [2, 3]$ E) $[-\infty, -2) \cup (2, 3]$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x-3=0 \Rightarrow x_1=3$

$x^2-4=0 \Rightarrow x_2=2,$

$x_3=-2$

x		−2		2		3	
$x-3$	−		−		−	○	+
x^2-4	+	○	−	○	+		+
ÇK(SS)	▨				▨		

$\Rightarrow ÇK(SS) = (-\infty, -2) \cup (2, 3)$

ÖRNEK / EXAMPLE 12

$x \in Z$,

$$\left.\begin{matrix} \dfrac{x^2+x-6}{|x-3|}<0 \\ \dfrac{x+3}{x^2-x}>0 \end{matrix}\right\} \Rightarrow \sum x = ?$$

A) −1 B) −2 C) −3 D) −4 E) −5

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x^2+x-6=0 \Rightarrow (x+3)\cdot(x-2)=0$

$x=-3\ ,\ x=2$

$|x-3|>0$

$x+3=0 \Rightarrow x=-3$

$x^2-x=0 \Rightarrow x(x-1)=0$

$x=0\ ,\ x=1$

x	−∞		−3		2		+∞
$\frac{x^2+x-6}{\lvert x-3\rvert}$		+		−		+	

x	−∞		−3		0		1		+∞
		−		+		−		+	

$ÇK(SS) = (-3, 0) \cup (1, 2)$

$\sum x = (-2)+(-1) = -3$

1. $3 - 6x \leq 0 \Rightarrow$ **ÇK (SS) = ?**

A) $\left[-\frac{1}{2}, \infty\right)$

B) $\left(\frac{1}{2}, \infty\right)$

C) $\left[\frac{1}{2}, \infty\right)$

D) $\left(-\frac{1}{2}, \infty\right)$

E) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

2. $\frac{x+1}{x^2+4} \geq 0 \Rightarrow$ **? ≤ x ≤ ?**

A) $-1 \leq x < \infty$

B) $-2 \leq x < \infty$

C) $-3 \leq x < \infty$

D) $-4 \leq x < \infty$

E) $-5 \leq x < \infty$

3. $x^2 - 6x + 5 > 0 \Rightarrow$ **? < x < ?**

A) $(-\infty, -2) \cup (3, \infty)$

B) $(-\infty, 1) \cup (3, \infty)$

C) $(-\infty, -2) \cup (6, \infty)$

D) $(-\infty, 1) \cup (5, \infty)$

E) $(-\infty, -1) \cup (4, \infty)$

4. $\frac{-x^2 + 3x + 4}{x^2 + 5} > 0 \Rightarrow$ **? < x < ?**

A) $-5 < x < 1$

B) $-4 < x < 2$

C) $-4 < x < 1$

D) $-1 < x < 5$

E) $-1 < x < 4$

5. $\frac{x+1}{x-1} - \frac{x+3}{x-2} > 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left(-\infty, \frac{1}{3}\right) \cup (1, 2)$

B) $(-\infty, -1) \cup (1, 2)$

C) $(-\infty, 0) \cup (2, 5)$

D) $(-\infty, -3) \cup (1, 5)$

E) $\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{1}{2}, 3\right)$

6. $x \in Z$,

$\frac{|x-3|-4}{x^2-x-12} \leq 0 \Rightarrow$ **$\sum x = ?$**

A) 12

B) 13

C) 14

D) 15

E) 16

7. $\dfrac{(x^2+1)(2-x)}{x^2-4x+4} < 0 \Rightarrow$ **? < x < ?**

A) $5 < x < \infty$

B) $4 < x < \infty$

C) $2 < x < \infty$

D) $1 < x < \infty$

E) $0 < x < \infty$

8. $x \in Z$,

$\dfrac{(x-1)^2}{x^2-6x+5} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) {1, 2, 3, 4}

B) {2, 3, 4}

C) {2, 3, 4, 5}

D) { –1, 1, 2, 3}

E) { – 2, –1, 0}

9. $x \in Z$,

$\dfrac{(x^2+5)\cdot(x-3)}{|3x+1|} < 0 \Rightarrow$ **max(x) = ?**

A) 0

B) 2

C) 4

D) 6

E) 8

10. $\left(\dfrac{2}{5}\right)^{x^2+4} > \left(\dfrac{25}{4}\right)^{2x} \Rightarrow$ **ÇK(SS) = ?**

A) ∅

B) (–2, 2)

C) (–2, 0)

D) (0, 2)

E) R

11. $\left.\begin{array}{l} x^2-3x-4<0 \\ x^2+7x+10<0 \end{array}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) ∅

B) (–∞, –5)

C) (–5, –2)

D) (–2, –1)

E) (–1, 4)

12. $(x^2-1)\cdot(x+4) > 0 \Rightarrow$ **ÇK(SS) = ?**

A) (–4 , – 2) ∪ (2, ∞)

B) (–4, –1) ∪ (1, ∞)

C) (–4, 1) ∪ (2, ∞)

D) (–2, 1) ∪ (2, ∞)

E) (–1, 4) ∪ (5 ,∞)

YANITLAR / ANSWERS

1. C	**2. A**	**3. D**	**4. E**	**5. A**	**6. D**	**7. C**	**8. B**
9. B	**10. A**	**11. A**	**12. B**				

1. $3 - 6x \leq 0$

$3 - 6x = 0$

$6x = 3$

$x = \frac{1}{2}$

x	$-\infty$		$\frac{1}{2}$		$+\infty$
$3-6x$		+	●	−	

$ÇK(SS) = \left\{ x \mid x \geq \frac{1}{2},\ x \in R \right\} = \left[\frac{1}{2}, \infty \right)$

Yanıt / The Answer is C

---

2. $\frac{x+1}{x^2+4} \geq 0$

$x + 1 = 0 \Rightarrow x = -1$

$x^2 + 4 = 0 \Rightarrow x \notin R$

x	$-\infty$		-1		$+\infty$
$x+1$		−	●	+	
x^2+4		+		+	
$\frac{x+1}{x^2+4}$		−		+	

$-1 \leq x < \infty$

Yanıt / The Answer is A

---

3. $x^2 - 6x + 5 > 0$

$x^2 - 6x + 5 = 0$

$(x - 5) \cdot (x - 1) = 0$

$x - 5 = 0$ veya (or) $x - 1 = 0$

$x = 5$ $x = 1$

x	$-\infty$		1		5		$+\infty$
x^2-6x+5		+	○	−	○	+	

$ÇK(SS) = \{ x \mid -\infty < x < 1$ veya (or) $5 < x < \infty,\ x \in R \}$

$= (-\infty, 1) \cup (5, \infty)$

Yanıt / The Answer is D

---

4. $\frac{-x^2+3x+4}{x^2+5} > 0$

$-x^2 + 3x + 4 = 0$

$(-x + 4)(x + 1) = 0$

$-x + 4 = 0$ veya (or) $x + 1 = 0$

$x = 4$ $x = -1$

$x^2 + 5 = 0 \Rightarrow x \notin R$

x	$-\infty$		-1		4		$+\infty$
$-x^2+3x+4$		−	○	+	○	−	
x^2+5		+		+		+	
$\frac{-x^2+3x+4}{x^2+5}$		−		+		−	

$-1 < x < 4$

Yanıt / The Answer is E

---

5. $\frac{x+1}{x-1} - \frac{x+3}{x-2} > 0$

$\frac{x^2-2x+x-2-x^2+x-3x+3}{(x-1)\cdot(x-2)} > 0$

$\frac{-3x+1}{(x-1)\cdot(x-2)} > 0$

$-3x+1=0 \Rightarrow x = \frac{1}{3}$

$x - 1 = 0 \Rightarrow x = 1,\ x - 2 = 0 \Rightarrow x = 2$

x	$-\infty$		$\frac{1}{3}$		1		2		$+\infty$
$-3x+1$		+	○	−		−		−	
$x-1$		−		−	○	+		+	
$x-2$		−		−		−	○	+	
$\frac{-3x+1}{(x-1)(x-2)}$		+		−		+		−	

$ÇK(SS) = \left\{ x \mid -\infty < x < \frac{1}{3} \vee 1 < x < 2,\ x \in R \right\} = \left(-\infty, \frac{1}{3} \right) \cup (1, 2)$

Yanıt / The Answer is A

---

6. $\frac{|x-3|-4}{x^2-x-12} \leq 0$

$|x - 3| - 4 = 0$ $x^2 - x - 12 = 0$

$|x - 3| = 4$ $(x - 4)\cdot(x + 3) = 0$

$x - 3 = 4$ $x - 3 = -4$ $x = 4,\ x = -3$

$x = 7$ $x = -1$

x		-3		-1		4		7	
$\frac{|x-3|-9}{x^2-x-12}$	+	○	−	●	+	○	−	●	+

$\sum x = -2 + (-1) + 5 + 6 + 7 = 15$

Yanıt / The Answer is D

7. $x^2+1=0 \Rightarrow x \notin R$

$2-x=0 \Rightarrow x=2$

$x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow x=2$

x	$-\infty$		2		$+\infty$
x^2+1		+		+	
$2-x$		+		−	
x^2-4x+4		+		+	
$\frac{(x^2+1)(2-x)}{x^2-4x+4}$		+		−	

$2 < x < \infty$

Yanıt / The Answer is C

---

8. $(x-1)^2=0 \Rightarrow x=1$

$x^2-6x+5=0$

$(x-5)(x-1)=0$

$x=5$ veya (or) $x=1$

x	$-\infty$		1		5		$+\infty$
$(x-1)^2$		+		+		+	
x^2-6x+5		+		−		+	
$\frac{(x-1)^2}{x^2-6x+5}$		+		−		+	

$1 < x < 5,\ x \in Z \Rightarrow$ ÇK (SS) $= \{2, 3, 4\}$

Yanıt / The Answer is B

---

9. $\frac{(x^2+5)\cdot(x-3)}{|3x+1|} < 0$

$\forall x \in R \Rightarrow x^2+5>0$

$\forall x \in R \Rightarrow |3x+1|>0$

$x-3<0$

$x<3$

$x \in Z \Rightarrow \max(x)=2$

Yanıt / The Answer is B

---

10. $\left(\frac{2}{5}\right)^{x^2+4} > \left(\frac{25}{4}\right)^{2x}$

$\left(\frac{2}{5}\right)^{x^2+4} > \left(\frac{2}{5}\right)^{-4x} \Rightarrow x^2+4<-4x$

$x^2+4x+4<0$

$x^2+4x+4=0$

$x_1=x_2=-2$

x	$-\infty$		-2		$+\infty$
x^2+4x+4		+		+	

$\Rightarrow$ ÇK(SS) $= \varnothing$

Yanıt / The Answer is A

---

11. $x^2-3x-4=0 \Rightarrow x_1=4,\ x_2=-1$

$x^2+7x+10=0 \Rightarrow x_1=-2,\ x_2=-5$

x	$-\infty$		-5		-2		-1		4		$+\infty$
x^2-3x-4		+		+		+		−		+	
$x^2+7x+10$		+		−		+		+		+	

$\Rightarrow$ ÇK(SS) $= \varnothing$

Yanıt / The Answer is A

---

12. $(x^2-1)\cdot(x+4)>0$

$x^2-1=0$

$(x-1)\cdot(x+1)=0$

$x-1=0$ veya (or) $x+1=0$

$x=1$ $x=-1$

$x+4=0 \Rightarrow x=-4$

x	$-\infty$		-4		-1		1		$+\infty$
x^2-1		+		+		−		+	
$x+4$		−		+		+		+	
$(x^2-1)(x+4)$		−		+		−		+	

ÇK(SS) $= (-4, -1) \cup (1, \infty)$

Yanıt / The Answer is B

1. $x^2 - 5x - 24 < 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: **(–3, 8)**

---

2. $x \in \mathbb{Z}$,

$x^2 - 6x - 16 < 0 \Rightarrow \sum x = ?$

Yanıt / The Answer is: **27**

---

3. $2^{2x} - 3 \cdot 2^{x+2} + 32 \leq 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: **[2, 3]**

---

4. $\dfrac{6-x}{x^2-7x+6} < 0 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: **(1, ∞) – {6}**

---

5. $x \in \mathbb{Z}$,

$$\frac{(2-x)\cdot(1-x)^2}{(x^2-6x+9)\cdot(x-3)} \geq 0 \Rightarrow \sum x = ?$$

Yanıt / The Answer is: **3**

---

6. $x \in \mathbb{Z}$,

$$\frac{x^2-9x+18}{(x+1)^2} < 0 \Rightarrow \sum x = ?$$

Yanıt / The Answer is: **9**

7. $x \in Z$,

$$\frac{(x-3)^2\cdot(2-x)}{(x+1)^3\cdot x^2} \ge 0 \;\Rightarrow\; \boldsymbol{\sum x = ?}$$

Yanıt / The Answer is: **6**

8. $x \in Z$,

$$\frac{(x^3+1)\cdot(x^2-4)}{|x-5|} > 0 \;\Rightarrow\; \mathbf{min(x) = ?}$$

Yanıt / The Answer is: **3**

9. $$\left.\begin{array}{l} \dfrac{x+1}{x-1} < 0 \\ x^2-4x<5 \end{array}\right\} \;\Rightarrow\; \textbf{ÇK(SS) = ?}$$

Yanıt / The Answer is: **(–1, 1)**

10. $x \in Z$,

$$\frac{(x-3)^2\cdot(x+5)}{x-6} < 0 \;\Rightarrow\; \mathbf{min(x) = ?}$$

Yanıt / The Answer is: **–4**

11. $x \in Z$,

$$\left.\begin{array}{l} \dfrac{x-4}{x+5} < 0 \\ x^2-3x-18<0 \end{array}\right\} \;\Rightarrow\; \boldsymbol{\sum x = ?}$$

Yanıt / The Answer is: **3**

12. $x \in Z$,

$$\frac{|x^2-16|\cdot(3^x-27)}{x^2-4x-12} > 0 \;\Rightarrow\; \mathbf{min(x) = ?}$$

Yanıt / The Answer is: **–1**

1. $3^{2x} - 10 \cdot 3^x + 9 \leq 0 \Rightarrow$ **ÇK(SS) = ?**

A) [0, 2]
B) [1, 2]
C) [1, 5]
D) [1, 9]
E) [–1, 4]

2. $\frac{4}{x} < \frac{3}{x+1} \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -1) \cup (0, 4)$
B) $(-\infty, 0) \cup (1, 4)$
C) $(-\infty, -4) \cup (-1, 0)$
D) $(-\infty, 4) \cup (5, \infty)$
E) $(-\infty, -1) \cup (2, 4)$

3. $\frac{6-x}{x-4} > 1 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, 4) \cup (5, 8)$
B) $(-\infty, -4) \cup (5, \infty)$
C) (–4, 5)
D) (4, 5)
E) (–5, 4)

4. $x \in Z$,

$$\frac{x^2 + x - 12}{-x^2 \cdot (4 - x^2)} \geq 0 \Rightarrow \textbf{min(x) = ?}$$

A) –6
B) –5
C) –4
D) –3
E) –2

5. $m \in N$,
$x_1 + x_2 > 0$,
$(m-2)x^2 - 3(m-1)x + 7 = 0 \Rightarrow$ **min(m) = ?**

A) 0
B) 1
C) 2
D) 3
E) 4

6. $x \in Z$,

$$\frac{x^3 - x}{2 - x} \geq 0 \Rightarrow \sum x = ?$$

A) –4
B) –3
C) –2
D) –1
E) 0

7. $x \in Z$,

$\dfrac{x-3}{-x^2-4} < 0 \Rightarrow$ **min(x) = ?**

A) –1

B) 1

C) 2

D) 3

E) 4

8. $x \in Z$,

$\dfrac{x^2+5x}{x^2+8x} < \dfrac{5x+64}{x^2+8x} \Rightarrow$ **min(x) = ?**

A) –2

B) –1

C) 0

D) 1

E) 2

9. $x \in Z^-$,

$\dfrac{x^2-x}{3} \geq 4 \Rightarrow$ **max(x) = ?**

A) –5

B) –3

C) –1

D) 1

E) 4

10. $x \in Z$,

$\dfrac{4}{x-2} + \dfrac{x}{x+2} < \dfrac{8}{x^2-4} \Rightarrow$ **$\sum x$ = ?**

A) 1

B) 5

C) 9

D) 15

E) 18

11. $x \in R^+$,

$\dfrac{(x^2-16)\cdot(x^2-8x+16)}{(x^2+6x+9)\cdot(x^2-9)} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) (–4, 3)

B) (–3, 4)

C) (–3, 3)

D) (3, 4)

E) (4, 5)

12. $\left(\dfrac{1}{2}\right)^{x^2-2x} \leq \dfrac{1}{8} \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -1] \cup [3, \infty)$

B) $(-\infty, -3] \cup [1, \infty)$

C) $(-3, 1]$

D) $[-1, 3]$

E) $[-4, 1)$

YANITLAR / ANSWERS

1. A 2. C 3. D 4. C 5. D 6. E 7. E 8. D
9. B 10. A 11. D 12. A

1. $x \in Z$,

$(4x-2)\cdot(3x-9) \le 0 \Rightarrow$ **min(x) = ?**

A) –5

B) –3

C) –1

D) 1

E) 3

2. $x \in Z$,

$$\frac{-x^2\cdot(x^2-5x+6)}{x^2-4} > 0 \Rightarrow \sum x = ?$$

A) 0

B) 3

C) 4

D) 6

E) 9

3. $x \in Z$,

$$\frac{(x^2-4x-5)\cdot(x+1)^2}{x^2-6x+9} \le 0 \Rightarrow \sum x = ?$$

A) 9

B) 10

C) 11

D) 12

E) 14

4. $x \in Z$,

$$\frac{(2-x)\cdot(x-2)}{x^2-4} \ge 0 \Rightarrow \mathbf{min(x) = ?}$$

A) 7

B) 4

C) 2

D) –1

E) –3

5. $$\frac{(-x+5)^2\cdot(x^4+5)}{x^2-6x+8} < 0 \Rightarrow \mathbf{ÇK(SS) = ?}$$

A) (2, 4)

B) (3, 7)

C) (4, 8)

D) (3, 5)

E) (2, 1)

6. $x \in Z$,

$x^2\cdot(7x-4) < x(x-2) \Rightarrow$ **max(x) = ?**

A) 6

B) 3

C) 2

D) –1

E) –5

7. $x \in Z$,

$\frac{4}{x} < x \Rightarrow$ **min(x) = ?**

A) –2

B) –1

C) 0

D) 1

E) 2

8. $\frac{(16-x^2)\cdot(x^3-1)}{x^2-1} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-4, -1) \cup (4, \infty)$

B) $(-\infty, -4) \cup [1, 4)$

C) $(-4, -1) \cup (1, 4)$

D) $(-4, 1) \cup (4, \infty)$

E) $(-\infty, -4) \cup (4, \infty)$

9. $x \in Z$,

$\frac{4^x\cdot(x^2-2x+5)}{x^2-5x-14} < 0 \Rightarrow$ **$\sum x$ = ?**

A) 15

B) 18

C) 20

D) 24

E) 27

10. $\frac{(x-2)^{2000}\cdot(x+1)^{1999}}{(x-1)^{1998}} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -1)$

B) $(0, 2)$

C) $(1, 2)$

D) $(-\infty, 1)$

E) $(-2, -1)$

11. $x \in Z$,

$\frac{|x-2|\cdot(x^2-4x+3)}{(2^x-4)} > 0 \Rightarrow$ **min(x) = ?**

A) 0

B) 1

C) 2

D) 3

E) 4

12. $\left.\begin{array}{l} x^2-x<12 \\ \frac{x+1}{x-1}<0 \end{array}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $(0, 4)$ B) $(-1, 4)$ C) $(1, 4)$

D) $(-3, -1)$ E) $(-1, 1)$

YANITLAR / ANSWERS

1. D	**2. A**	**3. C**	**4. D**	**5. A**	**6. D**	**7. B**	**8. A**
9. C	**10. A**	**11. E**	**12. E**				

İKİNCİ DERECEDEN FONKSİYONLAR
QUADRATIC FUNCTIONS

BÖLÜM
CHAPTER **4**

Tanım: a, b, c ∈ R ve a ≠ 0 olmak üzere,
$y = f(x) = ax^2 + bx + c$ şeklindeki ikinci dereceden fonksiyonların grafiğine parabol denir.

Definition: *Provided that a, b, c ∈ R and a ≠ 0, the graph of a quadratic function* $y = f(x) = ax^2 + bx + c$ *is called a parabola.*

NOT / NOTE

İkinci dereceden fonksiyonun grafiği, ∪ şeklindeki eğridir.

The graph of a quadratic function is a ∪ –shaped curve.

PARABOL ÇİZİMİ / DRAWING A PARABOLA

$y = f(x) = ax^2 + bx + c$ fonksiyonlarının grafiği çizilirken aşağıdaki sıra izlenir:

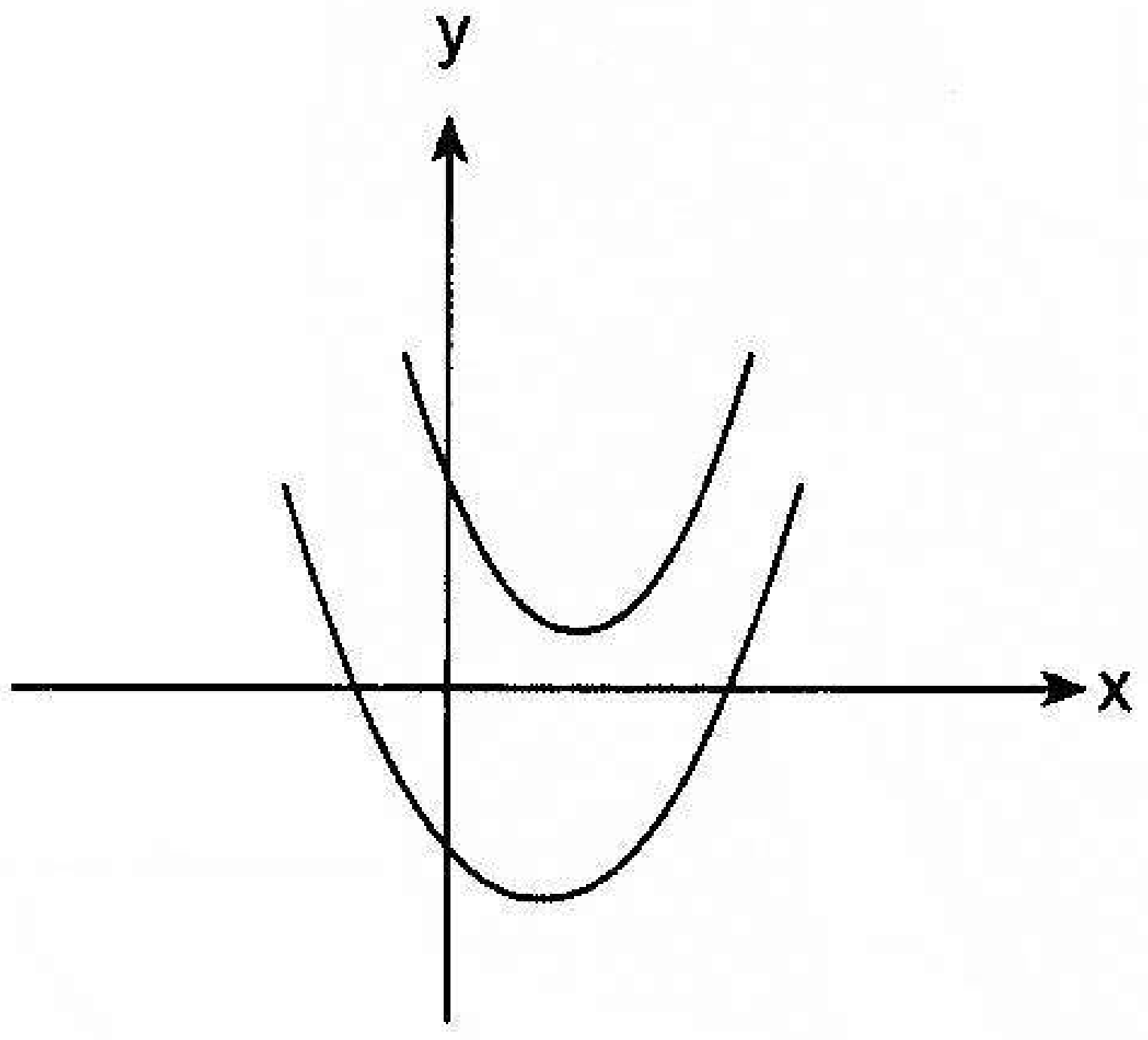
To draw the graph of a quadratic function, follow the steps below:

NOT / NOTE

a, ikinci dereceden fonksiyonun baş katsayısıdır.

a is called leading coefficient of a quadratic function.

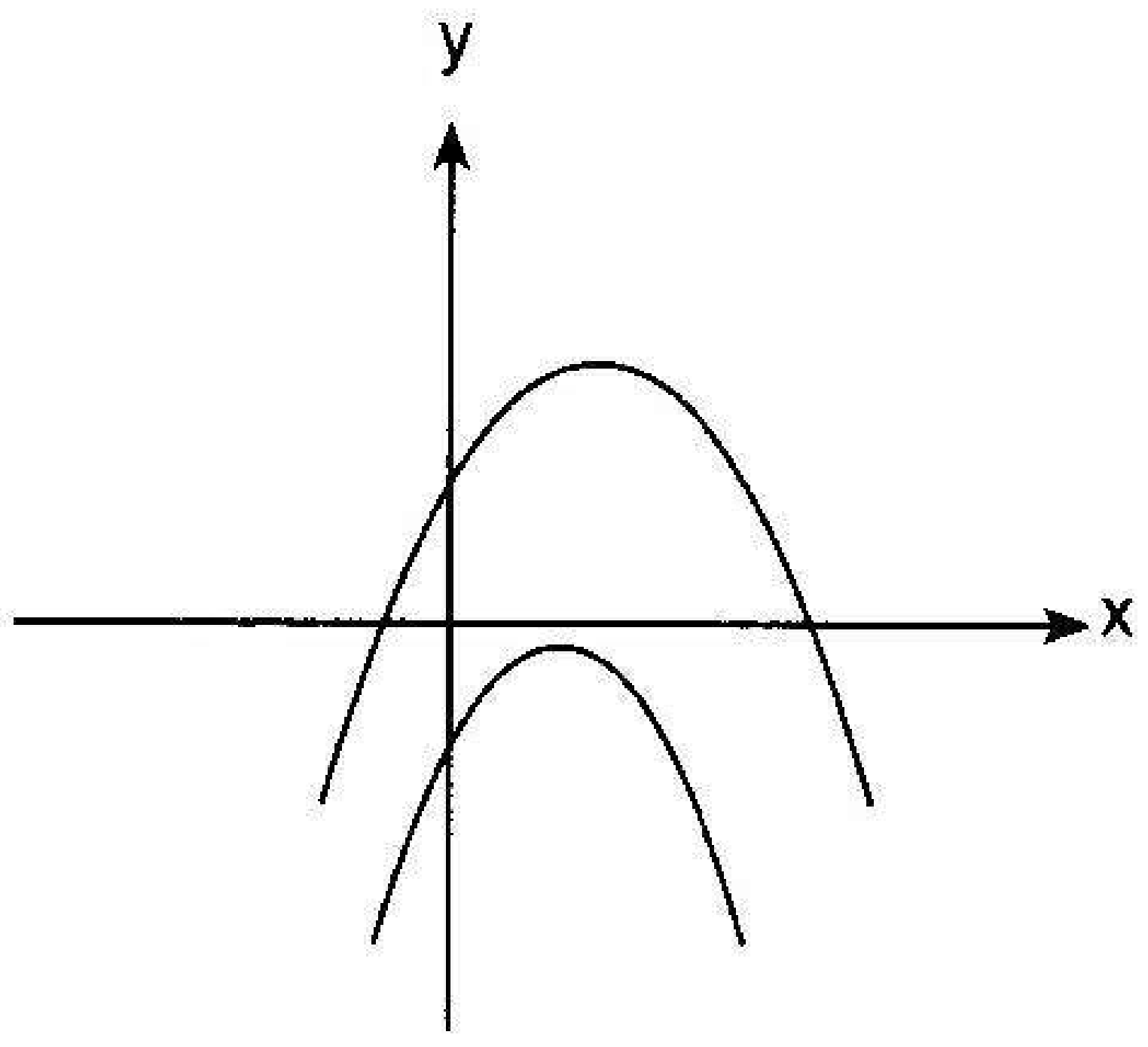
1 **a.**



a > 0 ise grafiğin kolları yukarı doğrudur.

If a > 0, then the curve opens upward.

b.



a < 0 ise grafiğin kolları aşağı doğrudur.

If a < 0, then the curve opens downward.

NOT / NOTE

Tüm paraboller, simetri ekseni olarak adlandırılan doğruya göre simetriktir.

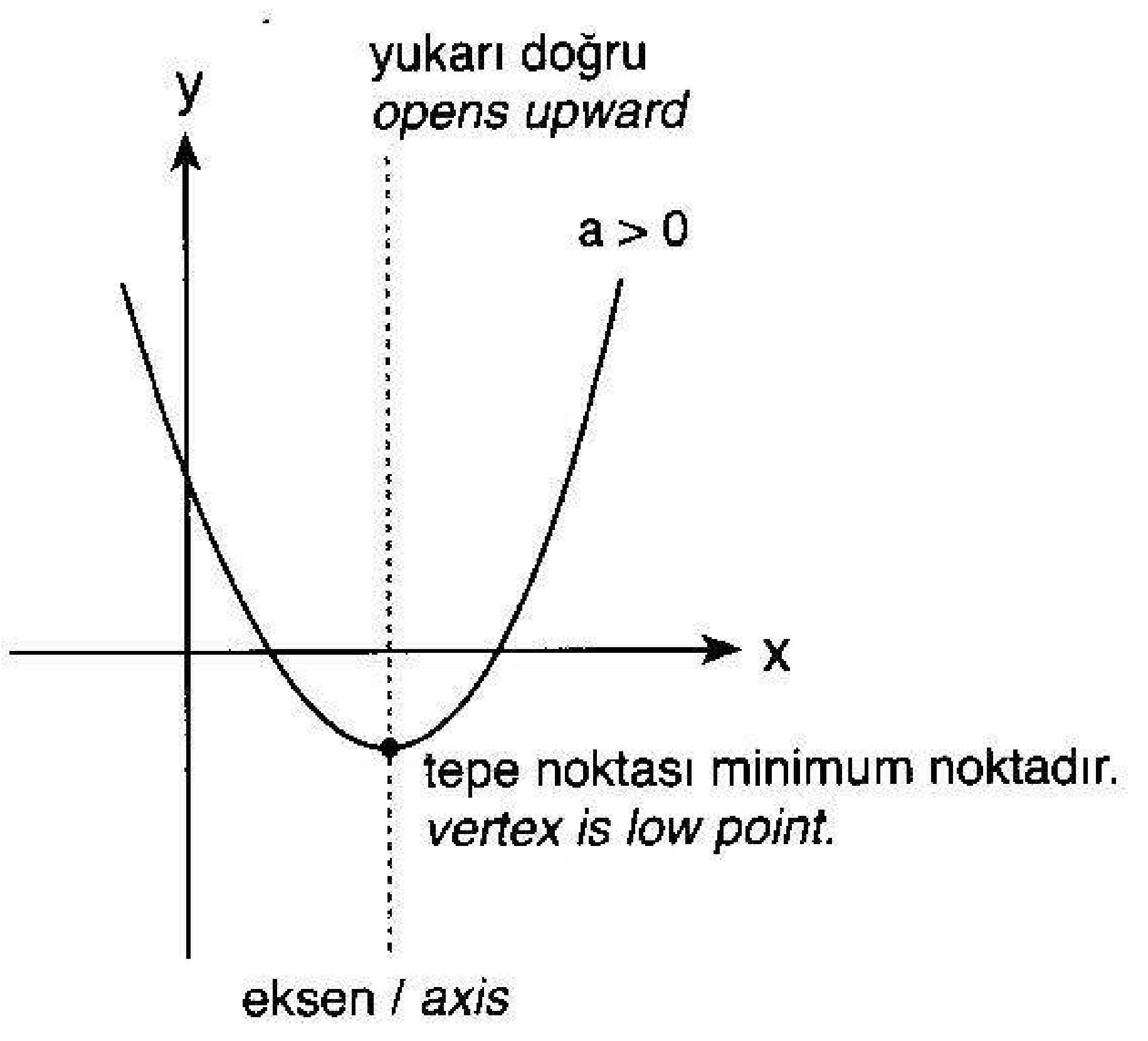
All parabolas are symmetric with respect to a line called the axis of symmetry.

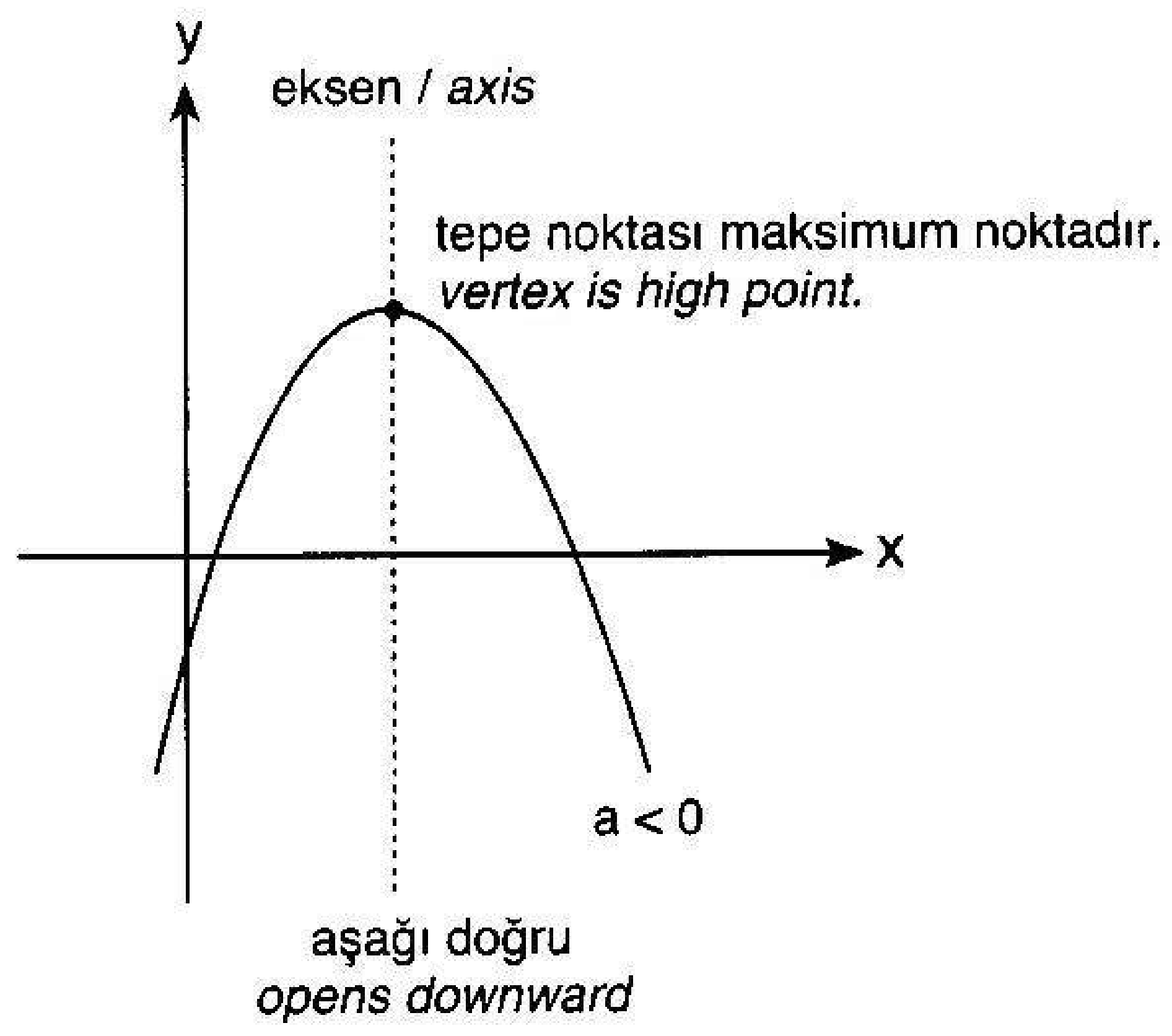
NOT / NOTE

Grafik, $x = -\frac{b}{2a}$ doğrusuna göre simetrik olduğundan $x = -\frac{b}{2a}$ doğrusuna parabolün simetri ekseni denir. k değerine; $a > 0$ ise parabolün en küçük değeri, $a < 0$ ise parabolün en büyük değeri denir.

Since the graph is symmetric with respect to the line $x = -\frac{b}{2a}$, then line $x = -\frac{b}{2a}$ is called symmetry axis. If $a > 0$, then k is called the minimum value of the parabola, and if $a < 0$, then k is called the maximum value of the parabola.

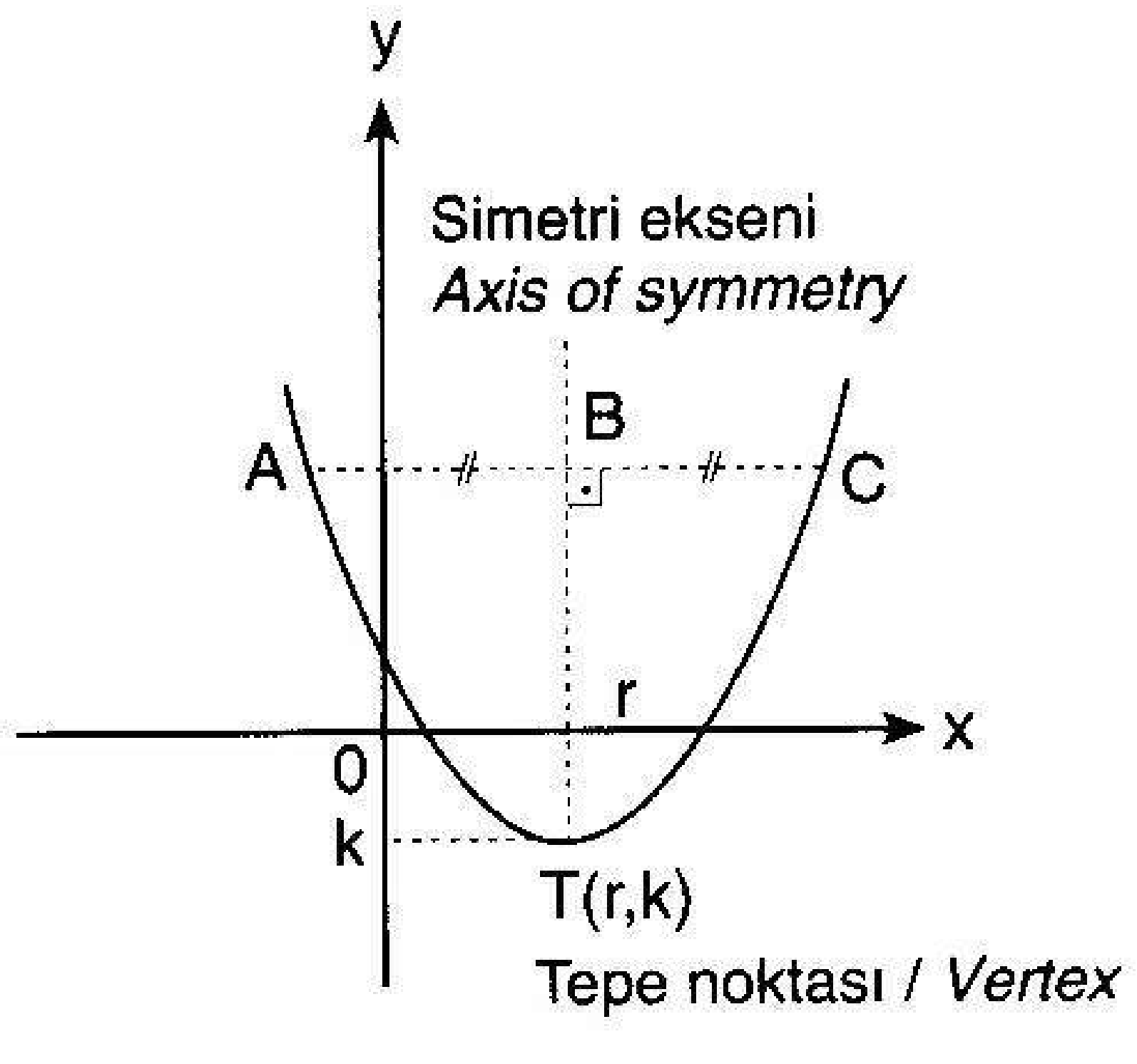
NOT / NOTE


y
yukarı doğru
opens upward
a > 0
x
tepe noktası minimum noktadır.
vertex is low point.
eksen / axis


y
eksen / axis
tepe noktası maksimum noktadır.
vertex is high point.
x
a < 0
aşağı doğru
opens downward

2 Tepe Noktası / *Vertex:*

$$(r,k) = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$$


y
Simetri ekseni
Axis of symmetry
A
B
C
r
x
0
k
T(r,k)
Tepe noktası / Vertex

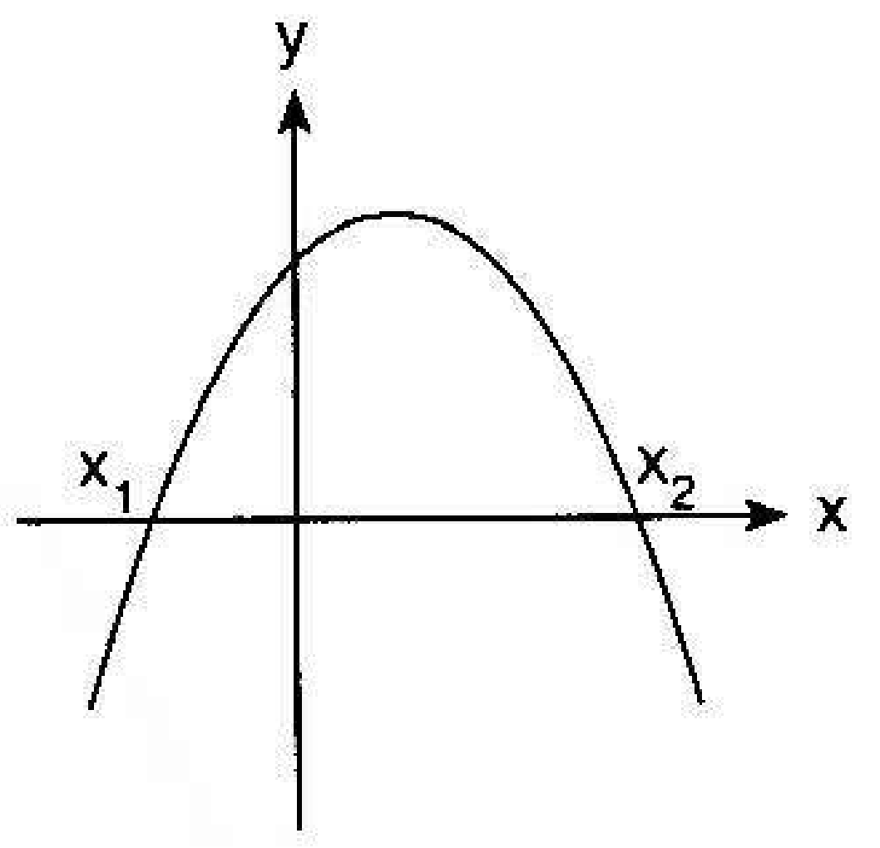
$|AB| = |BC|$

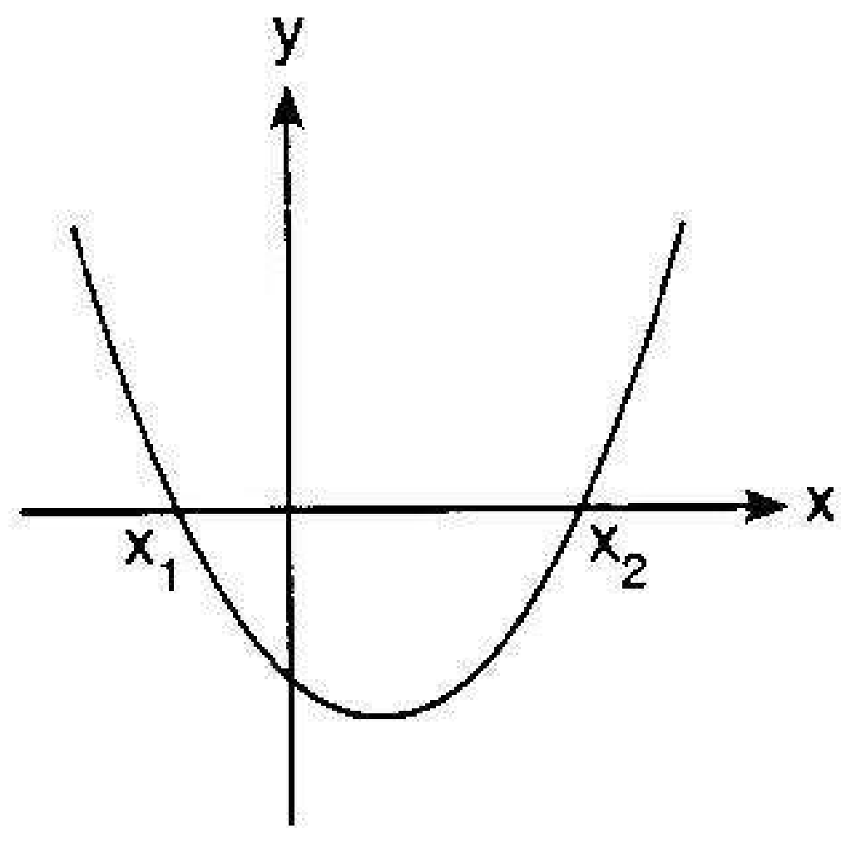
3 Parabolün x Eksenini Kestiği Noktalar / *x – Intercepts of a Parabola*

$y = f(x) = ax^2 + bx + c = 0$

a. $\Delta > 0 \Rightarrow$

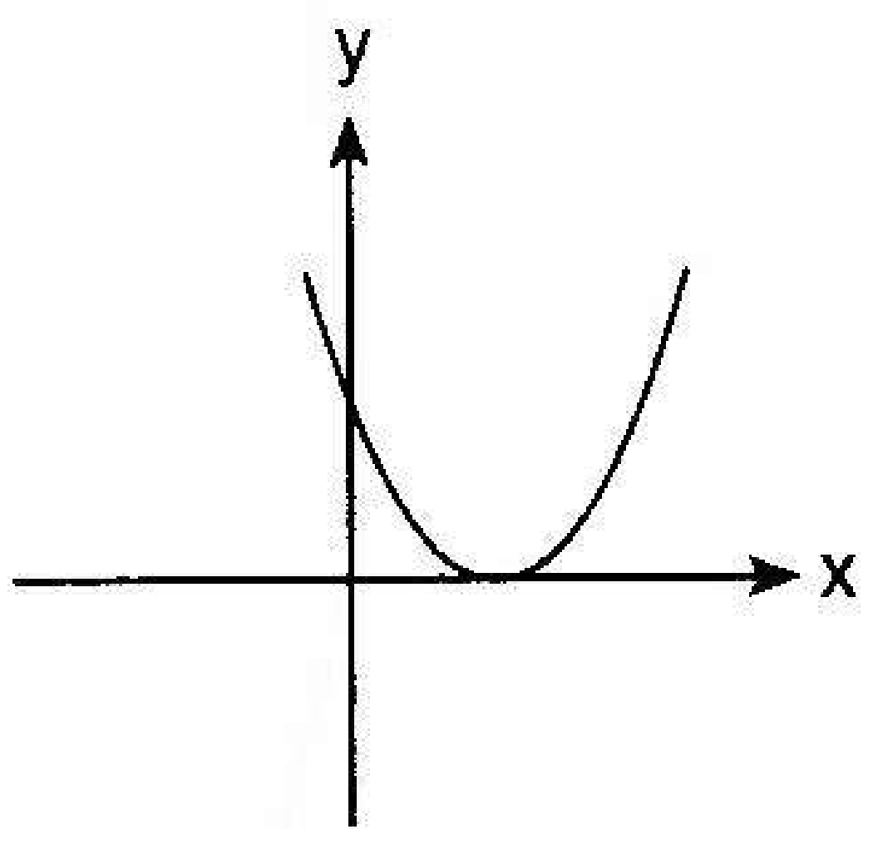
$\text{ÇK(SS)} = \{x_1, x_2\}$

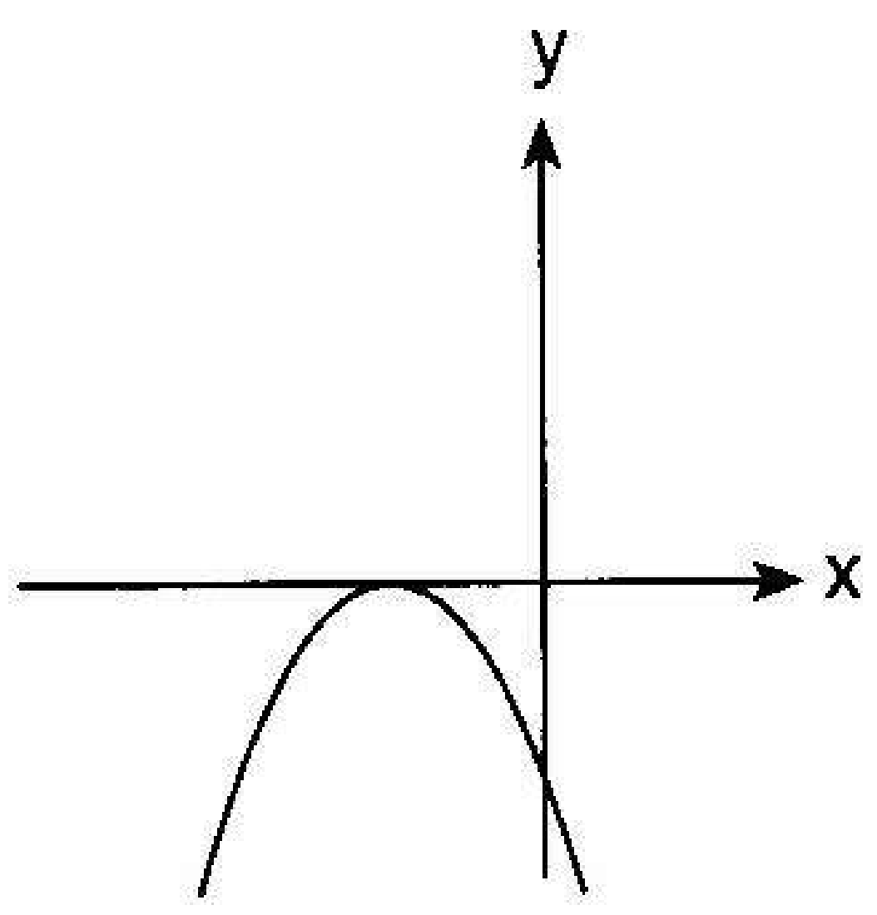




b. $\Delta = 0 \Rightarrow$

$\text{ÇK(SS)} = \{x_1, x_2\}, \; x_1 = x_2$



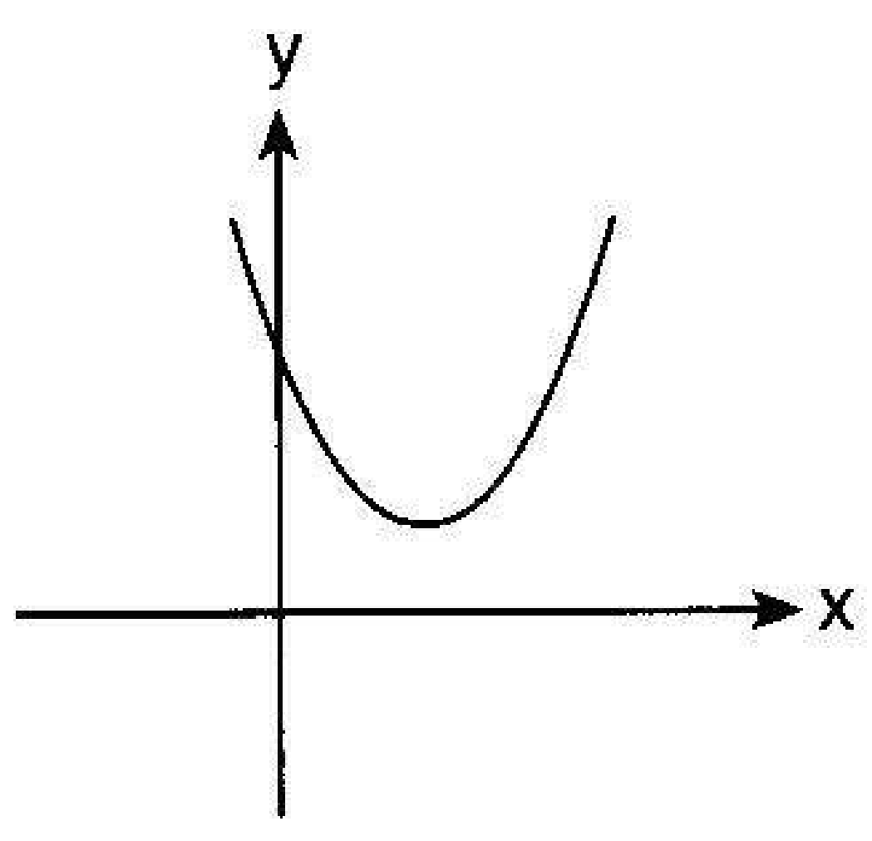


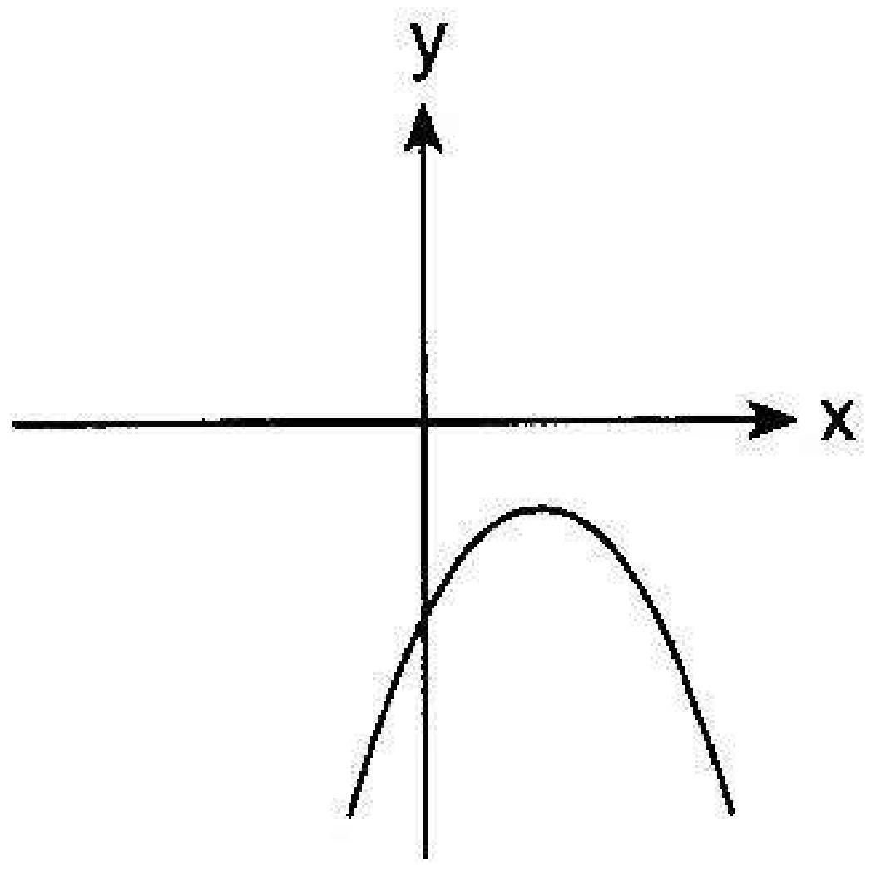
4 Parabolün y Eksenini Kestiği Noktalar / y – Intercept of a Parabola

$f(x) = ax^2 + bx + c$ parabolünün y eksenini kestiği nokta $x = 0$ için $f(0) = c$ dir.

y intercept of the parabola $f(x) = ax^2 + bx + c$, for $x = 0$, $f(0) = c$.

$\Delta < 0$

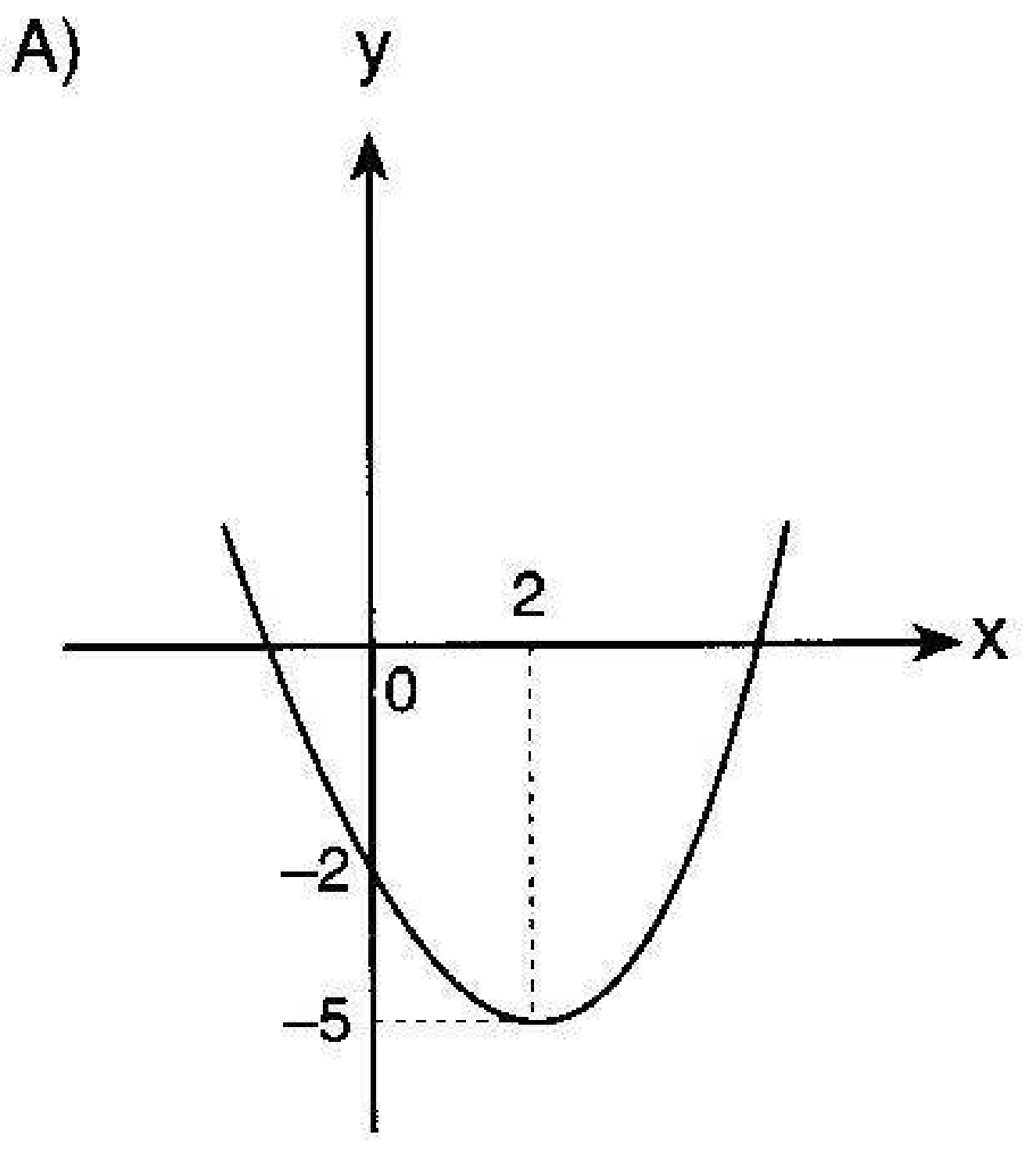




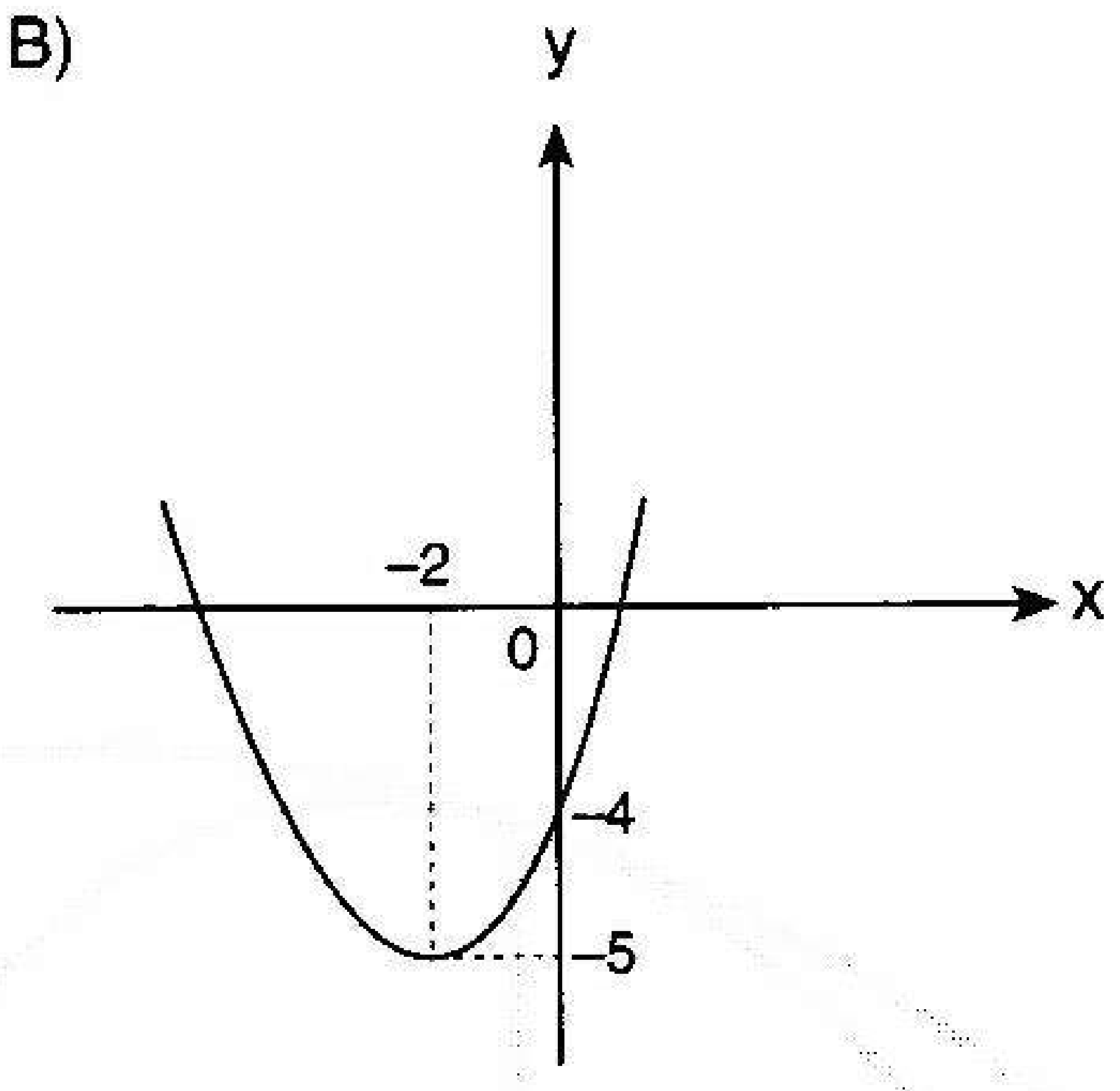
ÖRNEK / EXAMPLE 1

$y = (x-2)^2 - 5 \Rightarrow$ **Grafik** / ***Graph*** **= ?**

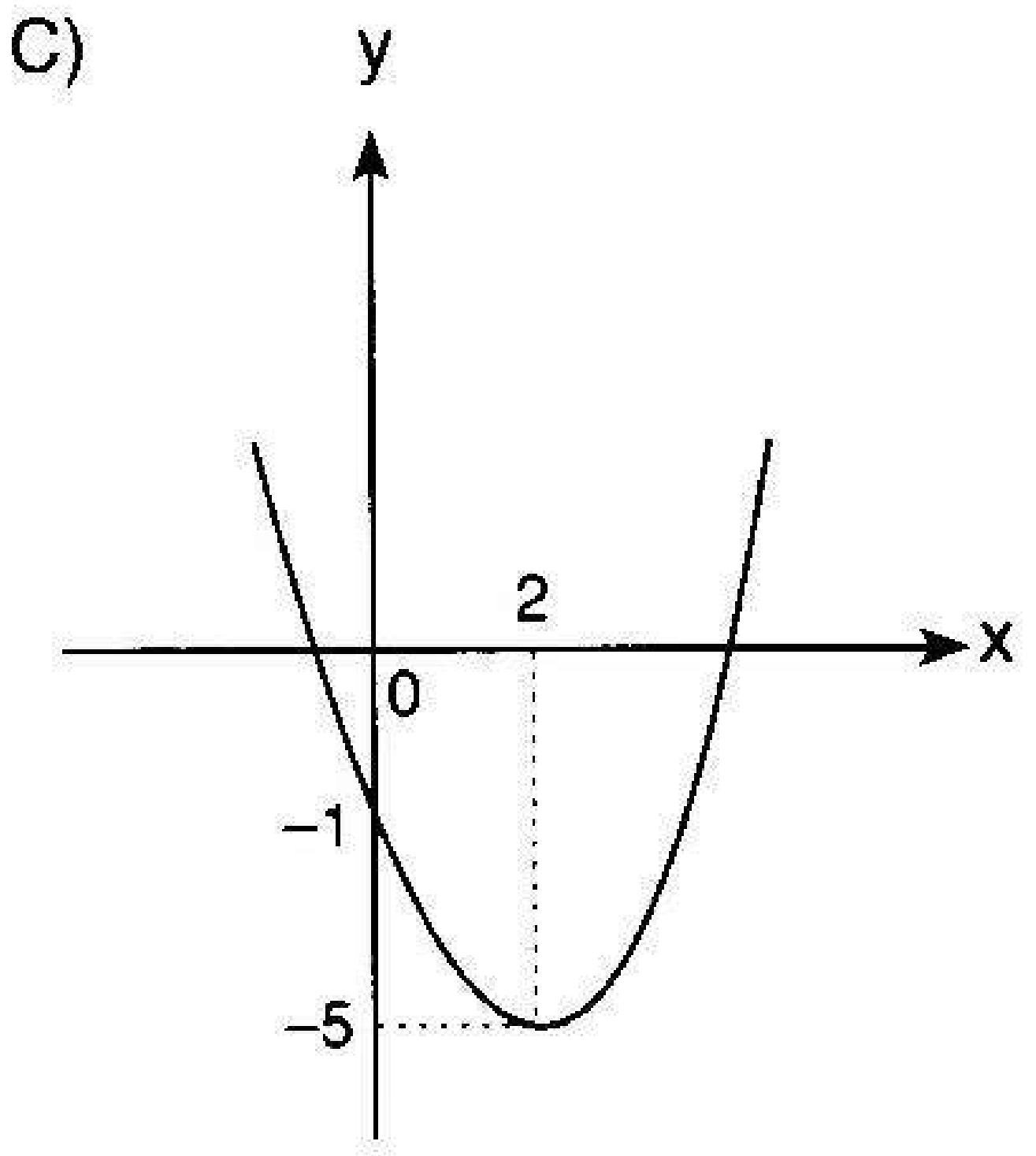
A)



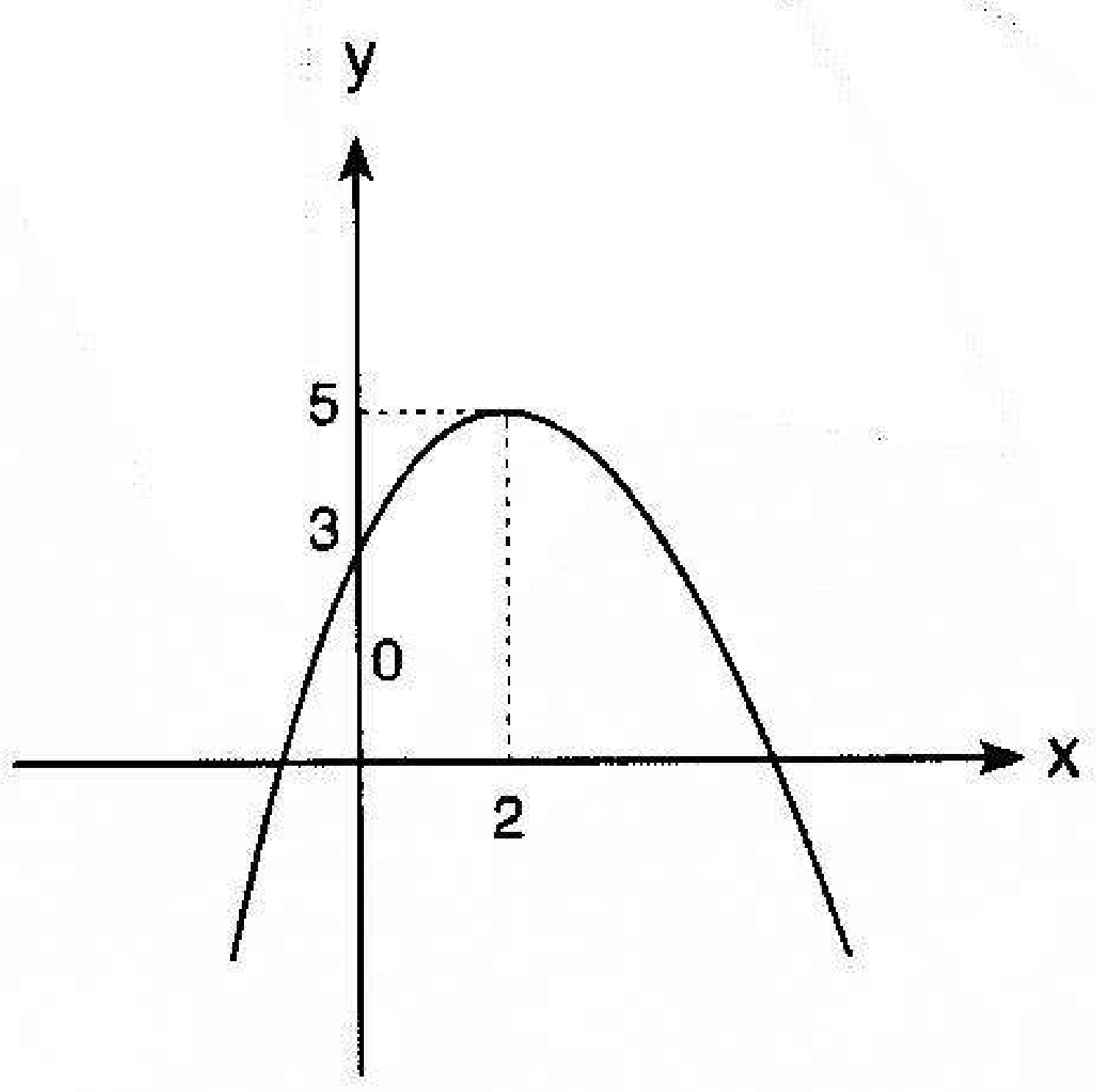
B)



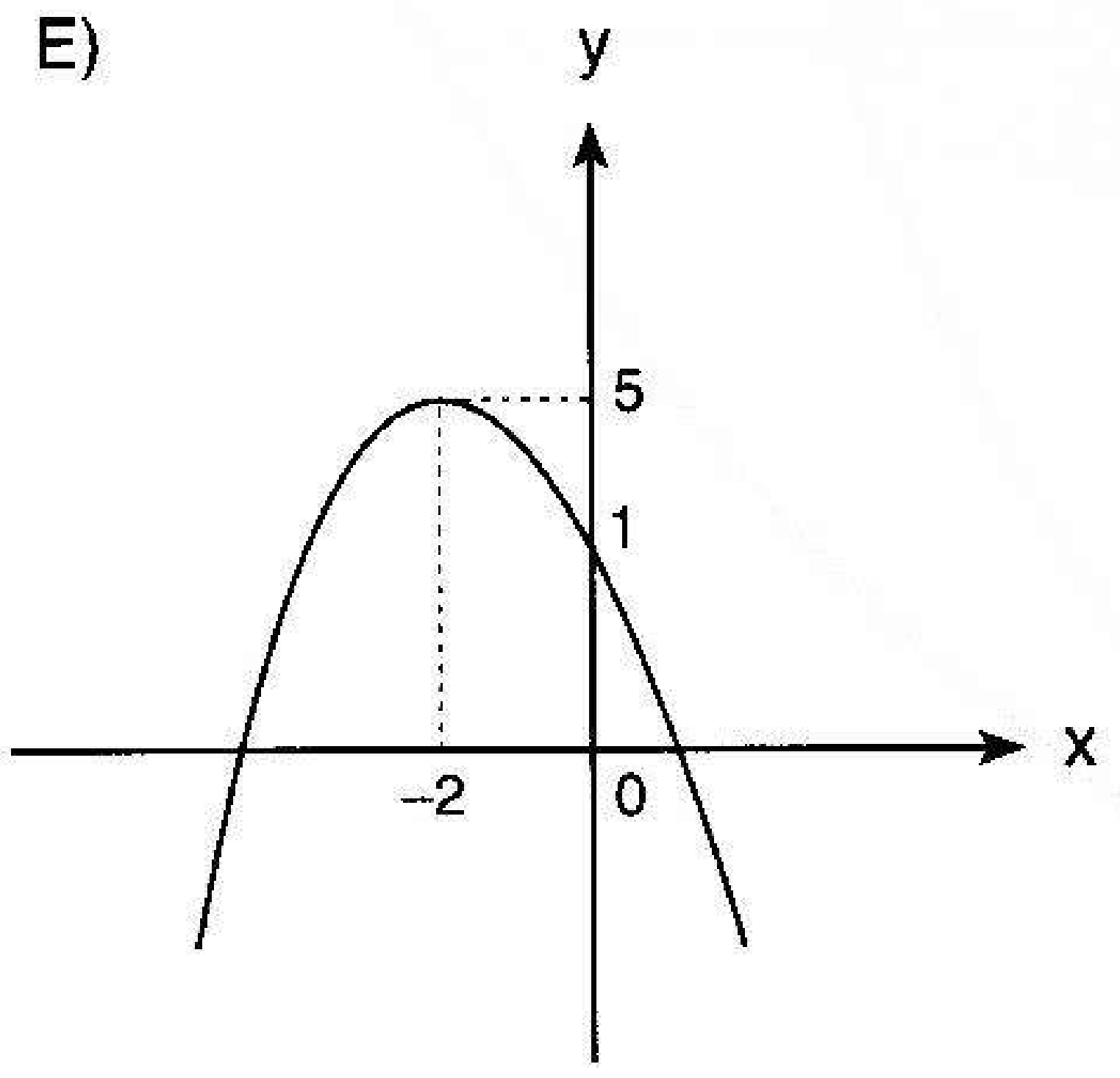
C)



D)



E)



ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$y = (x-2)^2 - 5$$

$$= x^2 - 4x + 4 - 5$$

$$= x^2 - 4x - 1$$

$$T\left(-\frac{b}{2a}, \frac{-\Delta}{4a}\right) = \left(\frac{4}{2}, -\frac{20}{4}\right)$$

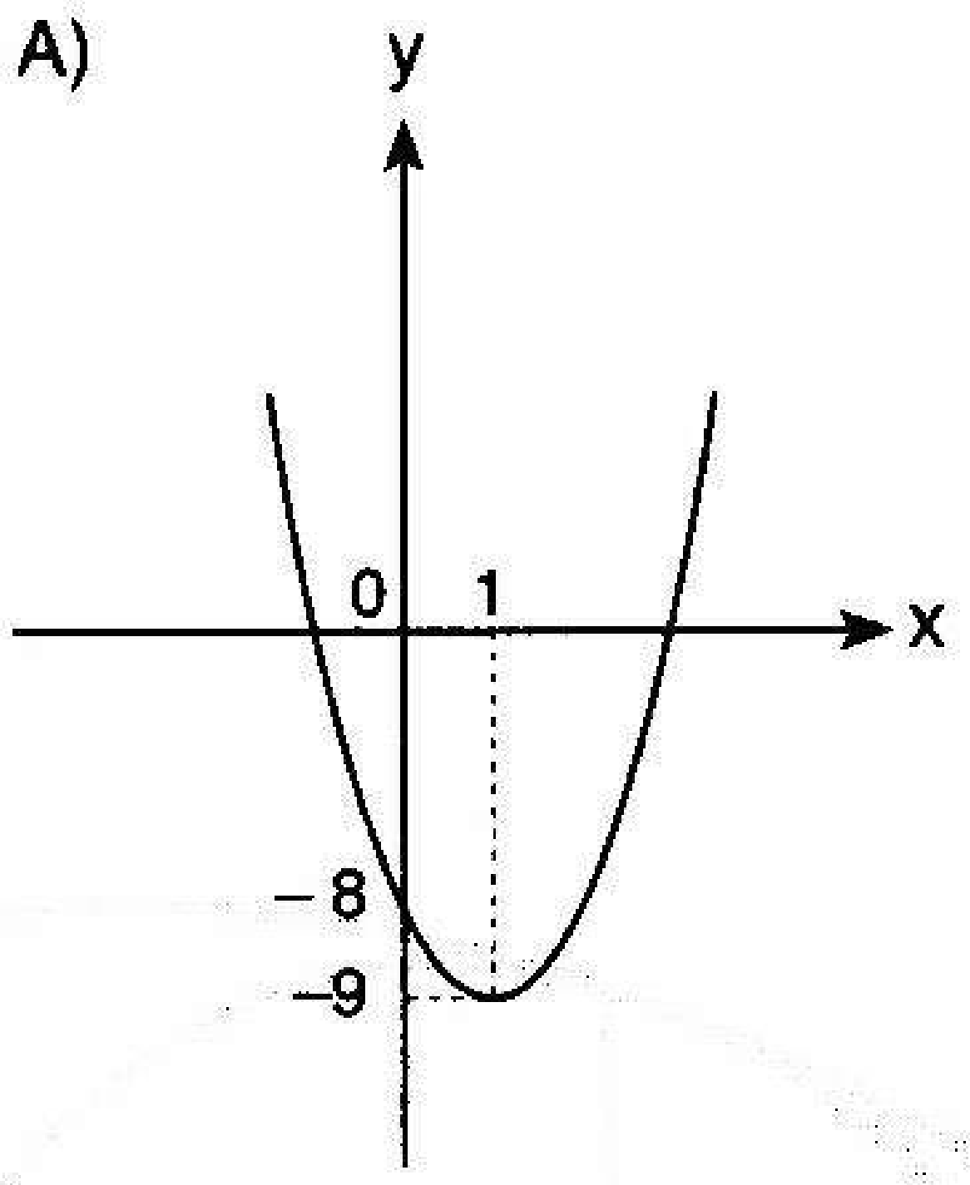
$$T(r, k) = (2, -5)$$

$x = 0 \Rightarrow f(0) = -1$

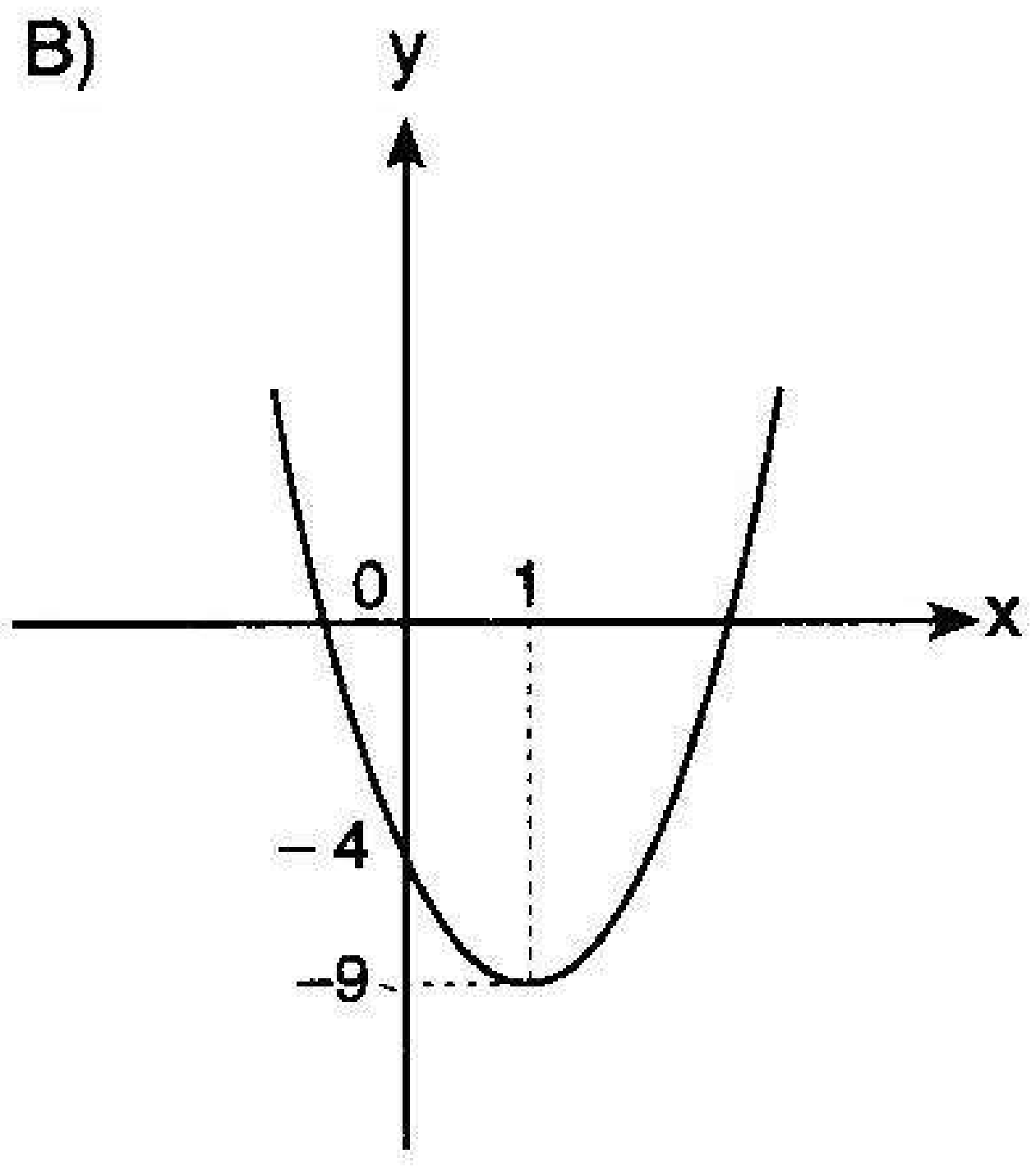
ÖRNEK / EXAMPLE 2

$y = x^2 - 2x - 8 \Rightarrow$ **Grafik /** ***Graph*** **= ?**

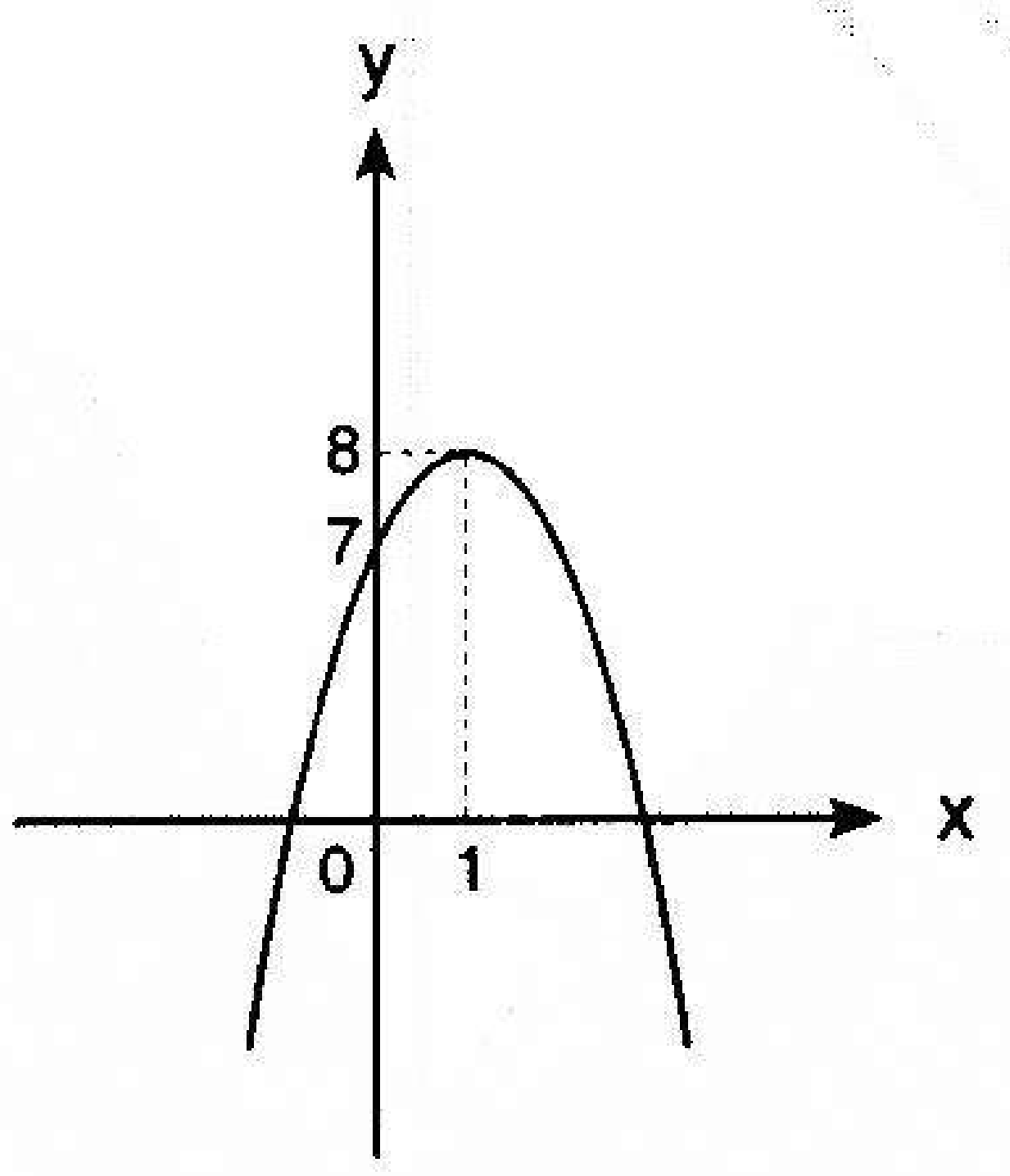
A)



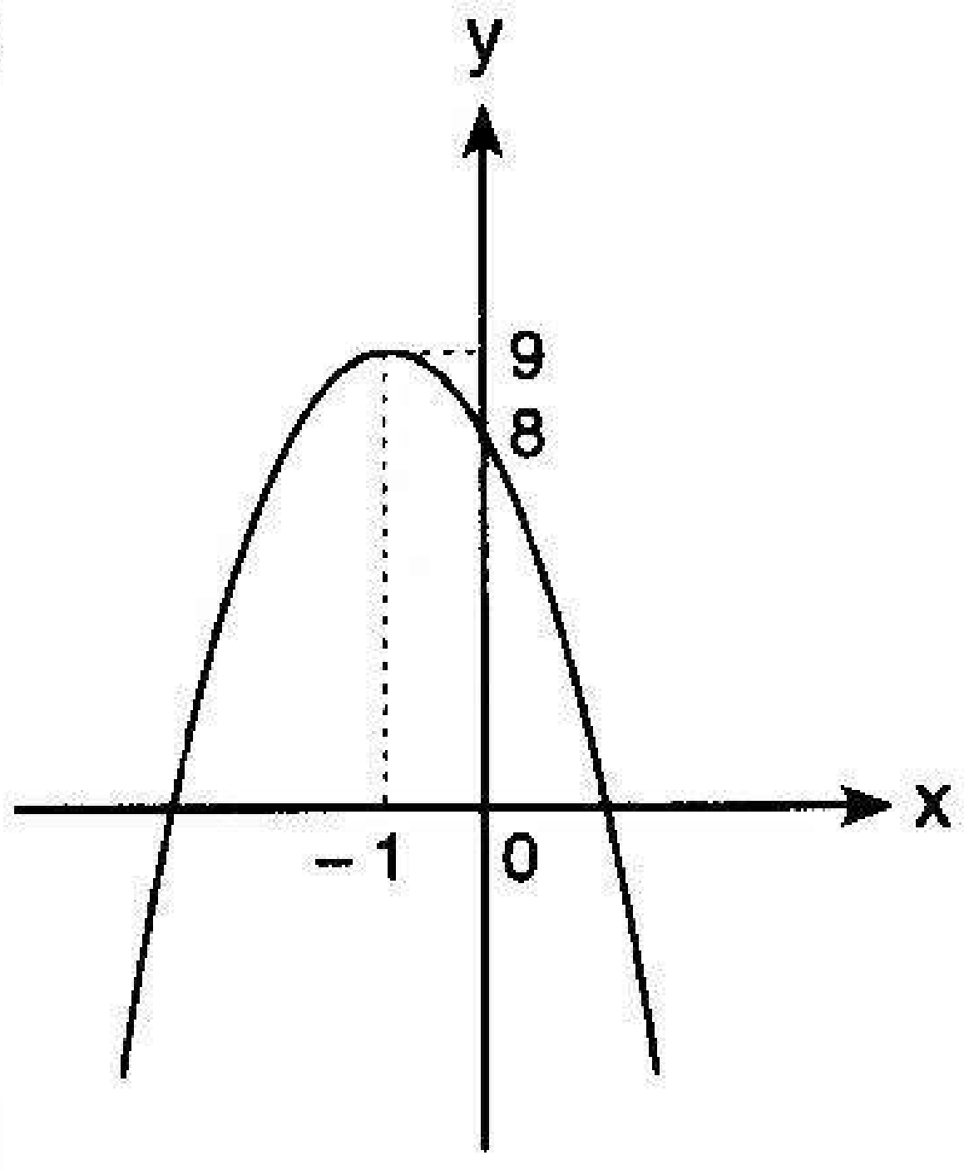
B)



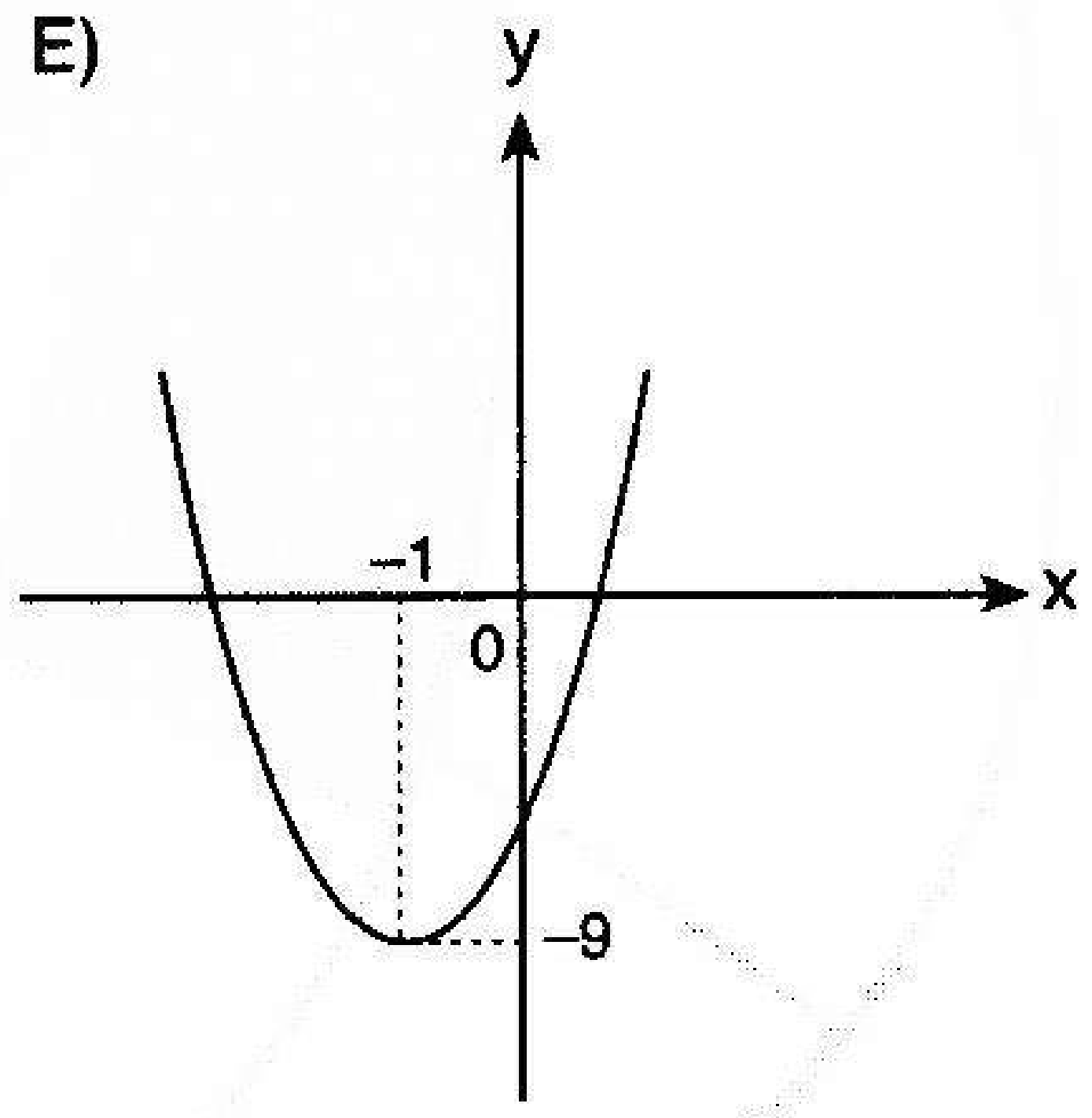
C)



D)



E)



ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$y = ax^2 + bx + c \Rightarrow T = (r,k) = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

$$y = x^2 - 2x - 8 \Rightarrow T = \left(-\frac{-2}{2\cdot 1}, \frac{4\cdot 1\cdot(-8) - (-2)^2}{4\cdot 1}\right)$$

$$= \left(1, \frac{-32-4}{4}\right)$$

$$= (1, -9)$$

$$x = 0 \Rightarrow y = 0^2 - 2\cdot 0 - 8$$

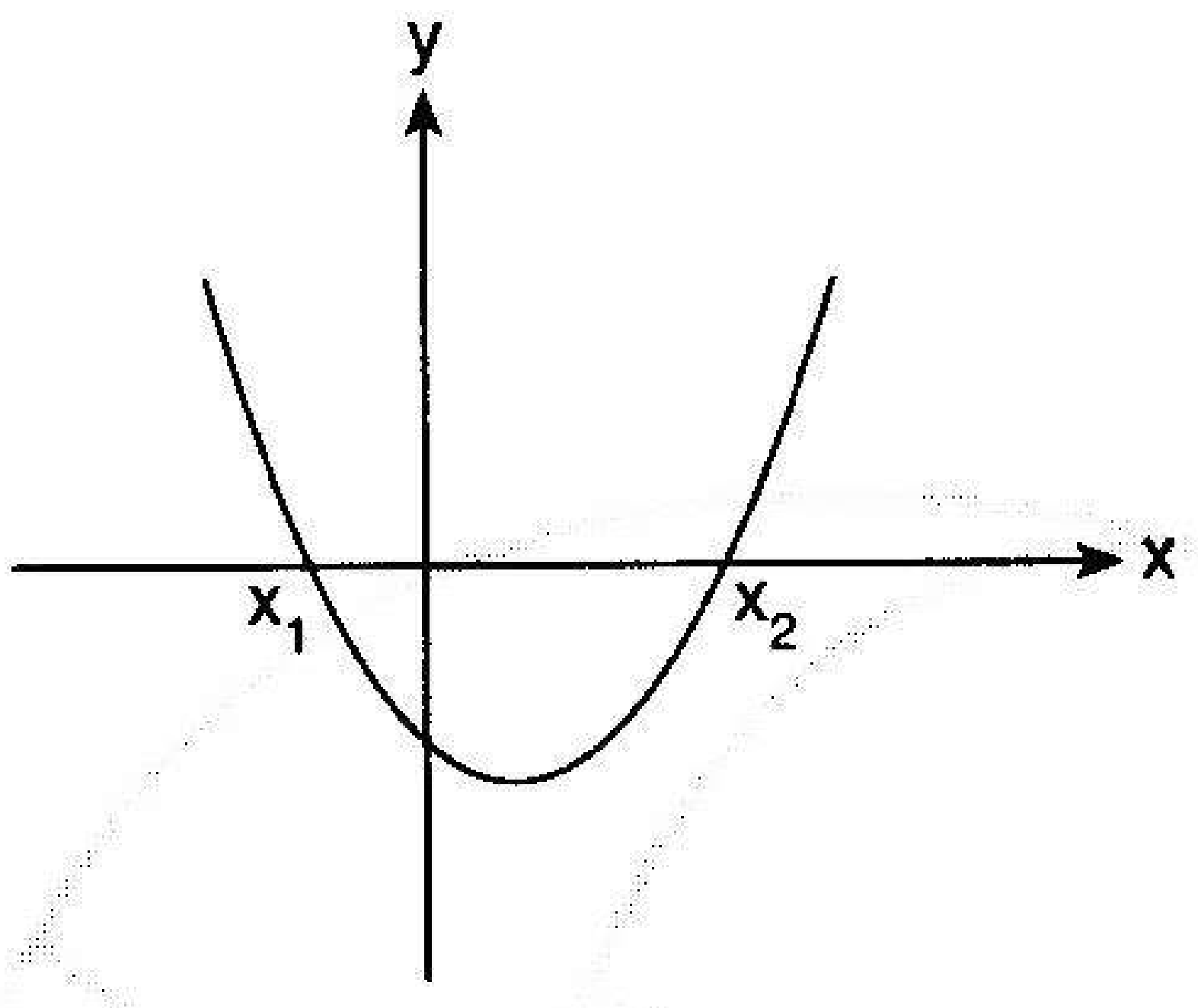
$$y = -8$$

PARABOL DENKLEMİNİN YAZILMASI / WRITING of a PARABOLA EQUATION

Eksenleri kestiği noktalar ile bir noktası bilinen parabolün denkleminin bulunması
Finding the Equation of the Parabola with one point and the x and y intercepts which are given

a. x eksenini $(x_1, 0)$, $(x_2, 0)$ noktalarında kesen parabolün denklemi:

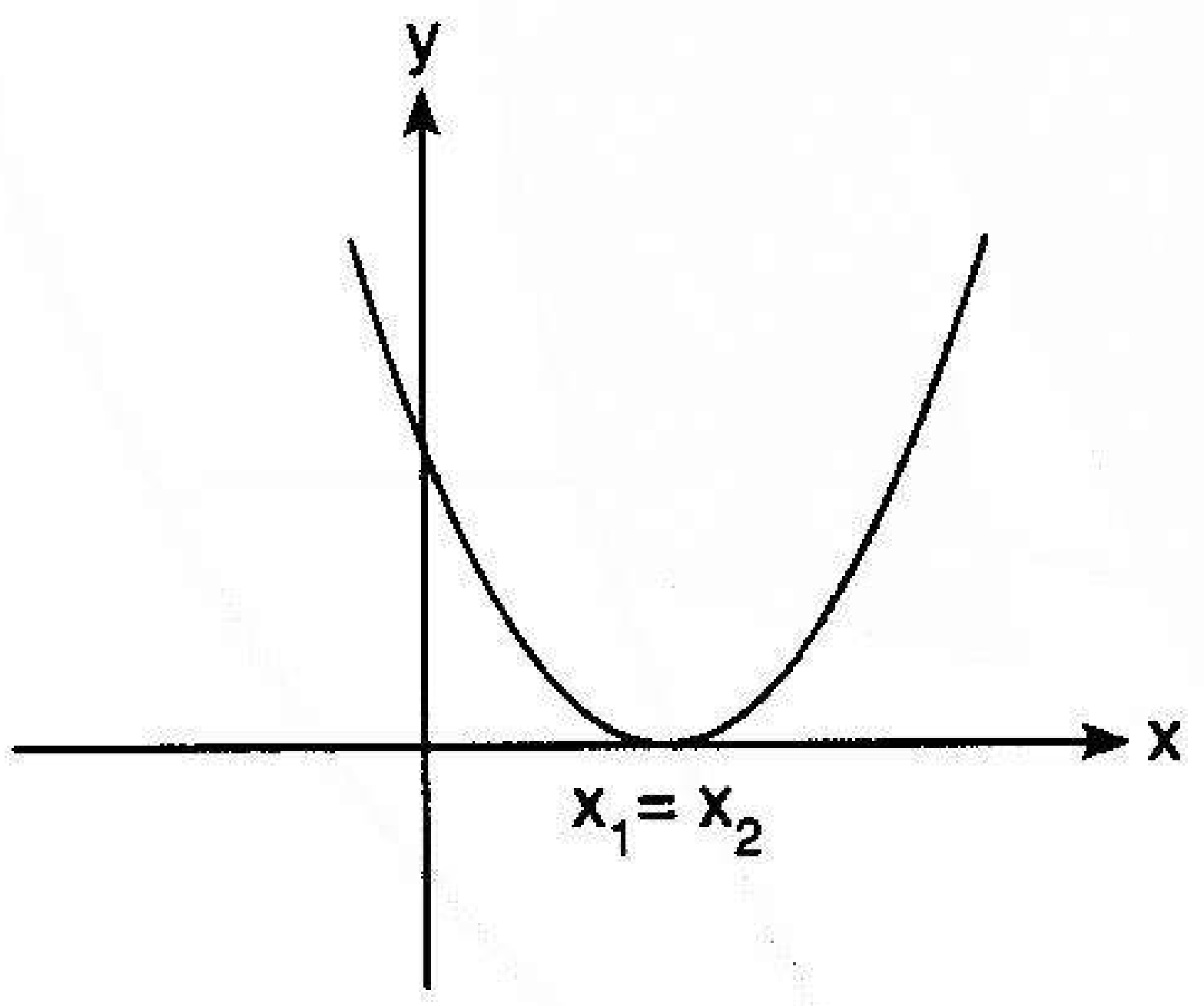
The equation of the parabola with x – intercepts; $(x_1, 0)$ *and* $(x_2, 0)$:



$$y = a\cdot(x - x_1)\cdot(x - x_2)$$

b. x eksenine $(x_1, 0) = (x_2, 0)$ noktasında teğet olan parabolün denklemi:

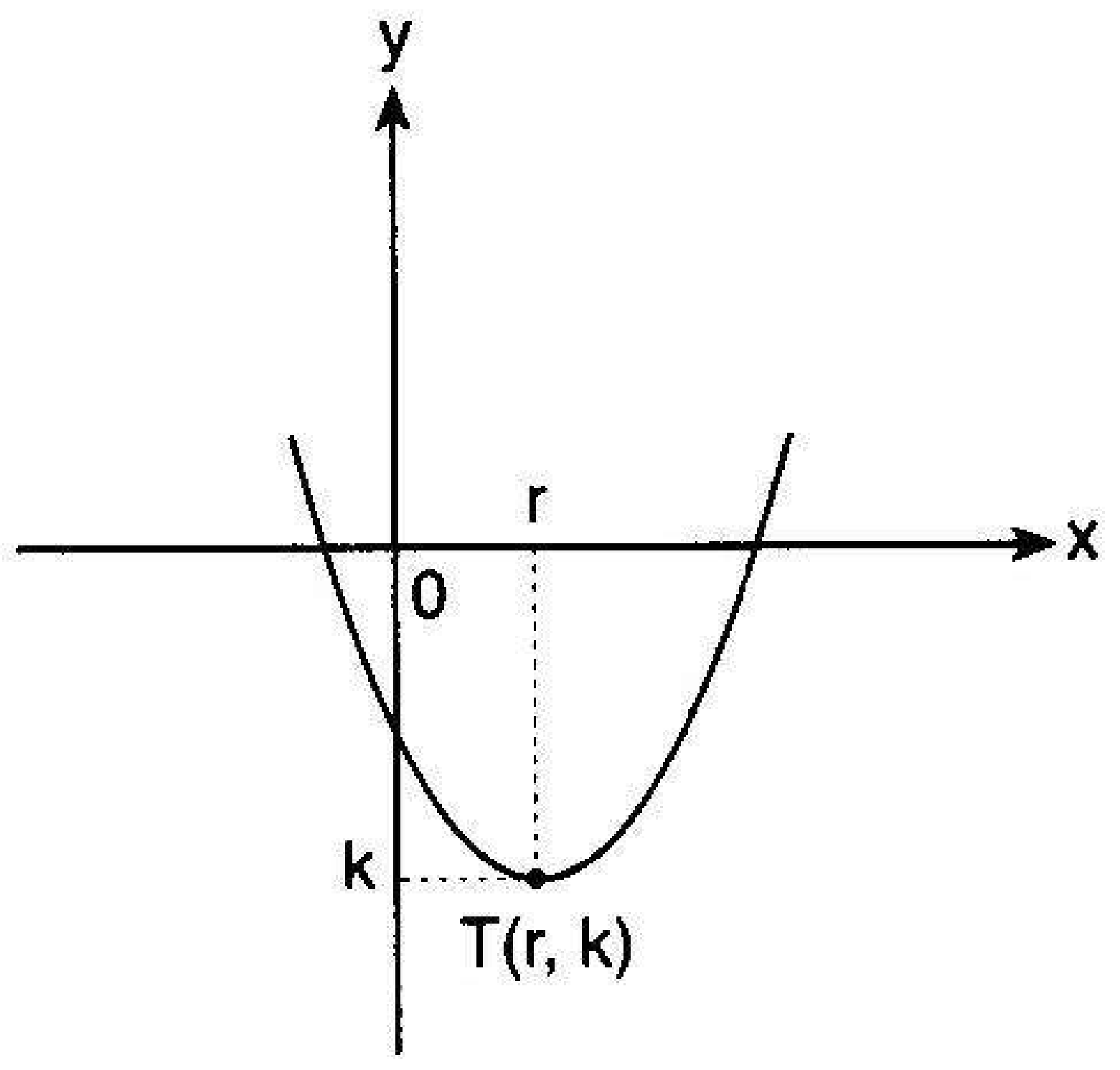
The equation of the parabola that is tangent to the x – axis at the point $(x_1, 0) = (x_2, 0)$:



$$y = a\cdot(x - x_1)^2$$

c. Tepe noktası T(r, k) olan parabolün denklemi:

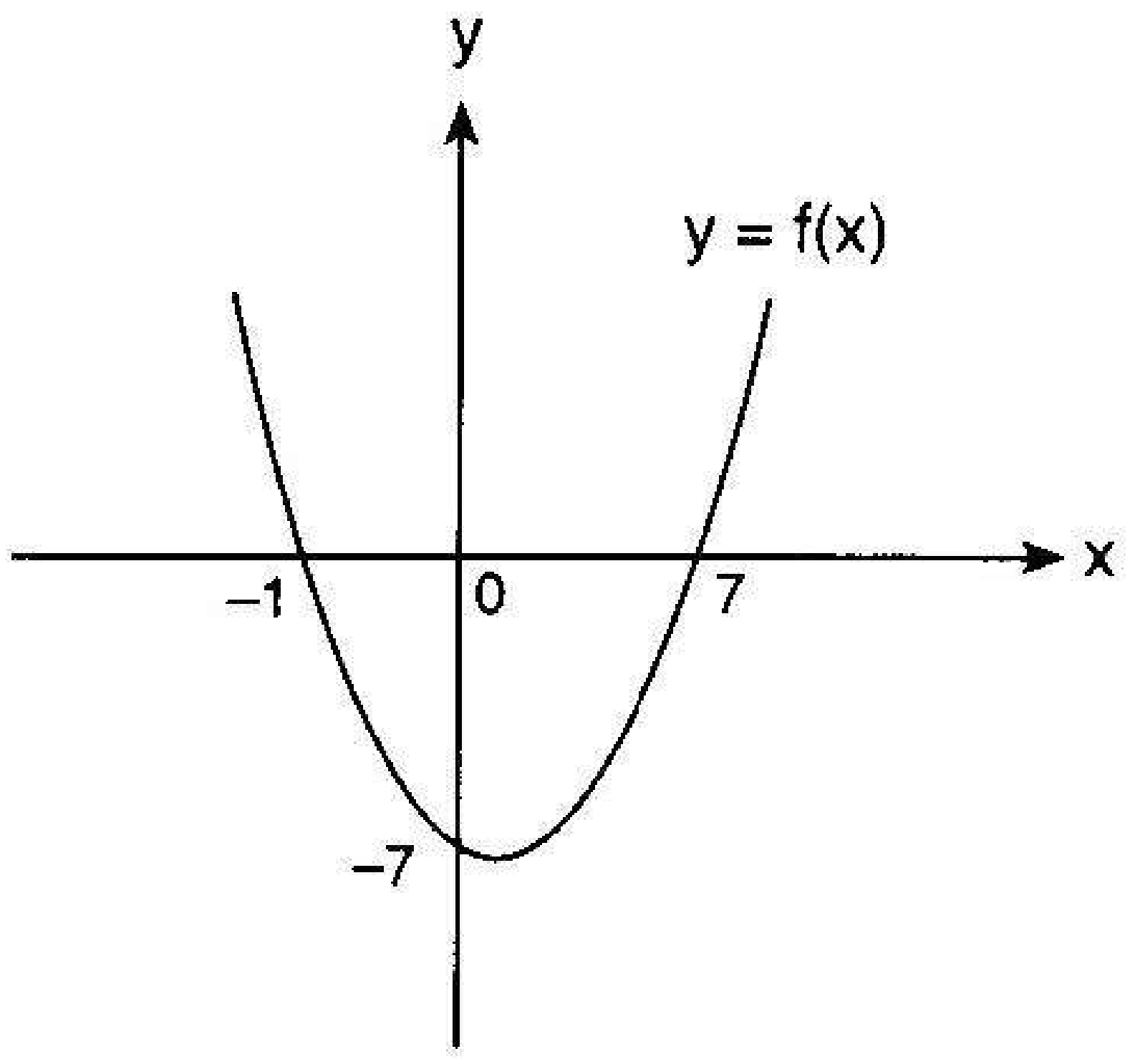
The equation of the parabola with a vertex T(r, k):



$$y = a(x - r)^2 + k$$

ÖRNEK / EXAMPLE 3

$y = f(x) = ?$



A) $x^2 - 6x + 7$

B) $x^2 - 6x - 7$

C) $x^2 + 6x - 7$

D) $x^2 - 4x - 7$

E) $x^2 + 4x - 7$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$y = a \cdot (x + 1) \cdot (x - 7)$

$-7 = a \cdot (0 + 1) \cdot (0 - 7)$ $[\,(x = 0 \Rightarrow y = -7\,]$

$-7 = a \cdot 1 \cdot (-7)$

$a = 1$

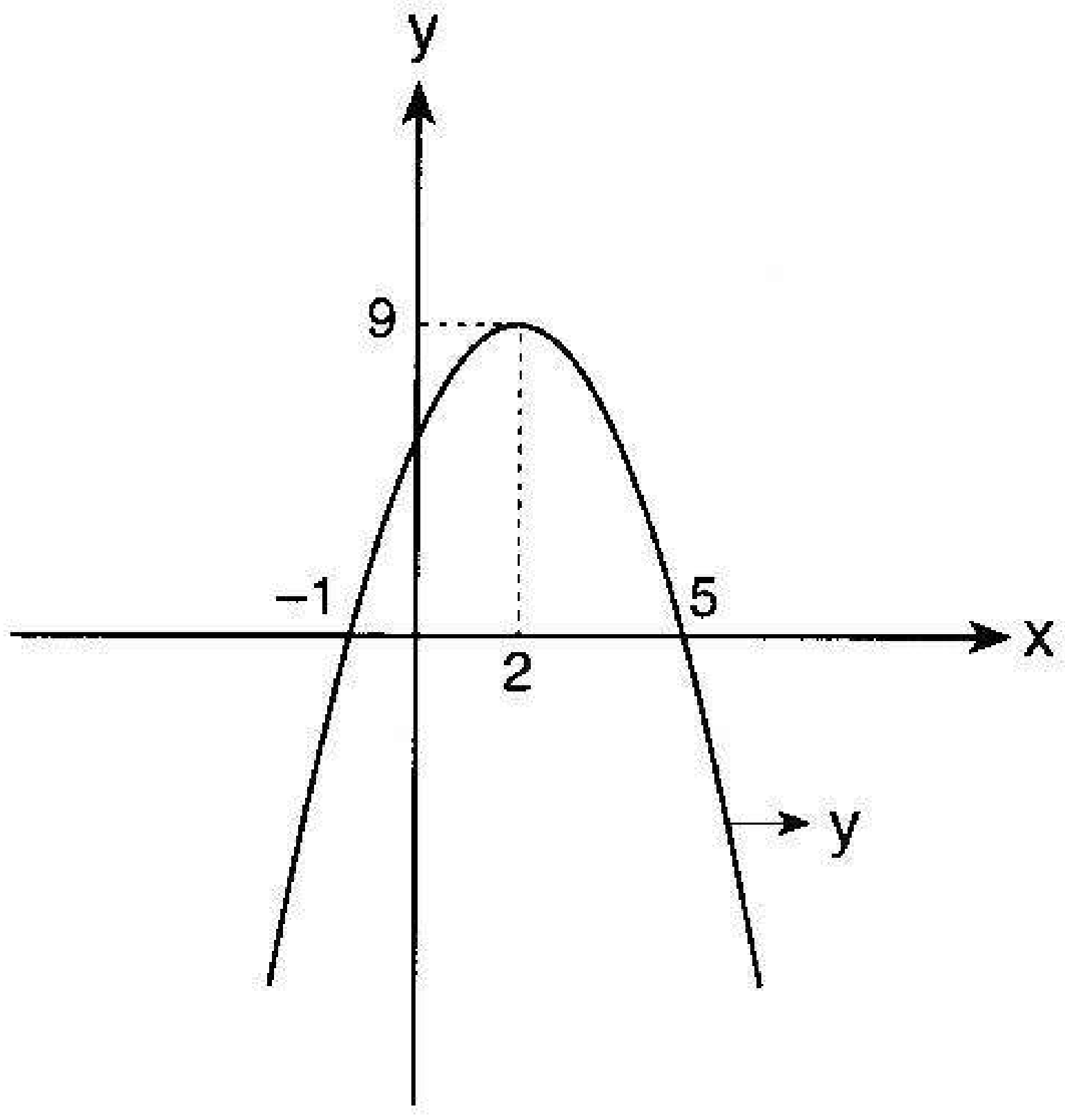
$y = 1 \cdot (x + 1) \cdot (x - 7)$

$y = x^2 - 7x + x - 7$

$y = x^2 - 6x - 7$

ÖRNEK / EXAMPLE 4

$y = ?$



A) $(x + 2)^2 - 5$

B) $(x - 3)^2 - 9$

C) $(x - 2)^2 + 9$

D) $-(x + 2)^2 - 9$

E) $-(x - 2)^2 + 9$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$T = (2\,,\,9)$

$y = a \cdot (x - 2)^2 + 9$

$0 = a \cdot (5 - 2)^2 + 9$ $[\,(5\,,\,0) \in y\,]$

$0 = a \cdot 9 + 9$

$-9 = a \cdot 9 \Rightarrow a = -1$

$y = -1 \cdot (x - 2)^2 + 9$

$y = -(x - 2)^2 + 9$

ÖRNEK / EXAMPLE 5

$y = ?$

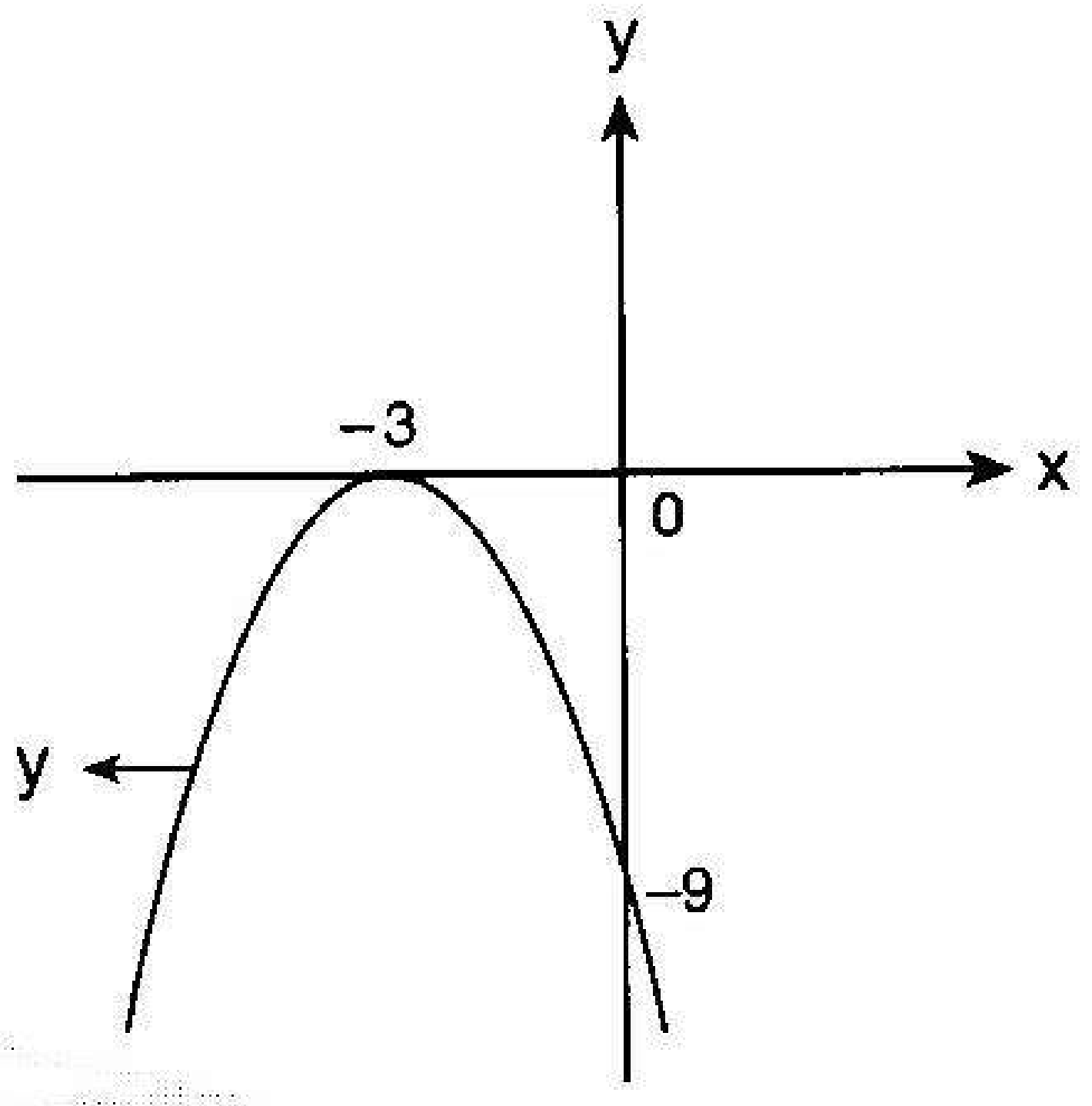
A) $-x^2 - 6x - 6$

B) $-x^2 - 9x - 3$

C) $-x^2 + 6x - 9$

D) $-x^2 - 6x - 9$

E) $-x^2 - 6x$



ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$y = a \cdot (x+3)^2$

$-9 = a \cdot (0+3)^2$ $[(0, -9) \in y]$

$-9 = a \cdot 9$

$a = -1$

$y = -1 \cdot (x+3)^2$

$y = -(x^2 + 6x + 9)$

$y = -x^2 - 6x - 9$

ÖRNEK / EXAMPLE 6

$T(r, k) \Rightarrow$ **$k = ?$**

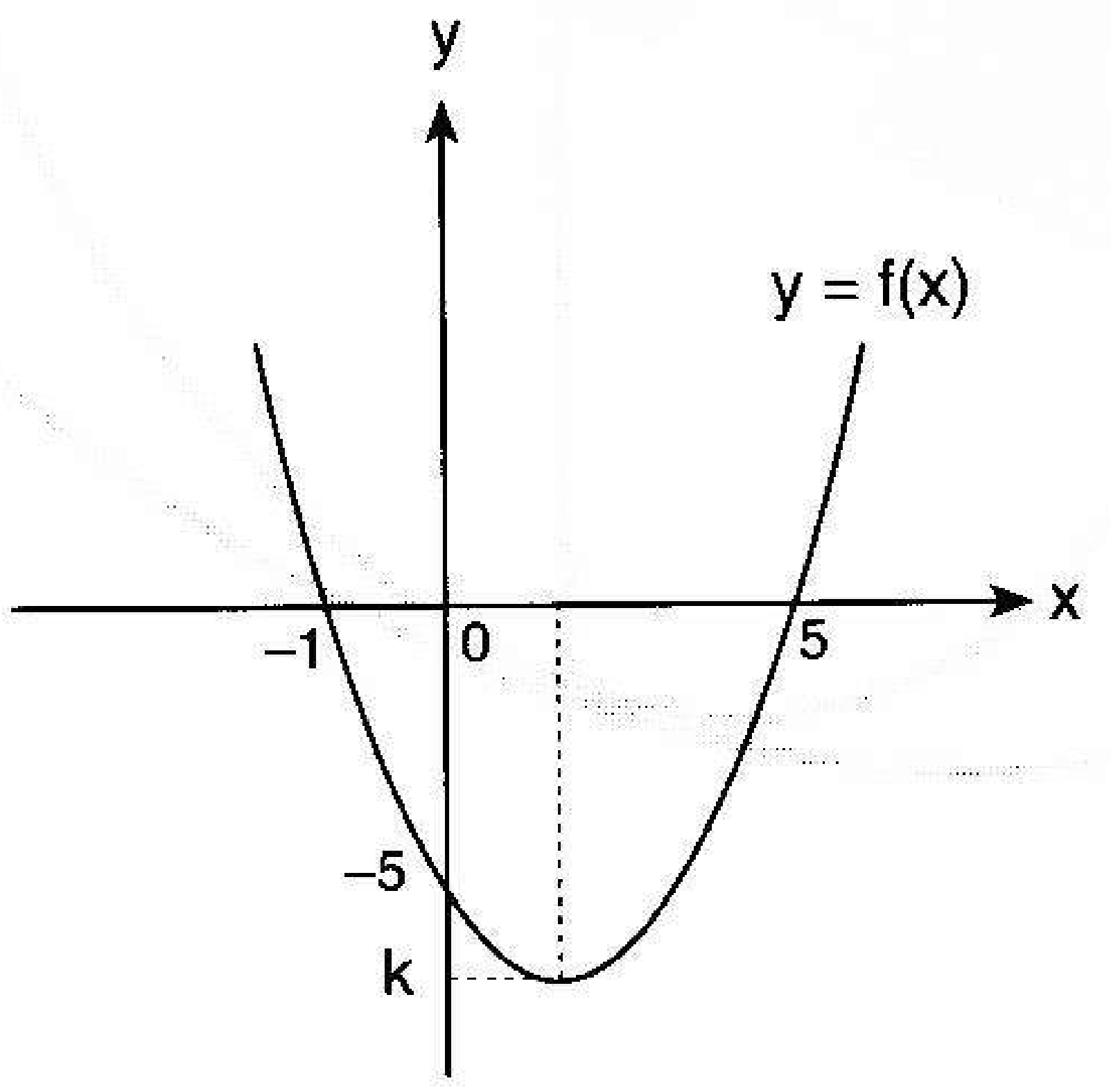
A) -6

B) $-\frac{13}{2}$

C) -7

D) $-\frac{15}{2}$

E) -9



ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$y = a \cdot (x+1) \cdot (x-5)$

$-5 = a \cdot (0+1) \cdot (0-5)$ $[(0, -5) \in y]$

$-5 = a \cdot 1 \cdot (-5) \Rightarrow a = 1$

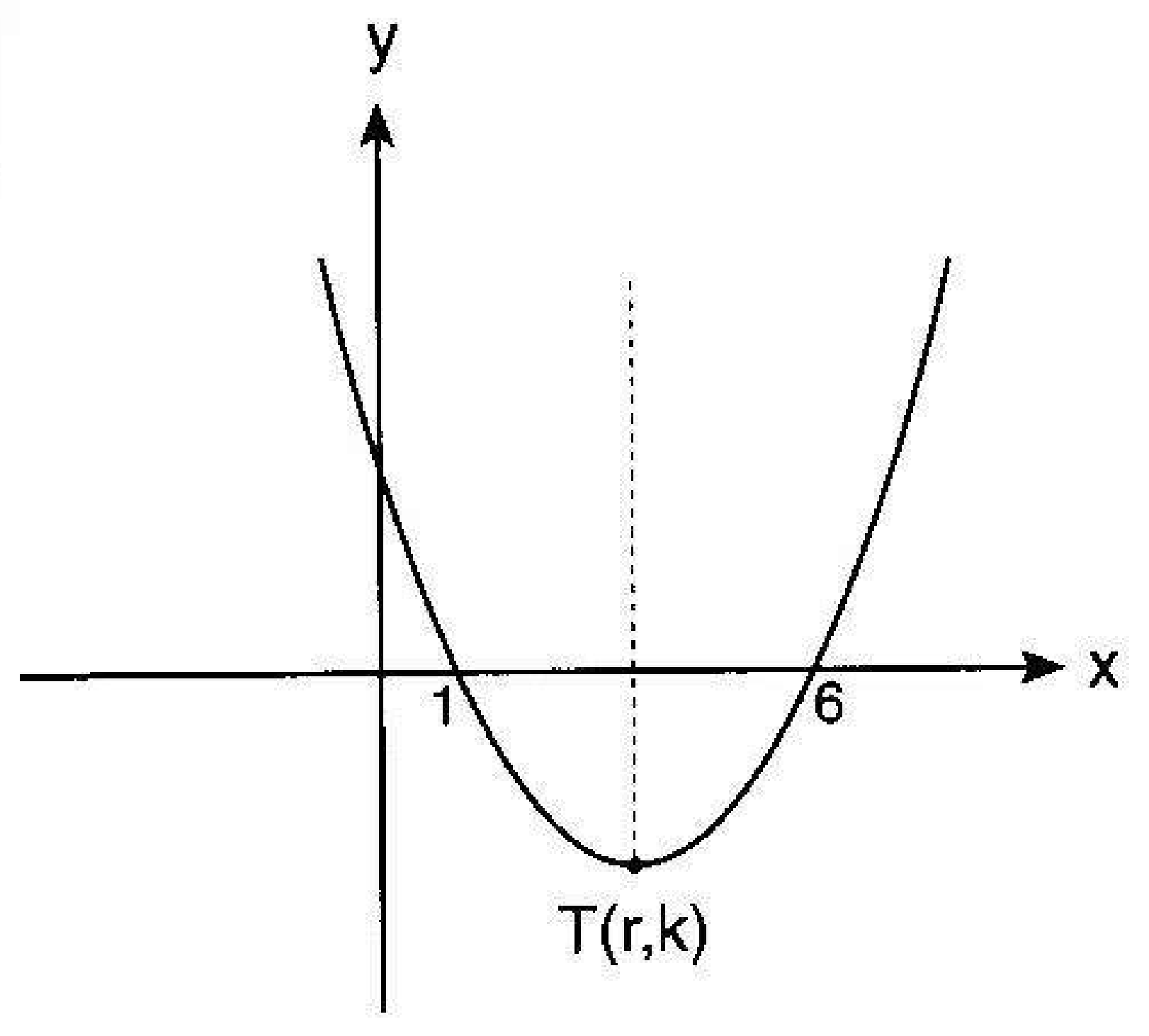
$y = 1 \cdot (x+1) \cdot (x-5)$

$y = x^2 - 4x - 5$

$$T = \left(\frac{-4}{-2}, \frac{4 \cdot 1 \cdot (-5) \cdot (-4)^2}{4} \right) = \left(2, \frac{-20-16}{4} \right) = (2, -9)$$

$T = (2, -9) \Rightarrow k = -9$

PEKİŞTİRME / REINFORCE



$r = ?$

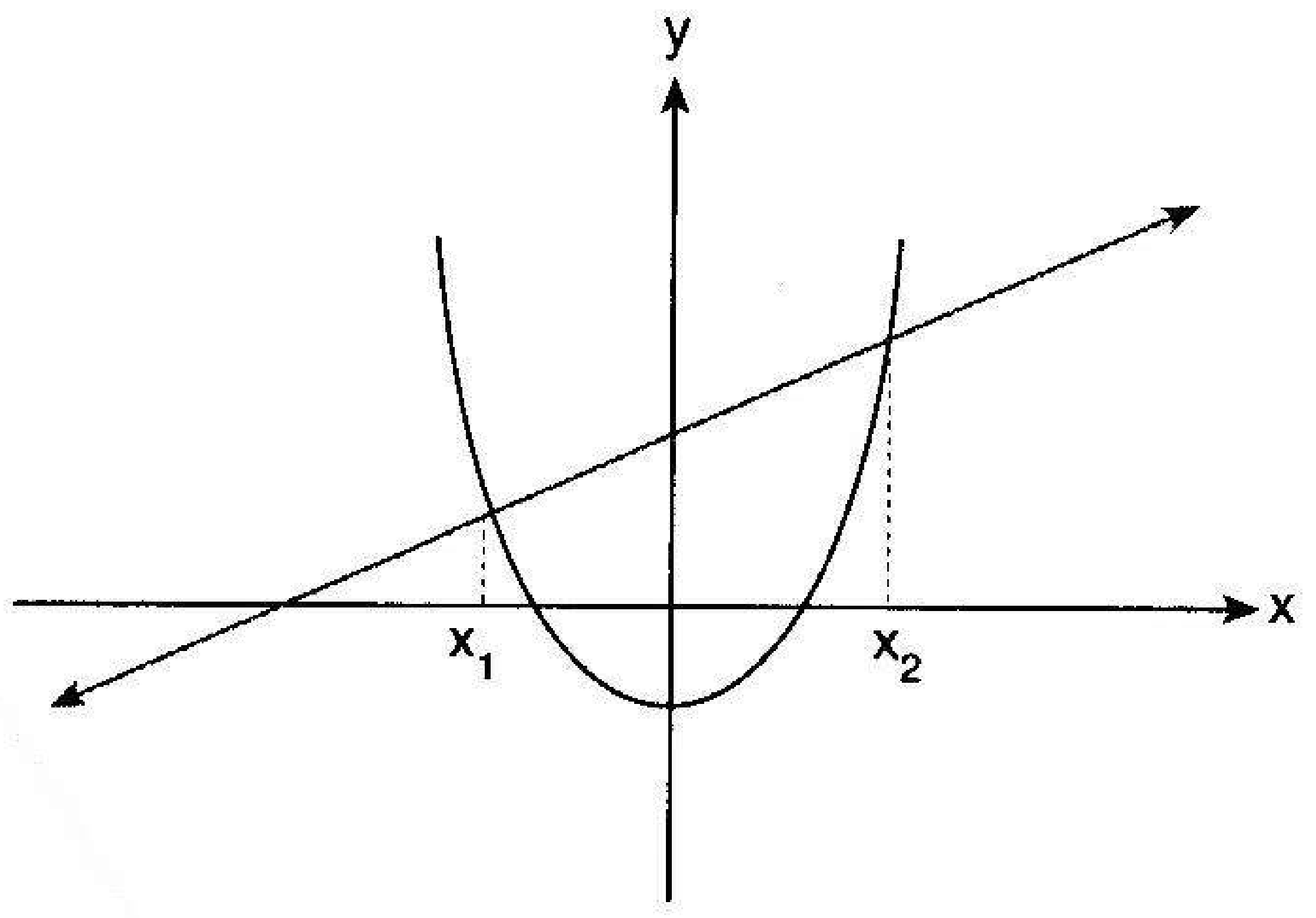
YANIT / ANSWER : 7/2

BİR PARABOLLE BİR DOĞRU ARASINDAKİ İLİŞKİ
RELATIONSHIP BETWEEN a PARABOLA and a LINE

$y = ax^2 + bx + c$ parabolü ile $y = mx + n$ doğrusunun durumu belirtmek için ortak çözümden yararlanılır.
We use common solution to the relationship between the parabola $y = ax^2 + bx + c$ and the line $y = mx + n$.

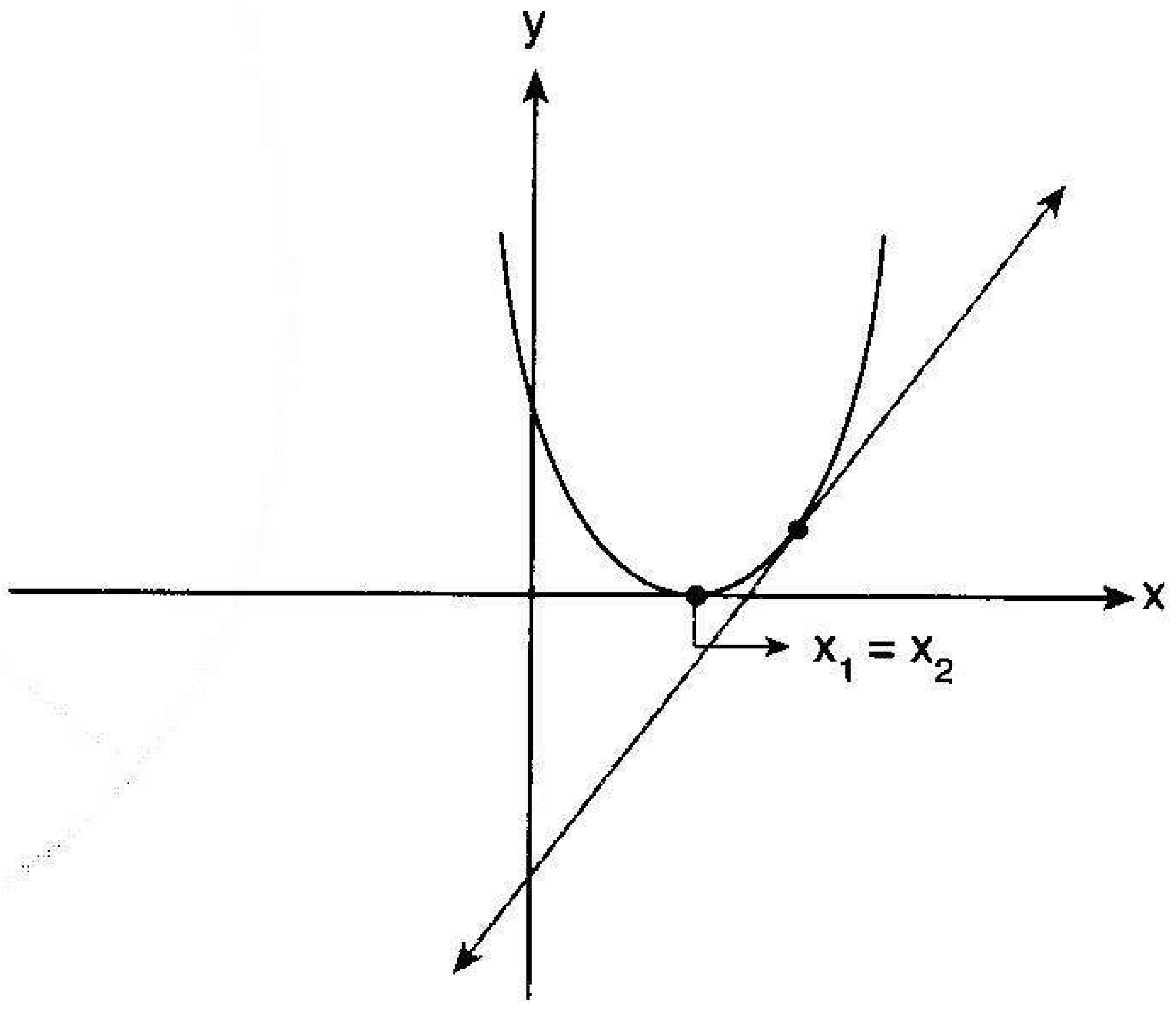
In equation **$ax^2 + bx + c = mx + n$ denkleminde,**

1 $\Delta > 0$ ise, doğru parabolü iki noktada keser.
If $\Delta > 0$, then line and parabola is cut by two points.



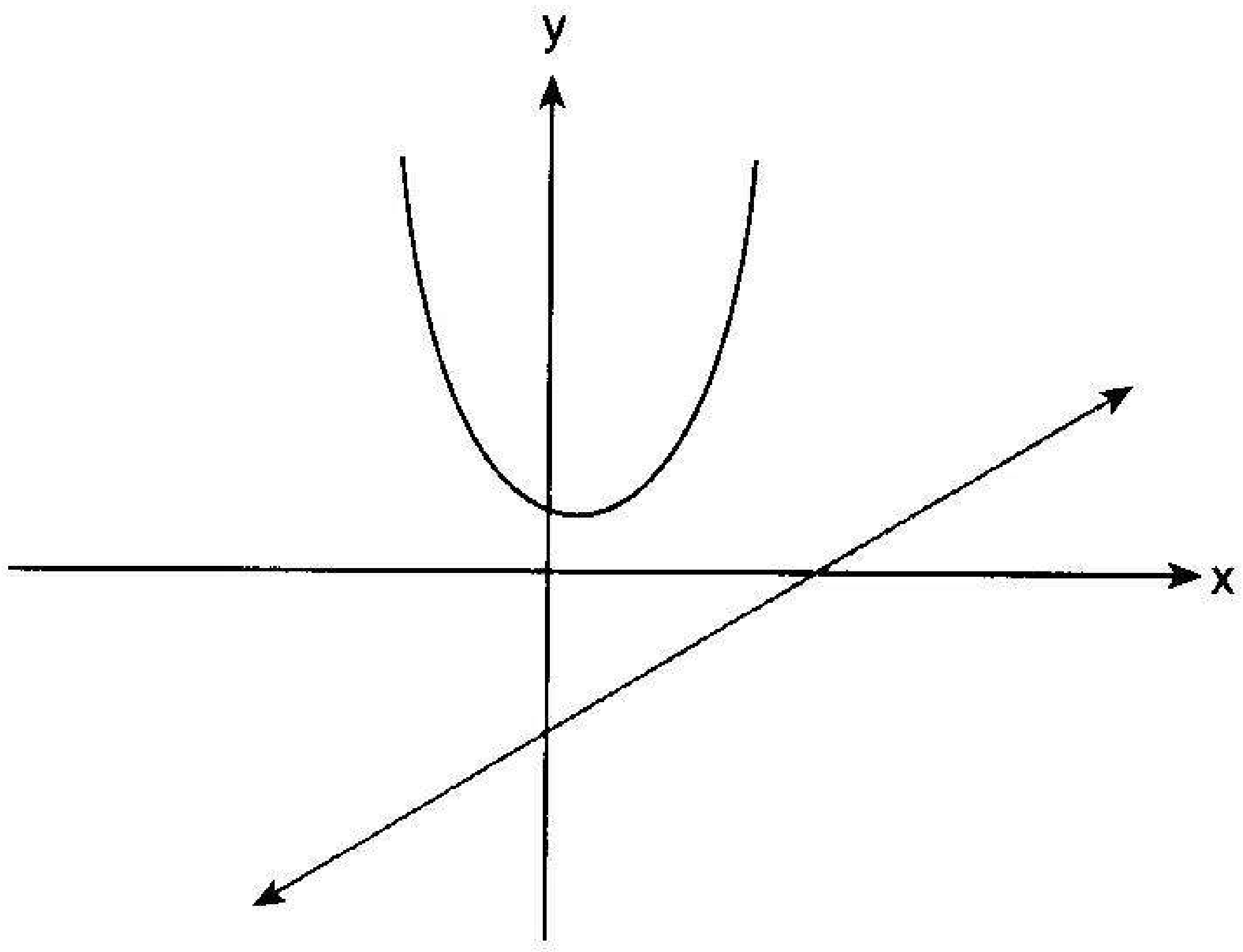
$\Delta > 0$

2 $\Delta = 0$ ise, doğru parabole bir noktada teğettir.
If $\Delta = 0$, line is tangent to the parabola at a point.



$\Delta = 0$

3 $\Delta < 0$ ise, parabol ile doğru kesişmez.
If $\Delta < 0$, parabola and line is not intersect.



$\Delta < 0$

ÖRNEK / EXAMPLE 7

A(m, n) ⇒ **m + n = ?**

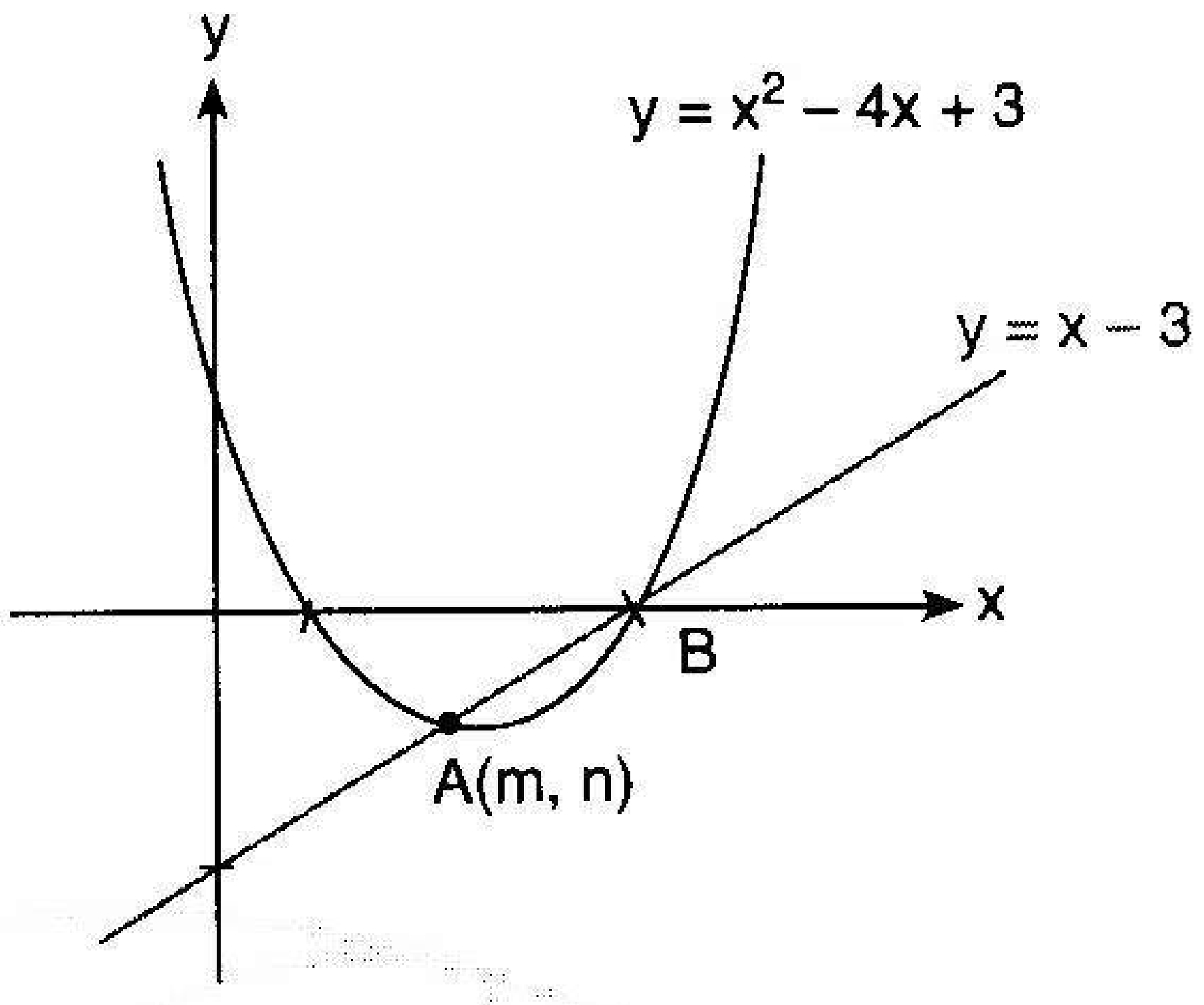
A) 0

B) 1

C) 2

D) 3

E) 4



ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left.\begin{matrix} y = x^2 - 4x + 3 \\ y = x - 3 \end{matrix}\right\} \Rightarrow x^2 - 4x + 3 = x - 3$$

$$x^2 - 5x + 6 = 0$$

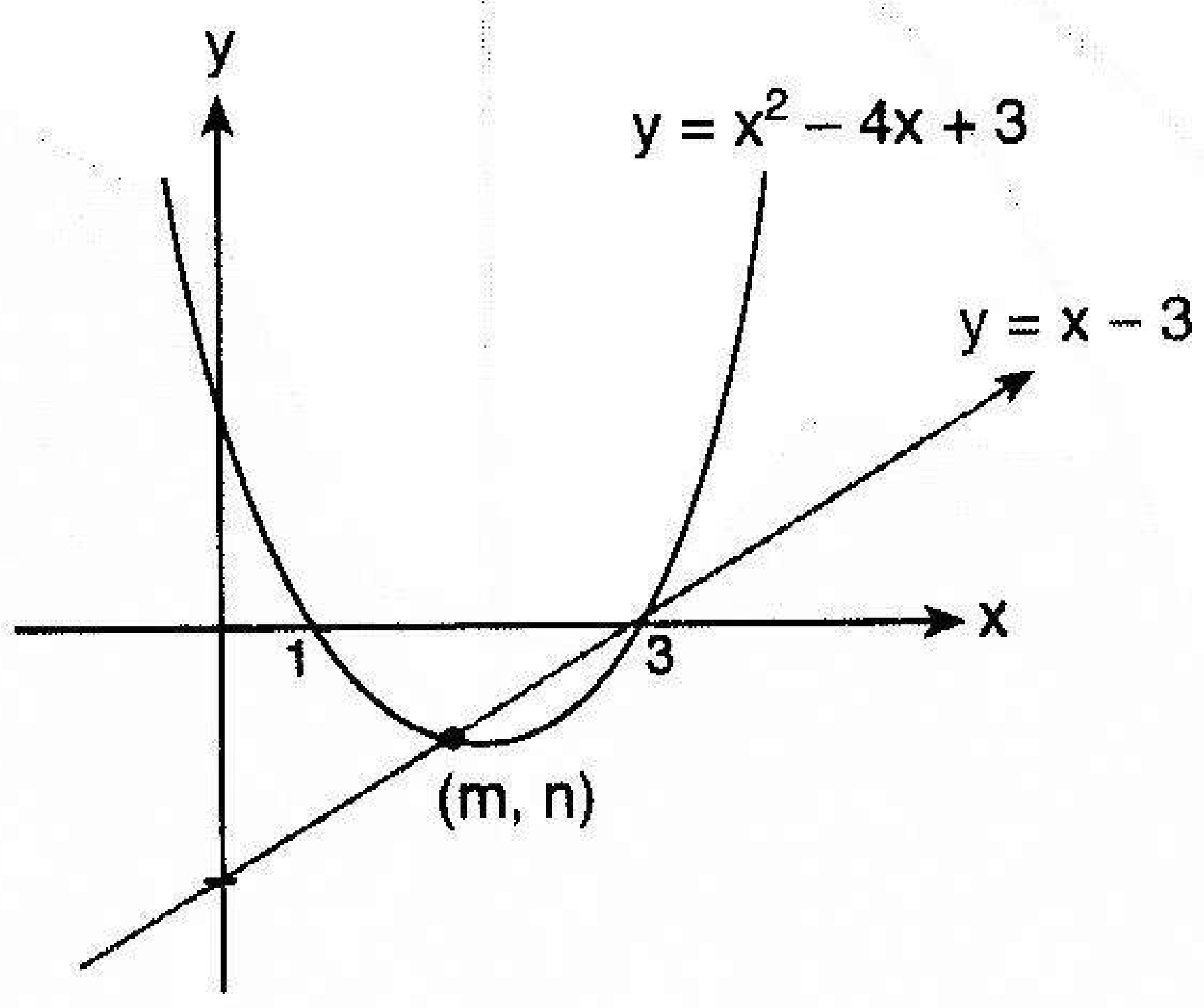
$$(x-2)\cdot(x-3) = 0$$

$$x_1 = 2, \quad x_2 = 3$$

$x = 2 \Rightarrow m = 2$

$m = 2 \Rightarrow n = 2 - 3 = -1$

$m + n = 1$



ÖRNEK / EXAMPLE 8

m = ?

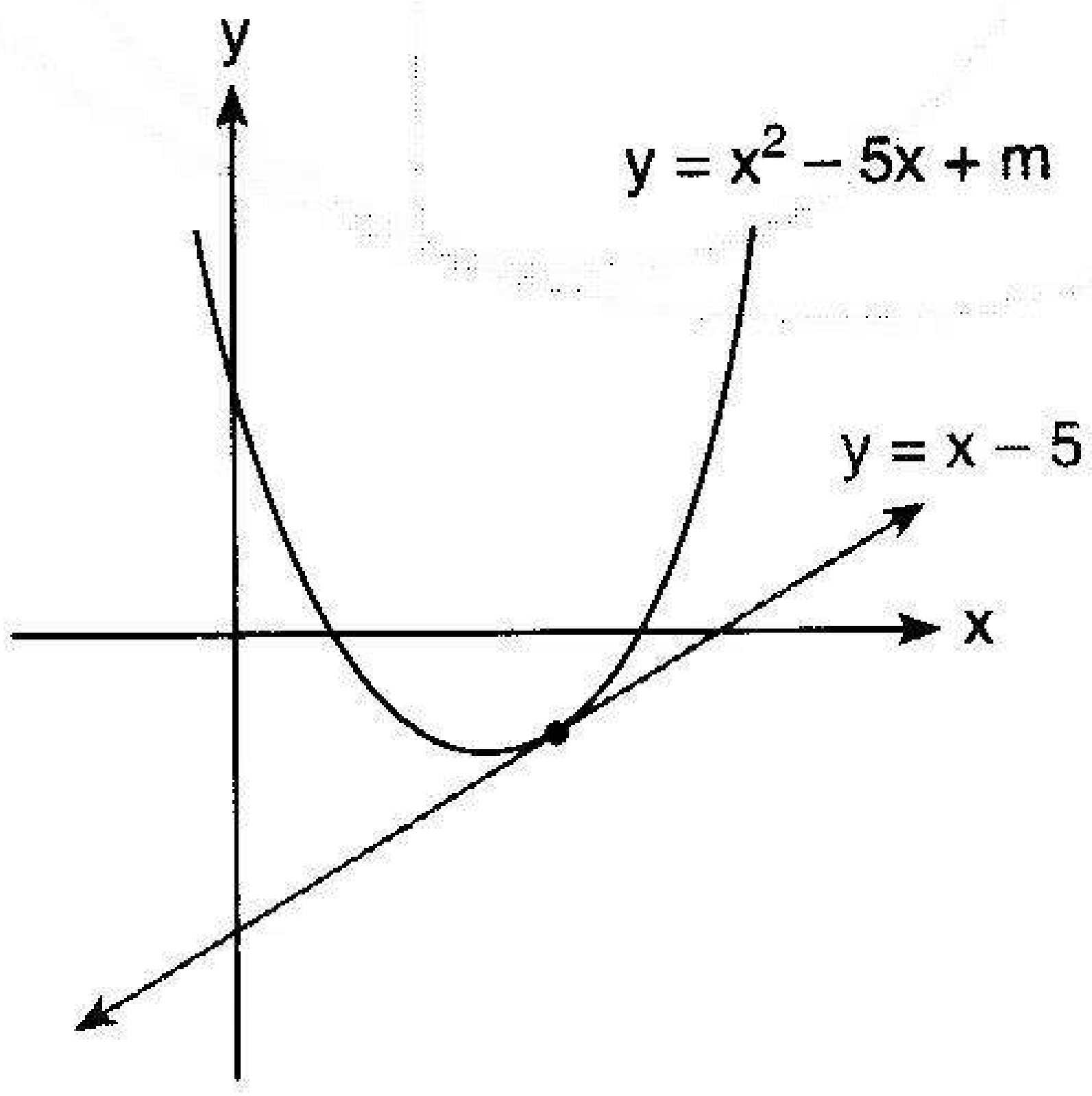
A) 2

B) 3

C) 4

D) 5

E) 6



ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\left.\begin{matrix} y = x^2 - 5x + m \\ y = x - 5 \end{matrix}\right\} \Rightarrow x^2 - 5x + m = x - 5$$

$$x^2 - 6x + m + 5 = 0$$

$\Delta = 0 \Rightarrow (-6)^2 - 4\cdot1\cdot(m + 5) = 0$

$36 - 4m - 20 = 0$

$16m = 4 \Rightarrow m = 4$

ÖRNEK / EXAMPLE 9

$m \cdot n = ?$

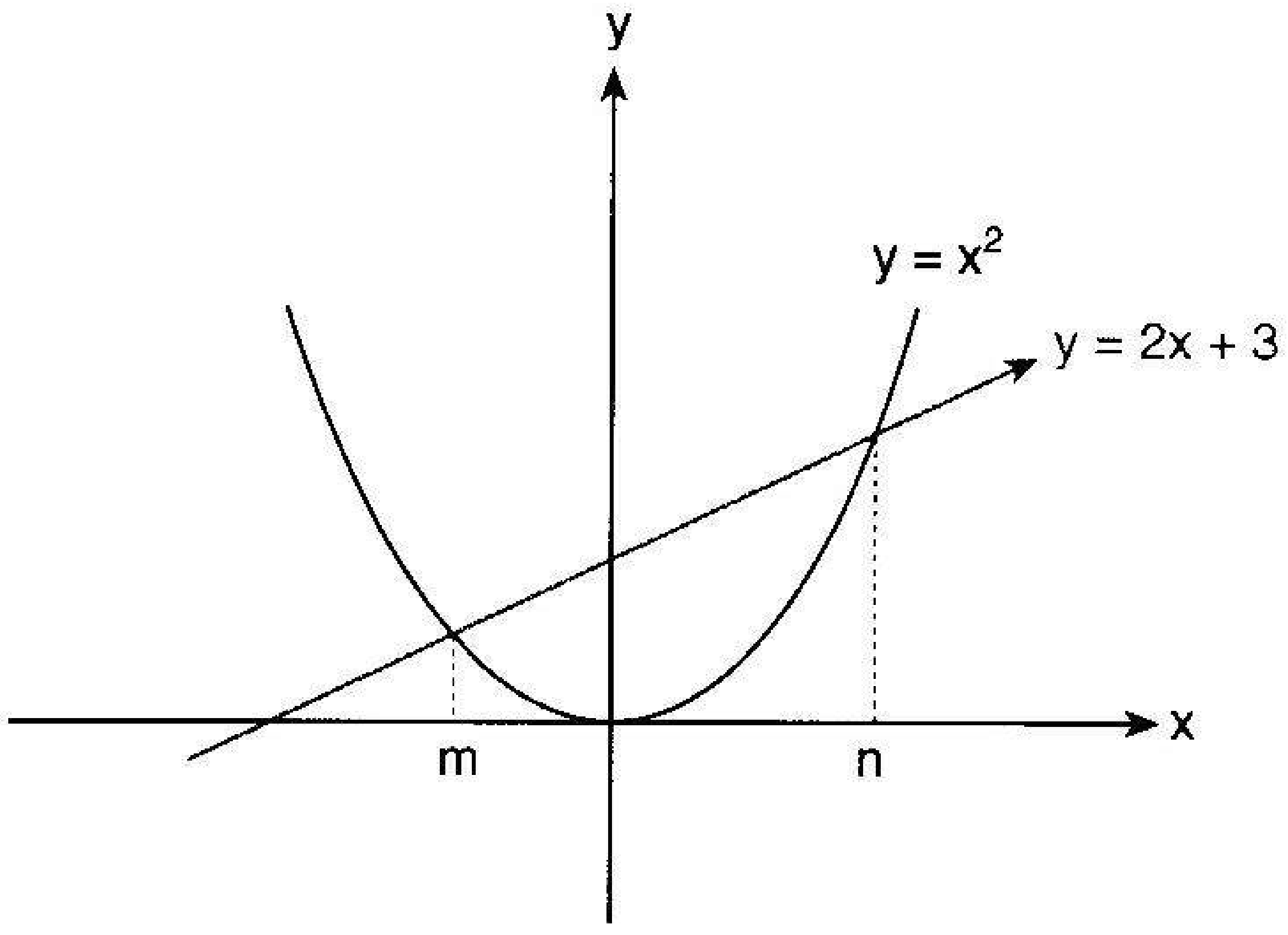
A) –3

B) –2

C) –1

D) 2

E) 3



ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

$(x - 3) \cdot (x + 1) = 0$

$x = 3 \quad , \quad x = -1$

$\Rightarrow \quad m = -1, \quad n = 3$

$m \cdot n = -3$

ÖRNEK / EXAMPLE 10

$m = ?$

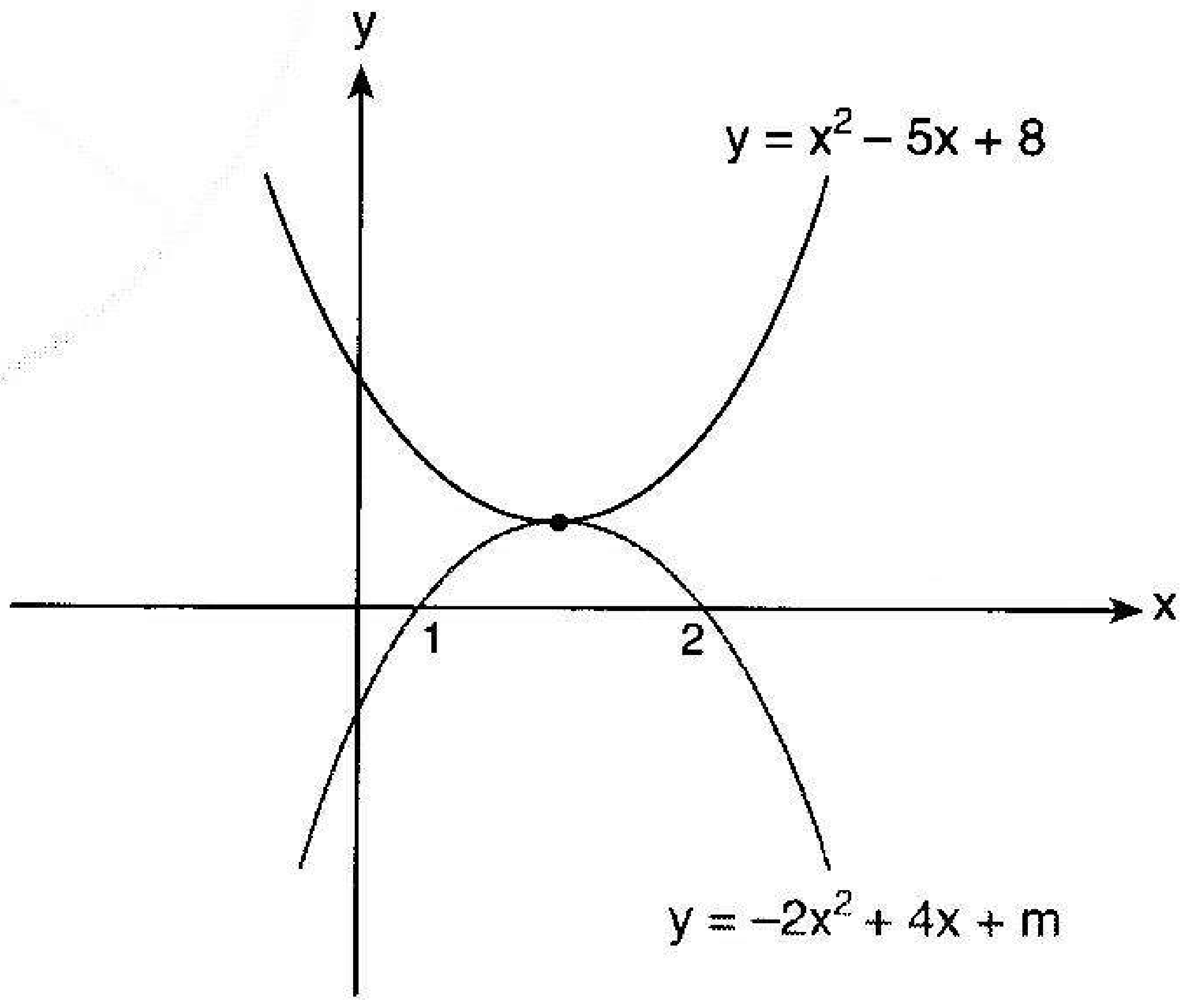
A) 1

B) $\frac{1}{4}$

C) $\frac{5}{4}$

D) $\frac{7}{4}$

E) $\frac{9}{4}$



ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$x^2 - 5x + 8 = -2x^2 + 4x + m$

$3x^2 - 9x + 8 - m = 0$

$\Delta = 81 - 4 \cdot 3 \cdot (8 - m) = 0$

$81 = 12(8 - m)$

$8 - m = \frac{27}{4} \Rightarrow m = \frac{5}{4}$

ÖRNEK / EXAMPLE 11

İSTANBUL ÜNİ. YÖS 2013

$y = ax^2 + bx + c \Rightarrow$ **b = ?**

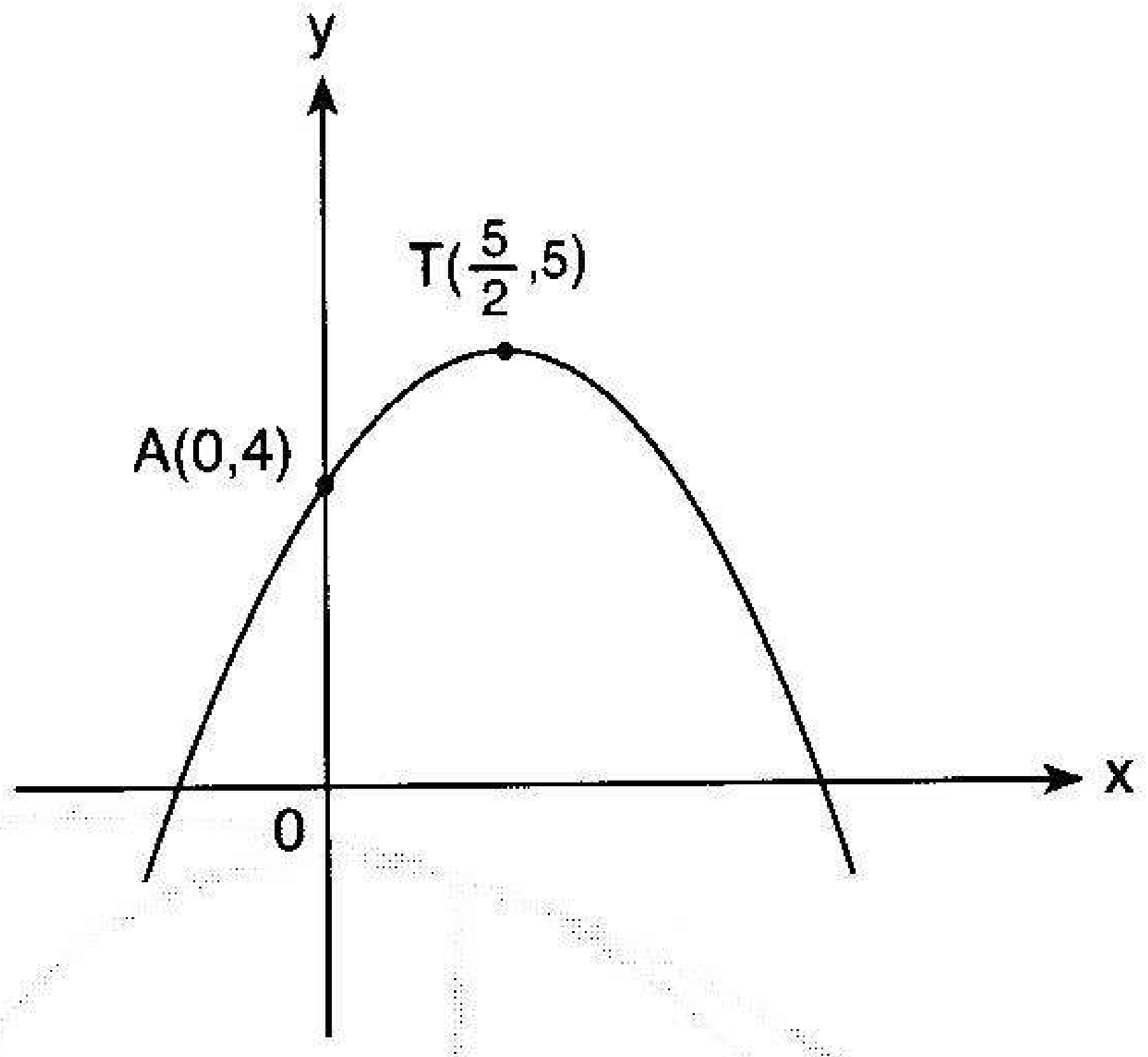
A) $-\frac{4}{25}$

B) $-\frac{4}{5}$

C) $\frac{4}{25}$

D) $\frac{4}{5}$

E) $\frac{5}{4}$



ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$f(x) = a(x-r)^2 + k$

$$f(x)=a\left(x-\frac{5}{2}\right)^2+5 \Rightarrow 4=a\left(-\frac{5}{2}\right)^2+5$$

$$\frac{25}{4}a=-1 \Rightarrow a=-\frac{4}{25}$$

$$-\frac{4}{25}(x-\frac{5}{2})^2+5=-\frac{4}{25}(x^2-5x+\frac{25}{4})+5$$

$$=\frac{-4x^2}{25}+\frac{4}{5}x+4 \Rightarrow b=\frac{4}{5}$$

ÖRNEK / EXAMPLE 12

KAYSERİ ERCİYES ÜNİV. YÖS 2015

f(–3) = ?

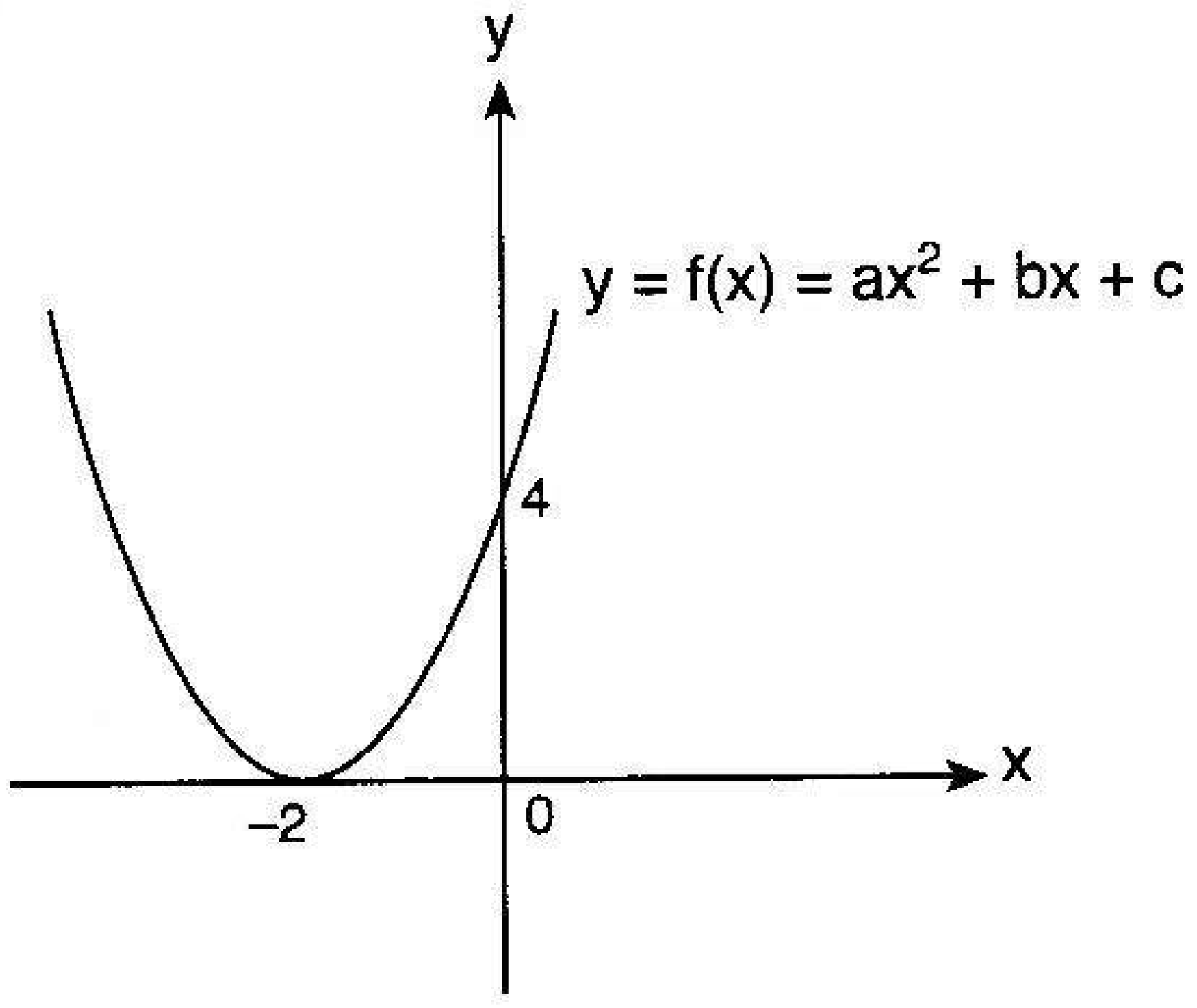
A) – 2

B) – 1

C) 0

D) 1

E) 2



ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

Tepe noktasının koordinatları / *Coordinates of vertex:* (r, k) = (– 2 , 0)

$y = a(x-r)^2 + k = a(x+2)^2 + 0$

$(0, 4) \Rightarrow 4 = a \cdot 4$

$a = 1$

$y = f(x) = (x+2)^2$

$f(-3) = (-3+2)^2 = 1$

ÖRNEK / EXAMPLE 13

MERSİN ÜNİV. YÖS 2015

a + b = ?

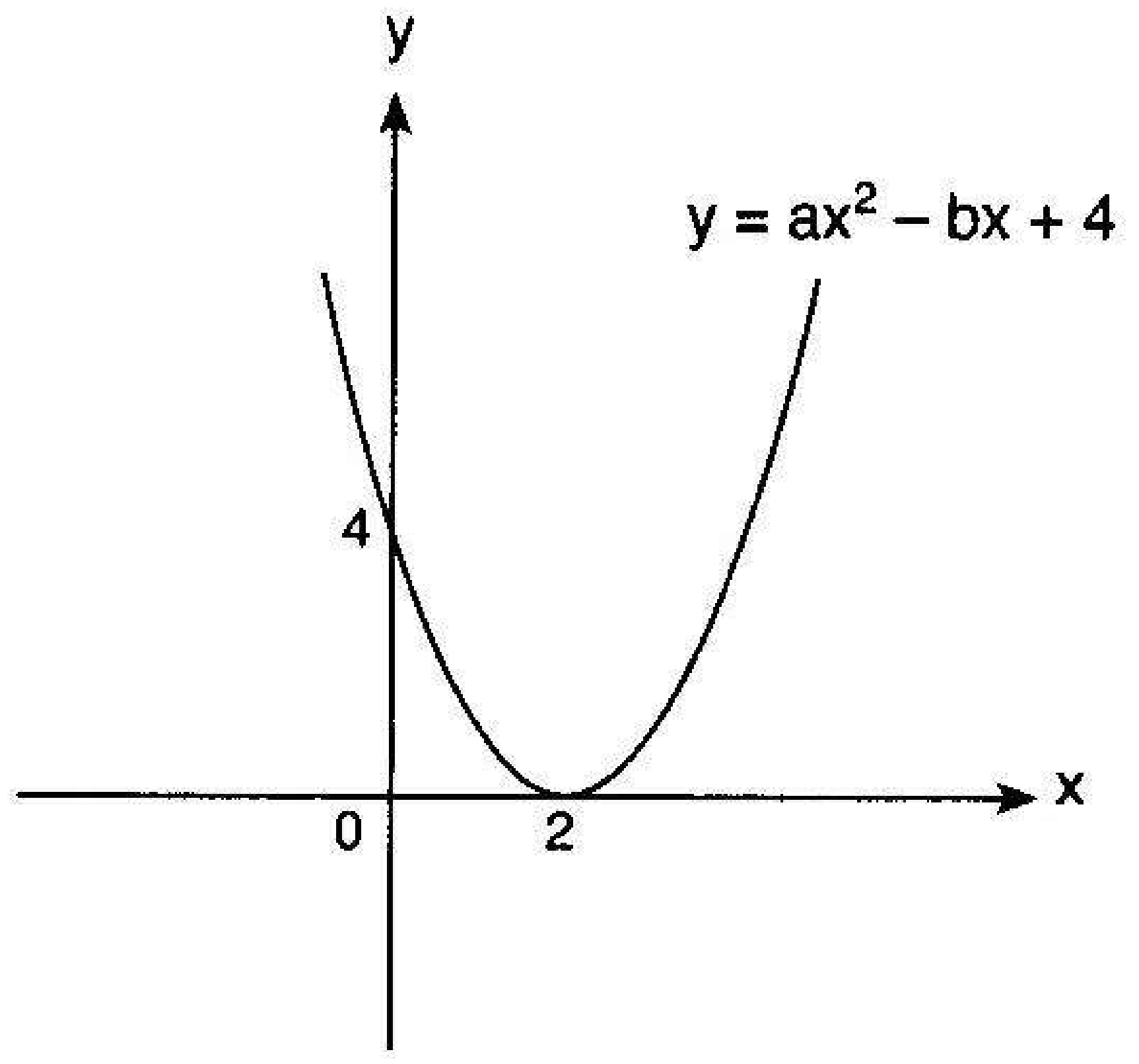
A) 4

B) 5

C) 6

D) 7

E) 8



ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

Tepe noktasının koordinatları / *Coordinates of vertex:* (r, k) = (2, 0)

$y = a(x-2)^2 + 0$

$(0, 4) \Rightarrow 4 = a\cdot(0-2)^2 \Rightarrow 4 = 4a$

$a = 1$

$y = 1(x-2)^2 = x^2 - 4x + 4 = a$

$x^2 - 4x + 4 = ax^2 + bx + 4$

$\Rightarrow a = 1$ ve (and) $b = 4$

$a + b = 5$

ÖRNEK / EXAMPLE 14

ANKARA ÜNİ. YÖS 2015

$f(x) = 2x^2 - 4x + c,$

$\min(f(x)) = 5 \Rightarrow$ **c = ?**

A) 0 B) 1 C) 2 D) 5 E) 7

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$y = ax^2 + bx + C \Rightarrow \min f(x) = \dfrac{4aC - b^2}{4a}$

$y = 2x^2 - 4x + C \Rightarrow$

$\min f(x) = \dfrac{4\cdot 2\cdot C - (-4)^2}{4\cdot 2}$

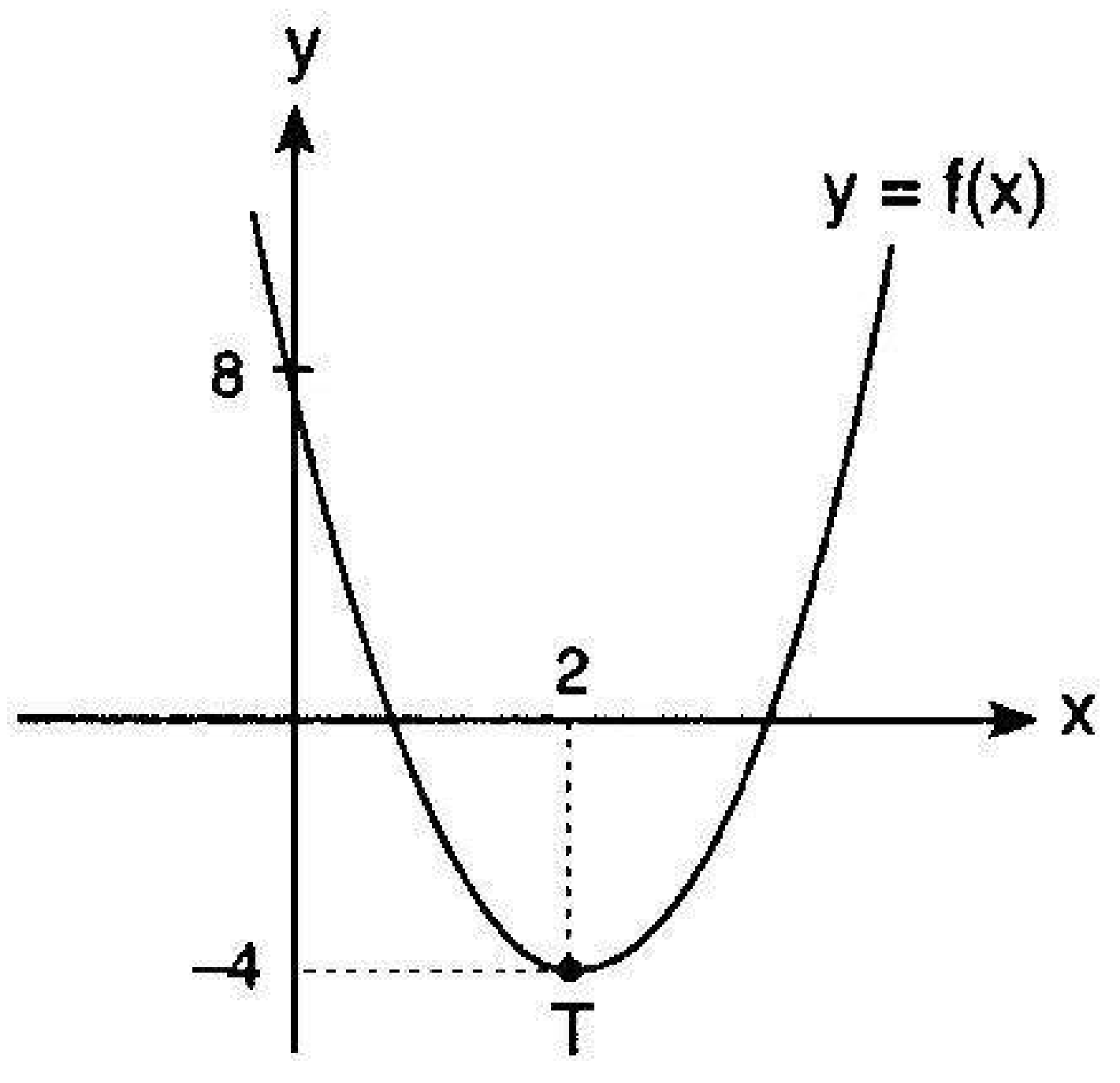
$5 = \dfrac{8\cdot C - 16}{8}$

$40 = 8\cdot C - 16$

$56 = 8\cdot C \Rightarrow C = 7$

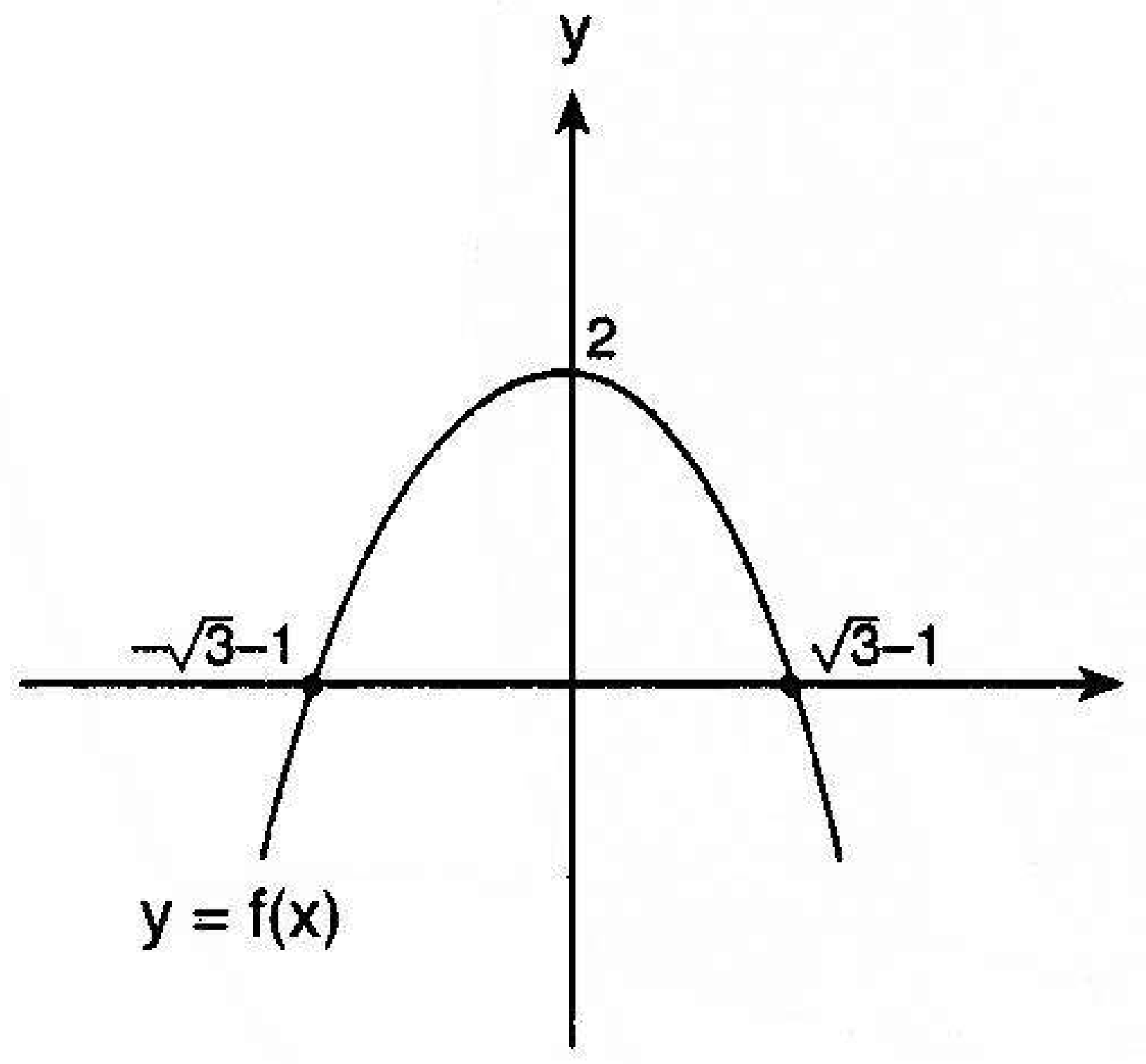
1. **$f(1) = ?$**

A) -1
B) -3
C) -5
D) -7
E) -9



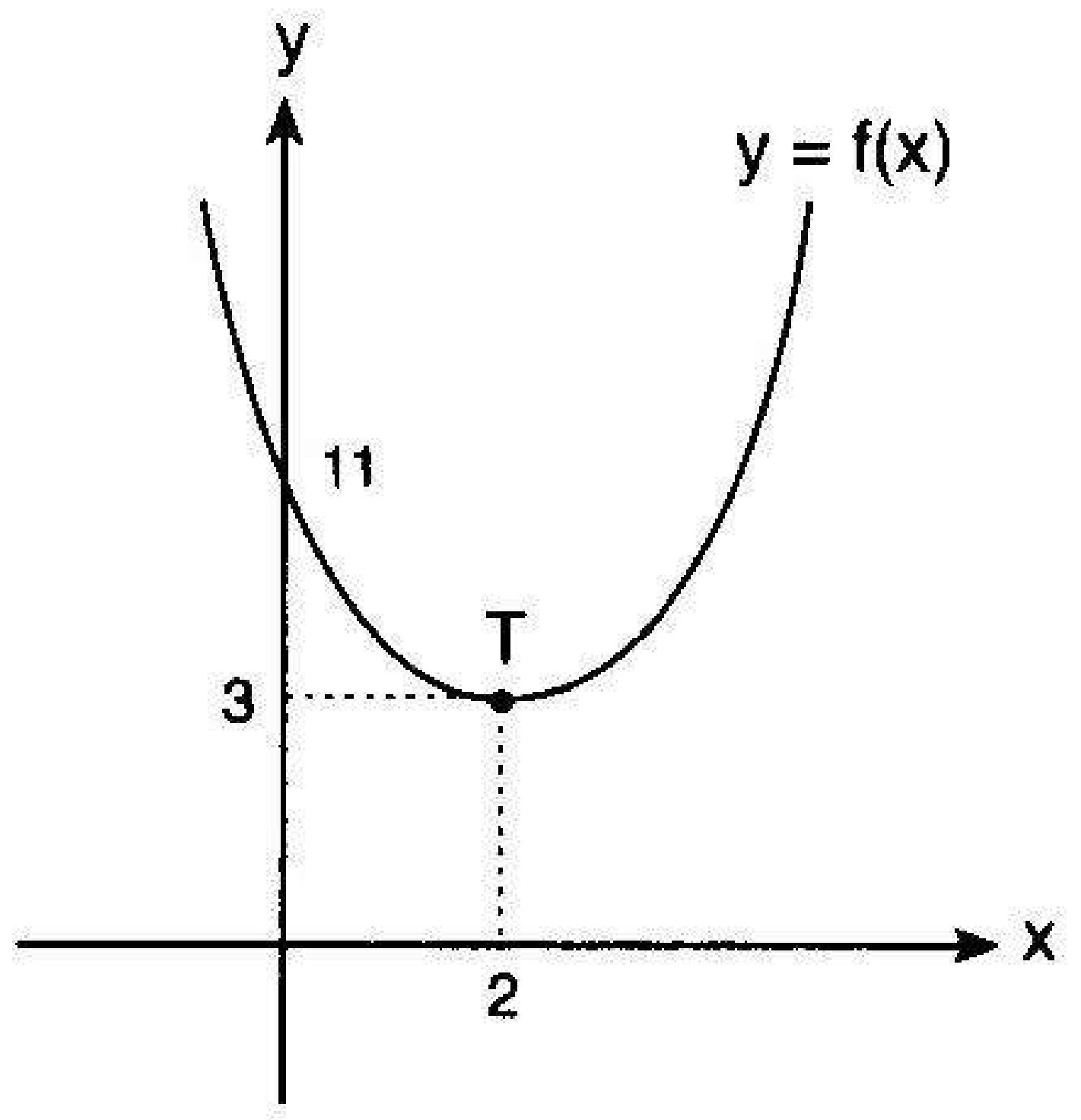
2. **$f(1) = ?$**

A) -1
B) -2
C) -3
D) -4
E) -5



3. **$f(3) = ?$**

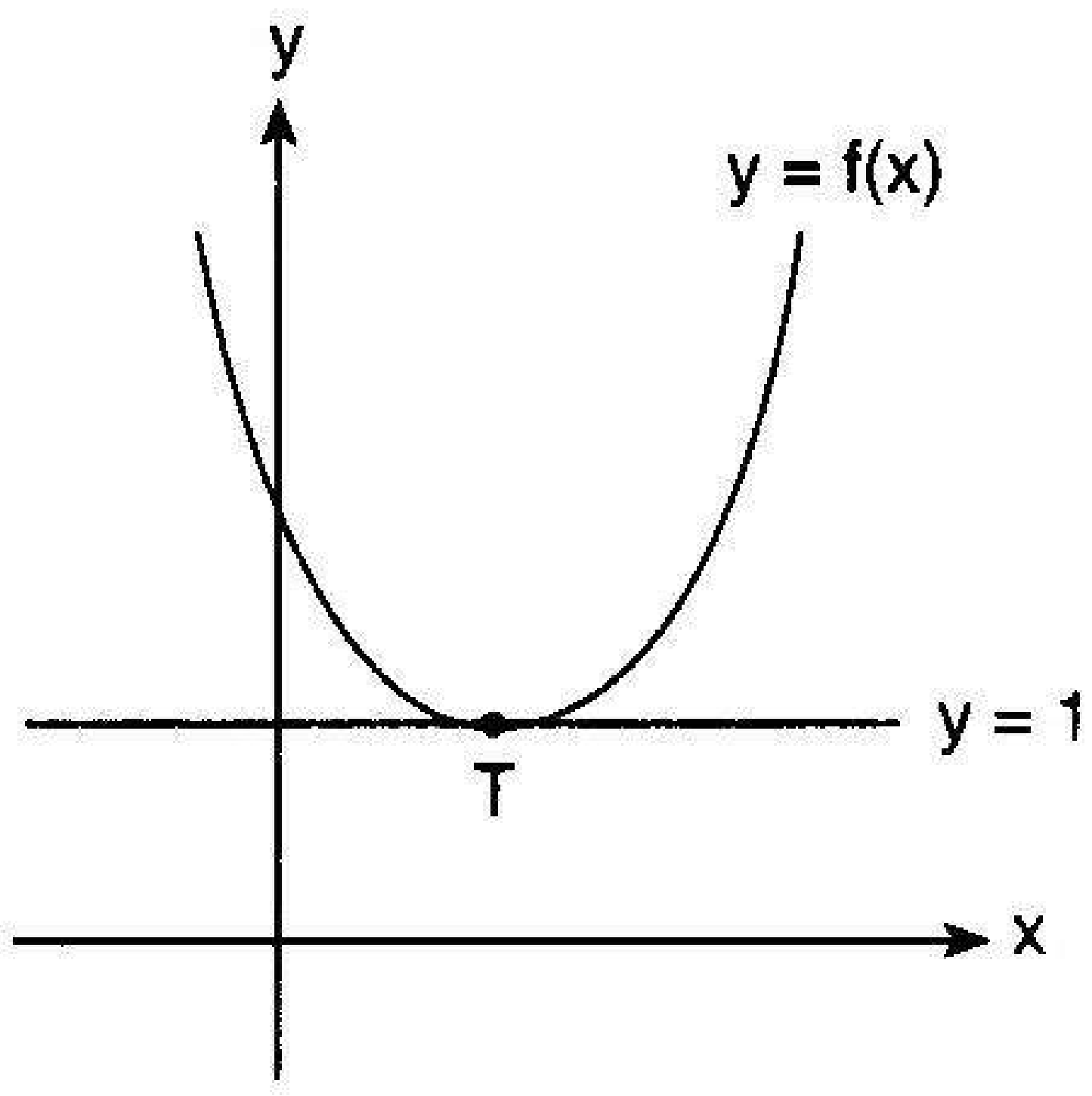
A) 4
B) 5
C) 6
D) 7
E) 8


y
y = f(x)
11
T
3
2
x

4. $f(x) = mx^2 - 2mx + 3$

$\Rightarrow$ **$m = ?$**

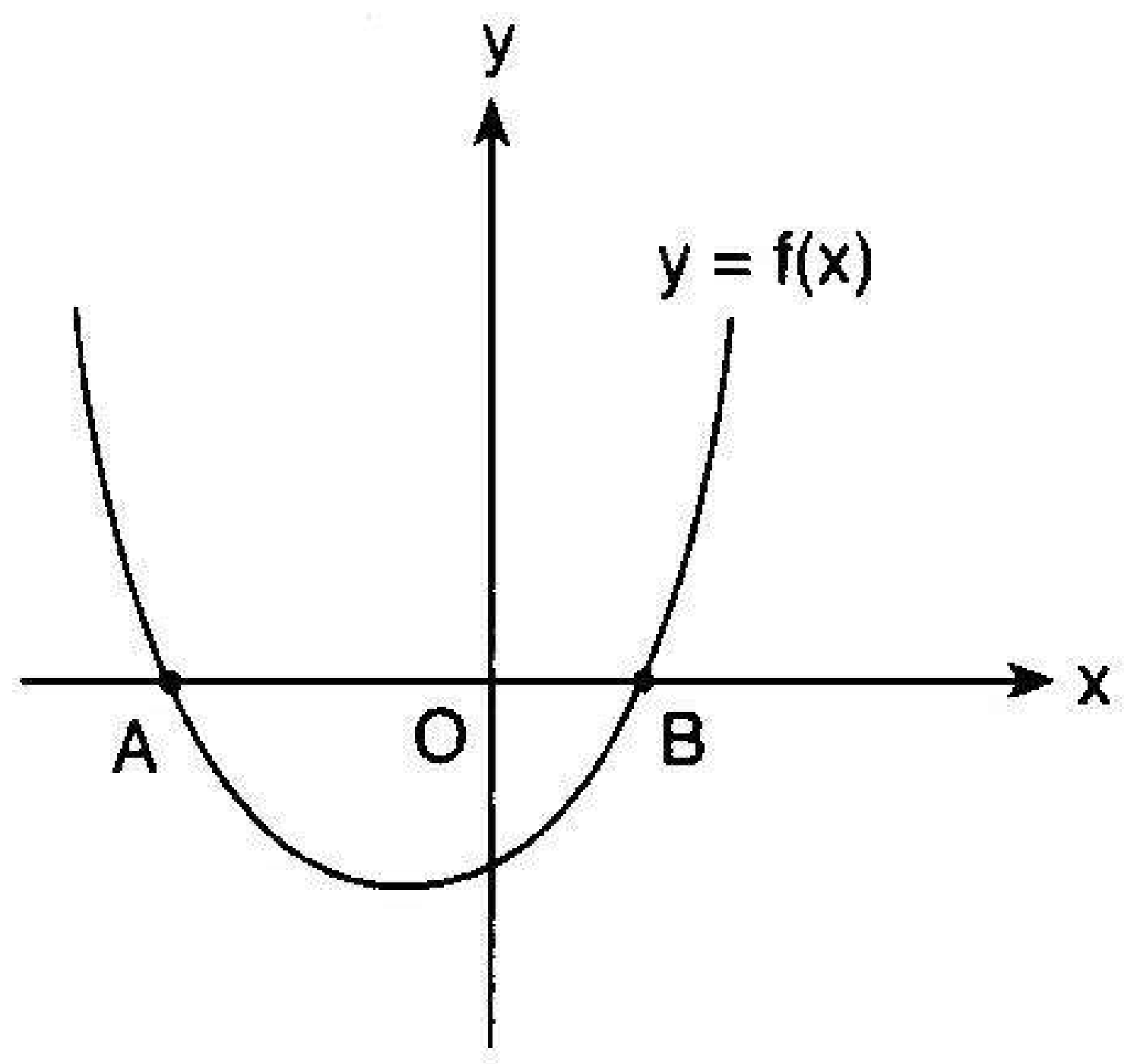
A) 2
B) 4
C) 6
D) 8
E) 10



5. $y = x^2 + 4x - 8m$,

$|AO| = 2|OB| \Rightarrow$ **$m = ?$**

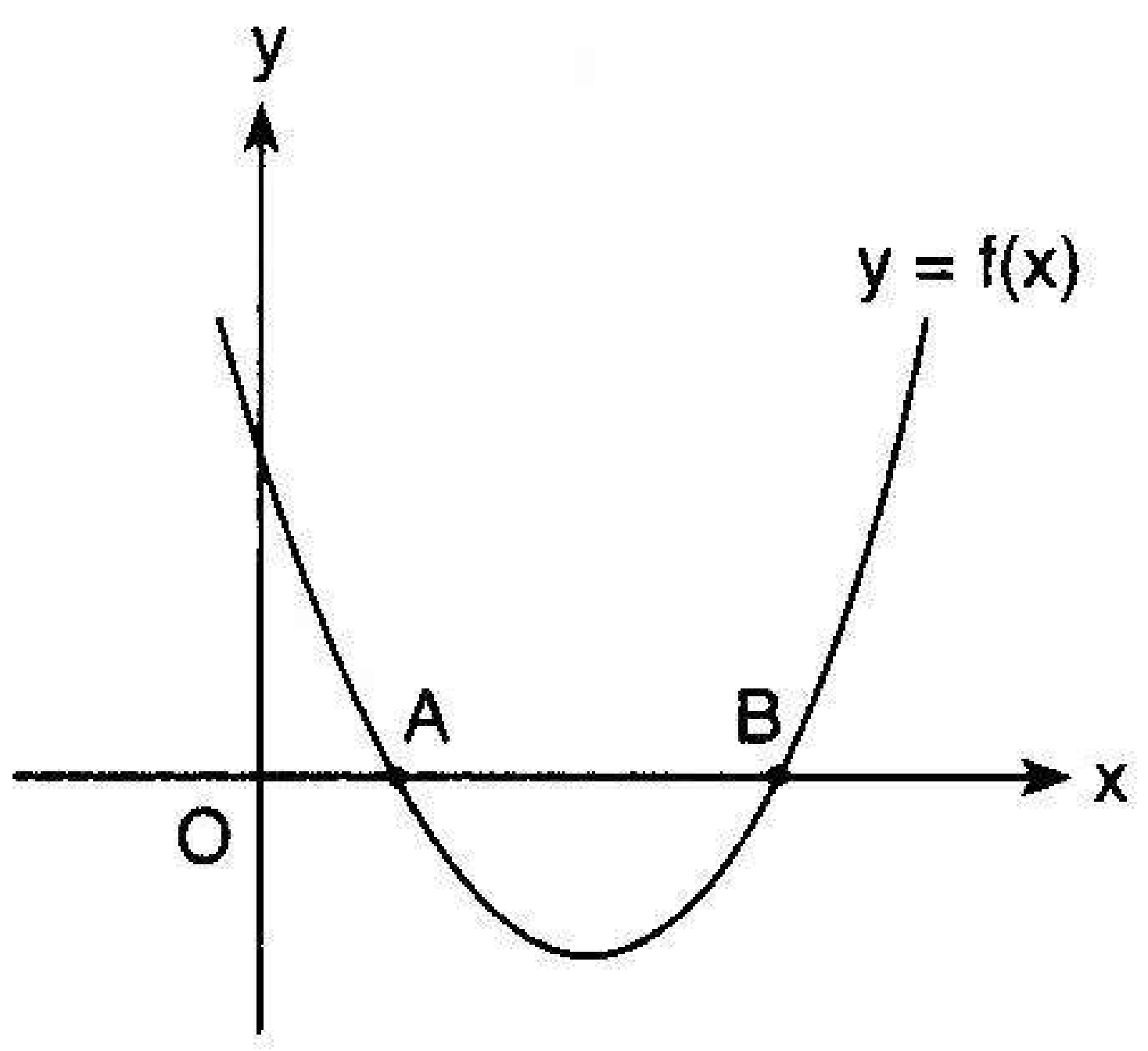
A) 1
B) 2
C) 3
D) 4
E) 5



6. $f(x) = 2x^2 - 16x + m$,

$2|OA| = 3|AB| \Rightarrow$ **$m = ?$**

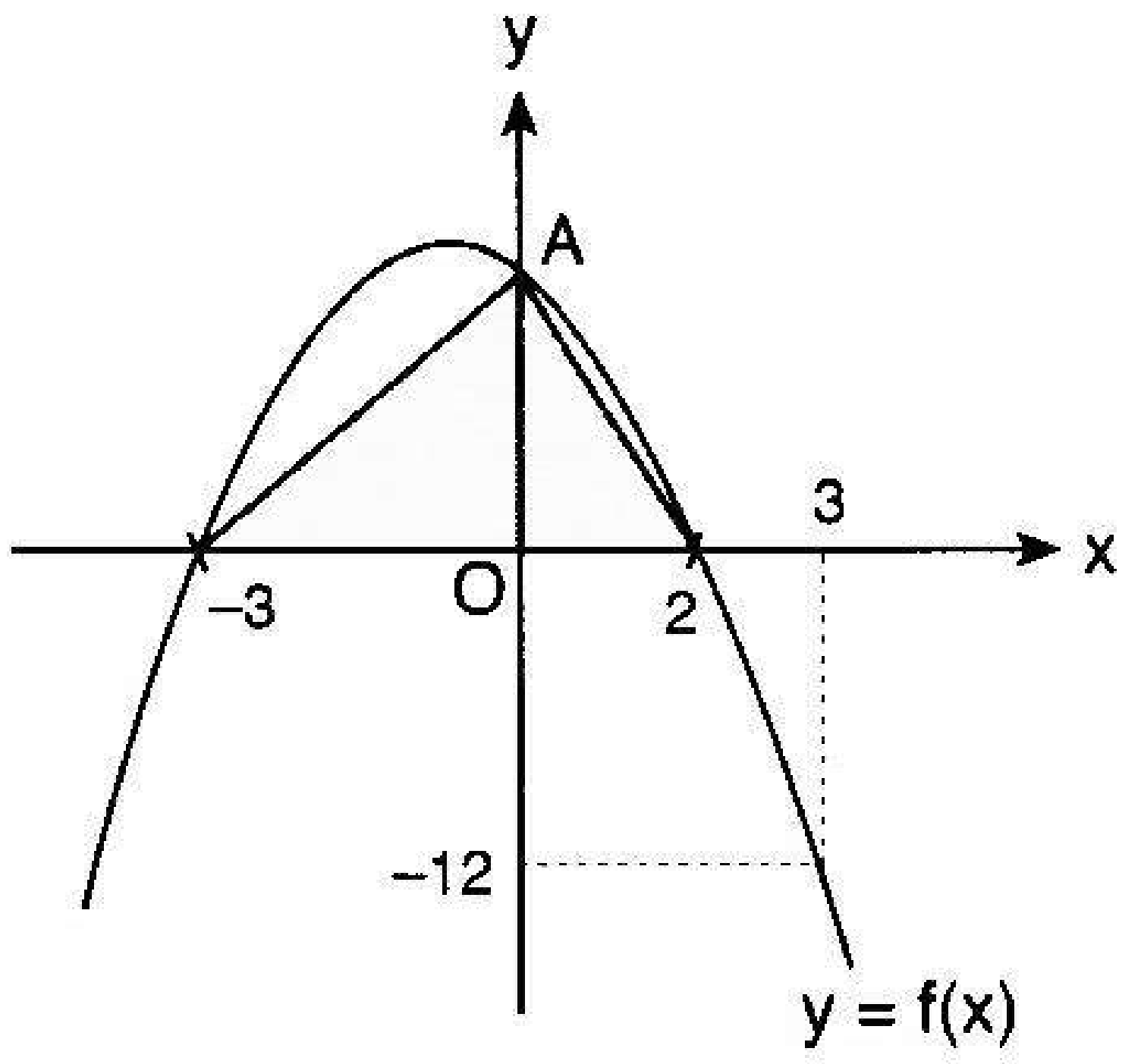
A) 27
B) 30
C) 32
D) 36
E) 39



7. **Taralı Alan**

The Shaded Area = ?

A) 24
B) 27
C) 28
D) 30
E) 32

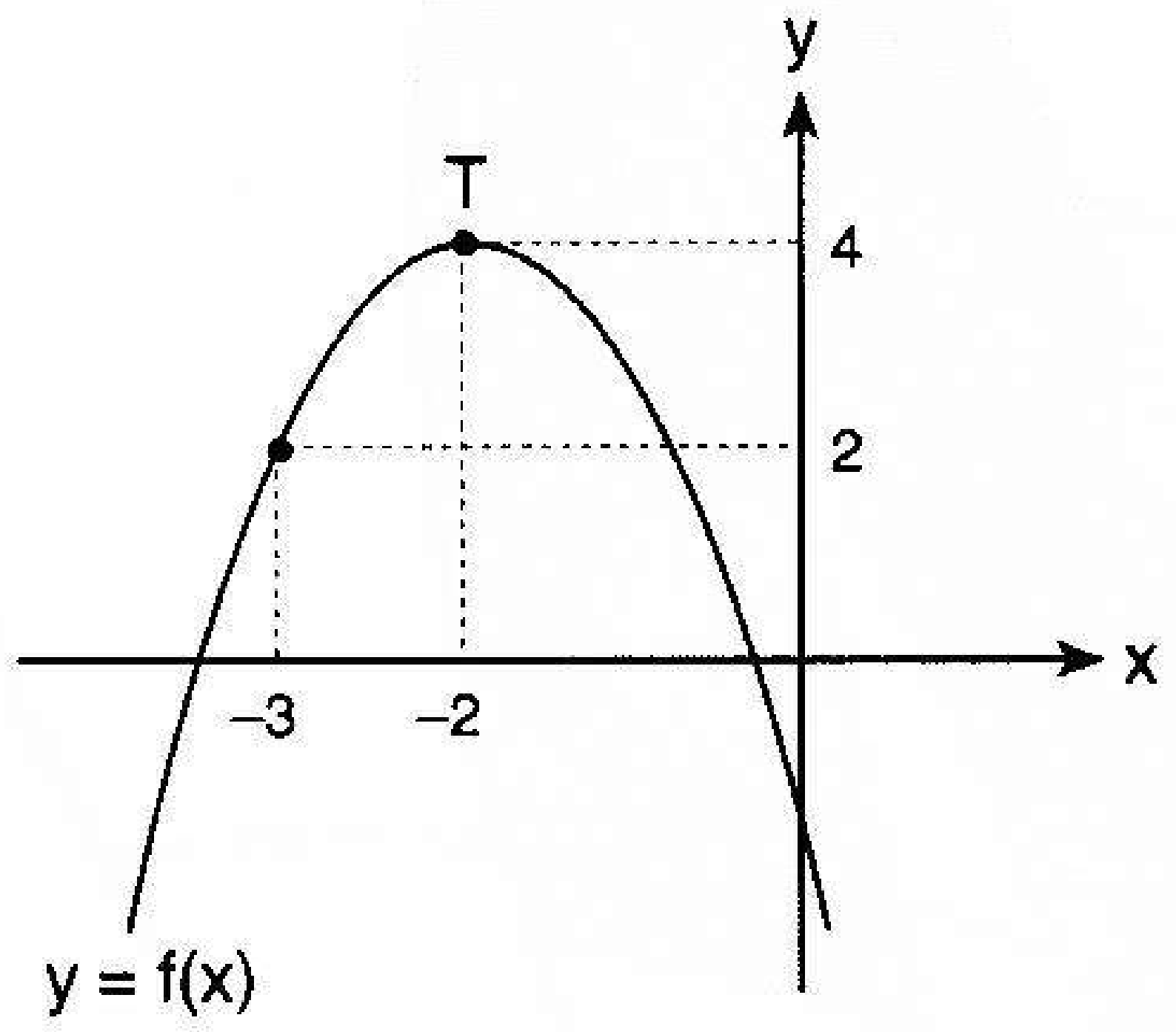

y
A
3
x
–3
O
2
–12
y = f(x)

---

8. $f(x) = ax^2 + bx + c$

$\Rightarrow$ **$a + b + c$ = ?**

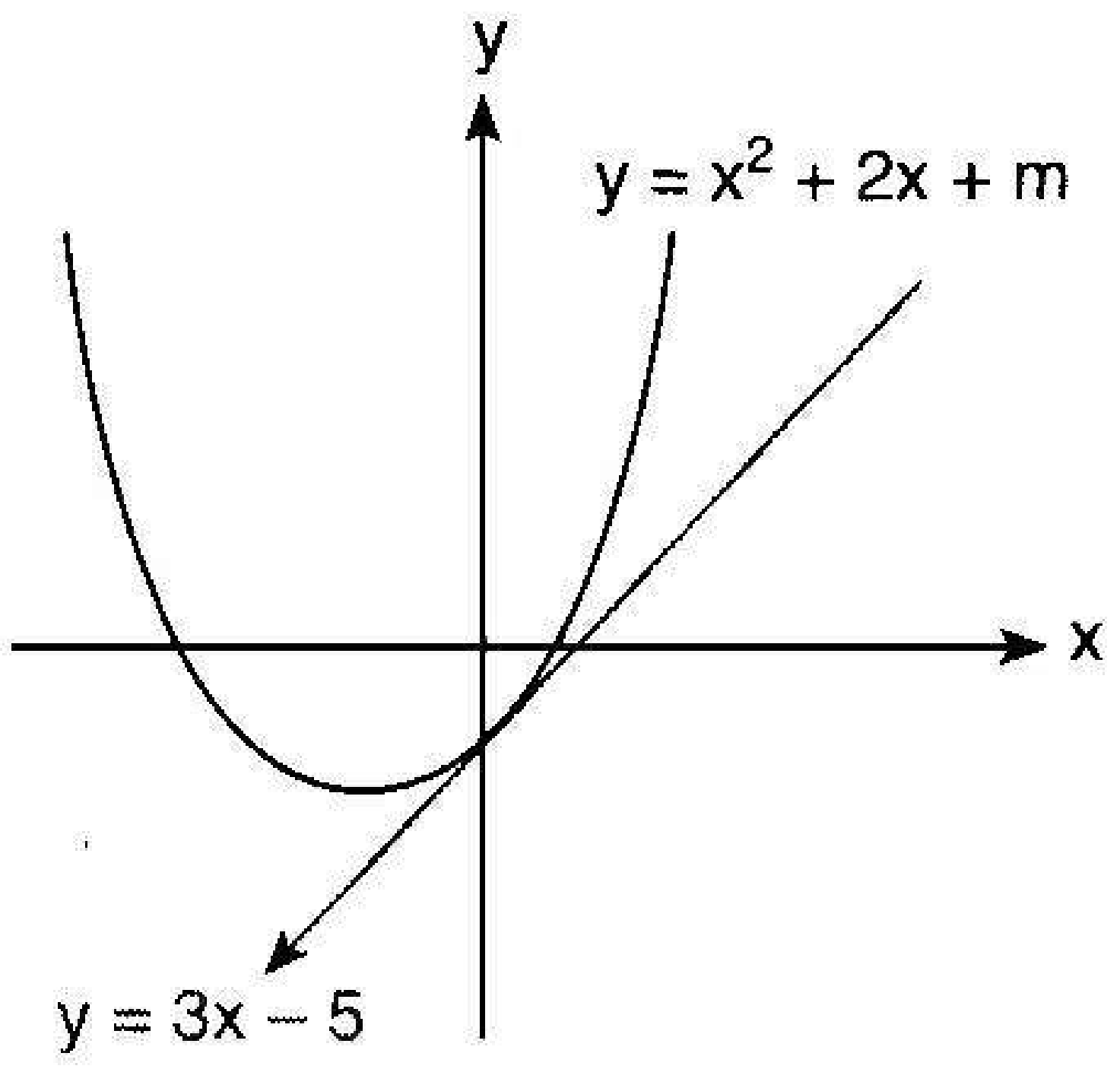
A) –6
B) –8
C) –10
D) –12
E) –14


y
T
4
2
x
–3
–2
y = f(x)

---

9. **m = ?**

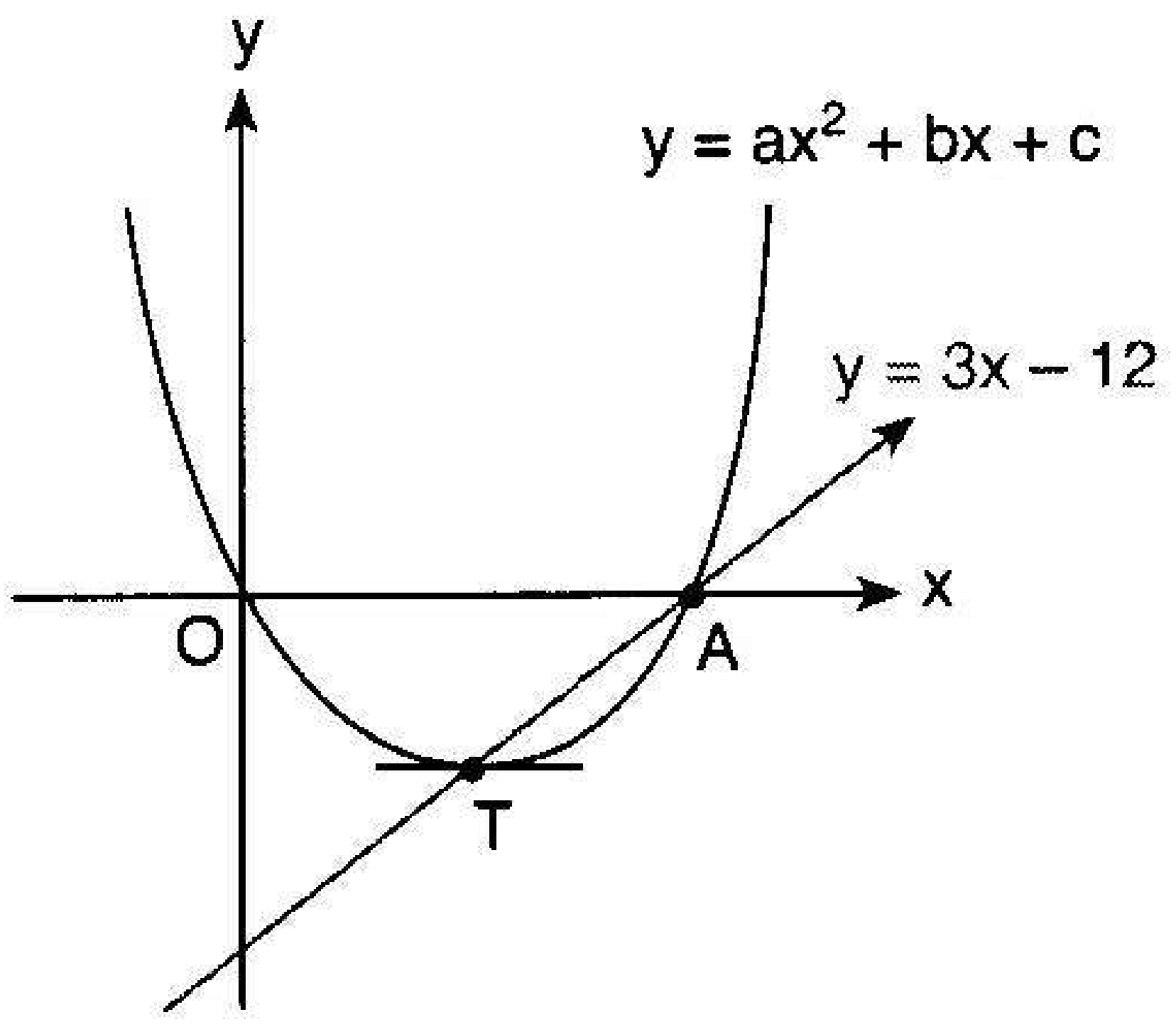
A) $-\frac{11}{4}$
B) $-\frac{13}{4}$
C) $-\frac{15}{4}$
D) $-\frac{17}{4}$
E) $-\frac{19}{4}$


y
y = x² + 2x + m
x
y = 3x – 5

---

10. **$a \cdot b$ = ?**

A) –9
B) –10
C) –11
D) –12
E) –13


y
y = ax² + bx + c
y = 3x – 12
x
O
A
T

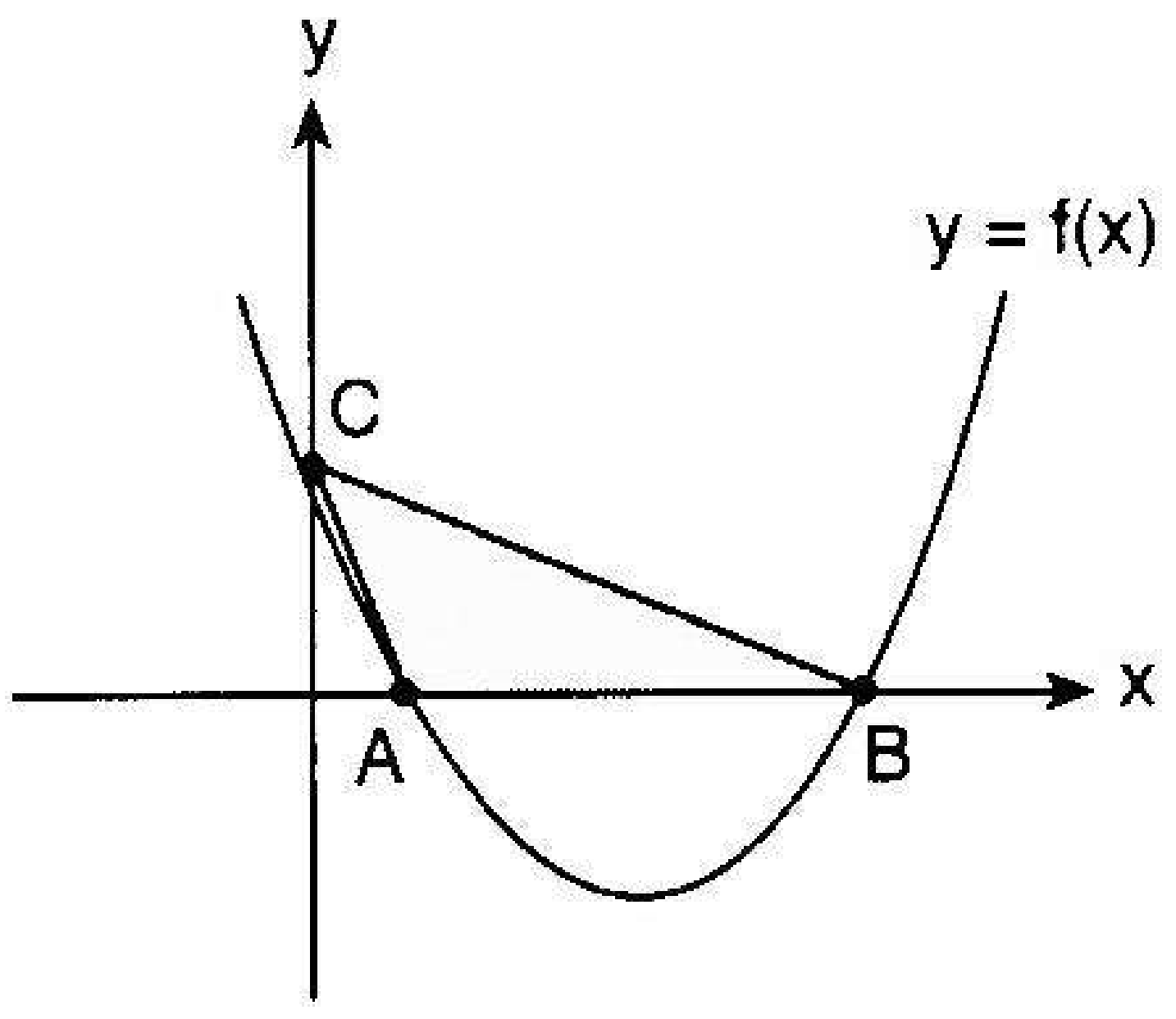
---

11. $y = 2x^2 - 12x + 10$

taralı alan

the shaded area **= ?**

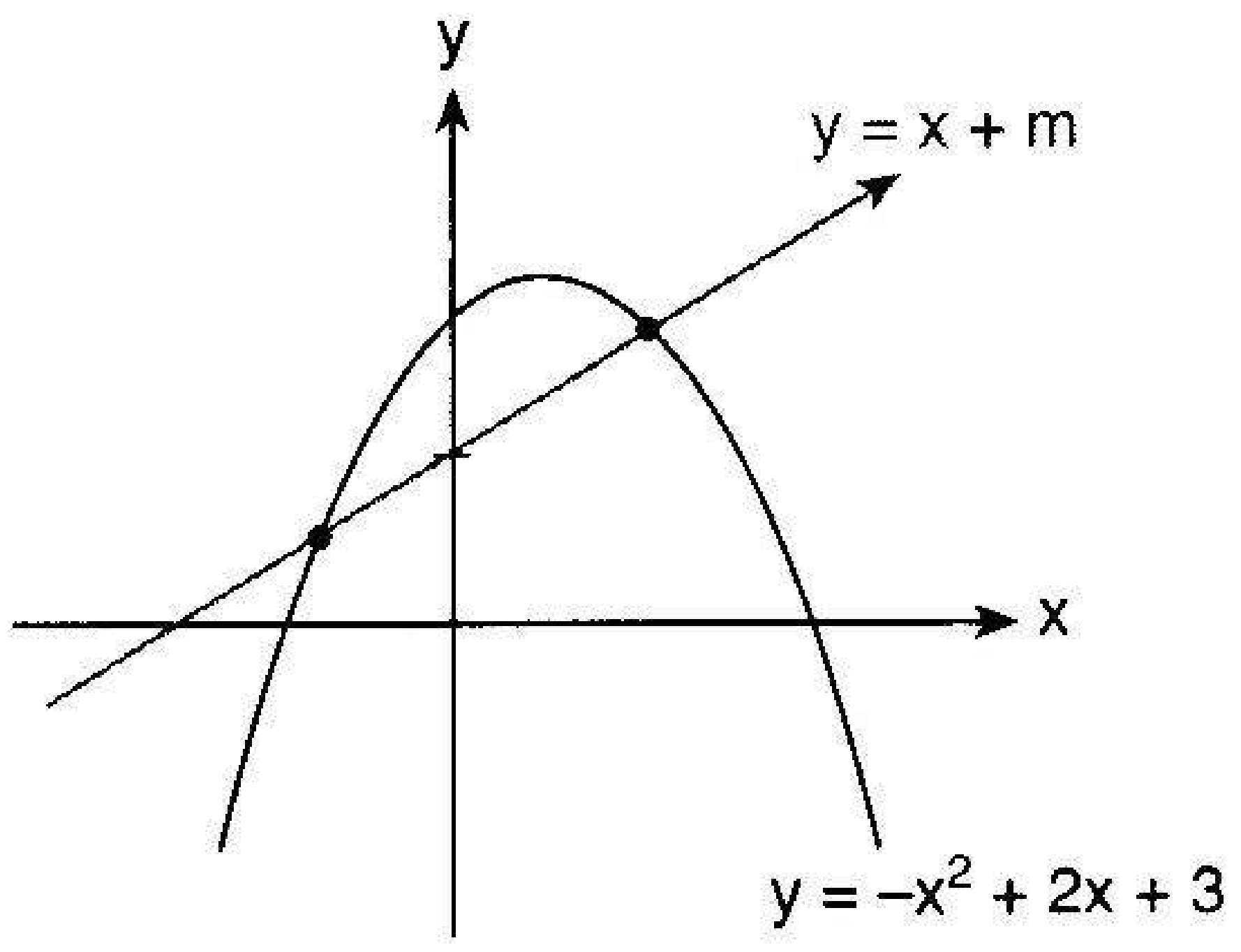
A) 20
B) 30
C) 40
D) 50
E) 60



---

12. $m \in Z \Rightarrow$ **max(m) = ?**

A) 1
B) 2
C) 3
D) 4
E) 5


y
y = x + m
x
y = –x² + 2x + 3

YANITLAR / ANSWERS

1. A	2. A	3. B	4. A	5. D	6. B	7. D	8. E
9. E	10. A	11. A	12. C				

1. $y = a\cdot(x-r)^2 + k$

$= a\cdot(x-2)^2 + (-4)$

$= a(x^2 - 4x + 4) + (-4)$

$= ax^2 - 4ax + 4a - 4$

$4a - 4 = 8 \Rightarrow a = 3$

$f(x) = 3x^2 - 12x + 8$

$f(1) = 3 - 12 + 8 = -1$

Yanıt / The Answer is A

2. $f(x) = a(x + (\sqrt{3}+1))\cdot(x - (\sqrt{3}-1))$

$= a(x^2 - x(\sqrt{3}-1) + x(\sqrt{3}+1) - 2)$

$= a(x^2 + 2x - 2)$

$= ax^2 + 2ax - 2a$

$-2a = 2 \Rightarrow a = -1$

$f(x) = -x^2 - 2x + 2$

$f(1) = -1 - 2 + 2 = -1$

Yanıt / The Answer is A

3. $f(x) = a\cdot(x-r)^2 + k$

$= a\cdot(x-2)^2 + 3$

$= ax^2 - 4ax + 4a + 3$

$4a + 3 = 11 \Rightarrow a = 2$

$f(3) = 5$

Yanıt / The Answer is B

4.
$$\left.\begin{aligned} y &= mx^2 - 2mx + 3 \\ y &= 1 \end{aligned}\right\} \Rightarrow \begin{aligned} &mx^2 - 2mx + 3 = 1 \\ &mx^2 - 2mx + 2 = 0 \\ &\Delta = 0 \Rightarrow 4m^2 - 4(m\cdot 2) = 0 \\ &\qquad 4m^2 - 8m = 0 \\ &\qquad m = 2 \end{aligned}$$

Yanıt / The Answer is A

5. $|AO| = 2\cdot|OB| \Rightarrow |AO| = 2k$

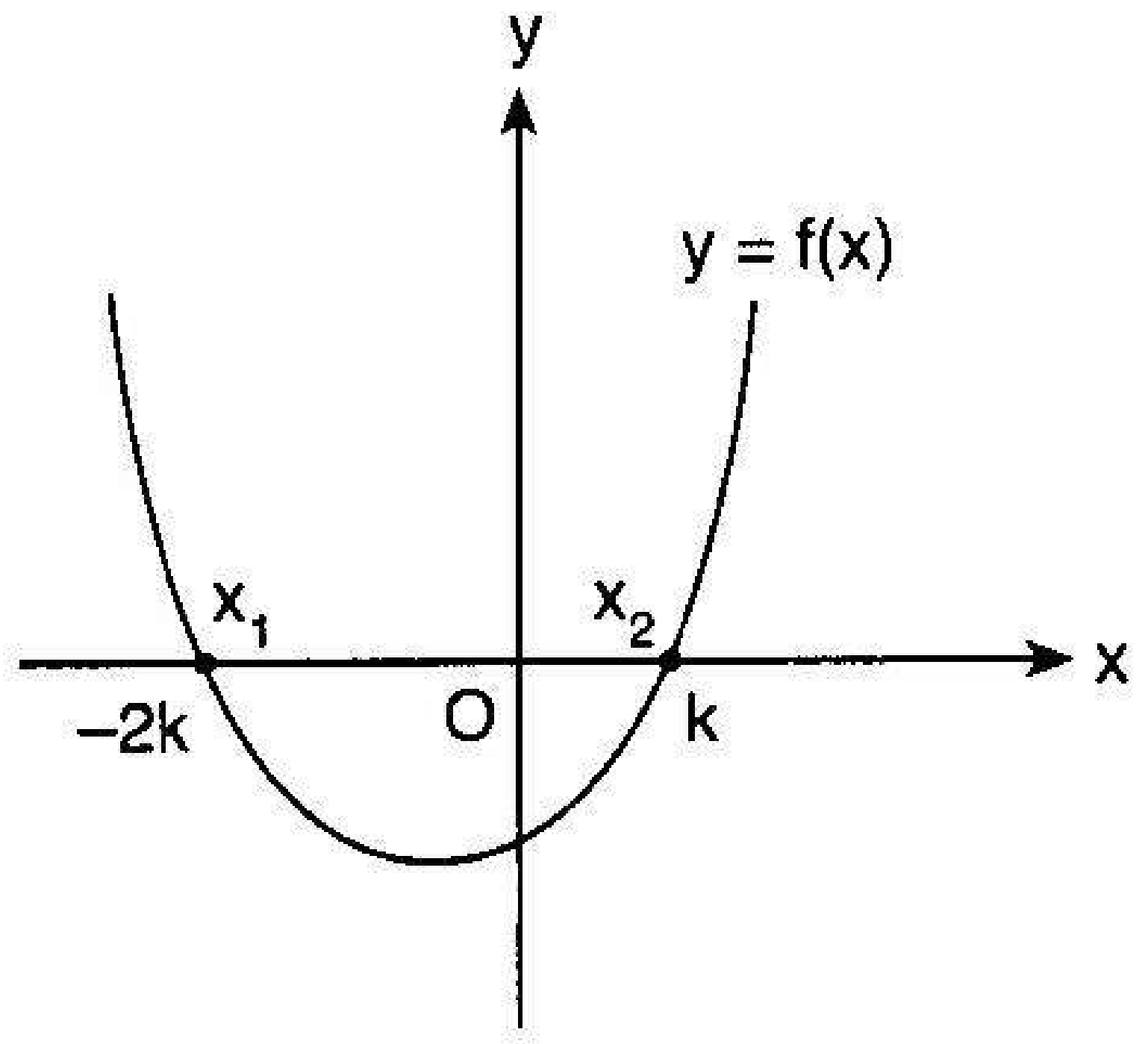
$|OB| = k$

$x_1 + x_2 = -2k + k = -\dfrac{b}{a}$

$x_1 \cdot x_2 = -2k\cdot k = -32$

$\dfrac{-8m}{1} = -32$

$m = 4$



Yanıt / The Answer is D

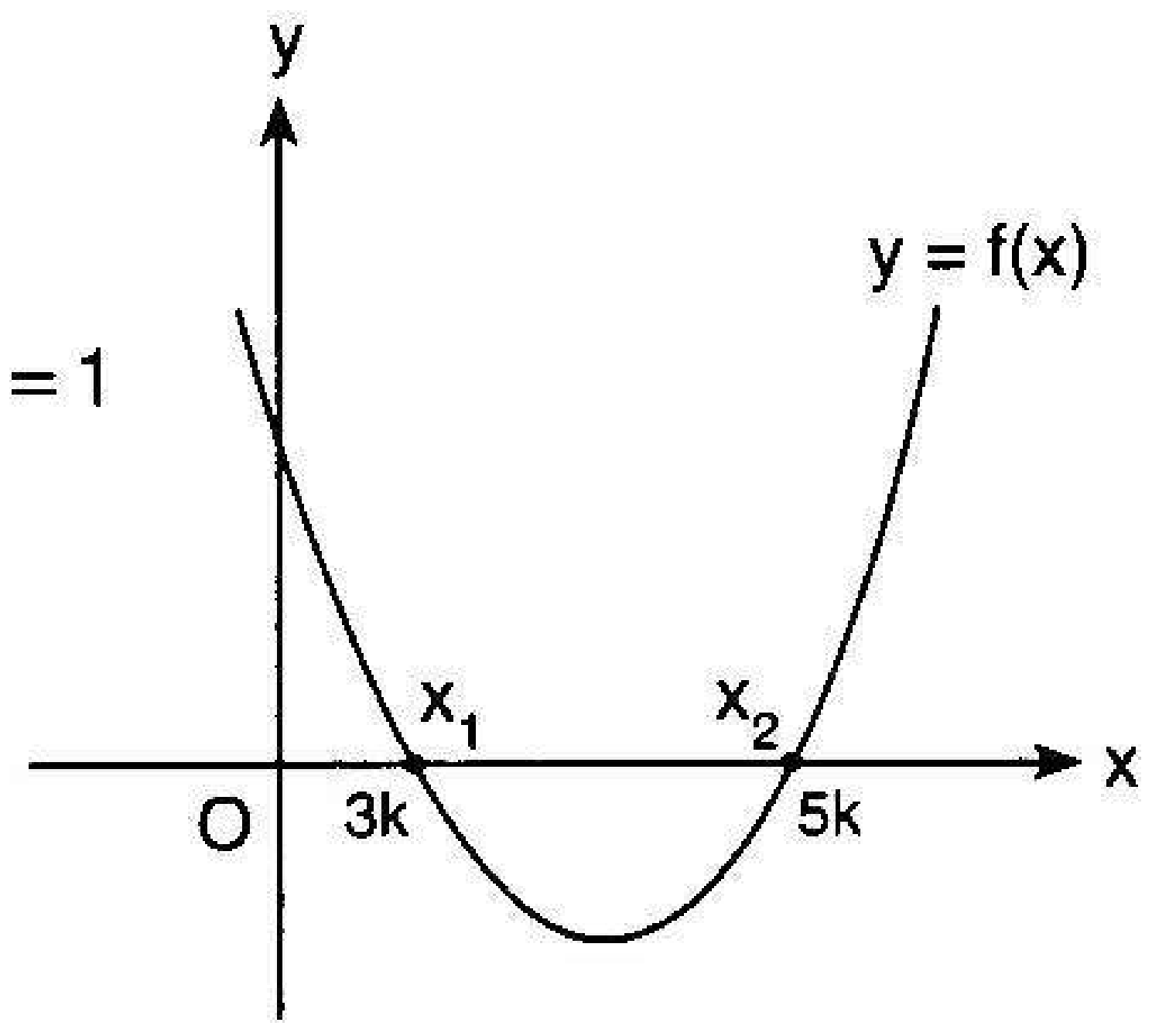
6. $2\cdot|OA| = 3\cdot|AB| \Rightarrow |AO| = 3k$

$|AB| = 2k$

$x_1 + x_2 = 3k + 5k = 8k = \dfrac{16}{2} \Rightarrow k = 1$

$x_1 \cdot x_2 = 3\cdot 5 = 15 = \dfrac{m}{2}$

$m = 30$



Yanıt / The Answer is B

7. $f(x) = a\cdot(x-2)\cdot(x+3)$

$= a\cdot(x^2 + x - 6)$

$= ax^2 + ax - 6a$

$(3,-12) \in f(x) \Rightarrow -12 = a\cdot 9 + 3a - 6a$

$f(x) = -2x^2 - 2x + 12 \Rightarrow |OA| = 12$

$TA/SA = \dfrac{12\cdot 5}{2} = 30\ br^2/u^2$

Yanıt / The Answer is D

8. $f(x) = a\cdot(x-r)^2 + k$

$= a\cdot(x+2)^2 + 4$

$= a\cdot(x^2 + 4x + 4) + 4$

$= ax^2 + 4ax + 4a + 4$

$(-3, 2) \in f(x) \Rightarrow 2 = 9a - 12a + 4a + 4$

$a = -2$

$f(x) = -2x^2 - 8x - 4$

$\Rightarrow a + b + c = -2 - 8 - 4 = -14$

Yanıt / The Answer is E

9. $\left.\begin{array}{l} y = x^2 + 2x + m \\ y = 3x - 5 \end{array}\right\} \Rightarrow \quad x^2 + 2x + m = 3x - 5$

$x^2 - x + m + 5 = 0$

$\Delta = 0 \Rightarrow (-1)^2 - 4\cdot 1\cdot(m+5) = 0$

$1 - 4m - 20 = 0$

$m = -\dfrac{19}{4}$

Yanıt / The Answer is E

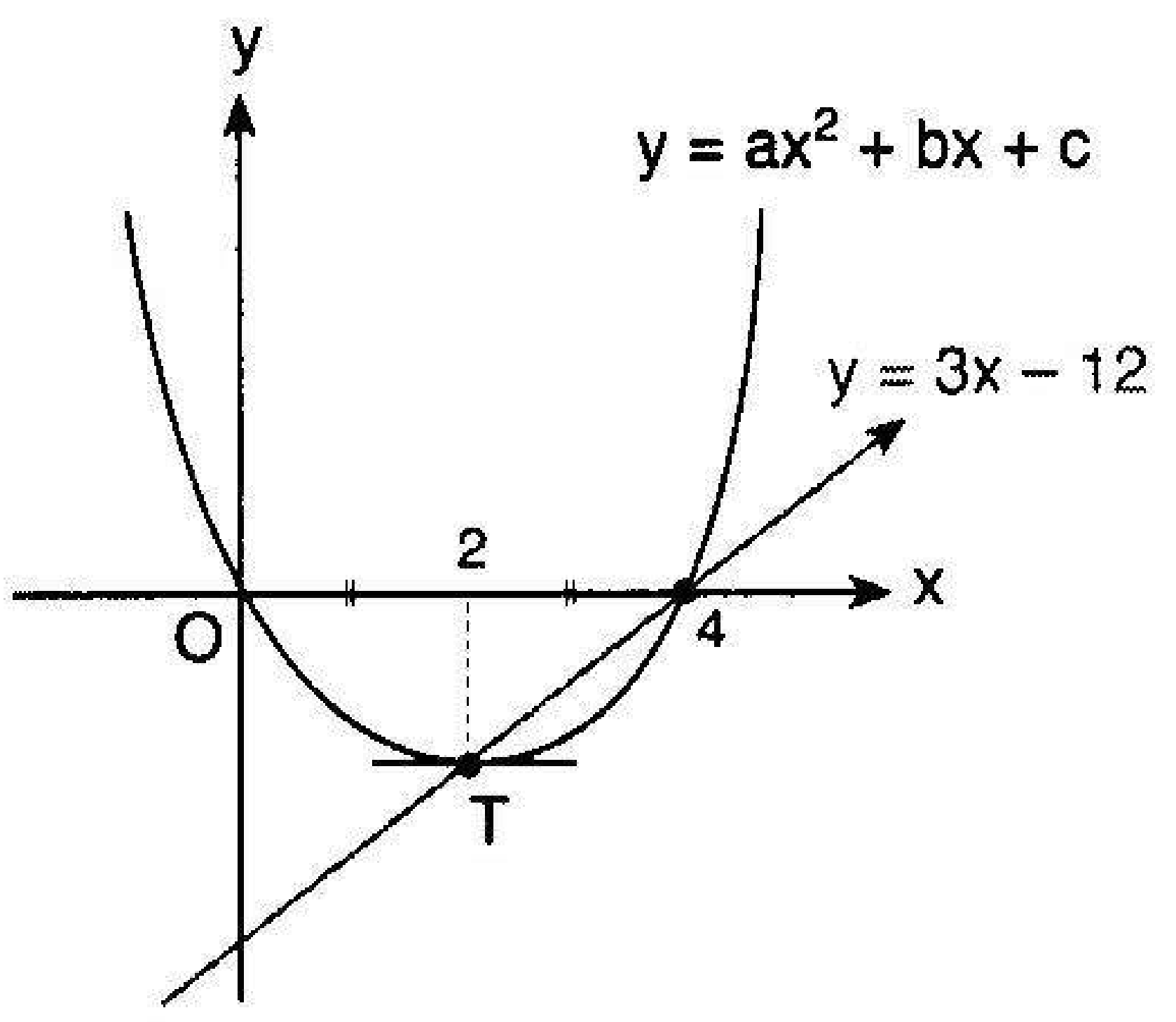
10.

$y = 3x - 12 = 0$

$x = 4$

$x = 4 \Rightarrow T(2, -6)$

$-\dfrac{b}{2a} = 2 \Rightarrow -b = 4a$



$f(x) = a(x)\cdot(x-4) = a(x^2 - 4x)$

$= ax^2 - 4ax \Rightarrow -6 = 4a - 8a \Rightarrow a = \dfrac{3}{2}$

$b = -4\cdot\dfrac{3}{2} = -6$

$a\cdot b = \dfrac{3}{2}\cdot -6 = -9$

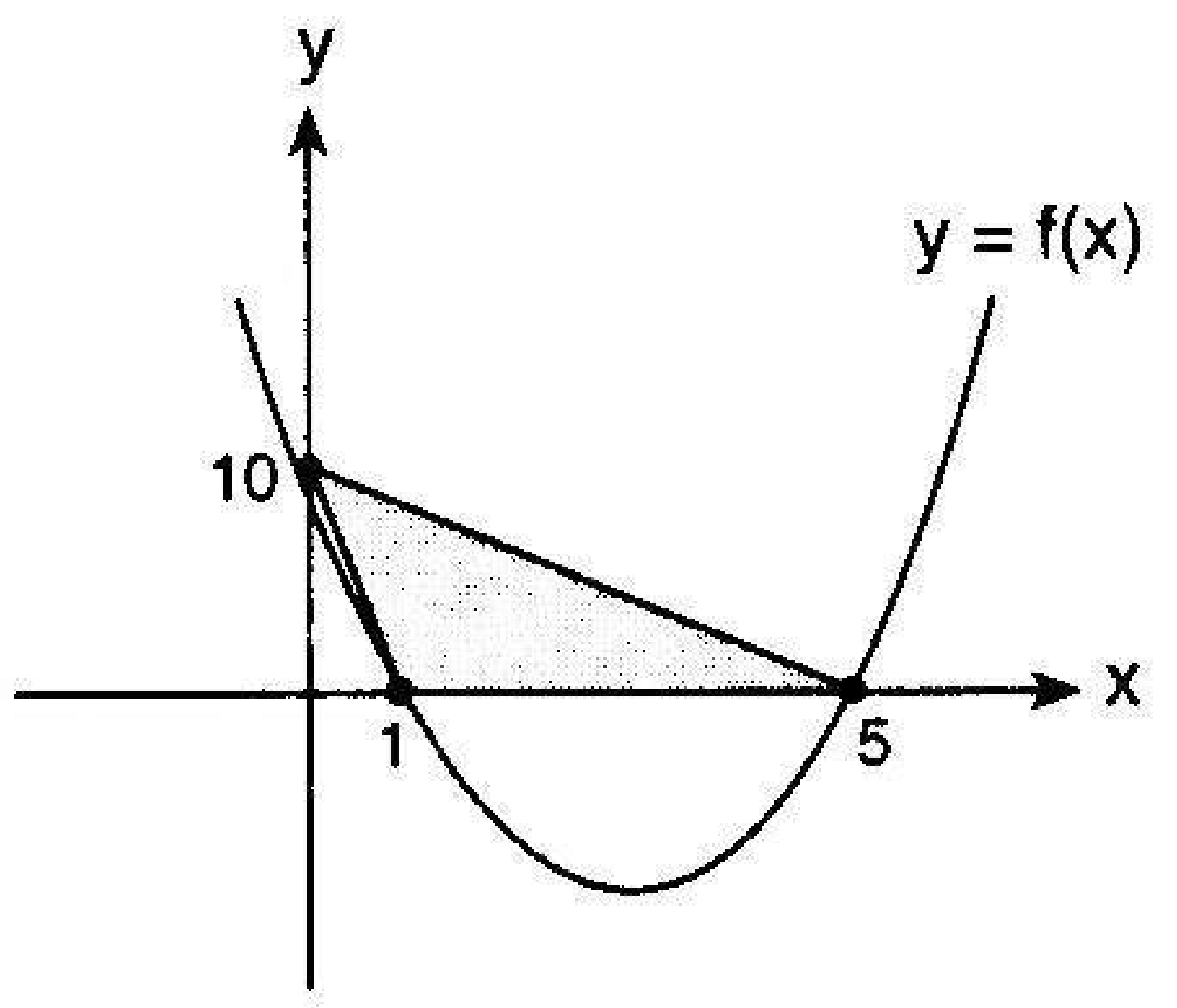
Yanıt / The Answer is A

11. $2x^2 - 12x + 10 = 0$

$x = 5,\ x = 1 \Rightarrow$

$A(1, 0)\ ,\ B(5, 0)\ ,\ C(10, 0)$

$TA/SA = \dfrac{10\cdot 4}{2} = 20\ br^2/u^2$



Yanıt / The Answer is A

12. $-x^2 + 2x + 3 = x + m$

$-x^2 - x + 3 - m = 0$

$\Delta > 0 \Rightarrow (-1)^2 - 4\cdot(-1)\cdot(3-m) > 0$

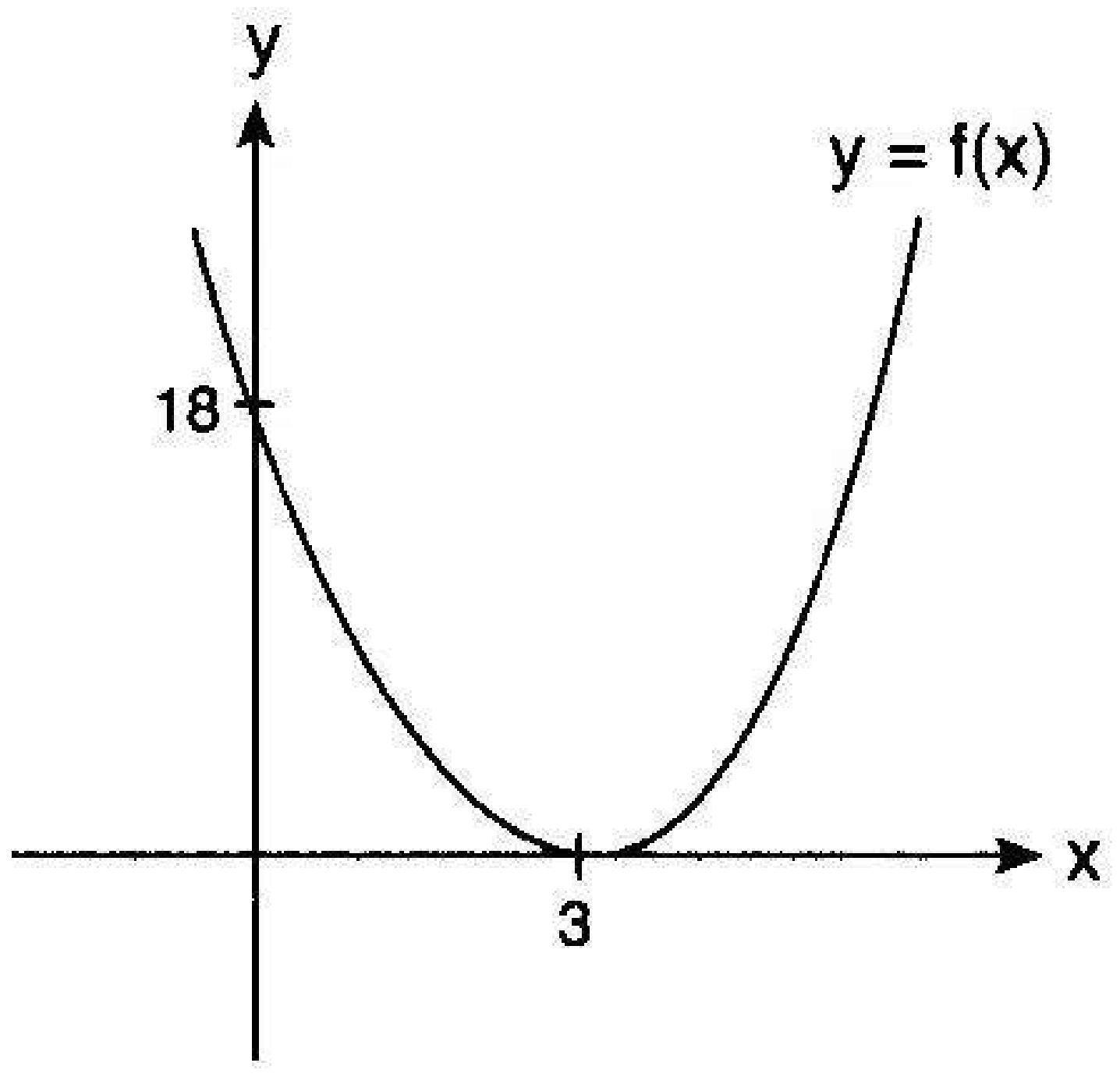
$1 + 12 - 4m > 0$

$13 > 4m$

$\Rightarrow \max(m) = 3$

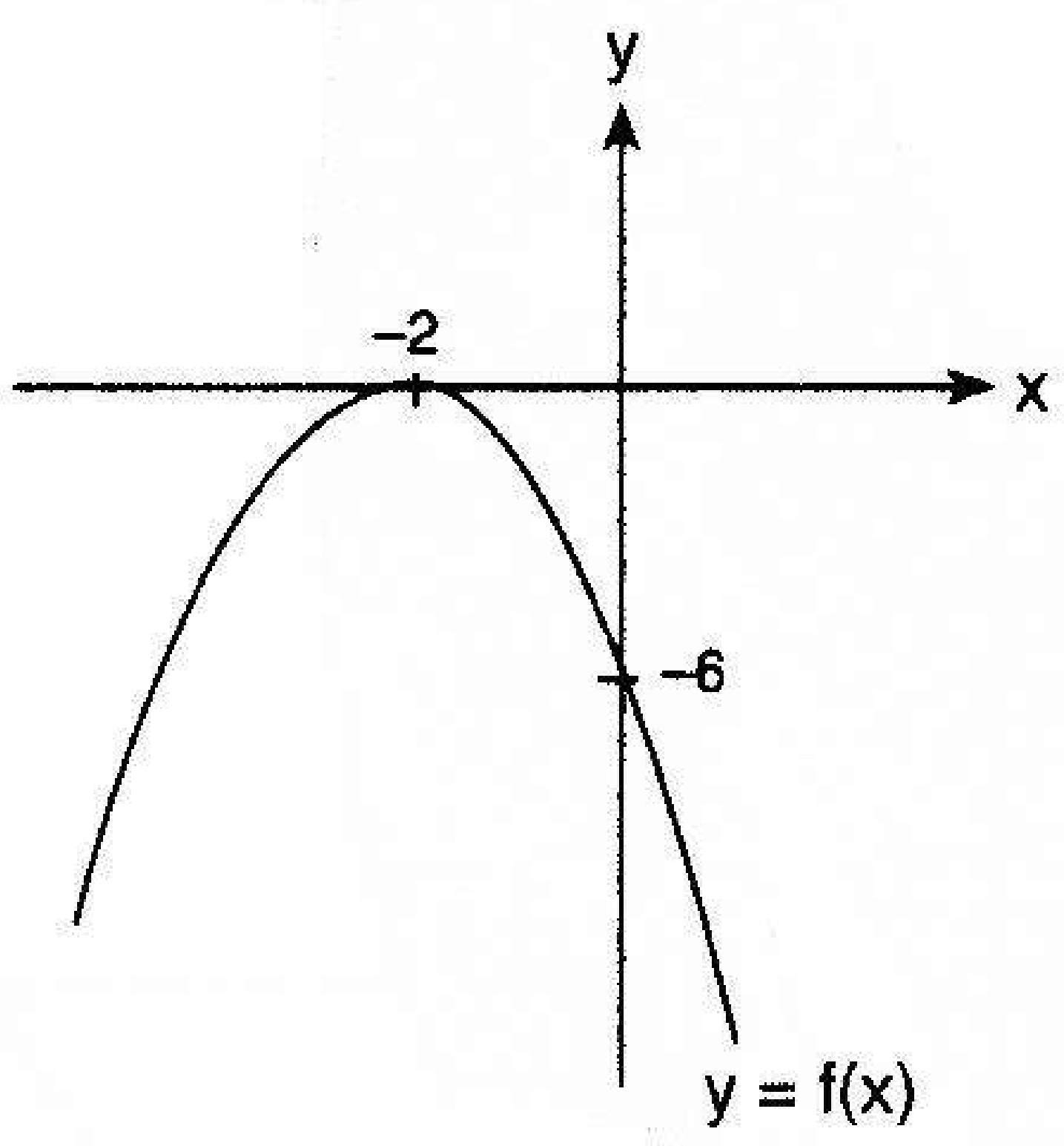
Yanıt / The Answer is C

1. $f(4) = ?$



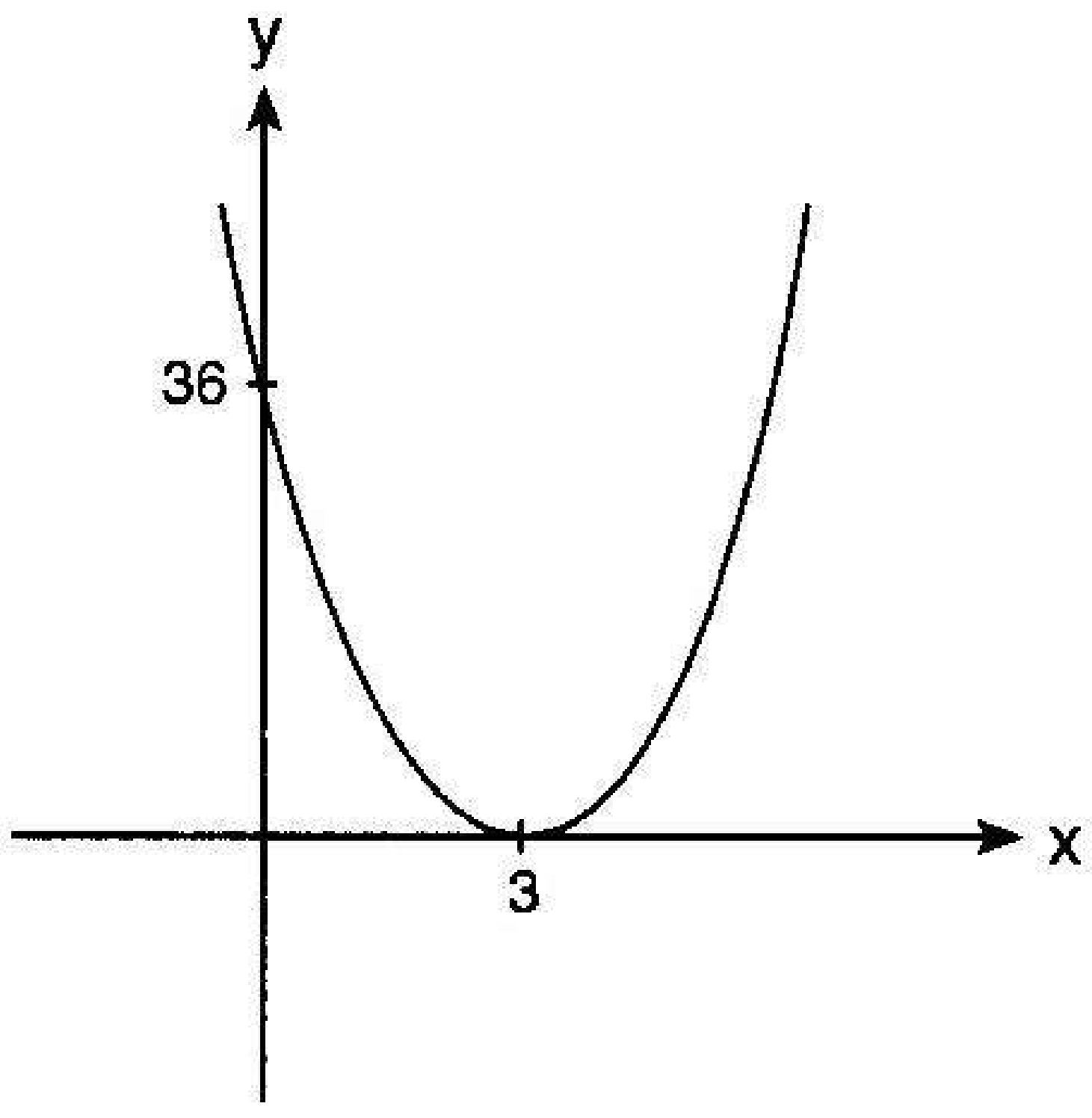
Yanıt / The Answer is: **2**

2. $f(2) = ?$



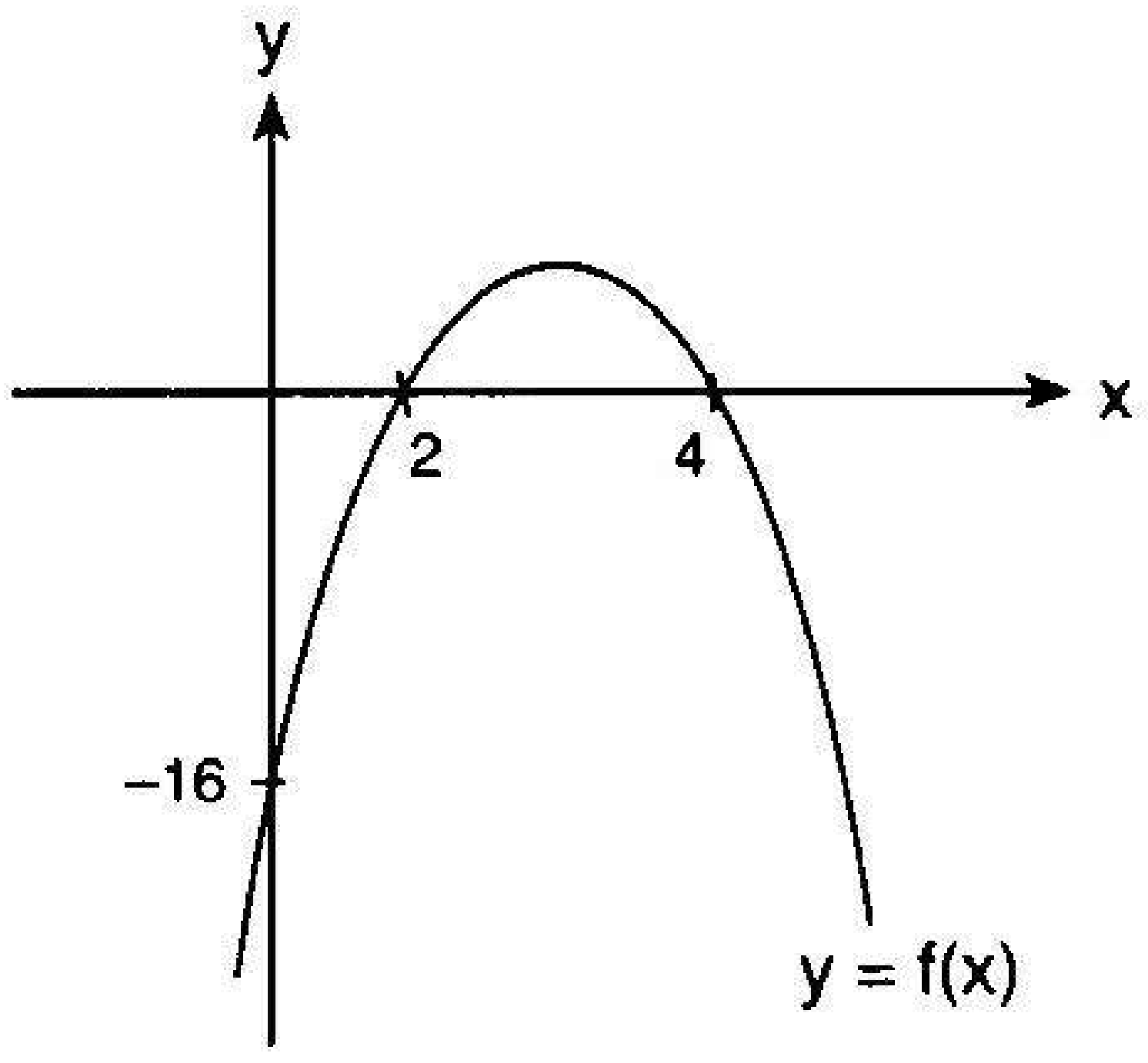
Yanıt / The Answer is: **–24**

3. $f(1) = ?$



Yanıt / The Answer is: **16**

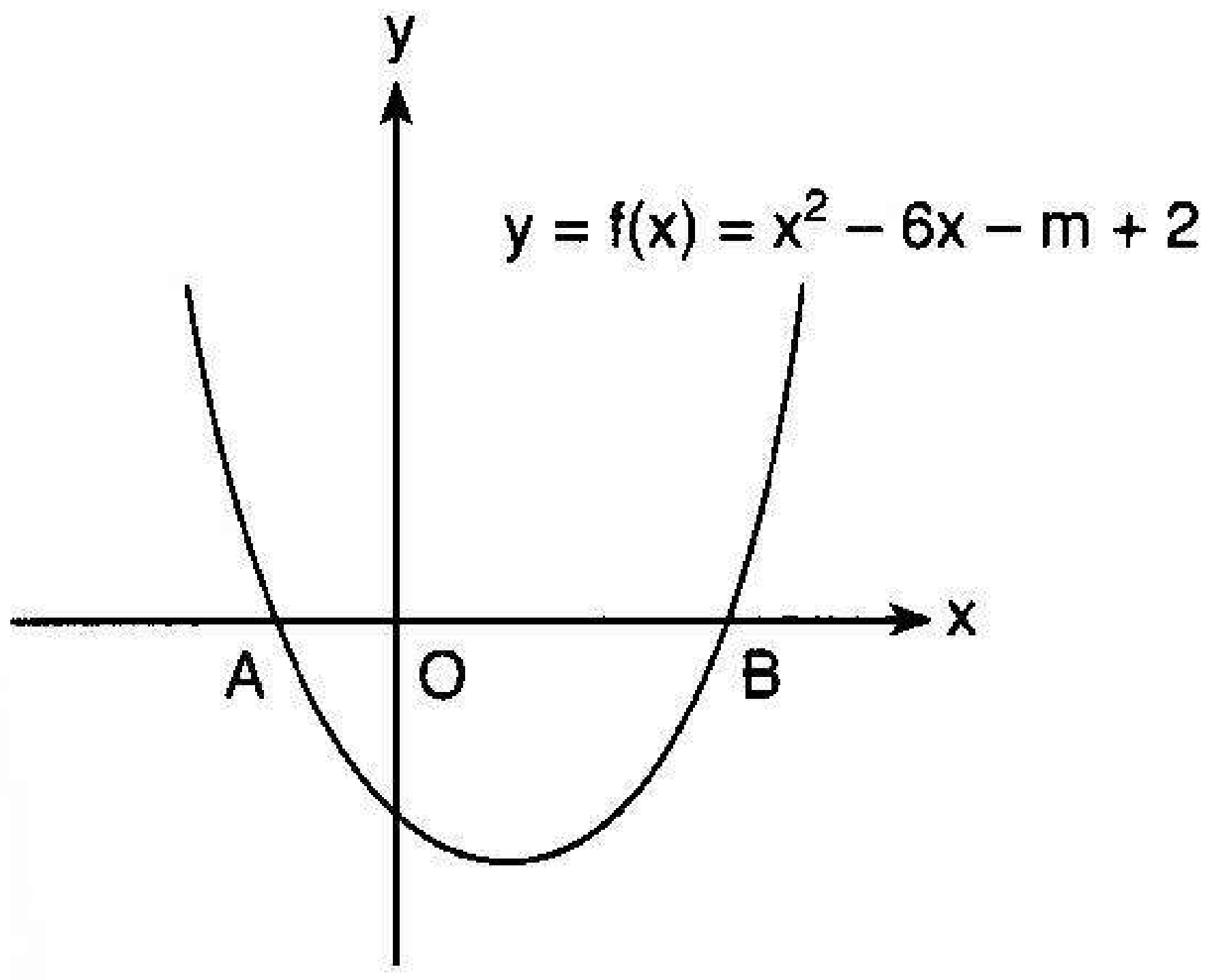
4. $\max(f(x)) = ?$



Yanıt / The Answer is: **2**

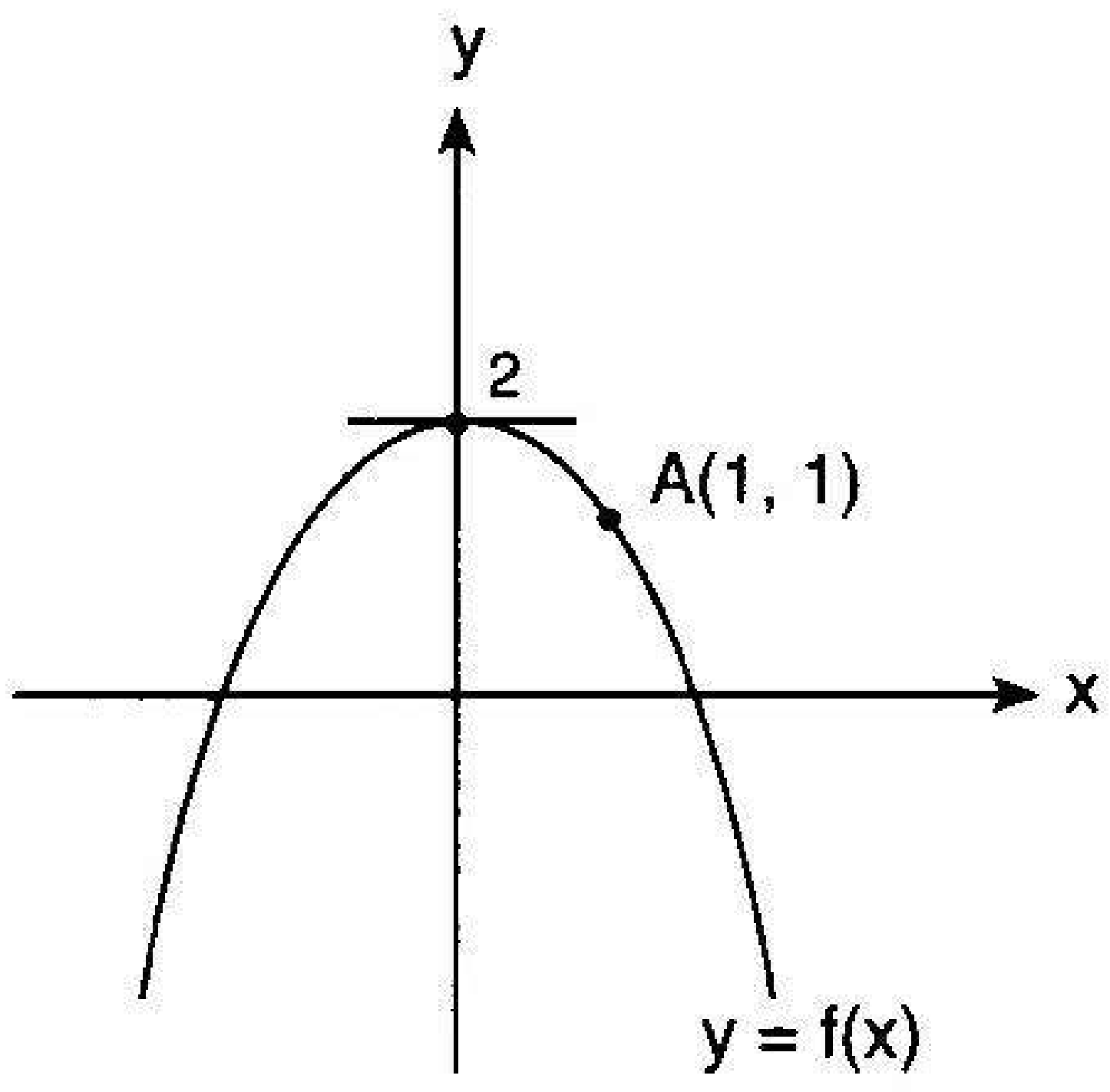
5. $|OB| = 7 \cdot |OA|$

$\Rightarrow$ **m = ?**


y
y = f(x) = x² – 6x – m + 2
x
A
O
B

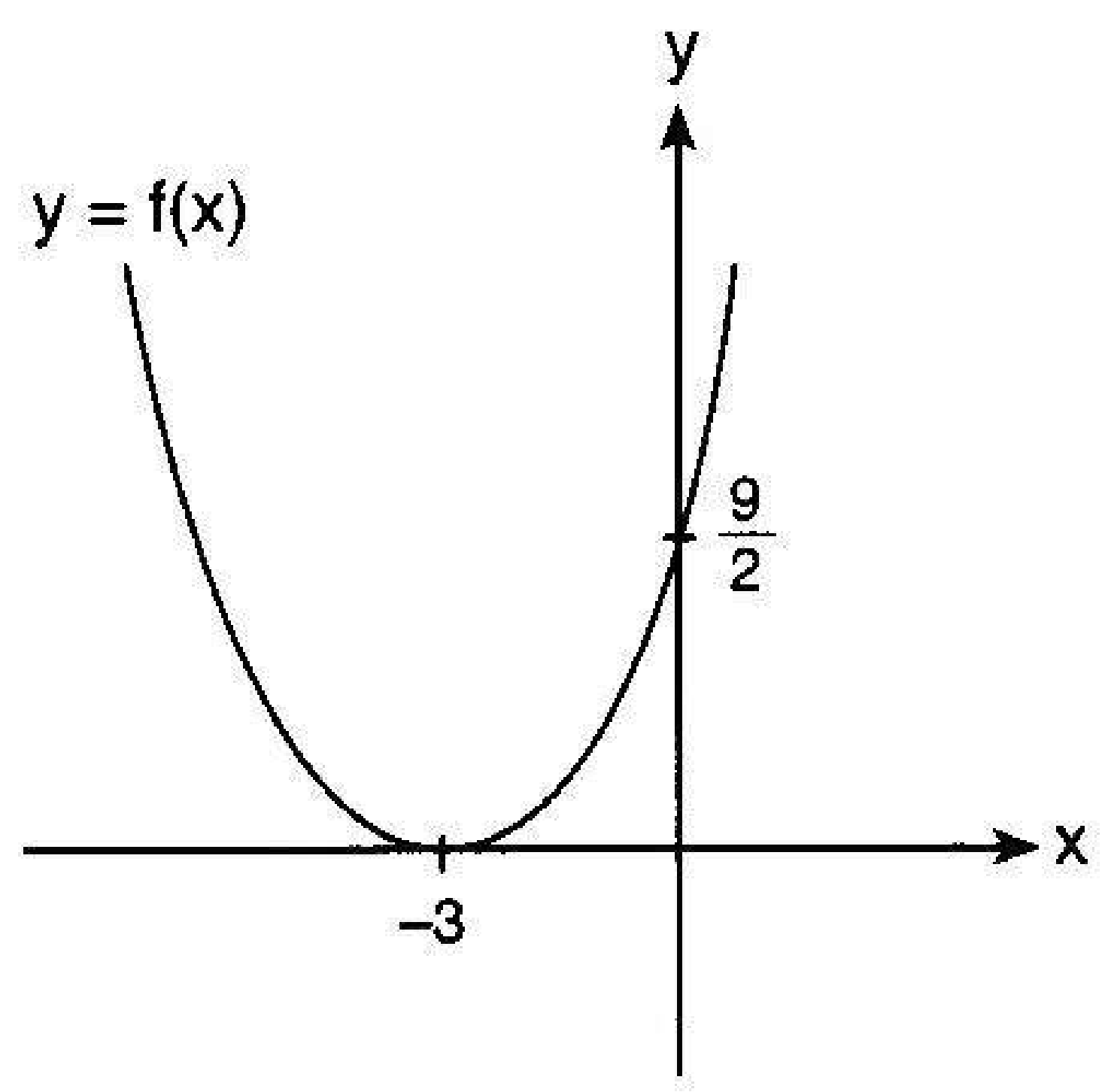
Yanıt / The Answer is: **9**

6. $f(2) = ?$



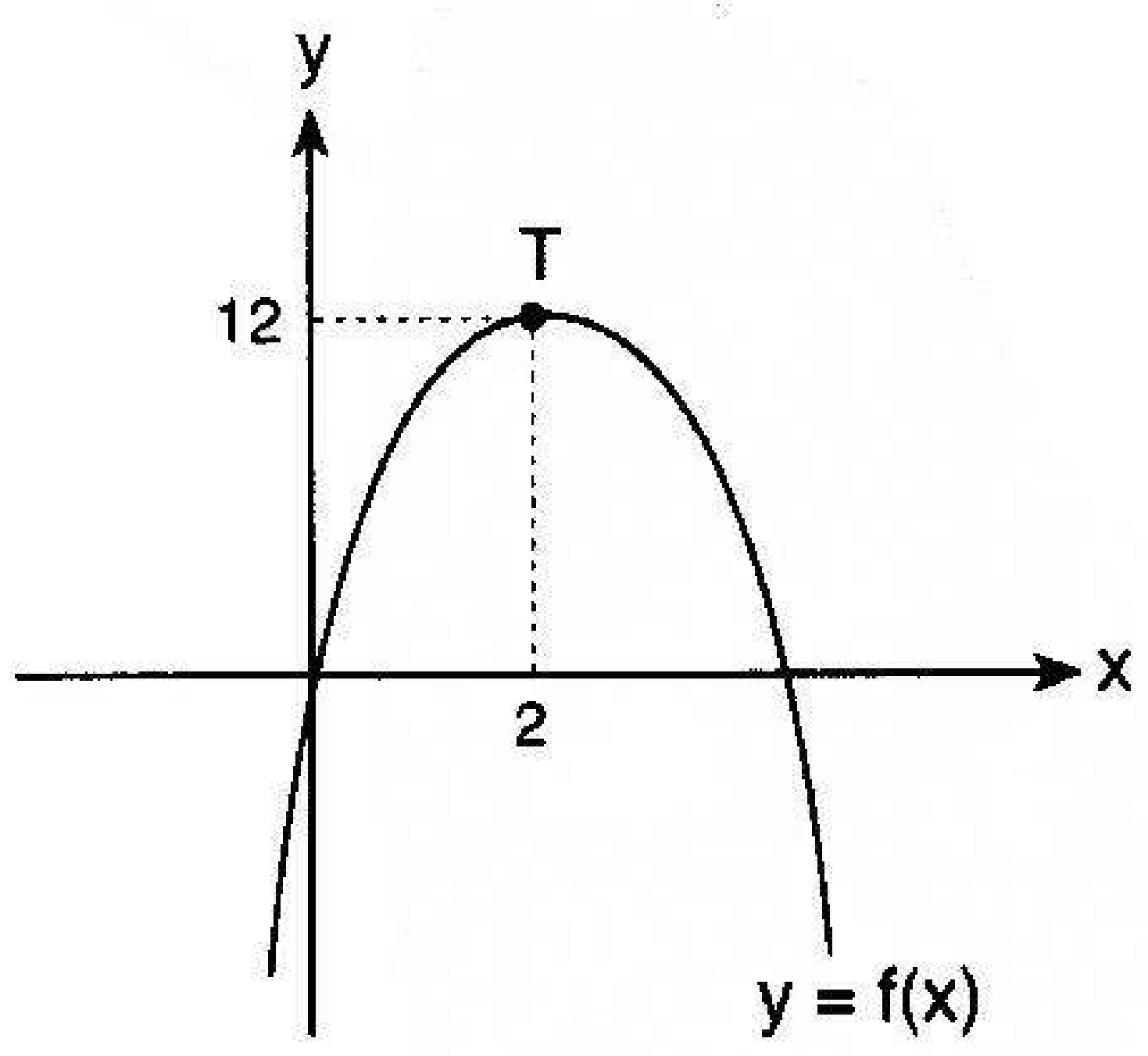
Yanıt / The Answer is: **–2**

7. **$f(1) = ?$**



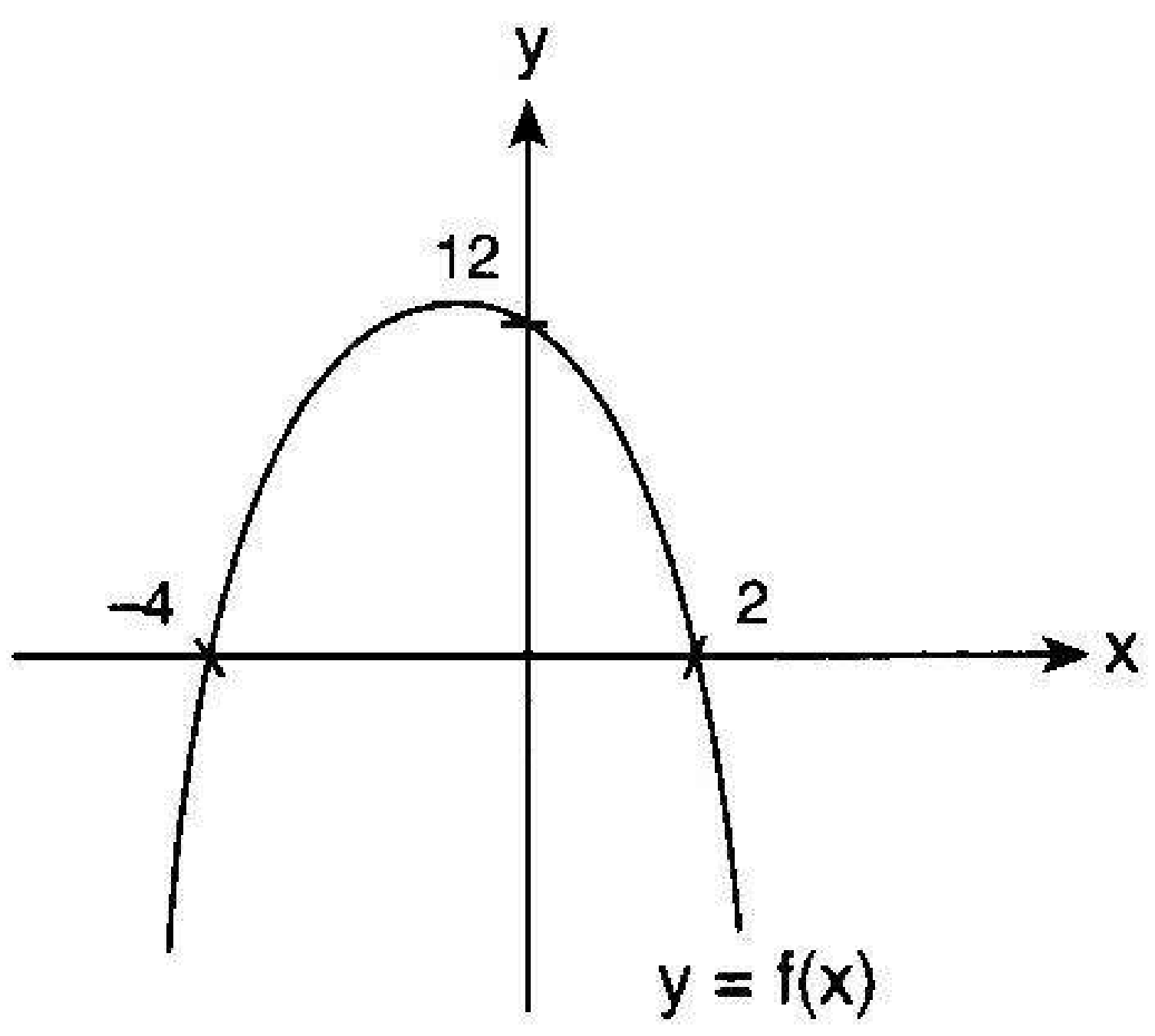
Yanıt / The Answer is: **8**

8. **$f(1) = ?$**



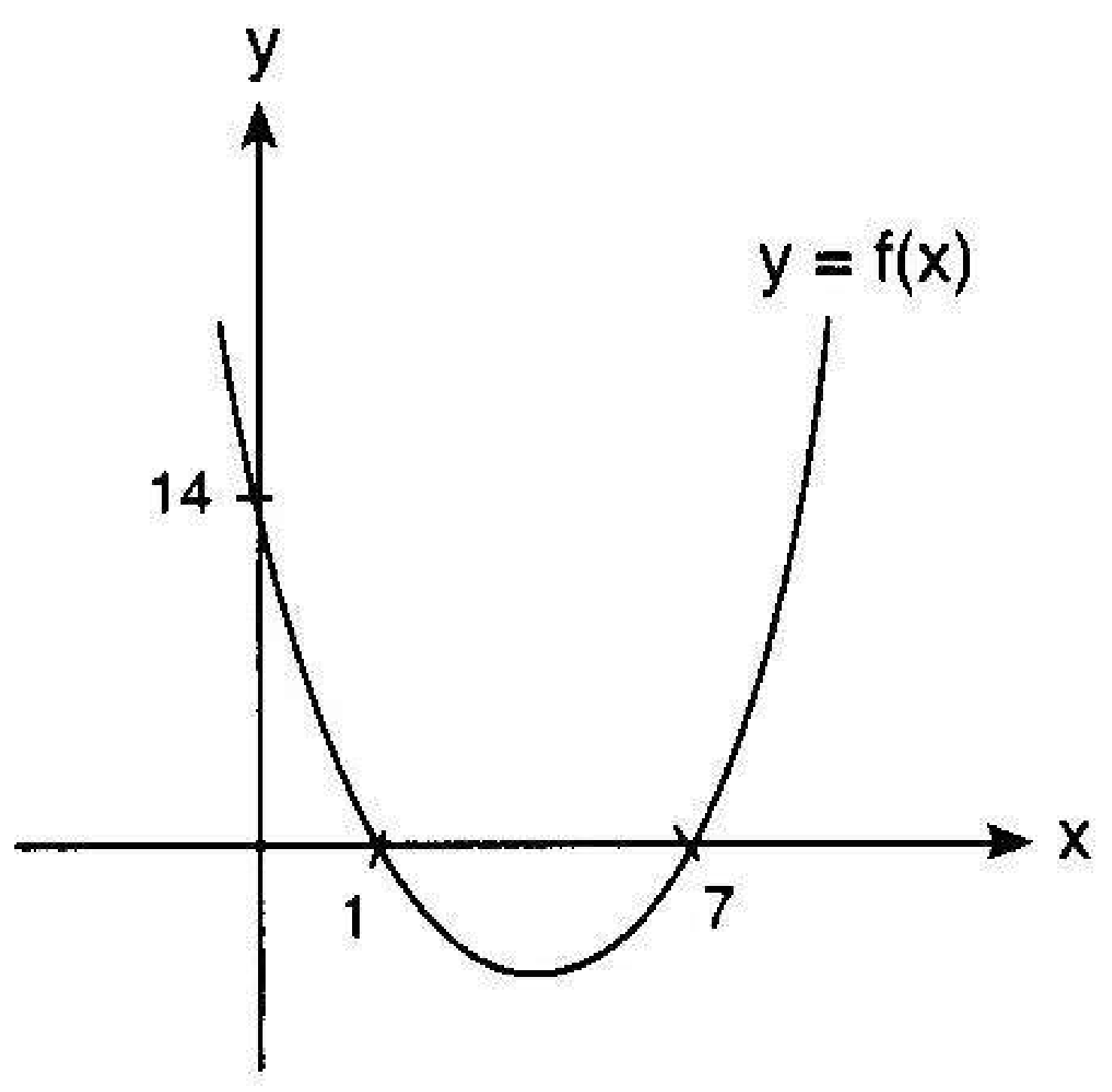
Yanıt / The Answer is: **9**

9. **$f(-1) = ?$**



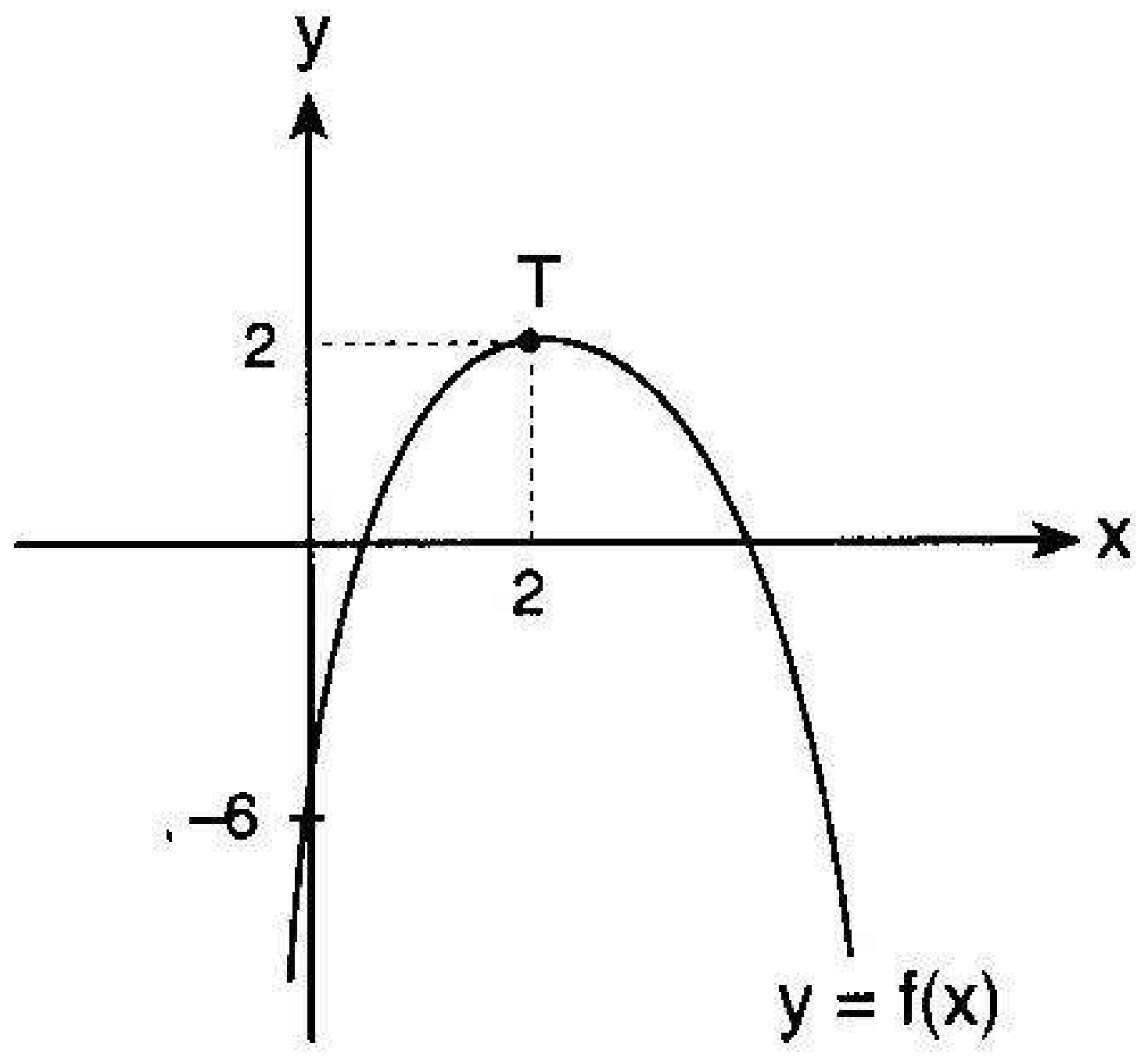
Yanıt / The Answer is: $\mathbf{\frac{27}{2}}$

10. **$\min(f(x)) = ?$**



Yanıt / The Answer is: **–18**

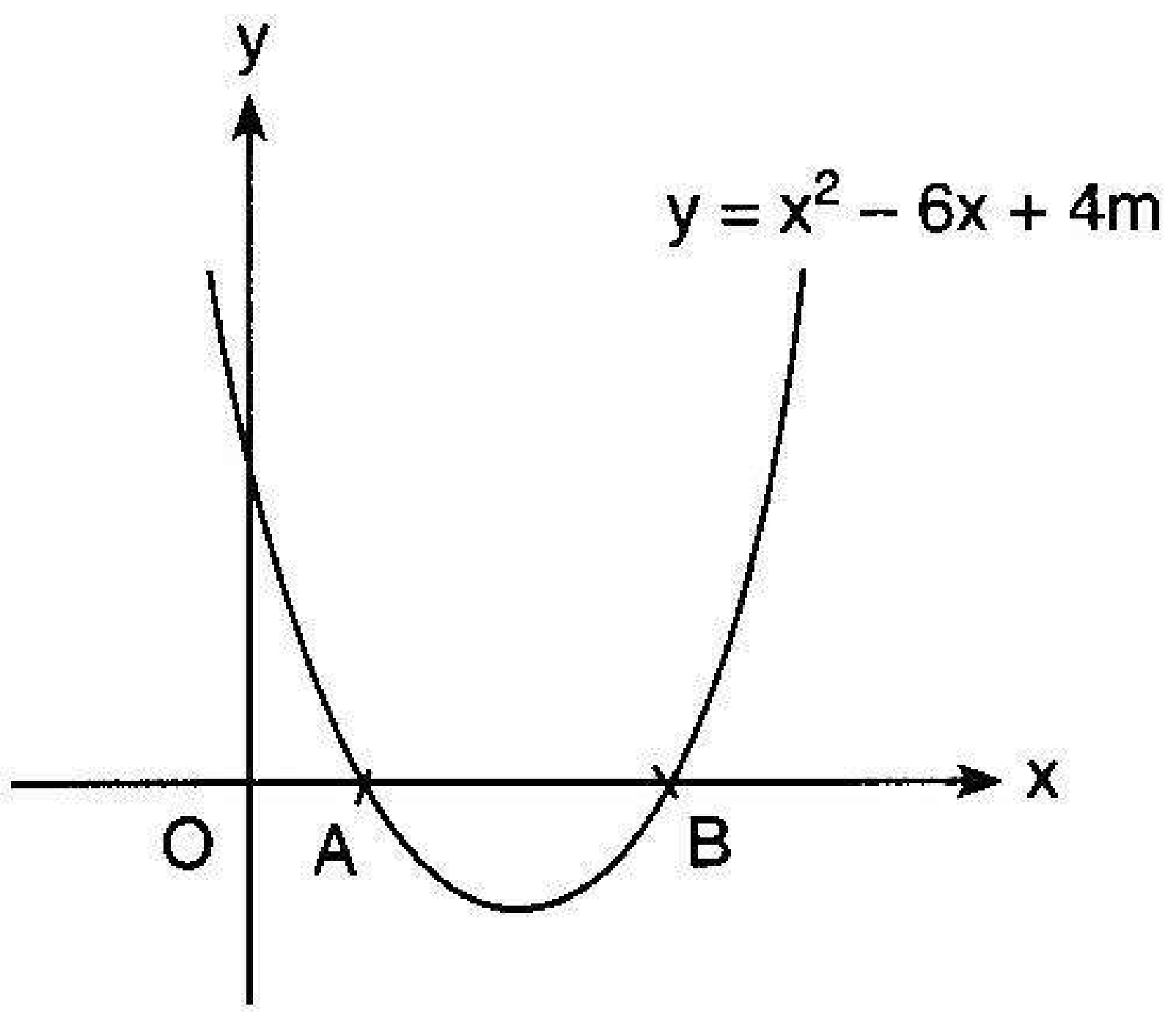
11. **$f(4) = ?$**



Yanıt / The Answer is: **–6**

12. $|OA| = m$

$\Rightarrow$ **$m = ?$**



Yanıt / The Answer is: **2**

1. $y = x^2 - 6x + 3 \Rightarrow$ **min(y) = ?**

A) 0

B) –2

C) –4

D) –6

E) –8

2. $y = x^2 + 4x - 2 \Rightarrow$ **min(y) = ?**

A) –6

B) –5

C) –4

D) –3

E) –2

3. $y = -x^2 + 2x + 5 \Rightarrow$ **max(y) = ?**

A) 3

B) 4

C) 6

D) 7

E) 9

4. $y = -x^2 + 4x - 1 \Rightarrow$ **max(y) = ?**

A) 0

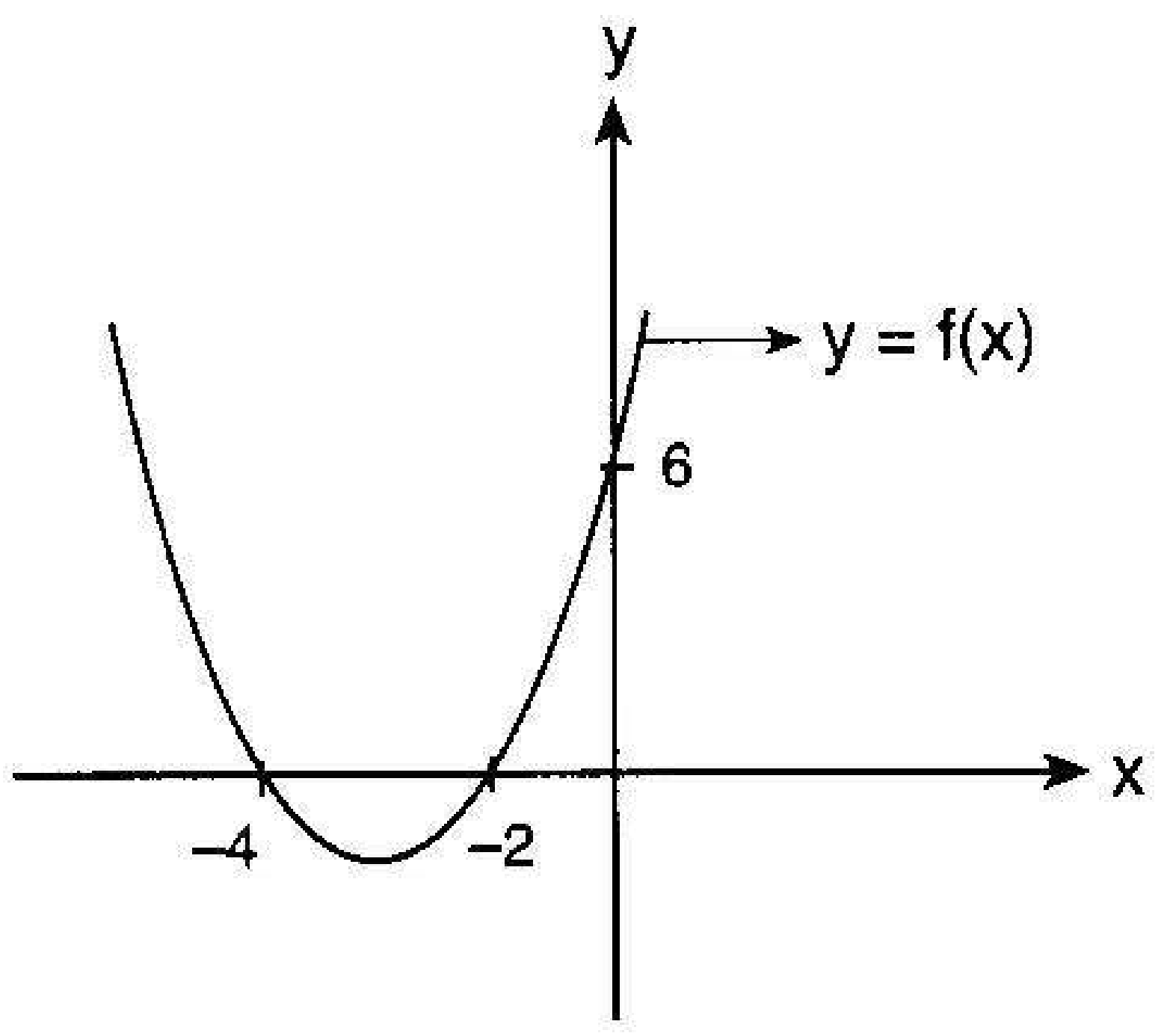
B) 1

C) 2

D) 3

E) 4

5. **min(y) = ?**



A) $\frac{1}{4}$

B) $-\frac{1}{4}$

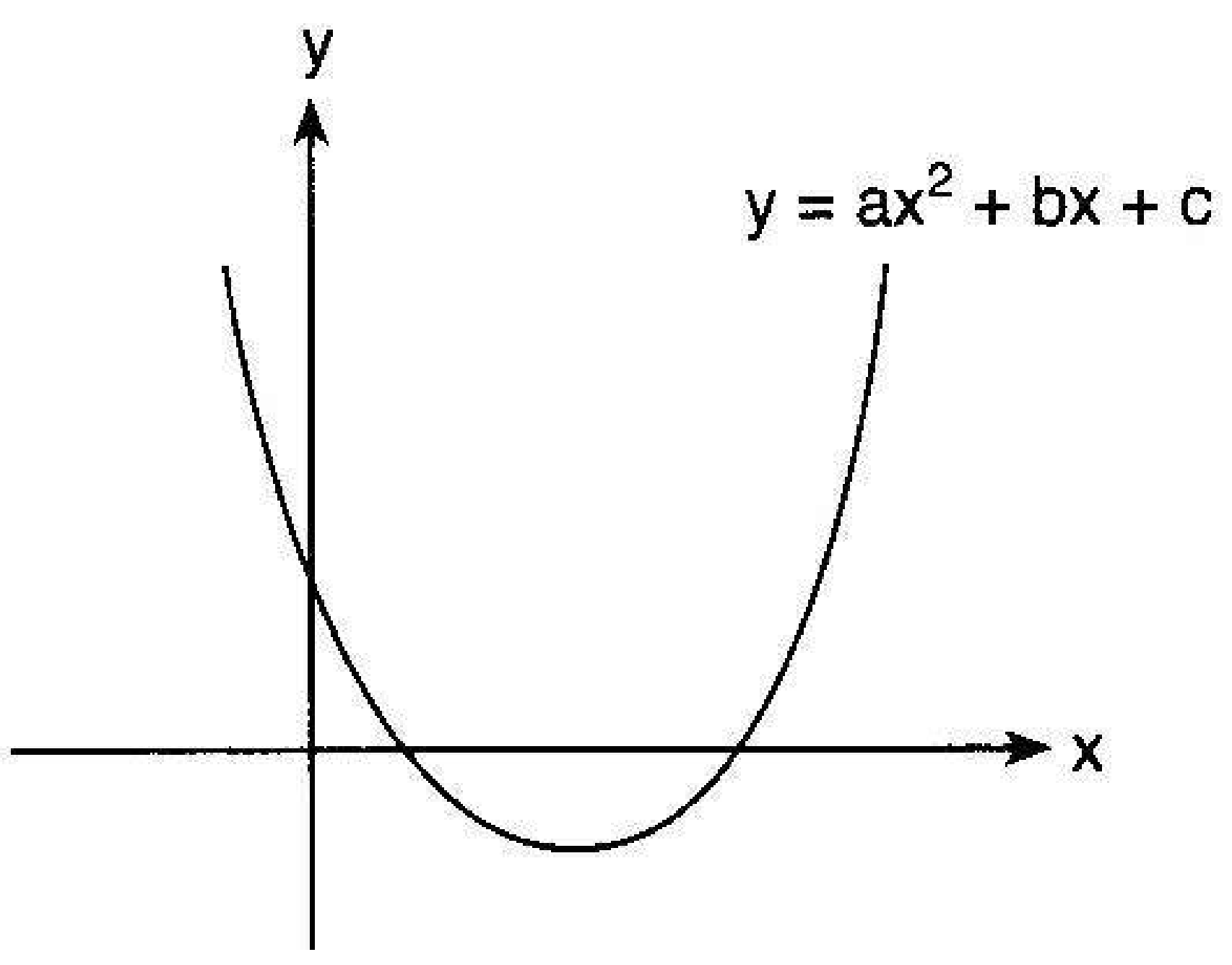
C) $-\frac{3}{4}$

D) $-\frac{5}{4}$

E) $-\frac{7}{4}$

6. **Aşağıdakilerden hangisi doğrudur?**

Which of the following is true?



A) $c < 0$

B) $\frac{b}{a} < 0$

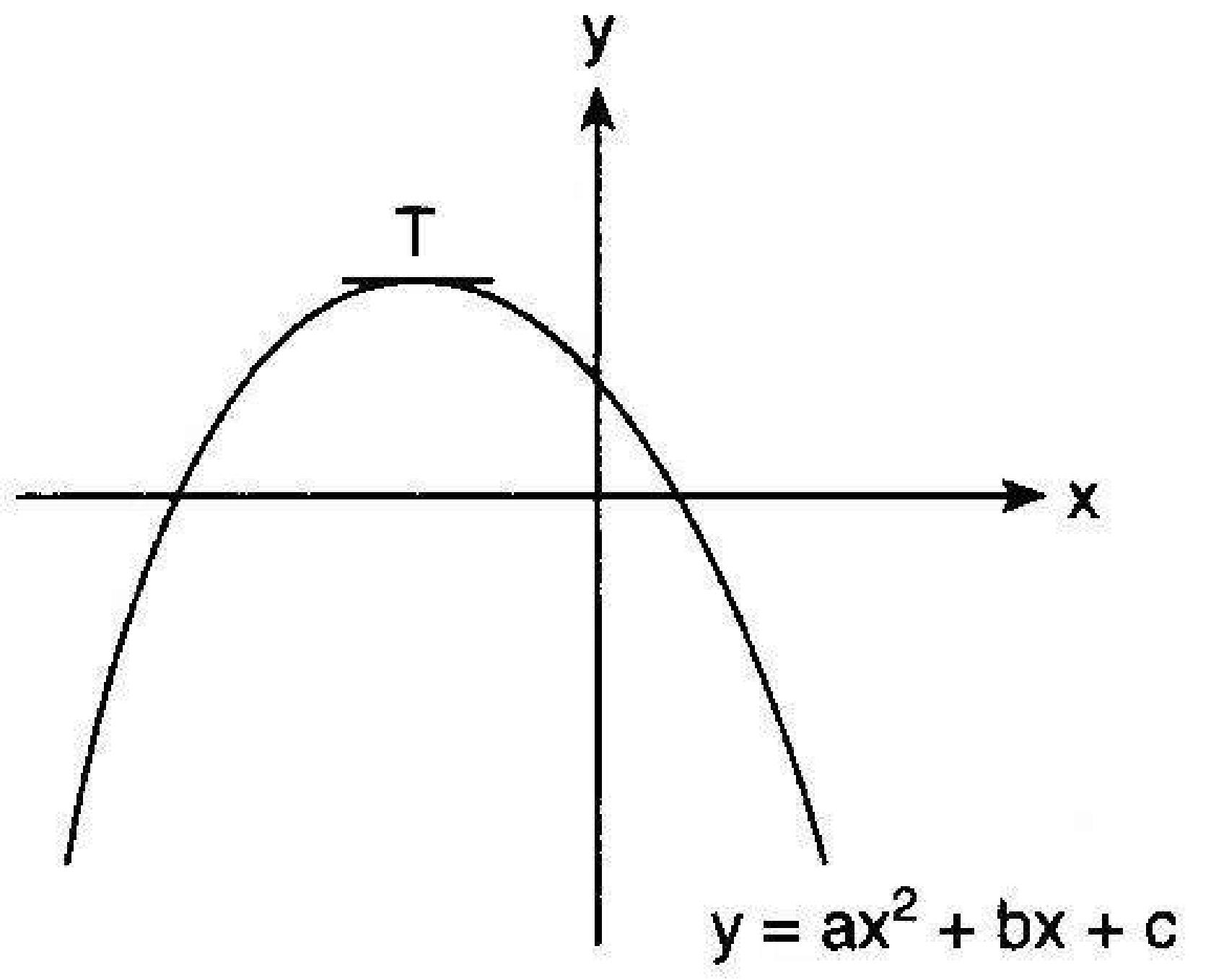
C) $a \cdot c < 0$

D) $\frac{c}{b} > 0$

E) $a + c < 0$

7. **Aşağıdakilerden hangisi doğrudur?**

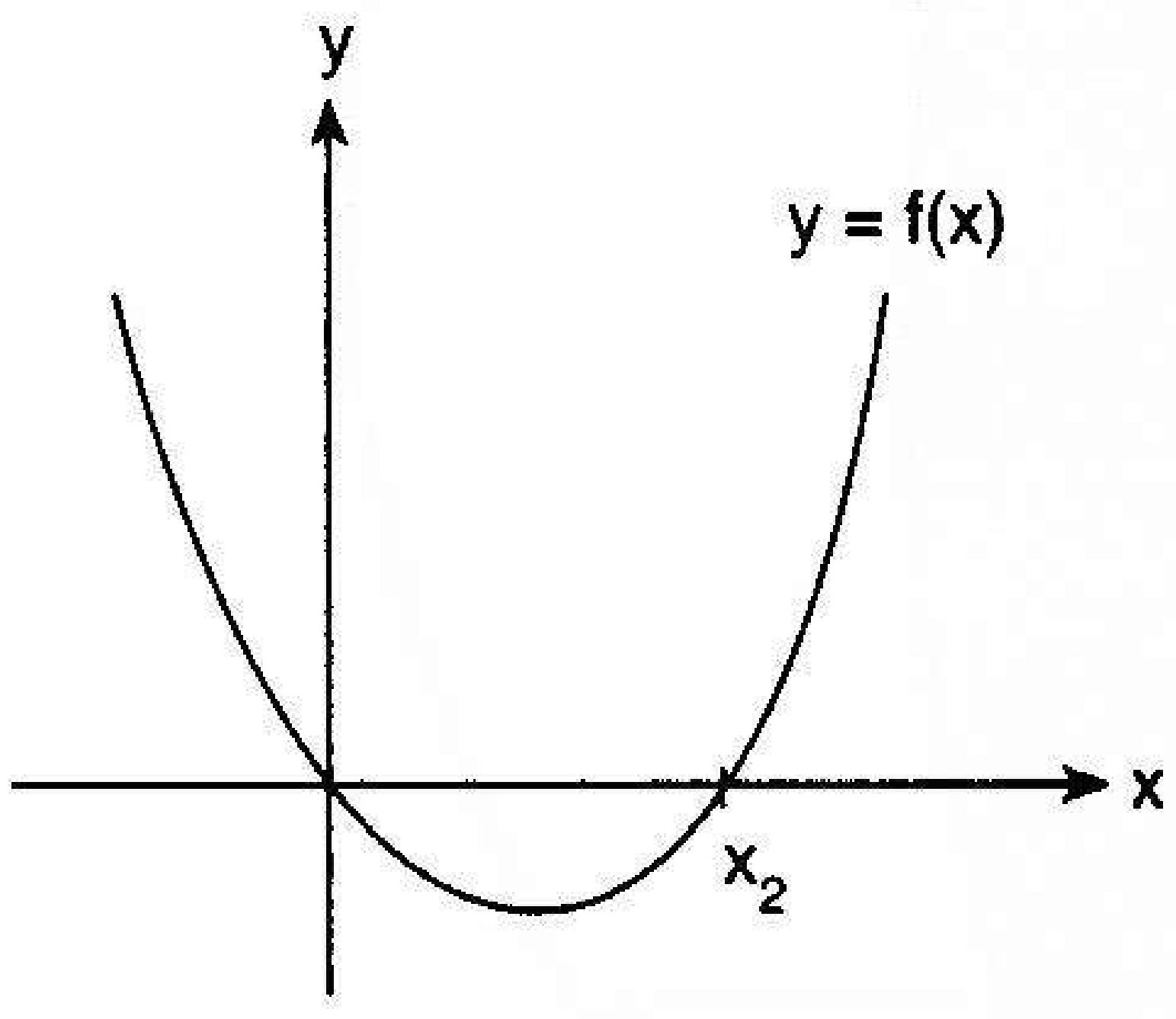
Which of the following is true?


y
T
x
y = ax² + bx + c

A) $c < 0$ B) $a \cdot b \cdot c < 0$ C) $\frac{b}{c} > 0$

D) $a \cdot c < 0$ E) $a + b > 0$

8.



$f(x) = (m - 1)x^2 - 4x + m - 2 \Rightarrow$ **x_2 = ?**

A) 1 B) 2 C) 3

D) 4 E) 5

9. $f(x) = mx^2 - 4x - 3m$

$\Rightarrow$ **c = ?**

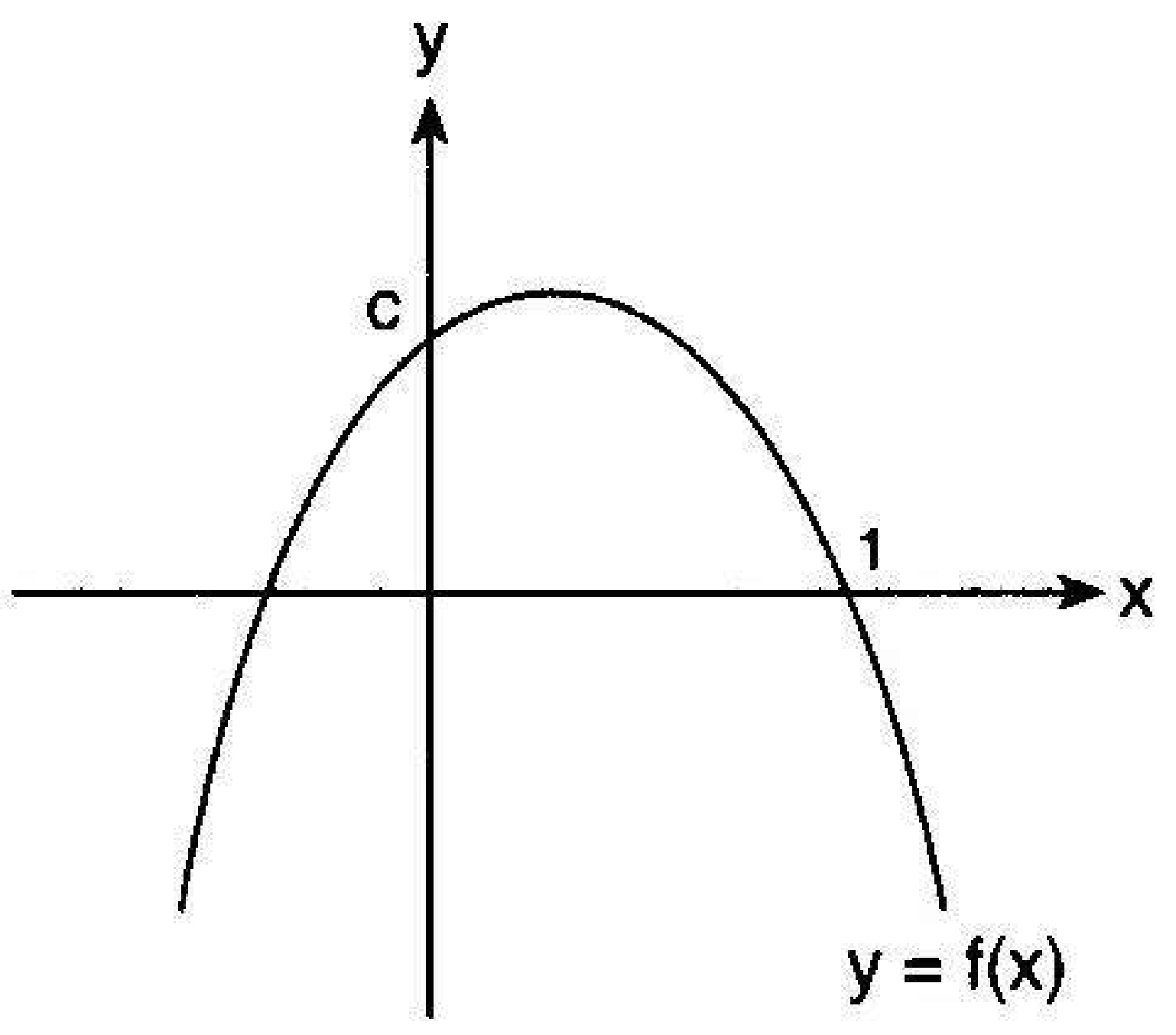
A) –4

B) –2

C) 2

D) 4

E) 6



10. **f(4) = ?**

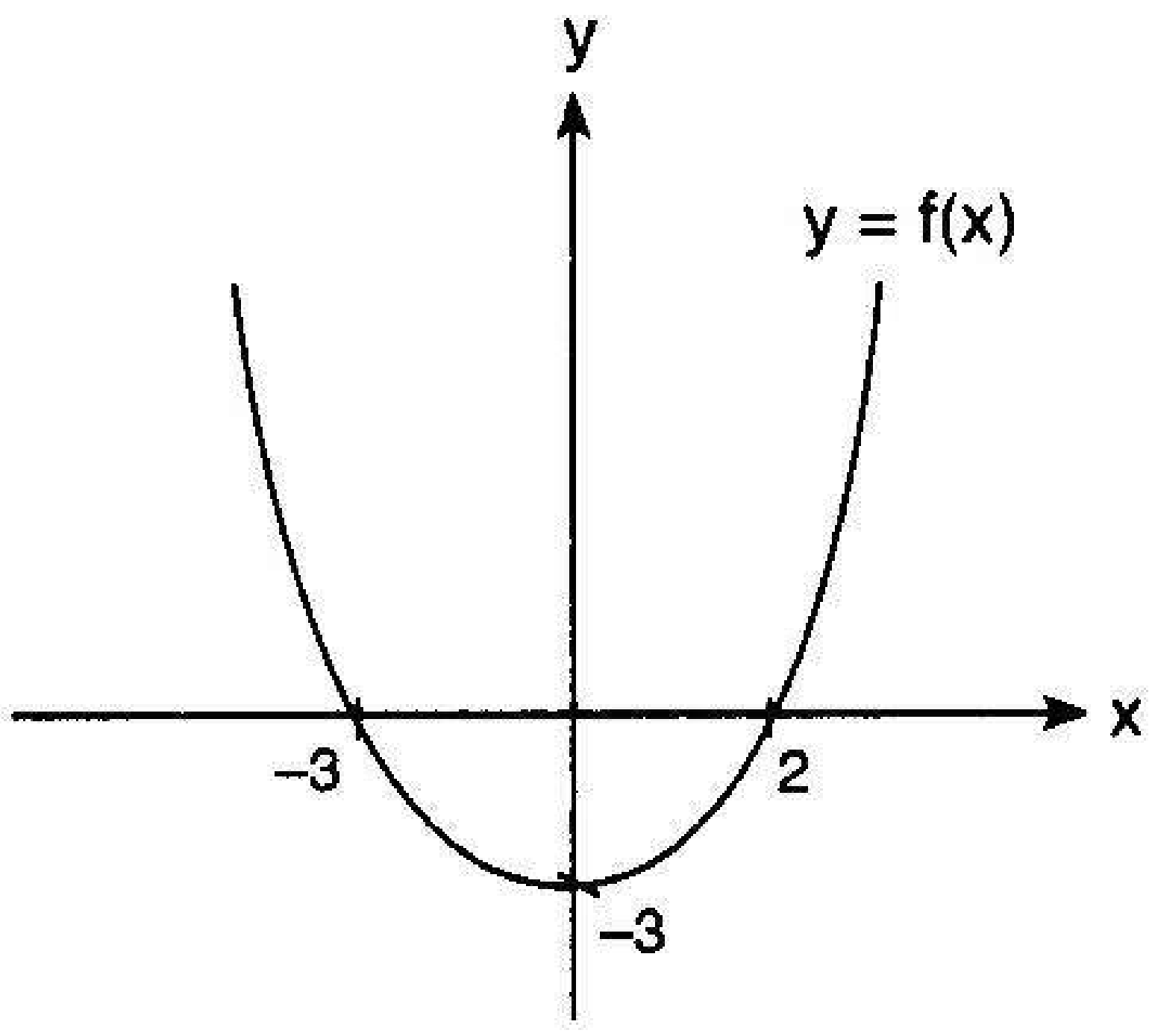
A) 6

B) 7

C) 8

D) 9

E) 10



11. $f(x) = mx^2 - 4x - 3m$

$\Rightarrow$ **m = ?**

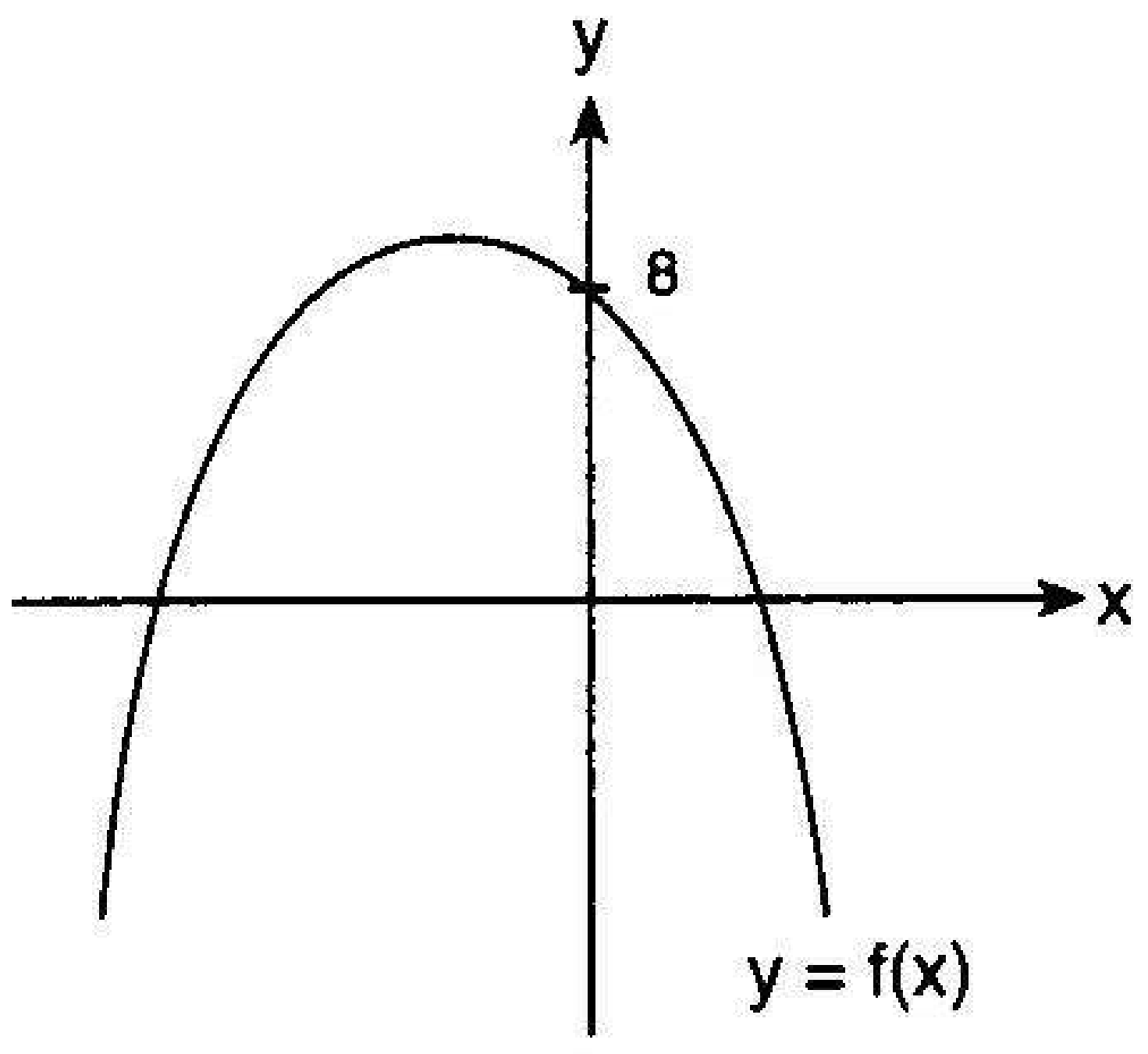
A) $\frac{8}{3}$

B) $\frac{7}{3}$

C) $-\frac{4}{3}$

D) -2

E) $-\frac{8}{3}$



12. **f(8) = ?**

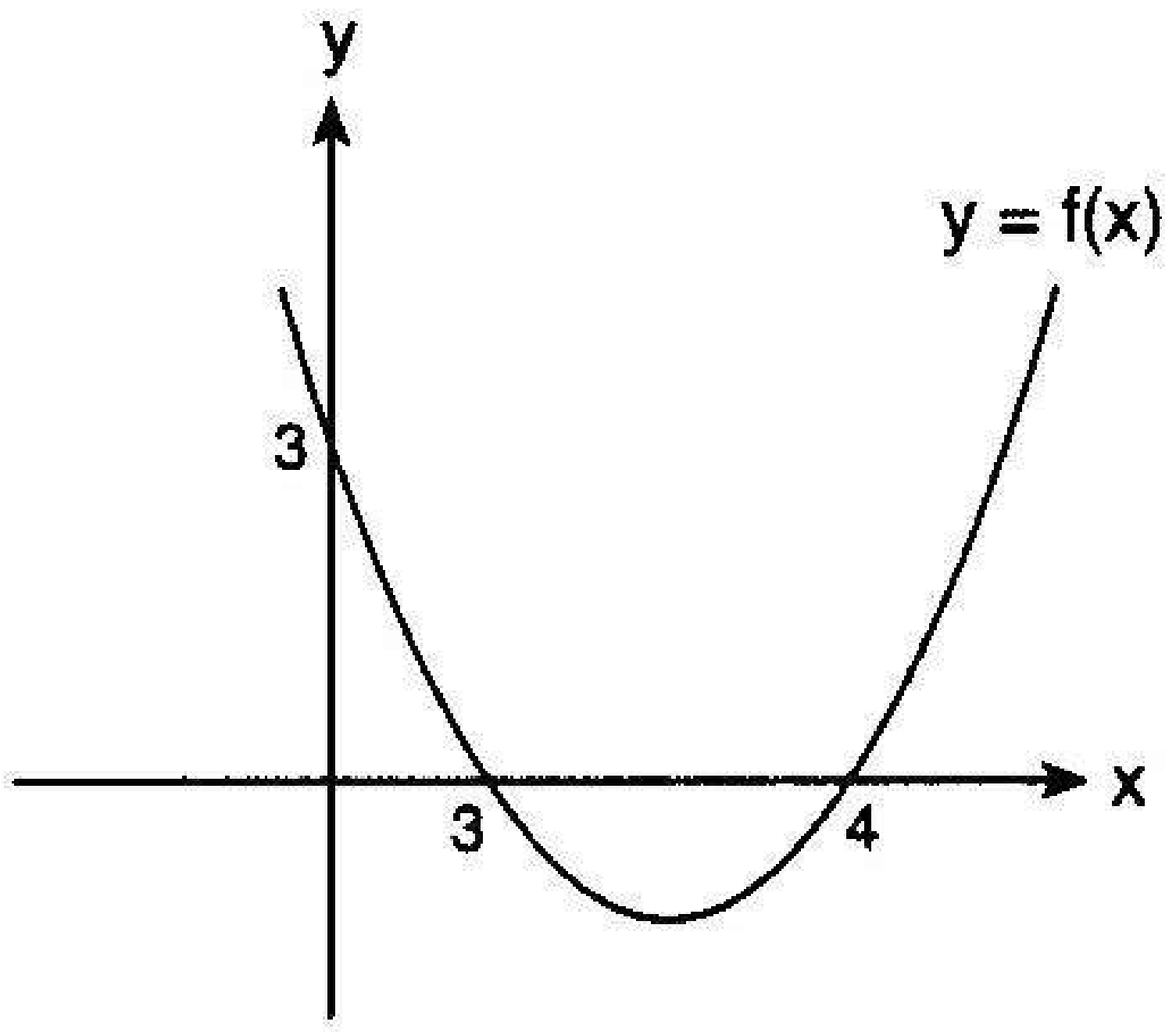
A) 2

B) 4

C) 5

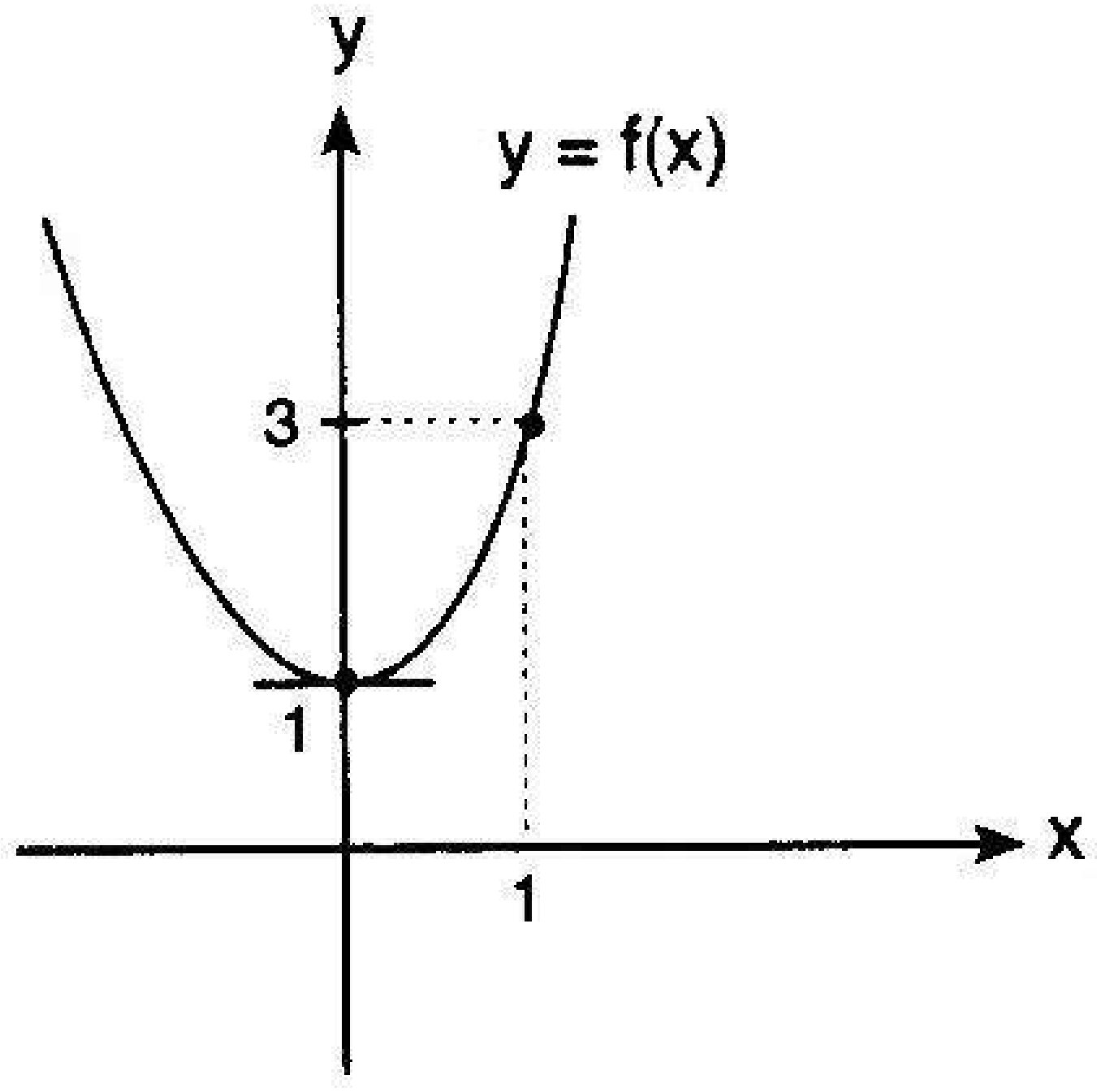
D) 8

E) 9



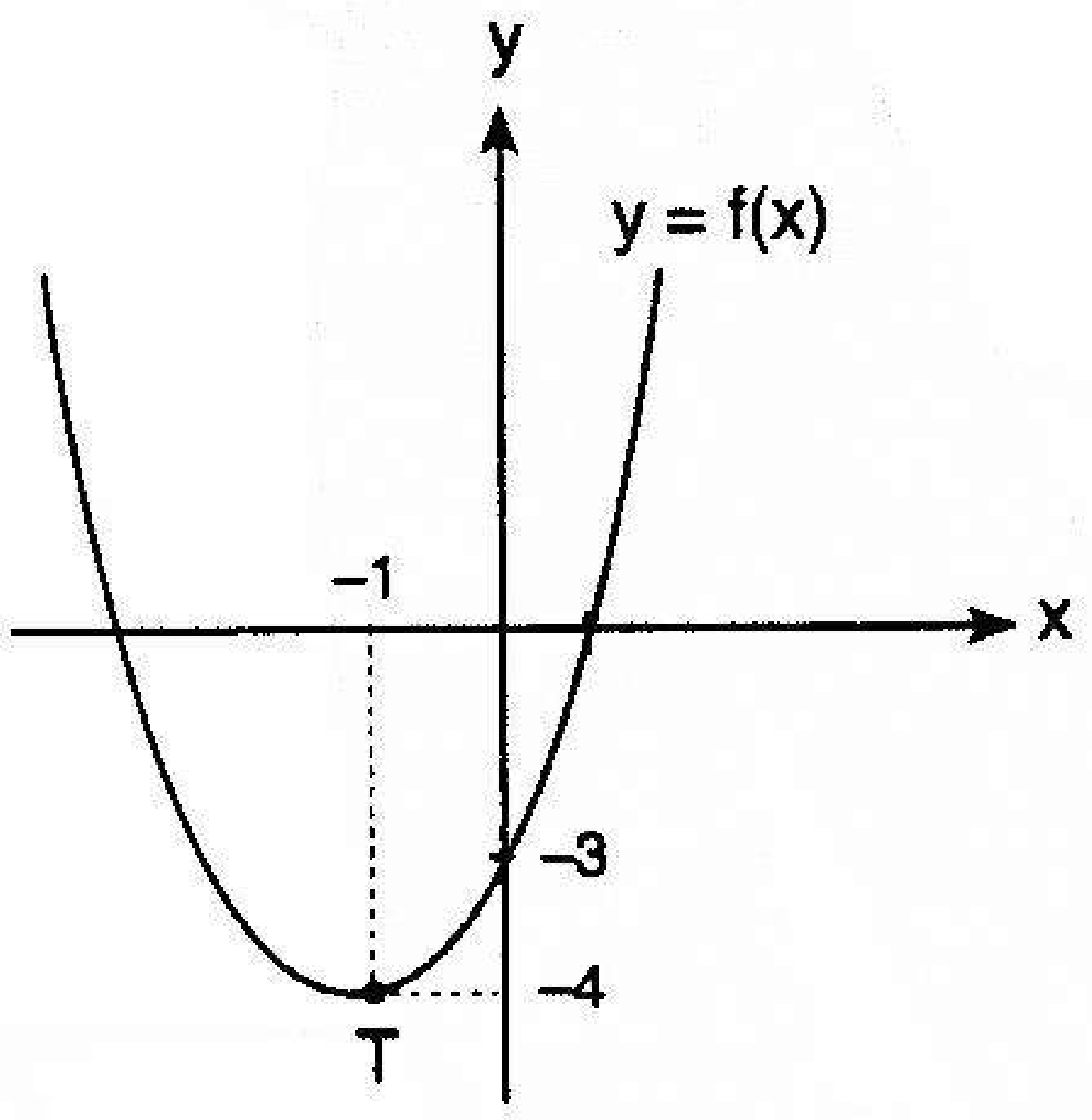
YANITLAR / ANSWERS							
1. D	2. A	3. C	4. D	5. C	6. B	7. D	8. D
9. E	10. B	11. E	12. C				

1. **$f(x) = ax^2 + bx + c = ?$**



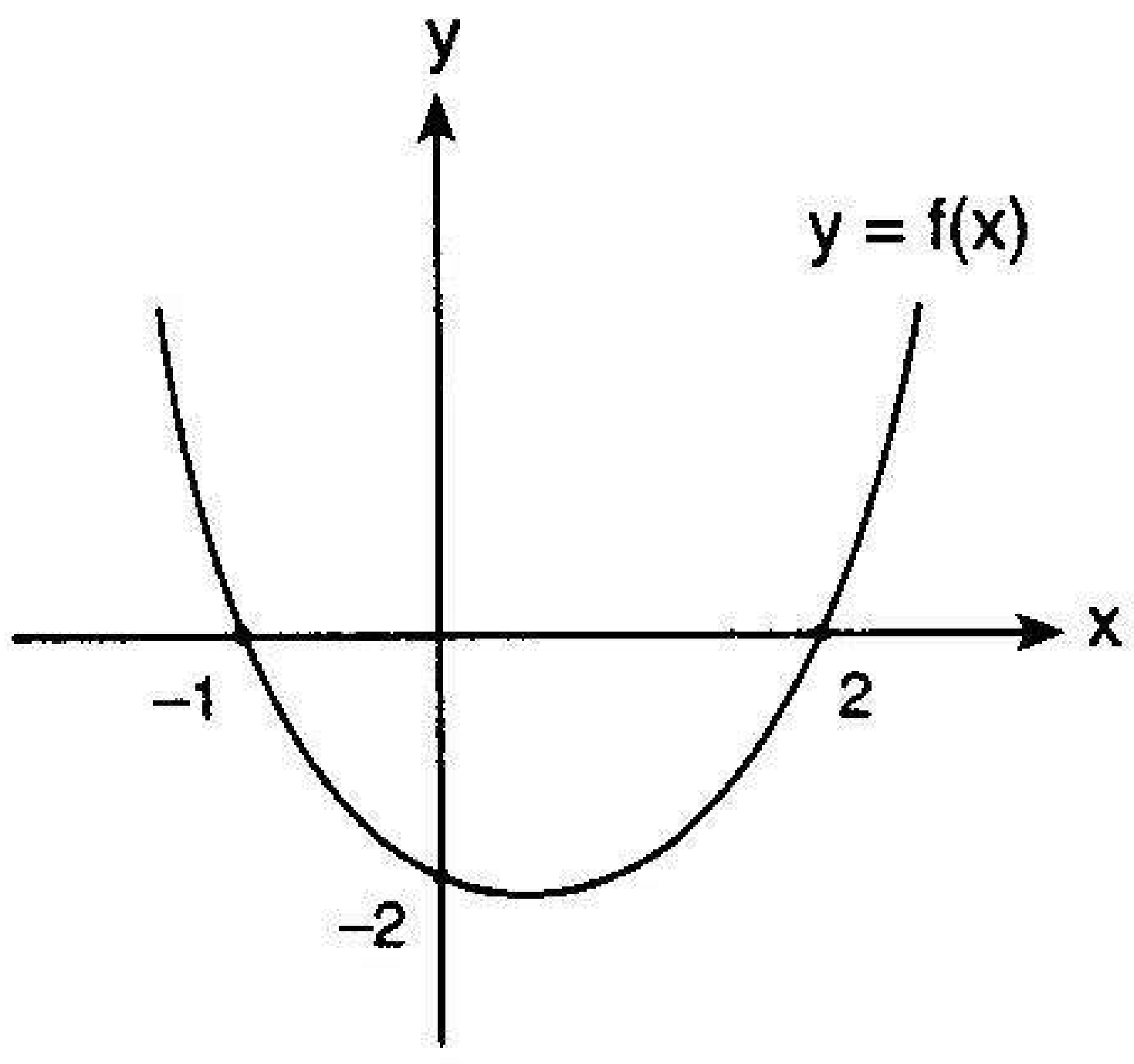
A) $3x^2$
B) $2x^2 + 1$
C) $4x^2 - 1$
D) $x^2 + 2x$
E) $2x^2 - x + 1$

2. $f(x) = ax^2 + bx + c$
$\Rightarrow a + b + c = ?$



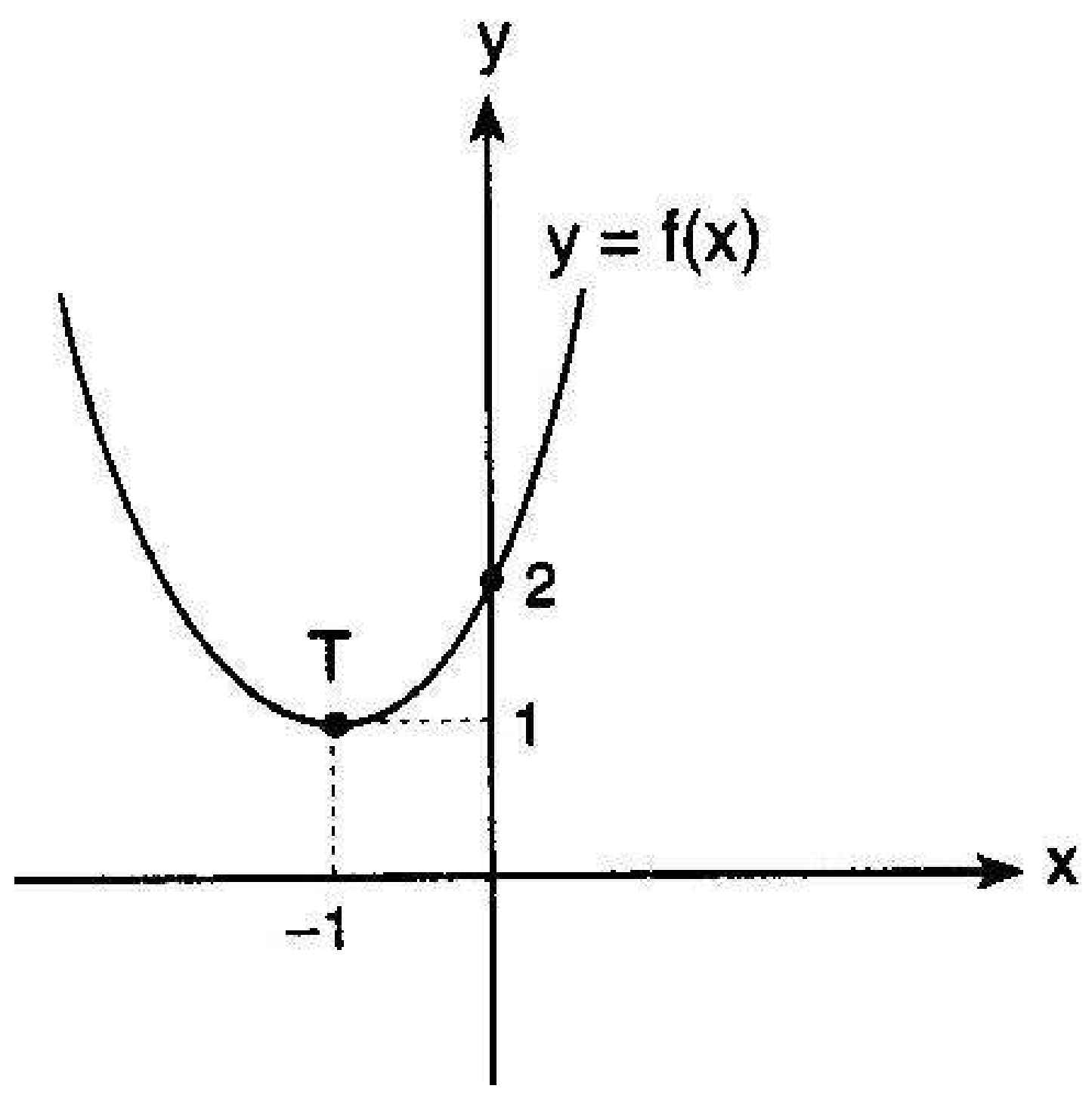
A) –2
B) –1
C) 0
D) 1
E) 2

3. **$f(-2) = ?$**



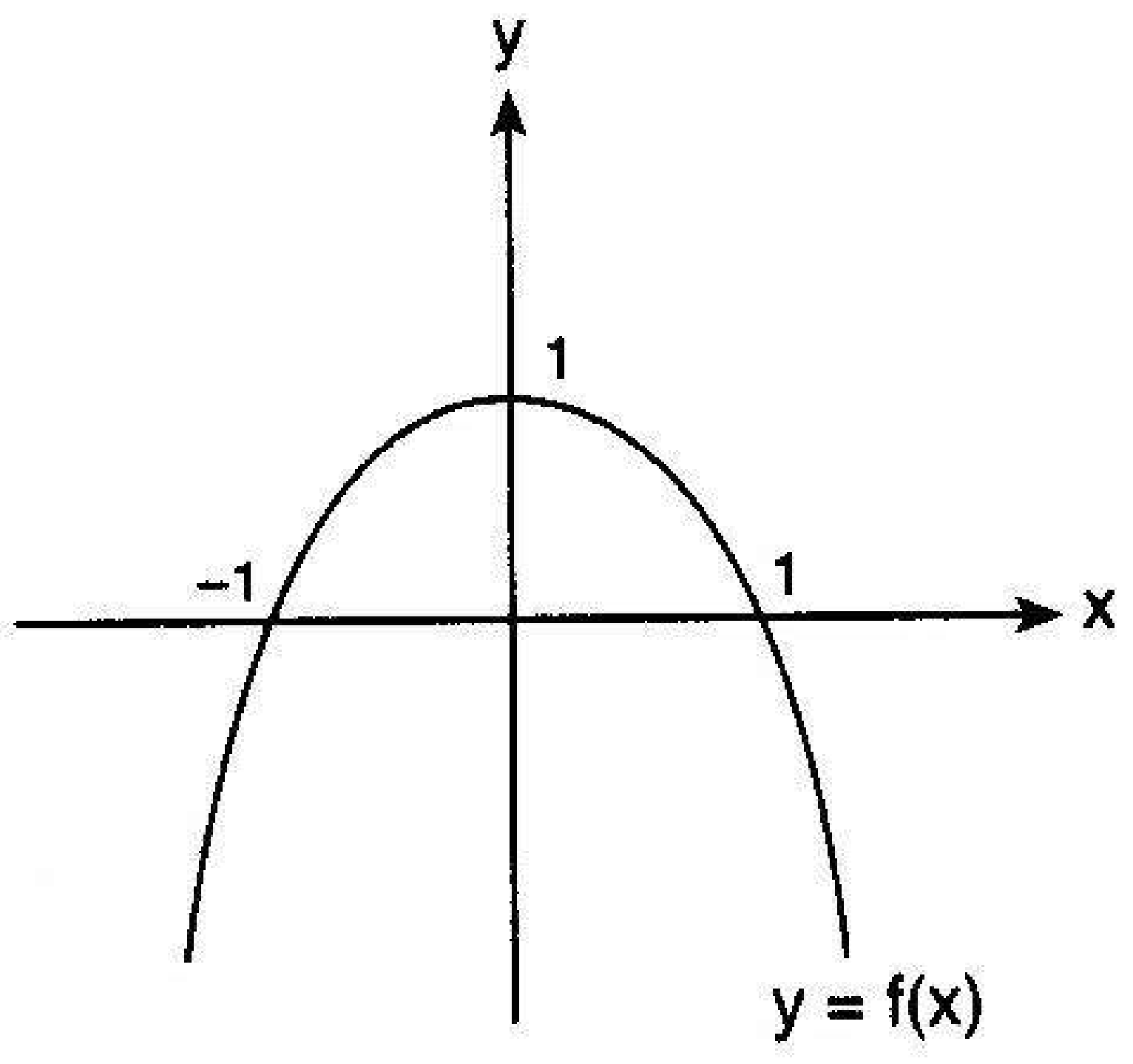
A) 0
B) 2
C) 4
D) 6
E) 8

4. **$f(1) = ?$**



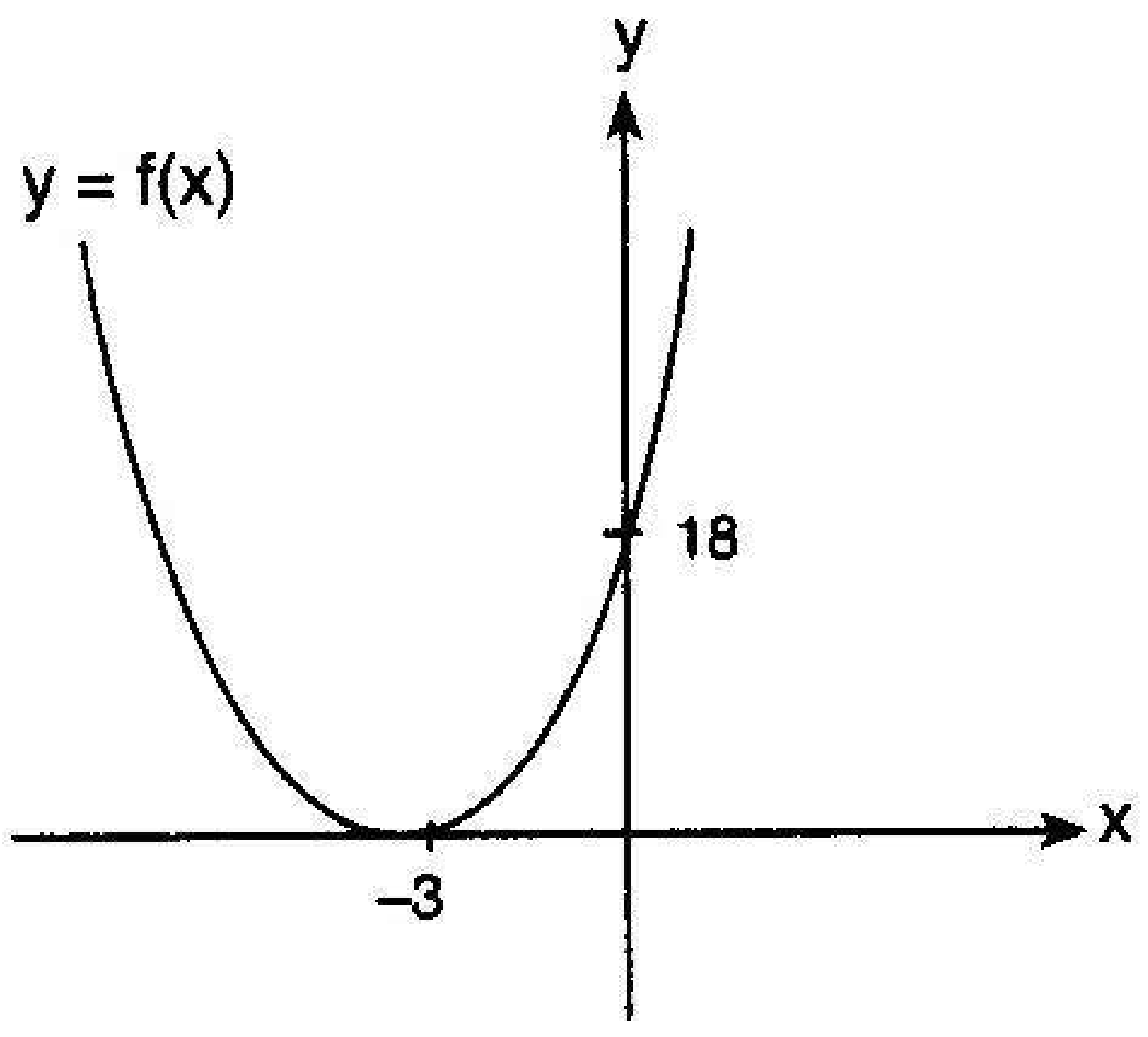
A) 2
B) 3
C) 4
D) 5
E) 6

5. **$f(3) = ?$**



A) –8
B) –10
C) –12
D) –14
E) –16

6. **$f(1) = ?$**



A) 19
B) 20
C) 24
D) 27
E) 32

7. **$f(3) = ?$**

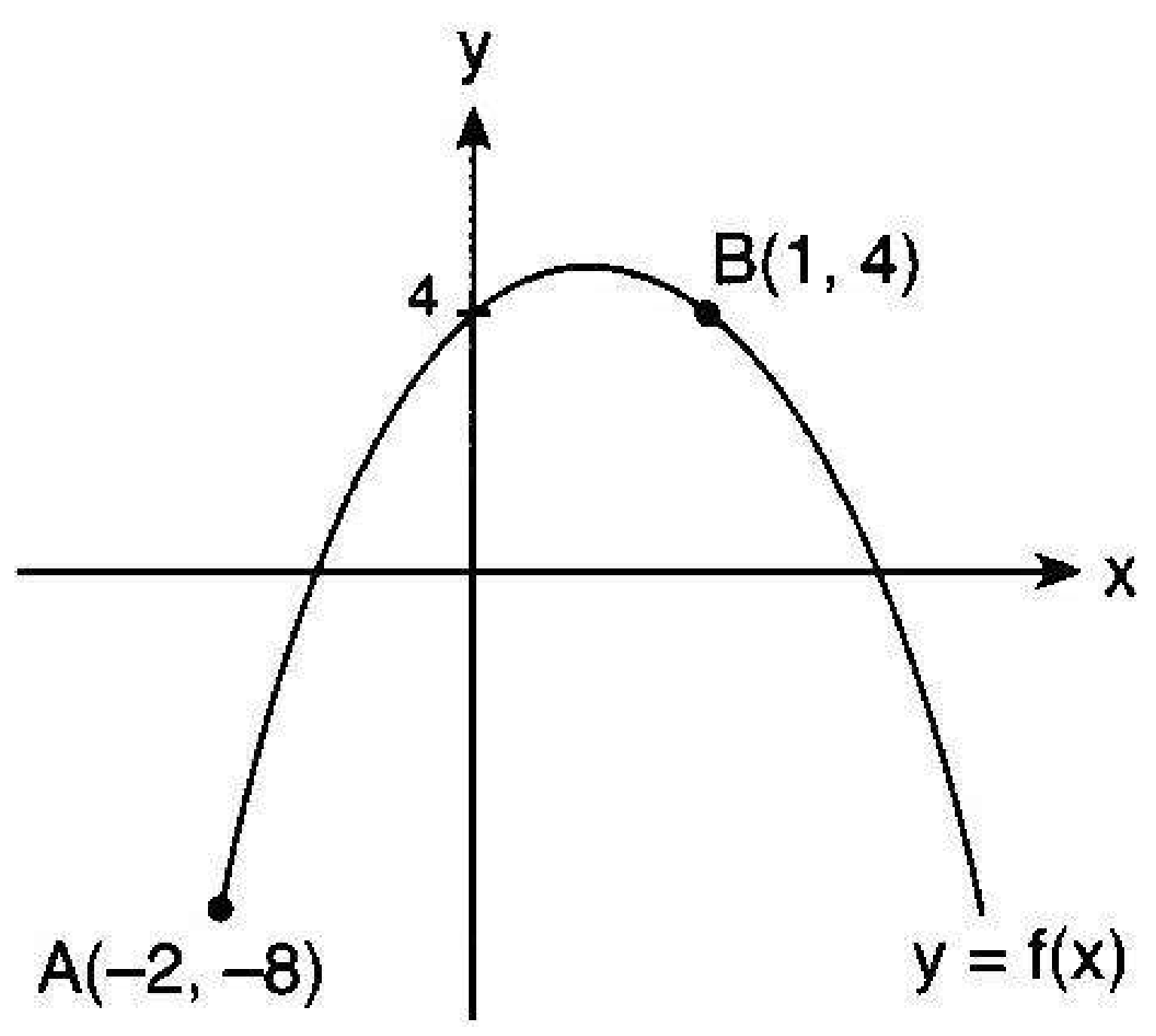
A) -6

B) -8

C) -10

D) -12

E) -14



8. **$f(-1) = ?$**

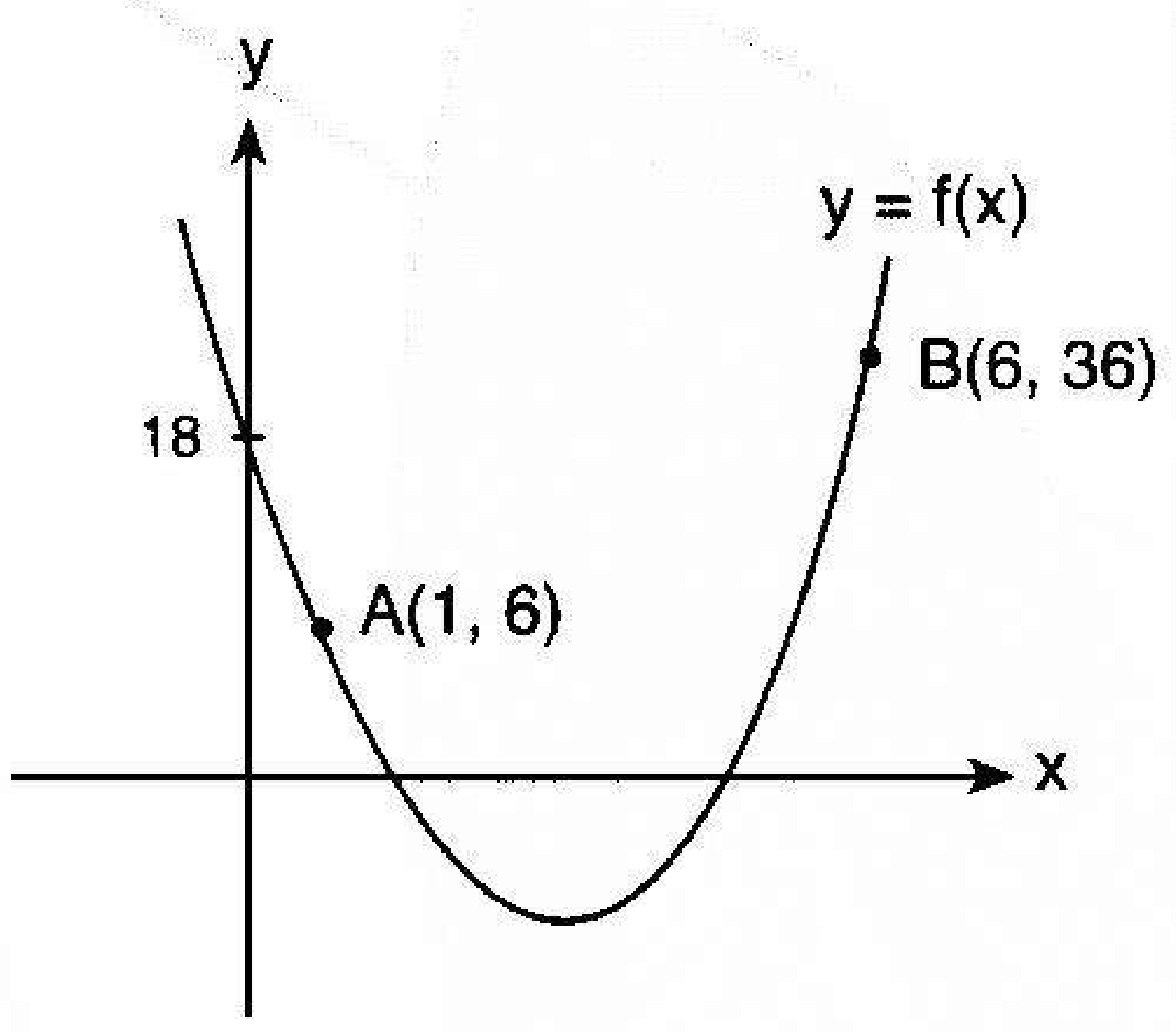
A) 32

B) 36

C) 38

D) 40

E) 42



9. $A(x_1, y_1)$, $B(x_2, y_2)$

$\Rightarrow$ **$x_1 + x_2 = ?$**

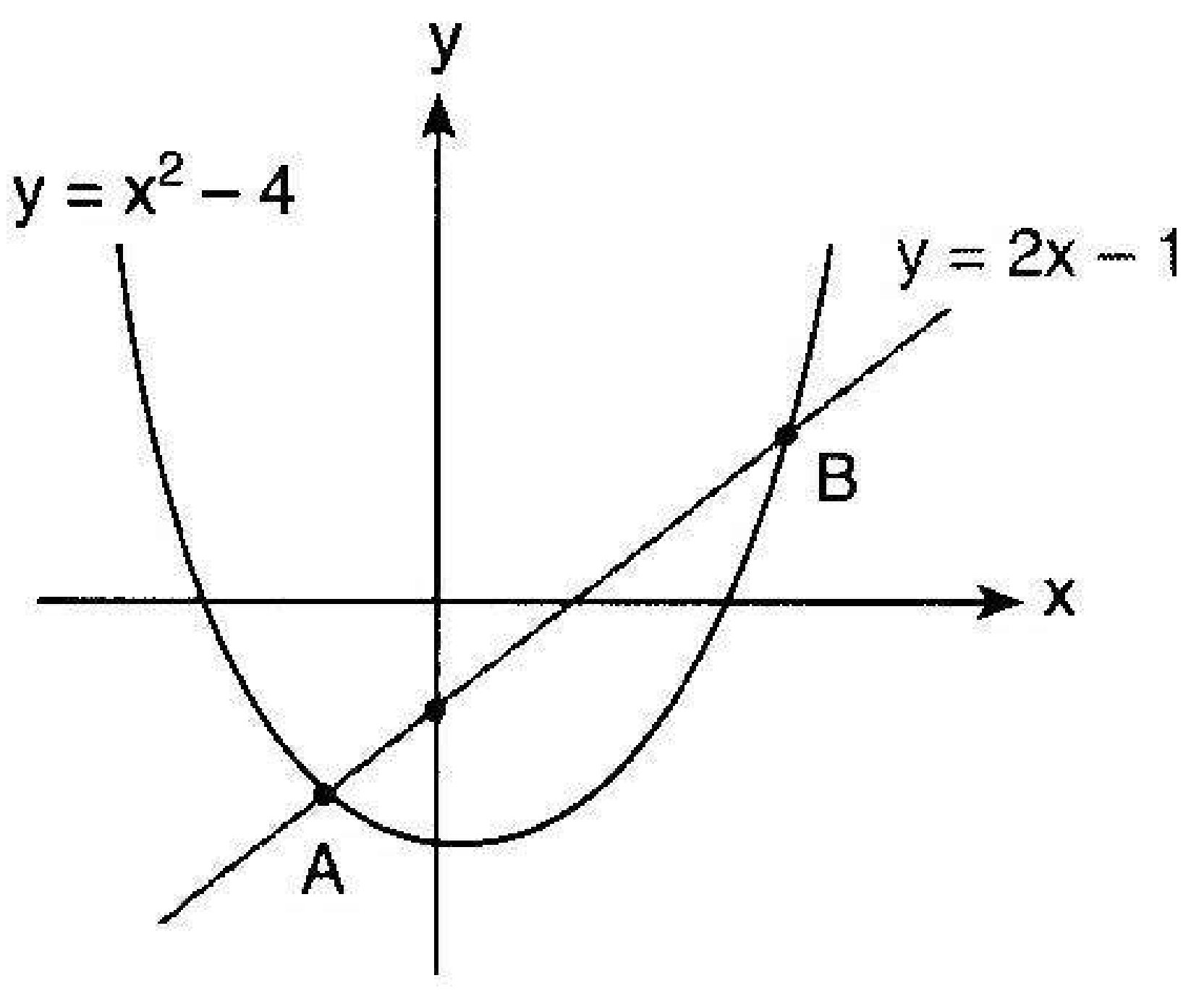
A) -1

B) 0

C) 1

D) 2

E) 3



10. **$m + n = ?$**

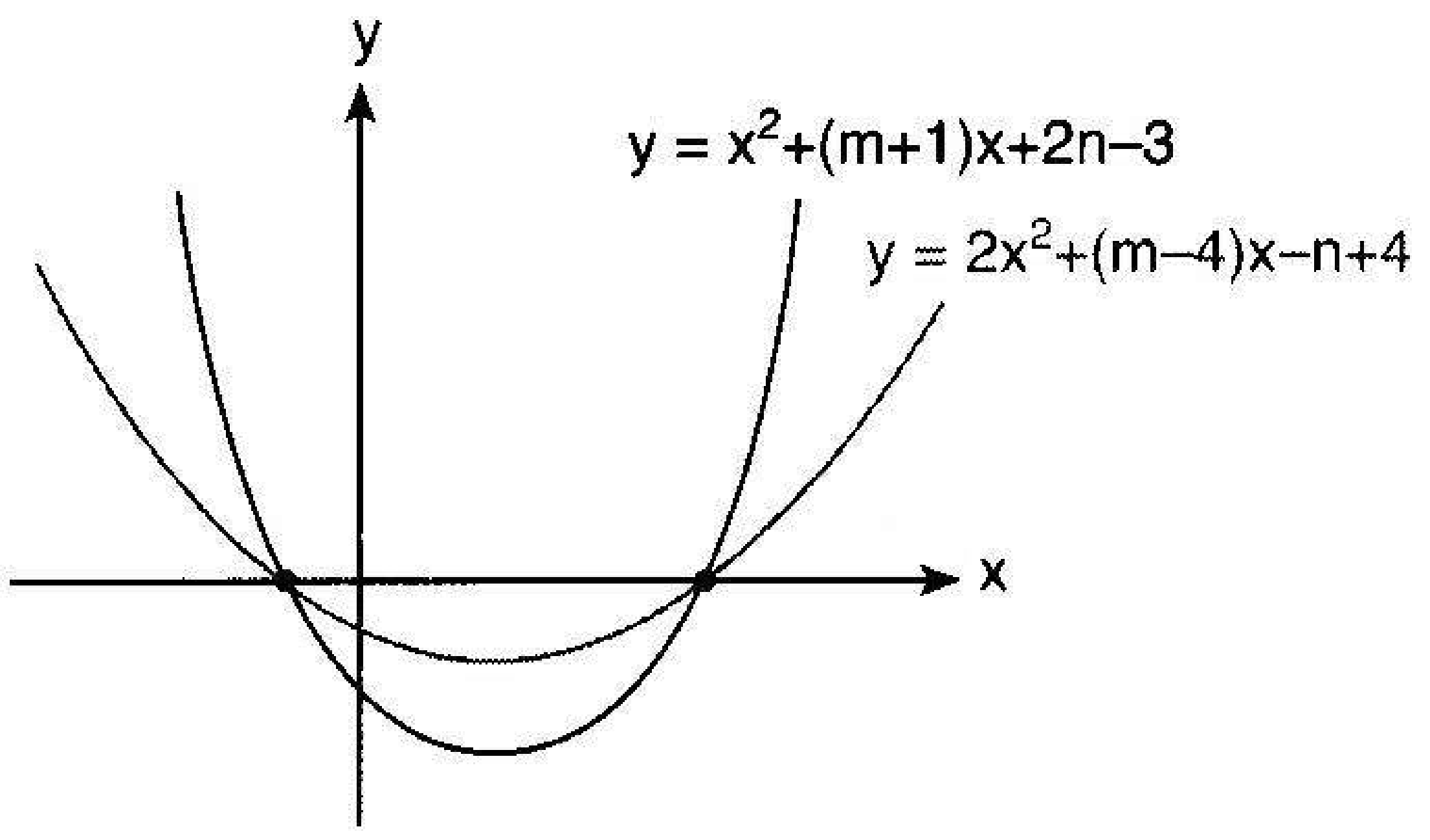
A) -7

B) -6

C) -5

D) -4

E) -3



11. **$m = ?$**

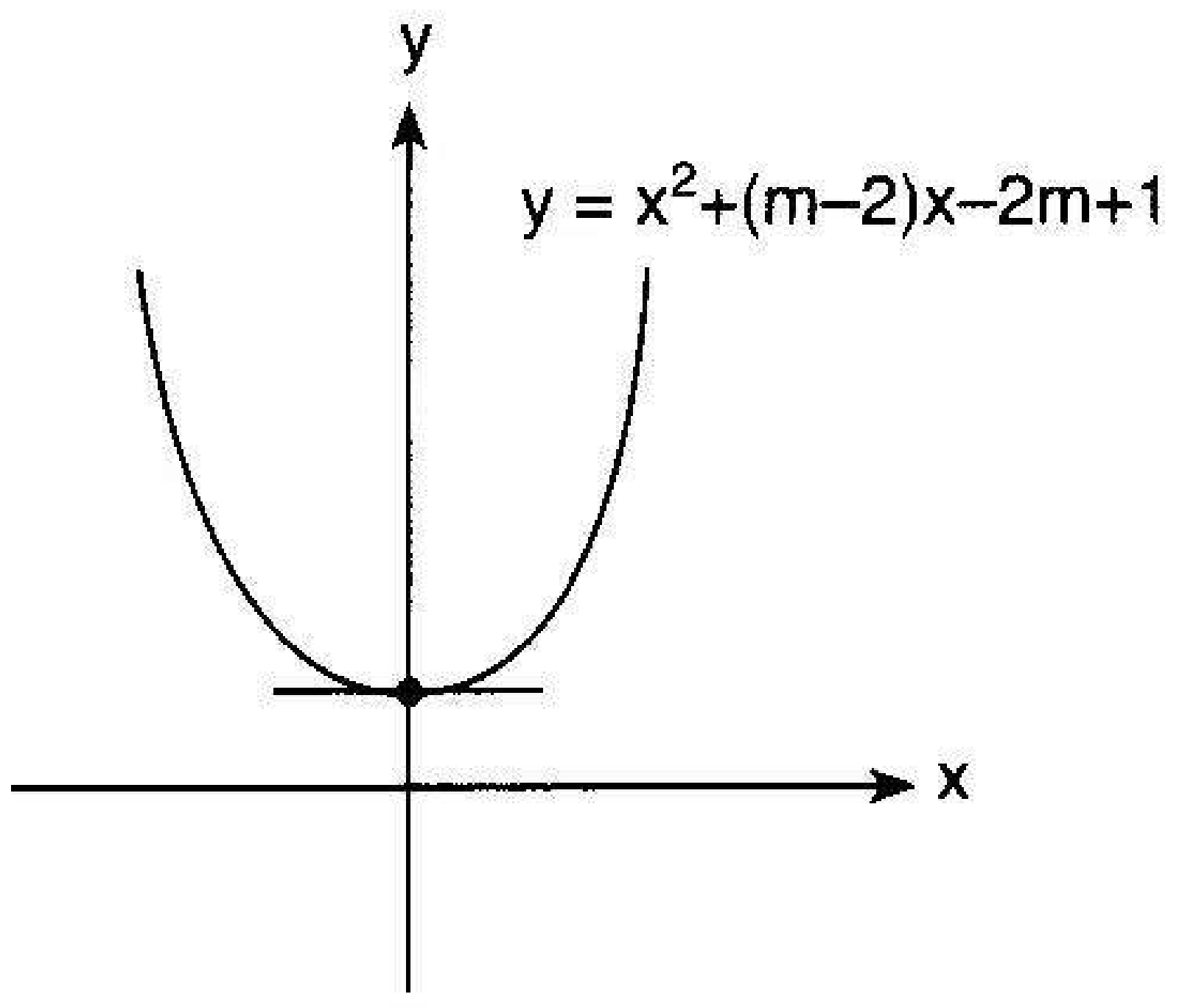
A) 2

B) 3

C) 4

D) 6

E) 8



12. **$m = ?$**

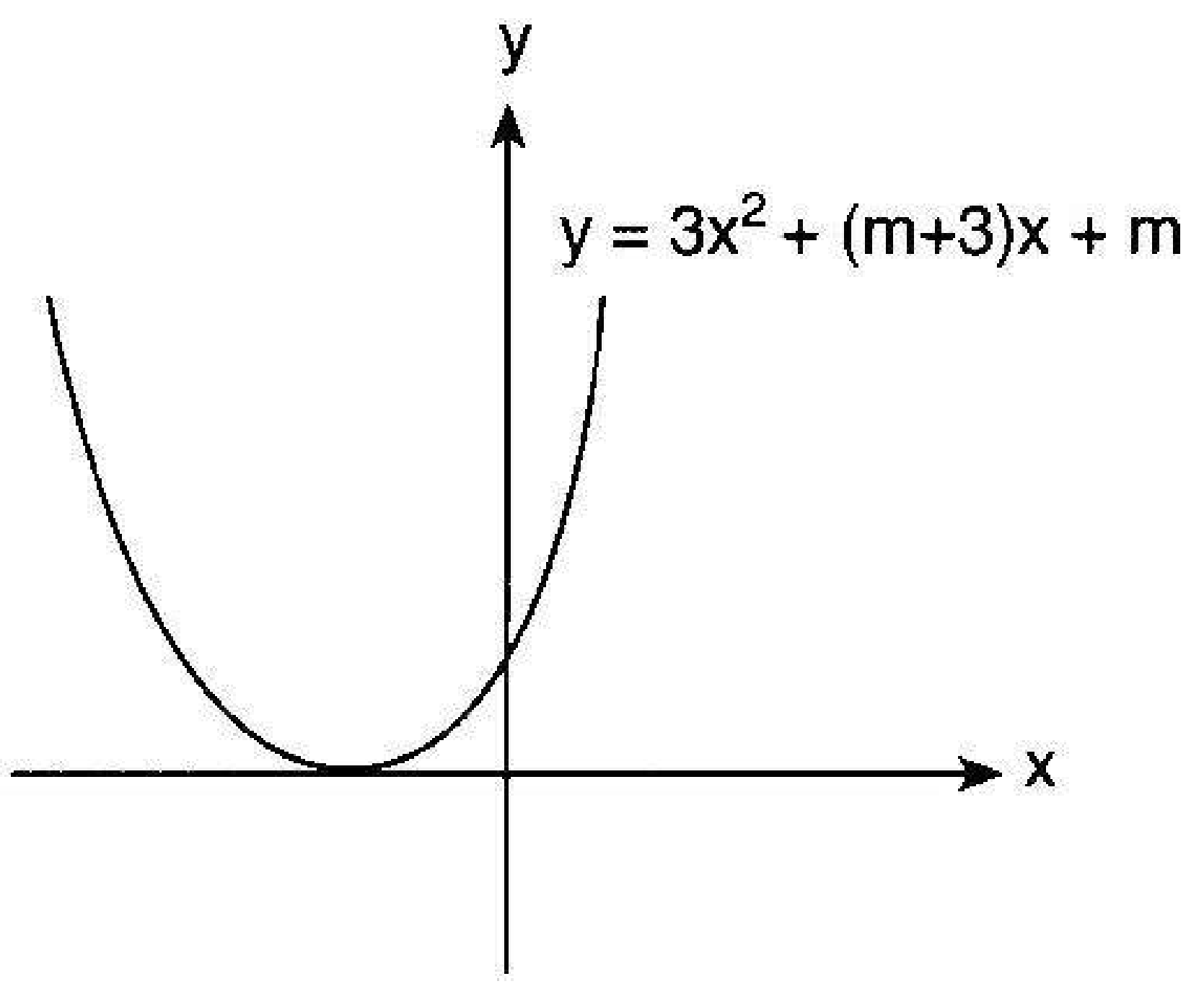
A) -3

B) -1

C) 1

D) 3

E) 5



YANITLAR / ANSWERS

1. B	2. C	3. C	4. D	5. A	6. E	7. B	8. B
9. D	10. D	11. A	12. D				

ÜNİTE 7

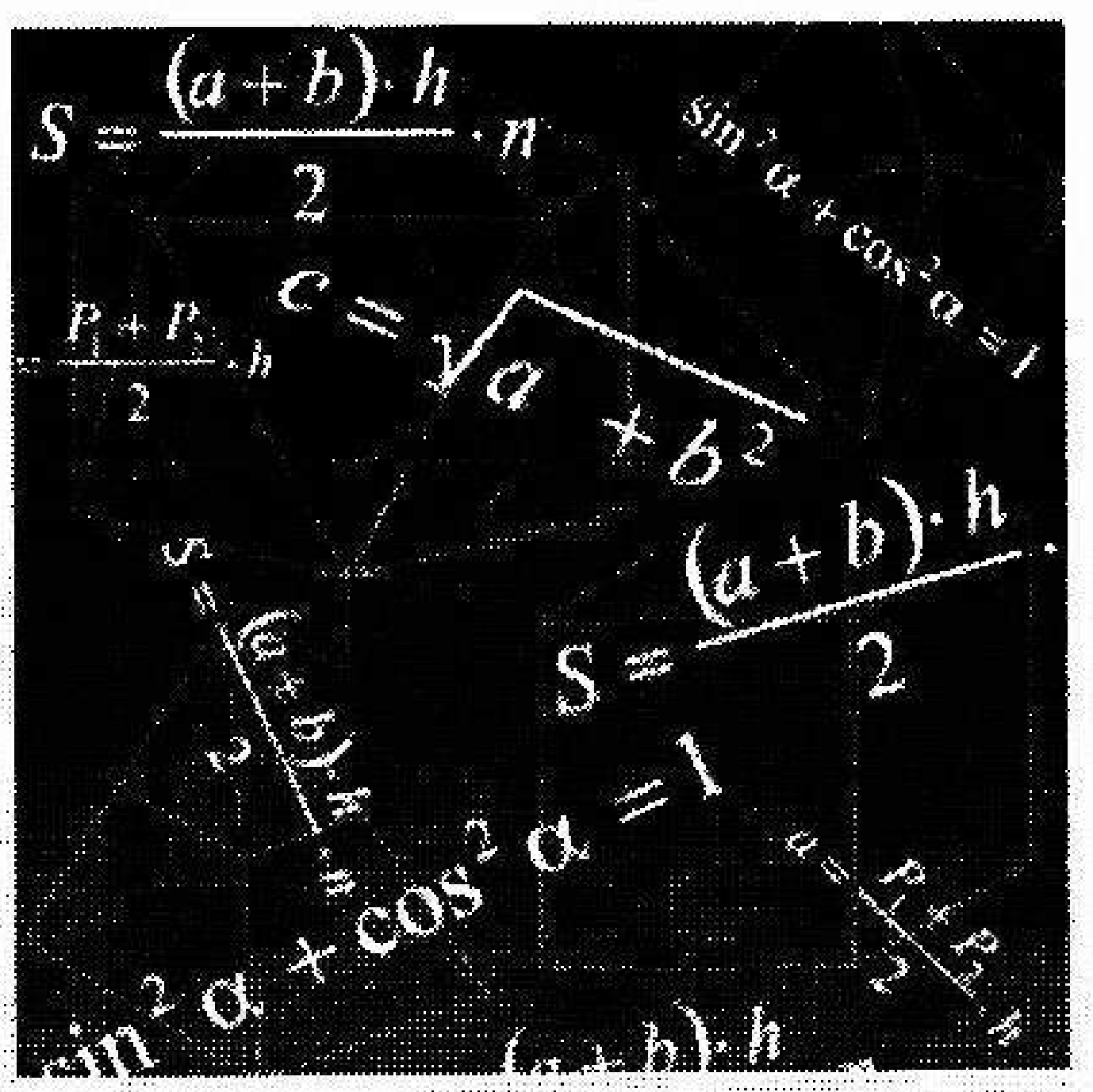
UNIT

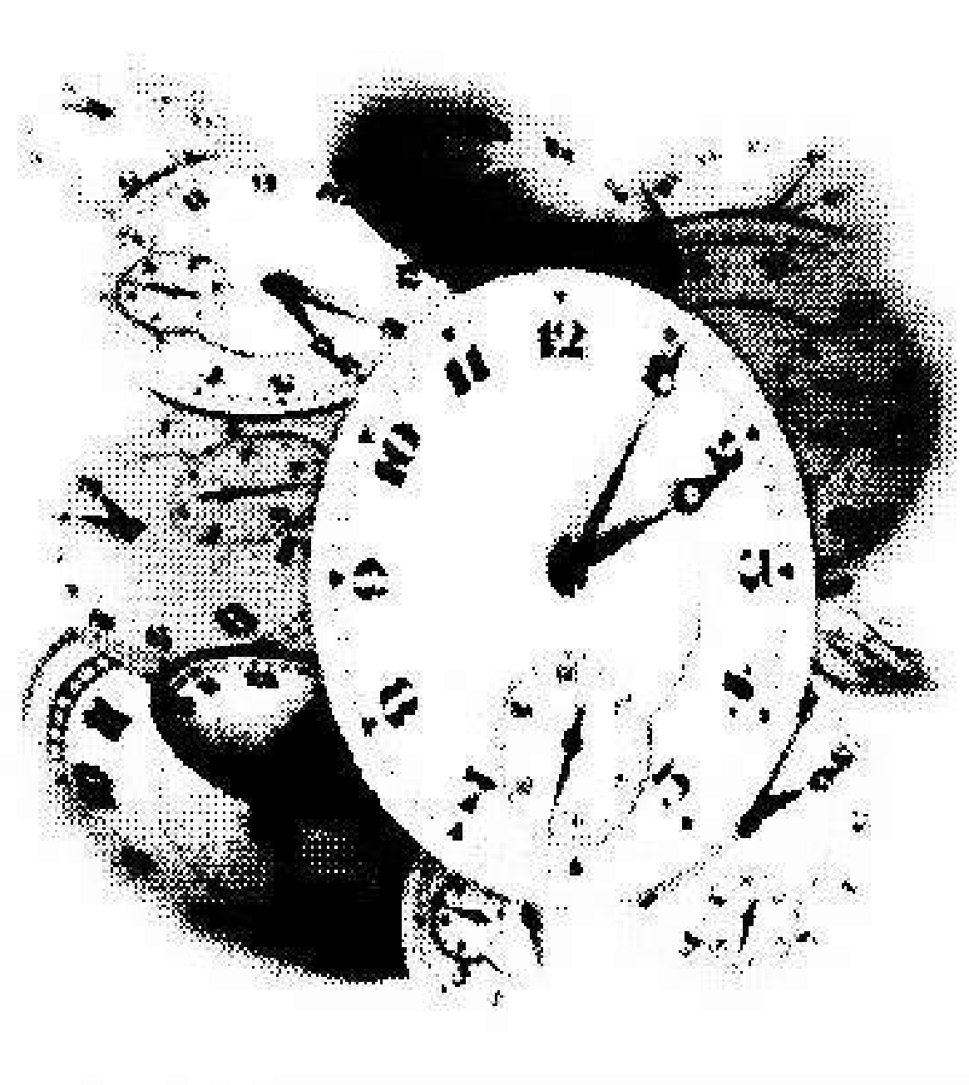
Bu ünitede, trigonometri, karmaşık sayılar ve logaritma ayrıntılı olarak anlatılmıştır. Trigonometri konusu zor olduğundan iki kısma ayrılmıştır. Birinci bölüm temel trigonometri kurallarından, ikinci bölüm ise trigonometrik formüllerden oluşmuştur.

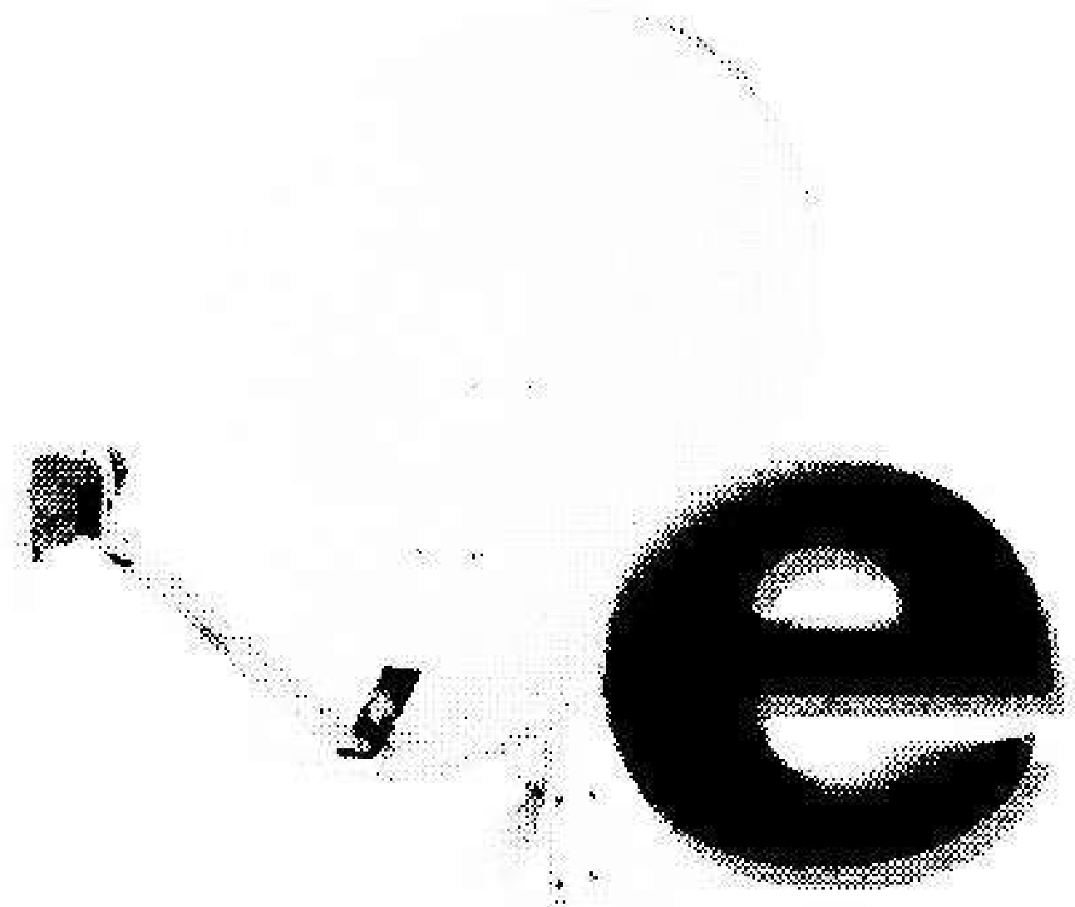
Bu ünite; 203 çözümlü örnek, 72 alıştırma ve 132 test sorusundan oluşmaktadır.

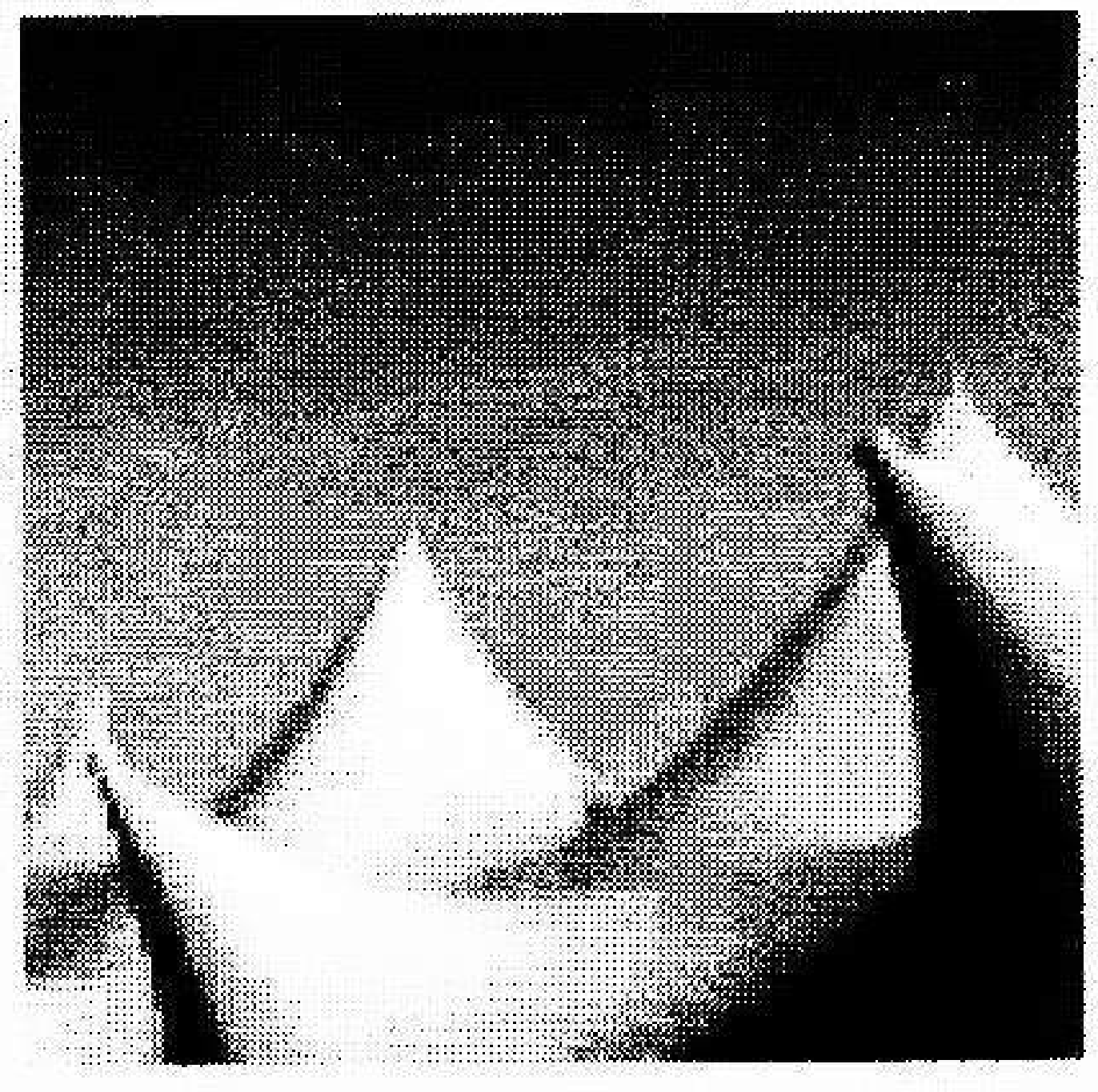
In this unit, trigonometry, complex numbers and logarithm are explained in detailed. Trigonometry is consist of two parts since this subject is very hardly. First chapter include basic rules of trigonometry and other chapter is consist of trigonometric formulas.

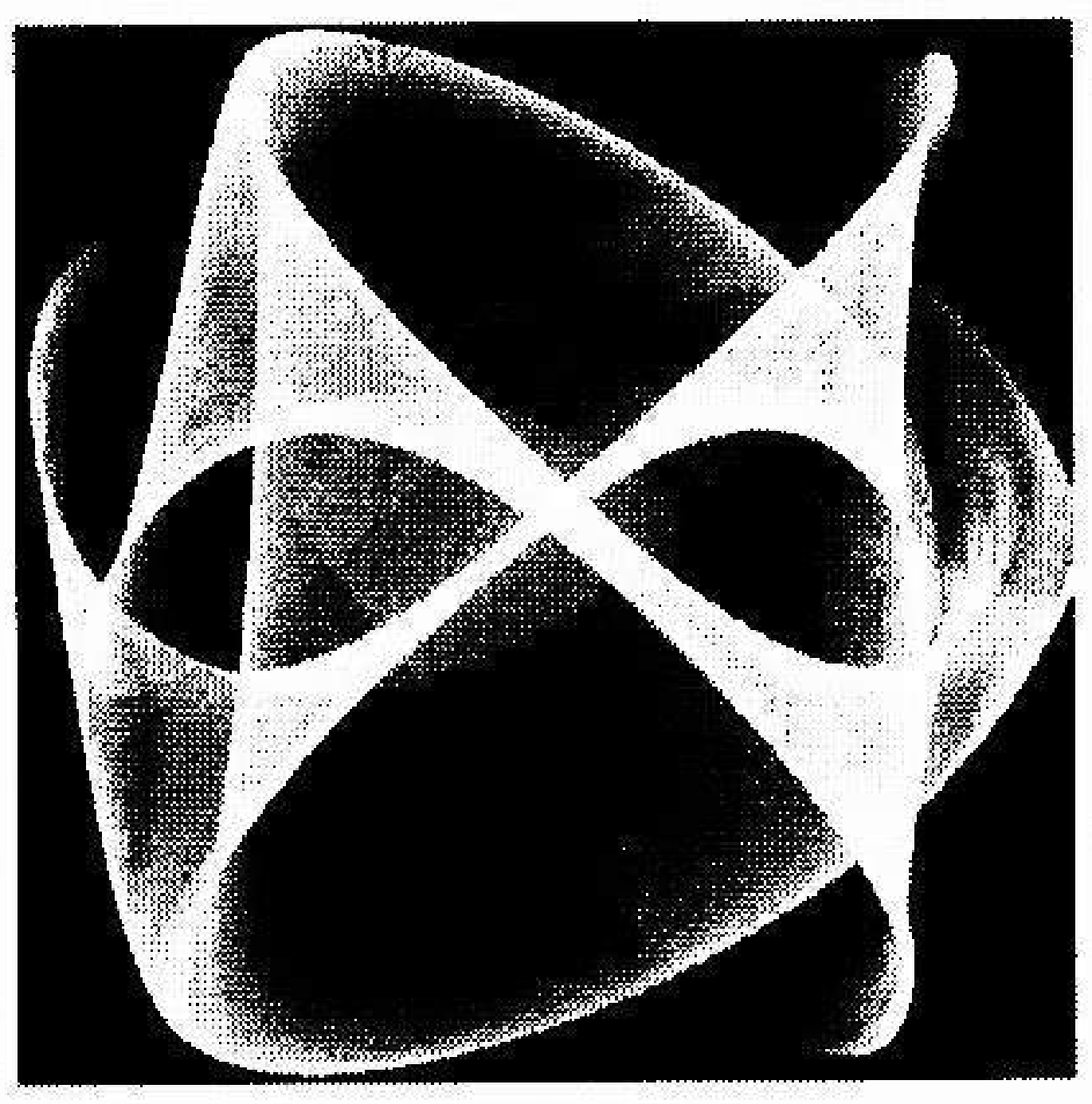
This unit includes; 203 examples, 72 exercises, and 132 test questions.


$S=\frac{(a+b)\cdot h}{2}\cdot n$
$\sin^2\alpha+\cos^2\alpha=1$
$c=\sqrt{a^2+b^2}$
$S=\frac{(a+b)\cdot h}{2}$



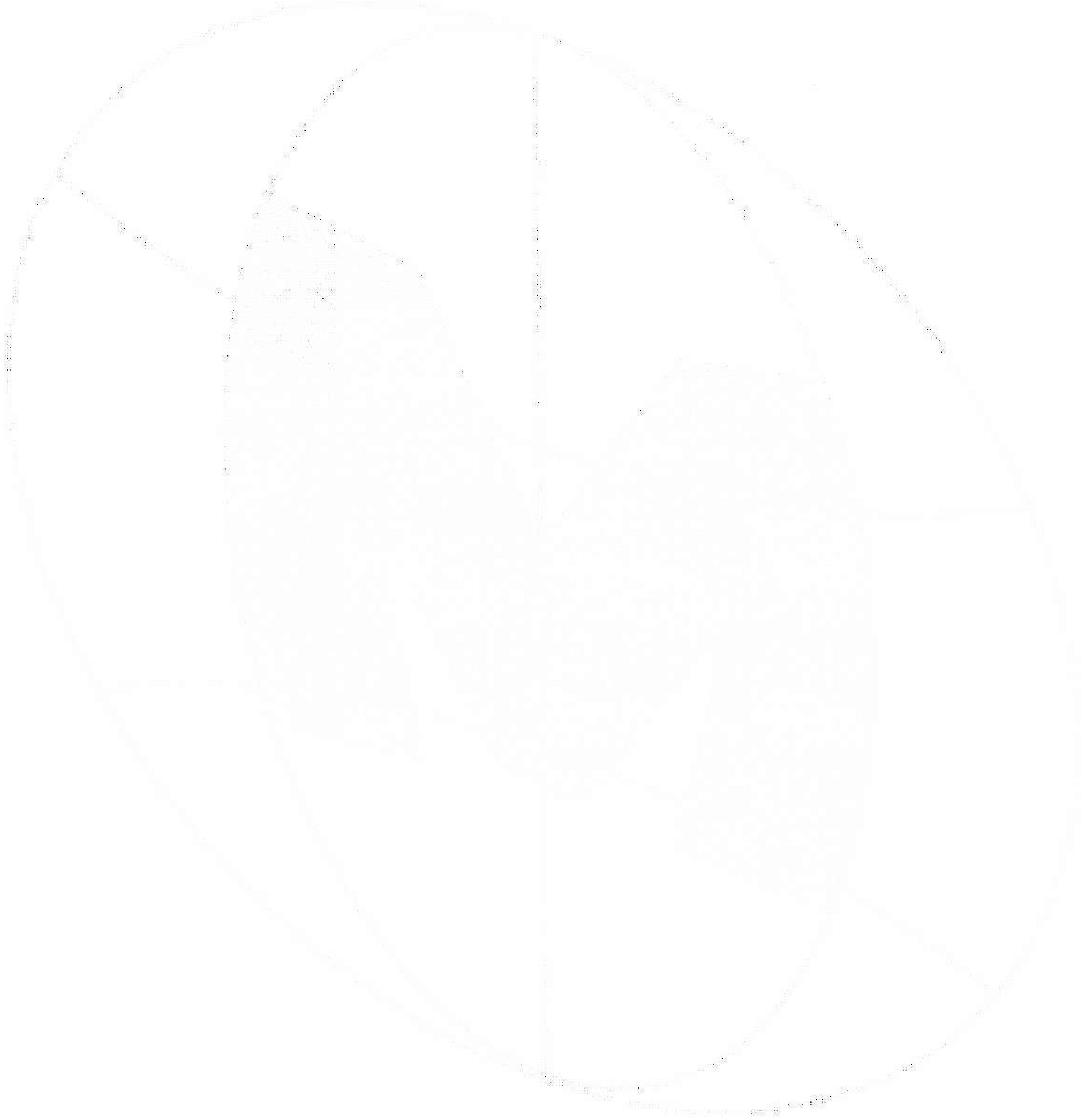






- **TRİGONOMETRİ I / *TRIGONOMETRY I*** . . . . 417 – 456
- **TRİGONOMETRİ II / *TRIGONOMETRY II*** . . . . 457 – 498
- **KARMAŞIK SAYILAR / *COMPLEX NUMBERS*** . . . . 499 – 526
- **LOGARİTMA / *LOGARITHM*** . . . . 527 – 554

BU SAYFADA KONU MATERYALİ YOKTUR.



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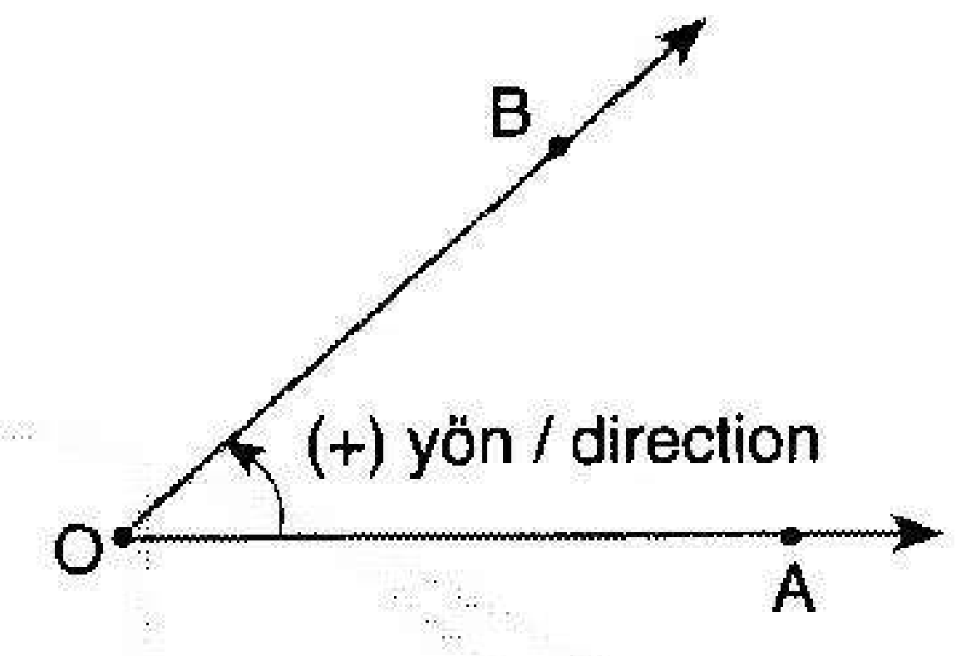
TRİGONOMETRİ I
TRIGONOMETRY I

BÖLÜM
CHAPTER
1

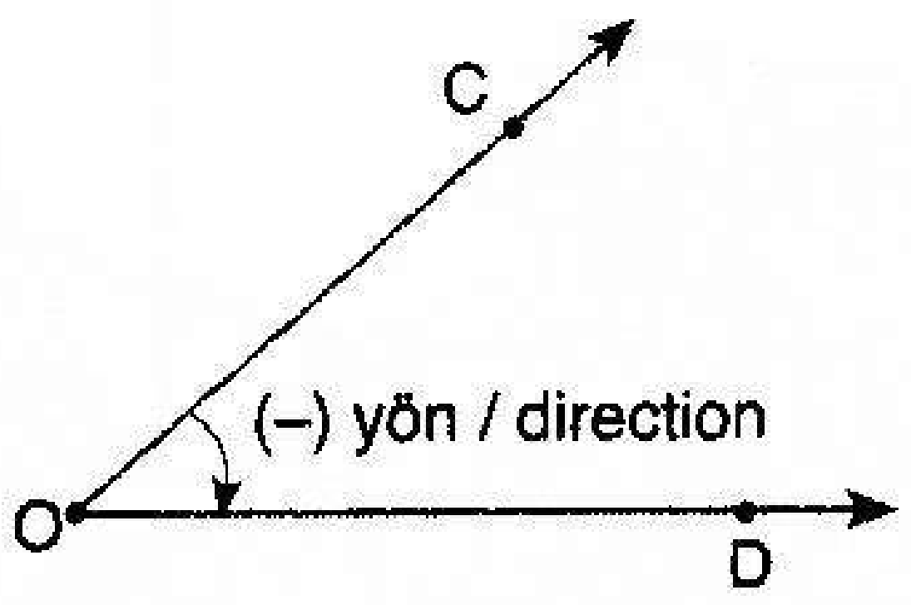
YÖNLÜ AÇILAR / DIRECTED ANGLES

Tanım / *Definition*



$\widehat{AOB}$ pozitif yönlü açı
$\widehat{AOB}$ is positive directional angle.

I. [OA, başlangıç kenarı; [OB, bitim kenarı olmak üzere saatin dönme yönünün tersi yönde alınan AOB açısına *pozitif yönlü açı* denir.
If the angle AOB rotates in a counter – clockwise direction, the rotation called positive where [OA is the initial side, [OB is the terminal side.



$\widehat{COD}$ negatif yönlü açı
$\widehat{COD}$ is negative directional angle.

II. [OC, başlangıç kenarı; [OD, bitim kenarı olmak üzere saatin dönme yönünde alınan COD açısına *negatif yönlü açı* denir.
If the angle COD rotates in a clockwise direction, the rotation is called negative where [OD is the initial side, [OC is the terminal side.

AÇI ÖLÇÜ BİRİMLERİ / UNITS of ANGLE

Derece: Tüm çember yayının 360 da birine 1 derecelik yay, bu yayı gören merkez açısının ölçüsüne de *1 derece (1°)* denir. Tüm çember yayının ölçüsü 360° dir.

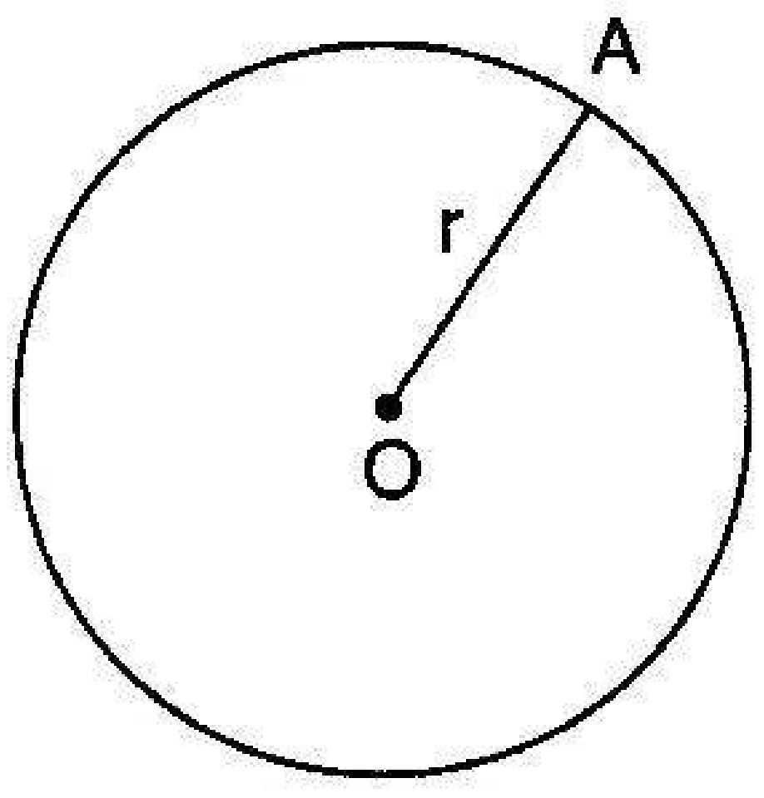
Degree: *Arc with a degree is* $\frac{1}{360°}$ *of the arc and the measure of the central angle is called a degree. One revolution is called 360°.*

Radyan: Yarıçap uzunluğundaki çember yayına bir radyanlık yay, bu yayı gören merkez açısının ölçüsüne de *1 radyan* denir.

Radian: *If the length of the arc is equal to radius, this arc is a radian and the measure of the central angle is called a radian.*

$$\frac{D}{180} = \frac{R}{\pi}$$

NOTLAR / NOTES



✓ Çevre uzunluğu / *The perimeter* = $2\pi r$
✓ Tüm çember yayının ölçüsü 2π radyandır.
The measure of a circular arc is 2π radii.

PEKİŞTİRME / REINFORCE

$\pi R = x \cdot D \Rightarrow x = ?$

YANIT / ANSWER : 180°

ÖRNEK / EXAMPLE 1

$\frac{3\pi}{2} R = x^\circ D \Rightarrow \mathbf{x = ?}$

A) 120° B) 150° C) 210° D) 240° E) 270°

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{D}{180^\circ} = \frac{R}{\pi} \Rightarrow \frac{x}{180^\circ} = \frac{\frac{3\pi}{2}}{\pi}$$

$$x = 180^\circ \cdot \frac{3}{2} = 270^\circ$$

ÖRNEK / EXAMPLE 2

$72^\circ = x\,R \Rightarrow \mathbf{x = ?}$

A) $\frac{\pi}{5}$ B) $\frac{2\pi}{5}$ C) $\frac{3\pi}{5}$ D) $\frac{5\pi}{3}$ E) $\frac{5\pi}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{D}{180^\circ} = \frac{x}{\pi} \Rightarrow \frac{72^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{72^\circ \pi}{180^\circ} = \frac{2\pi}{5}$$

ESAS ÖLÇÜ / REFERENCE MEASURE

Tanım: Verilen ölçülerin aşağıdaki denkliklerle hesaplanan pozitif yöndeki değerlerine *esas ölçü* denir.

Definition: *Values of the positive direction that calculated following equivalent is called reference measure of the given measure.*

I. $2\pi \cdot k + \theta \equiv \theta \pmod{2\pi} \quad (k \in Z)$

II. $360° \cdot k + \theta \equiv \theta \pmod{360°} \quad (k \in Z)$

ÖRNEK / EXAMPLE 3

$780° = \alpha + 360° \cdot k \Rightarrow \alpha = ?$

A) 20° B) 30° C) 40° D) 60° E)120°

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

Çözüm / *Solution* 1:

$780° = \alpha + 360° \cdot k$

$780° = 60° + 360° \cdot 2$

$\alpha = 60°$

Çözüm / *Solution* 2:

$$\begin{array}{r|l} 780 & 360 \\ \hline -\,720 & 2 \\ \boxed{60} \rightarrow \alpha & \end{array}$$

ÖRNEK / EXAMPLE 4

$\dfrac{27\pi}{5} = \alpha + 2k\pi \Rightarrow \alpha = ?$

A) $\dfrac{2\pi}{5}$ B) $\dfrac{4\pi}{5}$ C) π D) $\dfrac{6\pi}{5}$ E) $\dfrac{7\pi}{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

Çözüm / *Solution* 1:

$$\frac{27\pi}{5} = \alpha + 2k\pi$$

$$\frac{20\pi + 7\pi}{5} = \alpha + 2k\pi$$

$$4\pi + \frac{7\pi}{5} = \alpha + 2k\pi$$

$$\alpha = \frac{7\pi}{5}$$

Çözüm / *Solution* 2:

$$\begin{array}{r|l} 27 & 10 \\ \hline -\,20 & 2 \\ \boxed{7} & \end{array}$$

$$\Rightarrow \alpha = \frac{7\pi}{5}$$

NOTLAR / NOTES

- ✓ Verilen ölçü derece cinsinden ise 360° nin tam katları, radyan cinsinden ise 2π nin tam katları olabilir.
  The given angle can multiple of 360° in degrees or multiple of 2π in radii.
- ✓ Radyan cinsinden verilen kesirli ifadede esas ölçü aranırken paydanın çift katları paydan çıkarılır.
  In expression involving fraction in radii, reference measure is found subtract the numerator by the even multiples of denominator.
- ✓ Bir açının esas ölçüsü kesinlikle negatif(–) olamaz. Daima gördüğü yayın bitim noktası ile x ekseninin pozitif yönü arasındaki açı esas ölçü olarak alınır.
  The central angle of an angle is not negative. Central angle is between terminal side and positive direction of the x – axis.

PEKİŞTİRME / REINFORCE

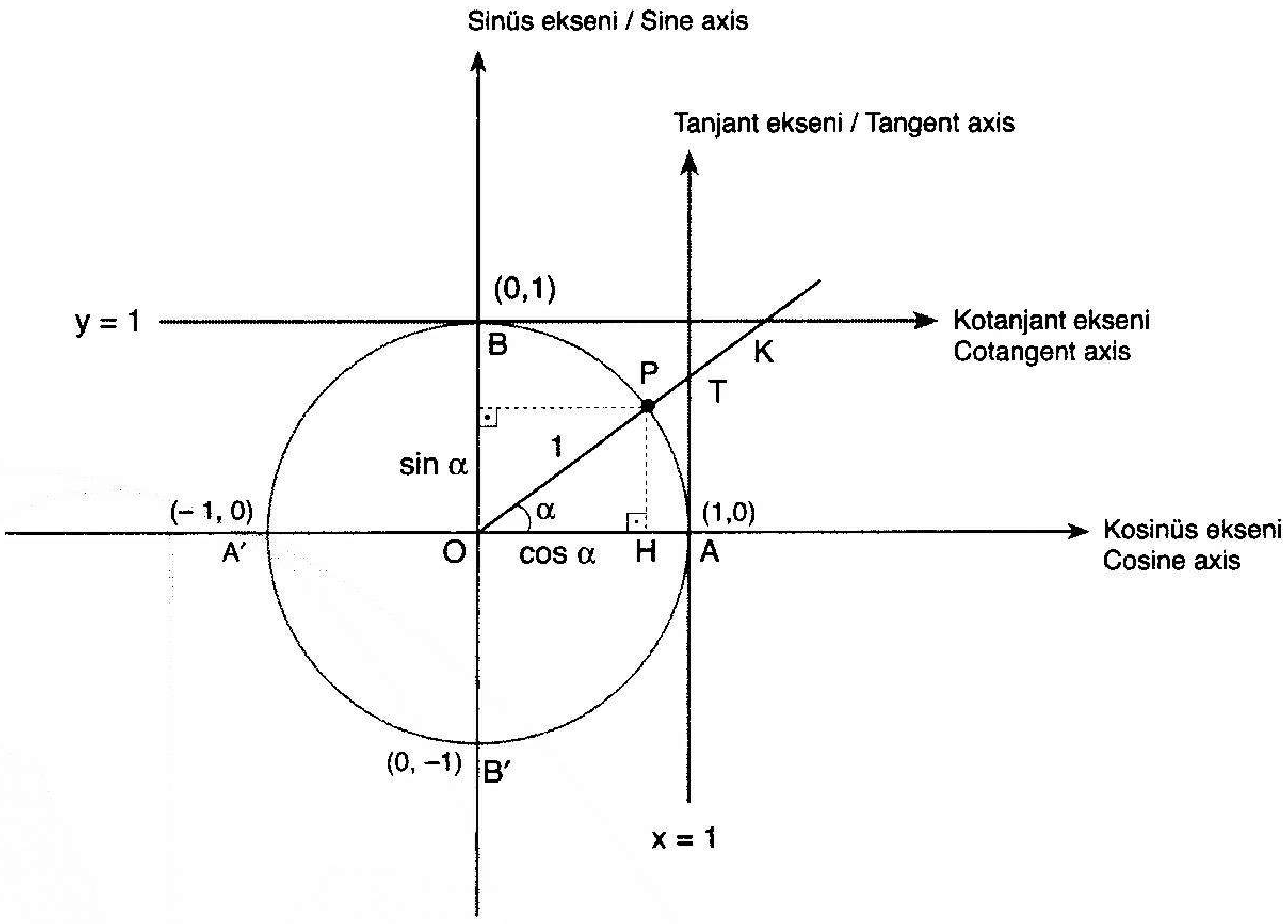
$-\dfrac{11\pi}{3} = \alpha + 2k\pi \Rightarrow \alpha = ?$

YANIT / ANSWER : $\pi/3$

Düşünün / Think about it

$2x^2 \le \sin^6 x + \cos^{10} x - 1 \Rightarrow$ ÇK(SS) = ?

TRİGONOMETRİK FONKSİYONLAR / TRIGONOMETRIC FUNCTIONS


Sinüs ekseni / Sine axis
Tanjant ekseni / Tangent axis
y = 1
(0,1)
B
P
K
T
Kotanjant ekseni
Cotangent axis
sin α
1
α
(– 1, 0)
A′
O
cos α
H
(1,0)
A
Kosinüs ekseni
Cosine axis
(0, –1)
B′
x = 1

Kosinüs Fonksiyonu / Cosine Function

Bir α açısına birim çember üzerinde karşılık gelen P noktasının apsisine *α açısının kosinüsü* denir.

Abscissa of point P is called cosine of angle α that on the unit circle of the angle α.

$$\cos(\widehat{AOP}) = |OH|$$

$$\cos: \mathbb{R} \rightarrow [-1, 1]$$

Sinüs Fonksiyonu / Sine Function

Bir α açısına birim çember üzerinde karşılık gelen P noktasının ordinatına *α açısının sinüsü* denir.

Ordinate of point P is called sine of angle α that on the unit circle of the angle α.

$$\sin(\widehat{AOP}) = |PH|$$

$$\sin: \mathbb{R} \rightarrow [-1, 1]$$

Yanıt / Answer

x ≤ 0

Tanjant Fonksiyonu / Tangent Function

Bir α açısına birim çember üzerinde karşılık gelen P noktasından geçen [OT ışınının tanjant eksenini kestiği T noktasının ordinatına *α açısının tanjantı* denir.

Ordinate of point T is tangent of the angle α that cut by tangent axis of ray [OT the passes through P.

$$\tan(\widehat{AOP}) = |TA|$$

$$\tan : R - \left\{\frac{\pi}{2} + 2k\pi , \frac{3\pi}{2} + 2k\pi\right\} \rightarrow R$$

Kotanjant Fonksiyonu / Cotangent Function

Bir α açısına birim çember üzerinde karşılık gelen P noktasından geçen [OK ışınının cotanjant eksenini kestiği K noktasının apsisine *α açısının cotanjantı* denir.

Abscissa of point K is cotangent of the angle α that cut by tangent axis of ray [OK the passes throung P.

$$\cot(\widehat{AOP}) = |BK|$$

$$\text{cotanjant: } R - \{\pi + 2k\pi , 2\pi + + 2k\pi\} \rightarrow R$$

NOT / NOTE

Fonksiyon *Function*	Tanım kümesi *Domain*	Görüntü kümesi *Range*
sin	R	[−1, 1]
cos	R	[−1, 1]
tan	$R - \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$	R
cot	$R - \{\pi, 2\pi\}$	R

TRİGONOMETRİK ÖZDEŞLİKLER / TRIGONOMETRIC IDENTITIES

i. $\sin^2 x + \cos^2 x = 1$

ii. $\tan x = \dfrac{\sin x}{\cos x}$

iii. $\cot x = \dfrac{\cos x}{\sin x}$

iv. $\sec x = \dfrac{1}{\cos x}$

v. $\text{cosec}\, x = \dfrac{1}{\sin x}$

vi. $\tan x \cdot \cot x = 1$

vii. $x + y = \dfrac{\pi}{2} \Rightarrow \sin x = \cos y \ , \ \tan x = \cot y$

ÖRNEK / EXAMPLE 5

$\dfrac{3\sin x - \cos x}{2\sin x + 5\cos x} = \dfrac{2}{3} \Rightarrow$ **tanx = ?**

A) 2 B) $\dfrac{11}{5}$ C) $\dfrac{13}{5}$ D) 3 E) $\dfrac{16}{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\dfrac{3\sin x - \cos x}{2\sin x + 5\cos x} = \dfrac{2}{3}$ [Verilen eşitlik / *Given equality*]

$9\sin x - 3\cos x = 4\sin x + 10\cos x$ [Çapraz çarpım / *Cross-multiply.*]

$5\sin x = 13\cos x$ [Benzer terimleri topla. / *Combine like terms.*]

$\dfrac{\sin x}{\cos x} = \dfrac{13}{5}$ [Sadeleştir. / *Simplify.*]

$\tan x = \dfrac{13}{5}$ [Sonuç / *Conclusion*]

PEKİŞTİRME / REINFORCE

$\tan x + \cot x = 2 \Rightarrow \tan^4 x + \cot^4 x = ?$

YANIT / ANSWER : 2

ÖRNEK / EXAMPLE 6

$\cos x + \sec x = \dfrac{7}{2} \Rightarrow$ **$\cos^2 x + \sec^2 x = ?$**

A) $\dfrac{41}{4}$ B) $\dfrac{43}{4}$ C) $\dfrac{47}{4}$ D) $\dfrac{49}{4}$ E) $\dfrac{51}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$\cos x + \sec x = \dfrac{7}{2}$ [Verilen eşitlik / *Given equality*]

$\cos x + \dfrac{1}{\cos x} = \dfrac{7}{2}$ [Trigonometrik özdeşlik / *Trigonometric identities*]

$\cos^2 x + 2\cdot\cos x\cdot\dfrac{1}{\cos x} + \dfrac{1}{\cos^2 x} = \dfrac{49}{4}$ [Tam kare / *Perfect square*]

$\cos^2 x + \dfrac{1}{\cos^2 x} = \dfrac{49}{4} - 2$ [Benzer terimleri topla. / *Combine like terms.*]

$\cos^2 x + \sec^2 x = \dfrac{41}{4}$ [Sonuç / *Conclusion*]

ÖRNEK / EXAMPLE 7

$A = 7\sin x - 6\cos y \Rightarrow$ **min(A) = ?**

A) 13 B) 1 C) 0 D) –1 E) –13

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{aligned} -1 \le \sin x \le 1 \quad &\Rightarrow \quad -7 \le 7\sin x \le 7 \\ -1 \le \cos x \le 1 \quad &\Rightarrow \quad + \; -6 \le -6\cos x \le 6 \\ \hline &-13 \le 7\sin x - 6\cos x \le 13 \\ &-13 \le A \le 13 \\ &\Rightarrow \min(A) = -13 \end{aligned}$$

PEKİŞTİRME / REINFORCE

$A = 3\sin x + 4\cos x \Rightarrow \max(A) = ?$

YANIT / ANSWER : 5

ÖRNEK / EXAMPLE 8

$\sin x + \cos x = \frac{3}{4} \Rightarrow$ **sinx·cosx = ?**

A) $\frac{-7}{16}$ B) $\frac{-7}{8}$ C) $\frac{-7}{32}$ D) $\frac{9}{16}$ E) $\frac{1}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\sin x + \cos x = \frac{3}{4}$ [Verilen eşitlik / *Given equality*]

$(\sin x + \cos x)^2 = \left(\frac{3}{4}\right)^2$ [Her iki tarafın karesine al. / *Square both sides.*]

$\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x = \frac{9}{16}$ [Tam kare / *Perfect square*]

$1 + 2\sin x \cdot \cos x = \frac{9}{16}$ [Trigonometik özdeşlik / *Trigonometric identities*]

$2\sin x \cdot \cos x = -\frac{7}{16}$ [Her iki taraftan 1 çıkar. / *Subtract 1 from both sides.*]

$\sin x \cdot \cos x = -\frac{7}{32}$ [Sonuç / *Conclusion*]

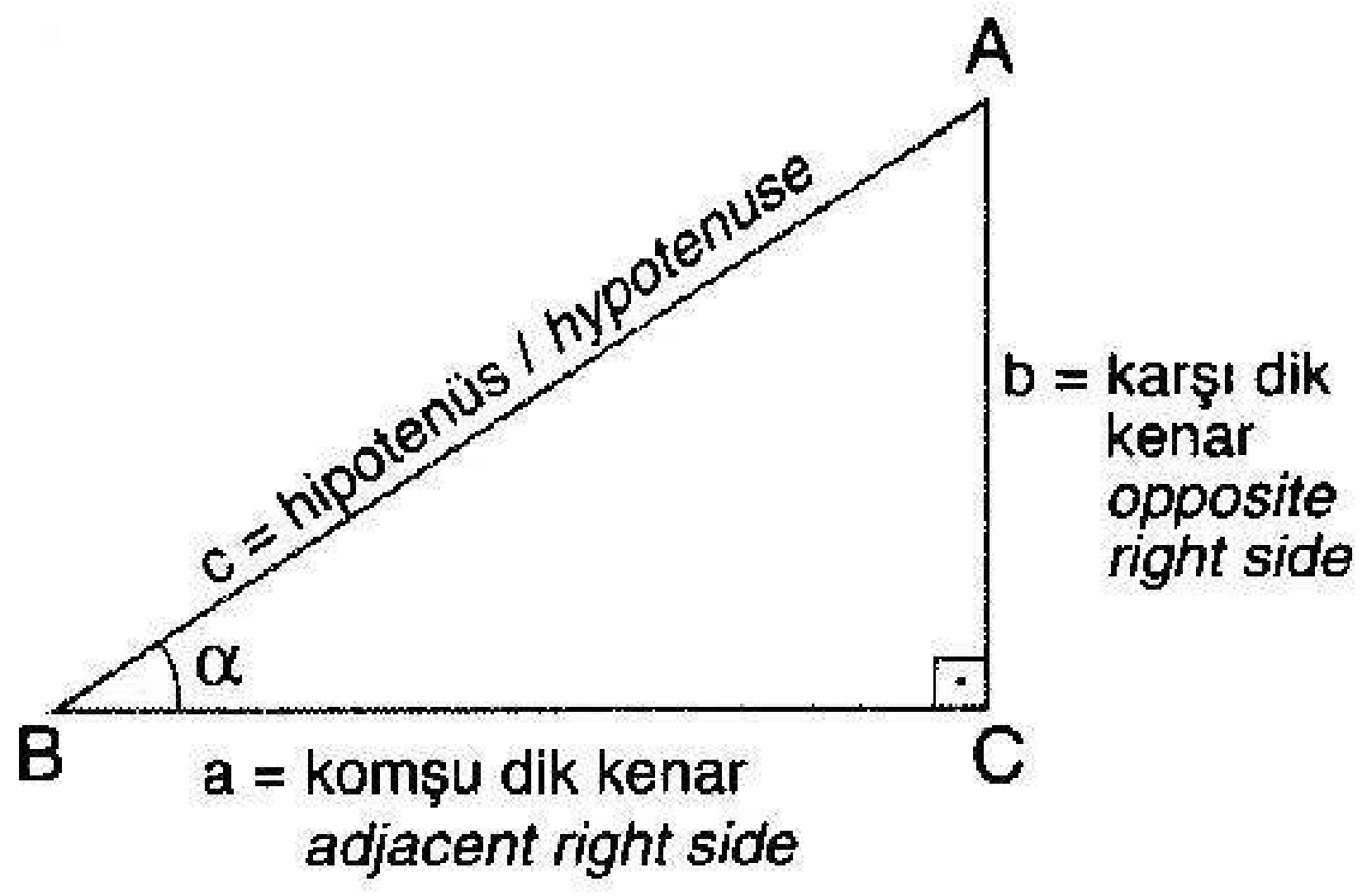
DİK ÜÇGENDE TRİGONOMETRİK ORANLAR
TRIGONOMETRIC RATIOS in a RIGHT TRIANGLE

$$\sin\alpha = \frac{\text{karşı dik kenar / }\textit{opposite right side}}{\text{hipotenüs / }\textit{hypotenuse}} = \frac{b}{c}$$

$$\cos\alpha = \frac{\text{komşu dik kenar / }\textit{adjacent right side}}{\text{hipotenüs / }\textit{hypotenuse}} = \frac{a}{c}$$

$$\tan\alpha = \frac{\text{karşı dik kenar / }\textit{opposite right side}}{\text{komşu dik kenar / }\textit{adjacent right side}} = \frac{b}{a}$$

$$\cot\alpha = \frac{\text{komşu dik kenar / }\textit{adjacent right side}}{\text{karşı dik kenar / }\textit{opposite right side}} = \frac{a}{b}$$


A
c = hipotenüs / hypotenuse
b = karşı dik kenar
opposite right side
α
B
a = komşu dik kenar
adjacent right side
C

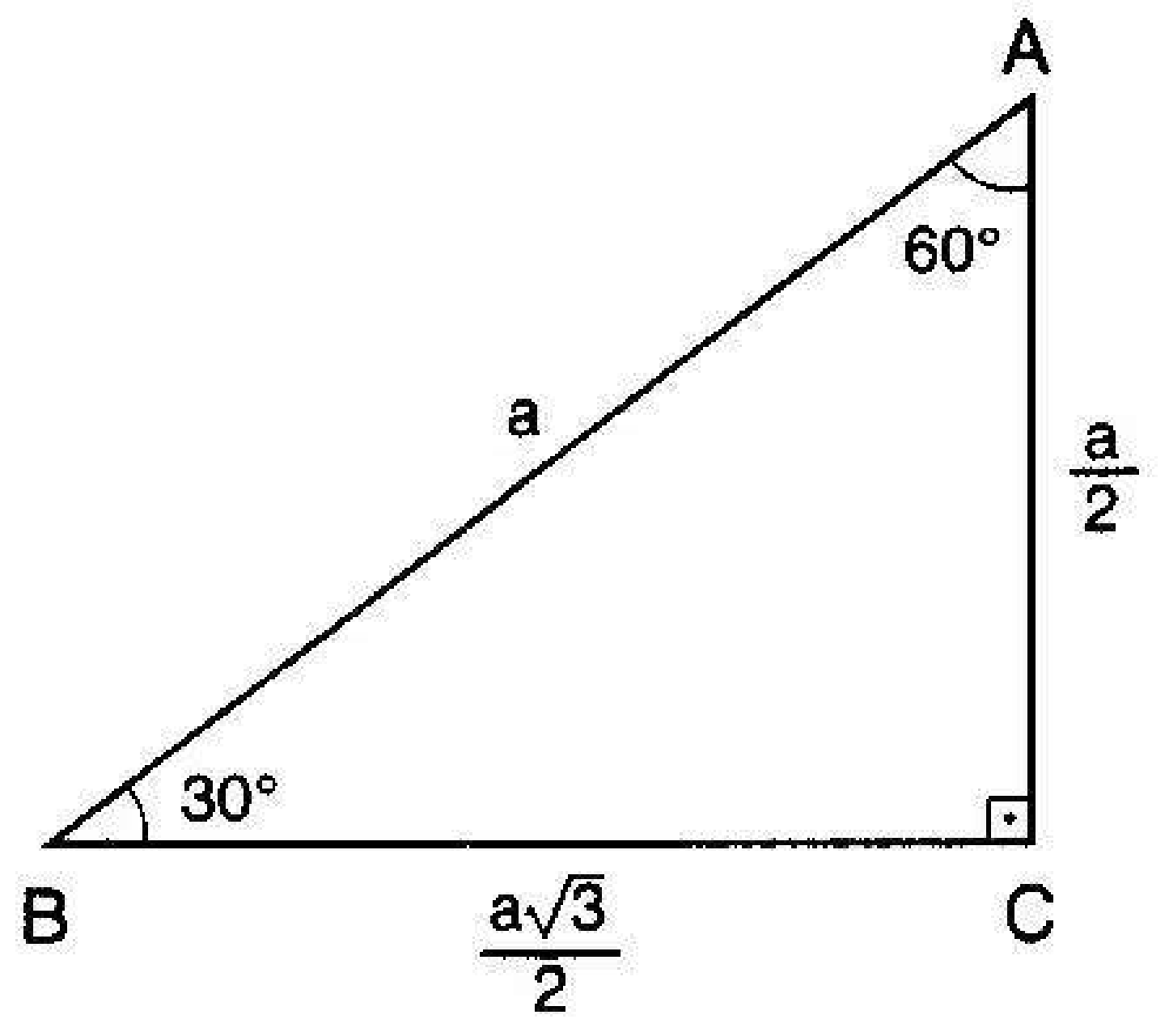
30° ve 60° nin Trigonometrik Değerleri / Trigonometric Values of 30° and 60°

$$\sin 30° = \frac{1}{2} \qquad \sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2}$$

$$\tan 30° = \frac{\sqrt{3}}{3} \qquad \tan 60° = \sqrt{3}$$

$$\cot 30° = \sqrt{3} \qquad \cot 60° = \frac{\sqrt{3}}{3}$$


A
60°
a
a/2
30°
B
a√3/2
C

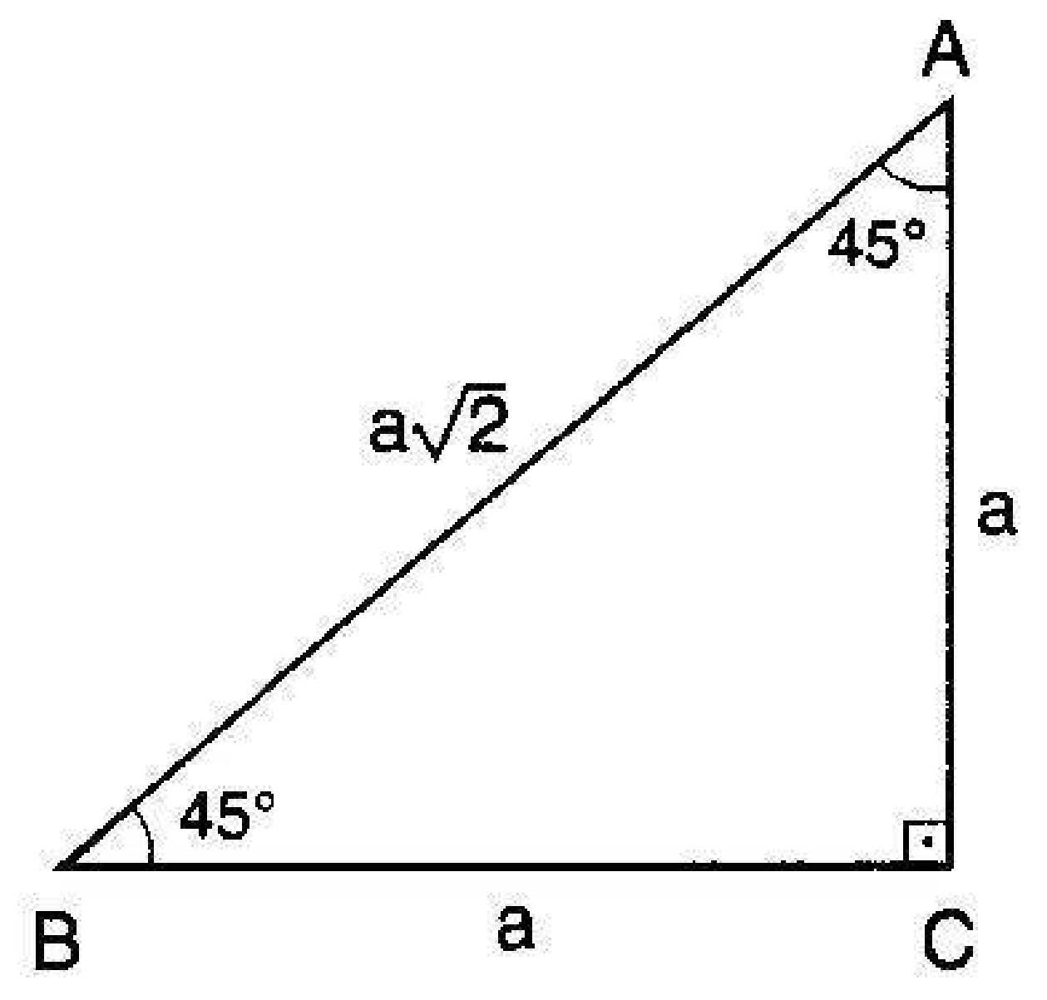
45° nin Trigonometrik Değerleri / Trigonometric Values of 45°

$$\sin 45° = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{\sqrt{2}}{2}$$

$$\tan 45° = 1$$

$$\cot 45° = 1$$


A
45°
a√2
a
45°
B
a
C

TEMEL AÇILARIN TRİGONOMETRİK DEĞERLERİ
TRIGONOMETRIC VALUES of the BASIC ANGLES

x (radyan radian)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x (derece degree)	0°	30°	45°	60°	90°	180°	270°	360°
sinx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cosx	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
tanx	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Tanımsız *Indefinite*	0	Tanımsız *Indefinite*	0
cotx	Belirsiz	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Tanımsız *Indefinite*	0	Tanımsız *Indefinite*

ÖRNEK / EXAMPLE 9

$$\frac{\cos\frac{\pi}{2}+\cos\pi}{\sin\frac{3\pi}{2}-\tan\frac{\pi}{4}}=?$$

A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) $\frac{1}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\left.\begin{array}{r}\cos\frac{\pi}{2}=0\\ \cos\pi=-1\\ \sin\frac{3x}{2}=-1\\ \tan\frac{\pi}{4}=1\end{array}\right\}\Rightarrow\frac{\cos\frac{\pi}{2}+\cos\pi}{\sin\frac{3\pi}{2}-\tan\frac{\pi}{4}}=\frac{0-1}{-1-1}=\frac{1}{2}$$

ÖRNEK / EXAMPLE 10

[AB] ⊥ [AC],

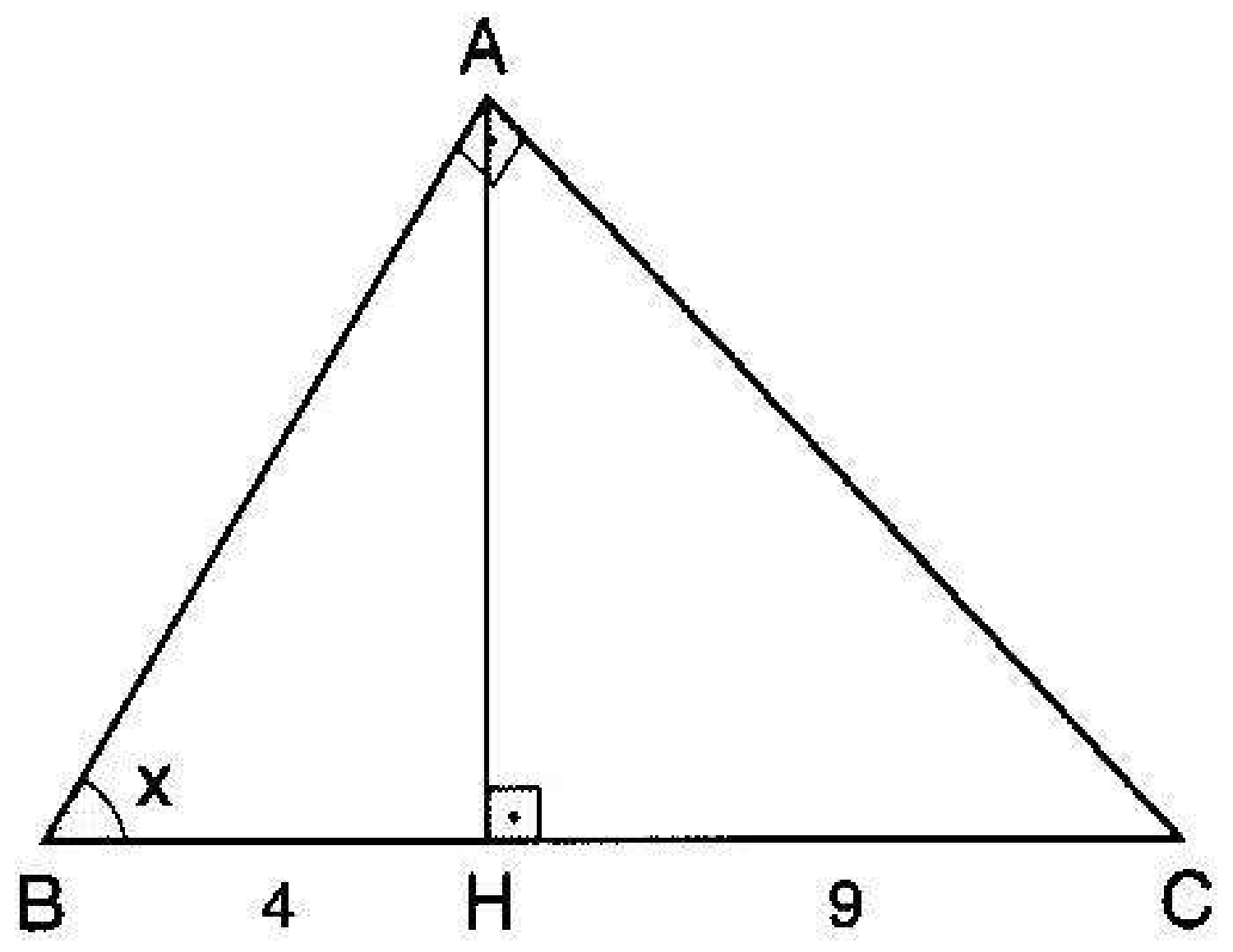
[AH] ⊥ [HC]

|BH| = 4 br,

|HC| = 9 br,

$m(\widehat{ABC}) = x$

⇒ **tanx = ?**



A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

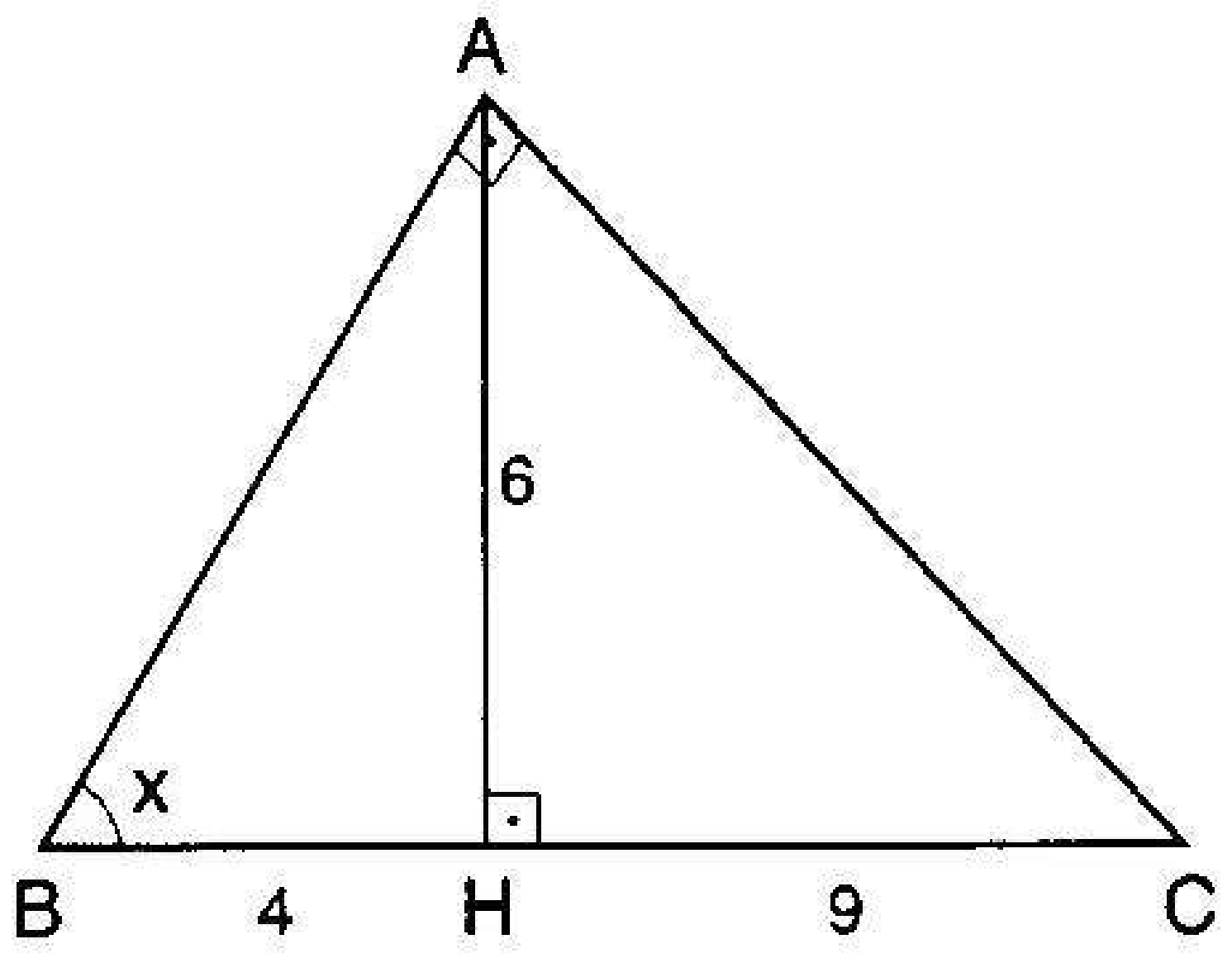
$|AH|^2 = |BH| \cdot |HC|$ [Öklid / *Euclid*]

$= 4 \cdot 9$

$= 36$

$\Rightarrow |AH| = 6$ br

$$\tan x = \frac{|AH|}{|BH|} = \frac{6}{4} = \frac{3}{2}$$



ÖRNEK / EXAMPLE 11

$$\frac{\tan x}{1-\cos x} - \frac{\tan x}{1+\cos x} = 6 \Rightarrow \mathbf{\sin x = ?}$$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{\tan x}{\underset{(1+\cos x)}{1-\cos x}} - \frac{\tan x}{\underset{(1-\cos x)}{1+\cos x}} = 6$$ [Verilen eşitlik / *Given equality*]

$$\frac{\tan x(1+\cos x) - \tan x(1-\cos x)}{1-\cos^2 x} = 6$$ [Topla. / *Combine.*]

$$\frac{\tan x + \tan x \cdot \cos x - \tan x + \tan x \cdot \cos x}{\sin^2 x} = 6$$ [Dağılma özelliği / *Distributive property*]

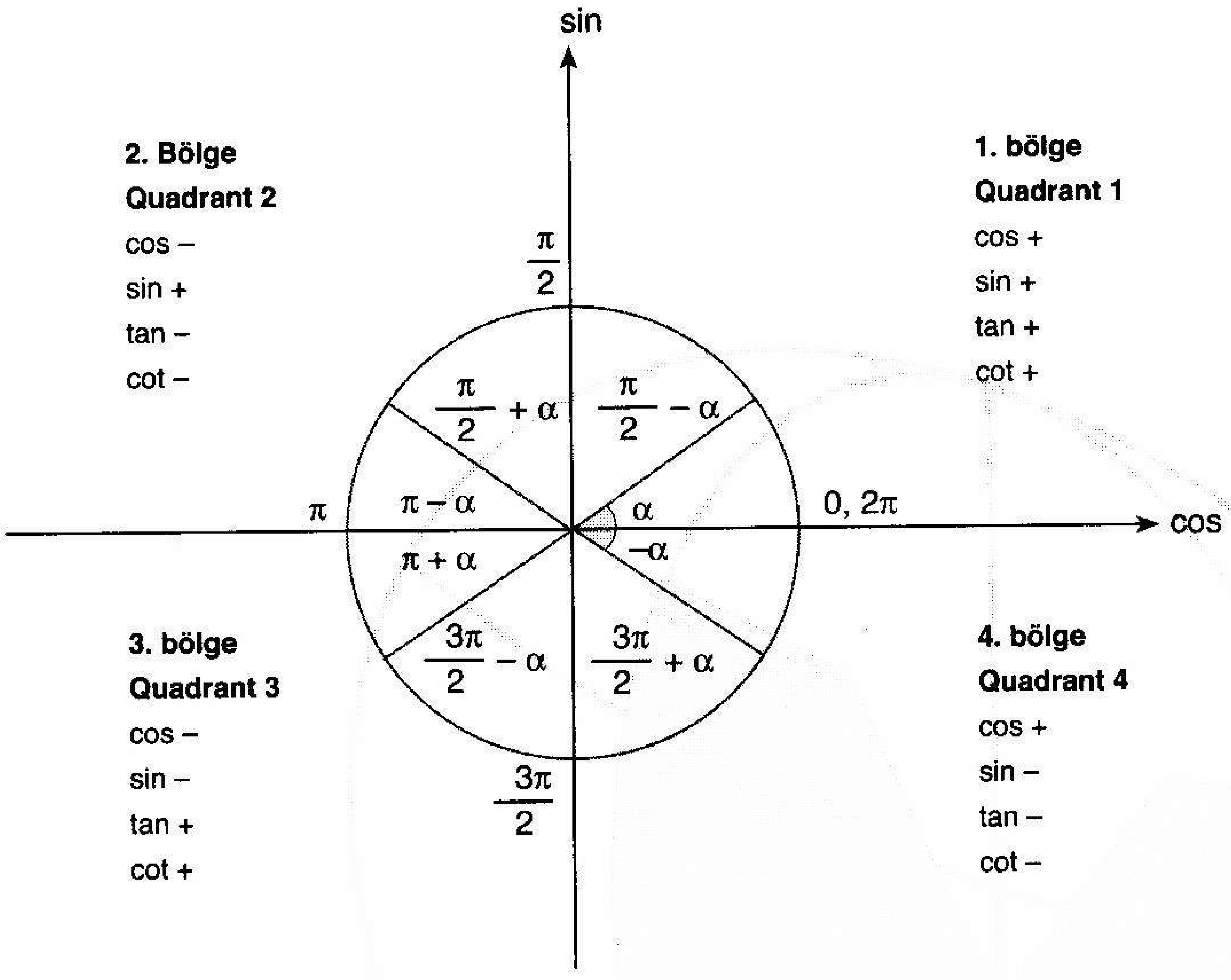
$$\frac{2\tan x \cdot \cos x}{\sin^2 x} = 6$$ [Benzer terimleri topla. / *Combine like terms.*]

$$\frac{2\sin x}{\sin^2 x} = 6$$ [Sadeleştir. / *Simplify.*]

$$\sin x = \frac{1}{3}$$ [Sonuç / *Conclusion*]

ÖLÇÜSÜ 90° den BÜYÜK OLAN AÇILARIN TRİGONOMETRİK DEĞERLERİ
TRIGONOMETRIC VALUES of ANGLES with GREATER THAN 90°

$0 < \alpha < \frac{\pi}{2}$


sin
2. Bölge
Quadrant 2
cos –
sin +
tan –
cot –
1. bölge
Quadrant 1
cos +
sin +
tan +
cot +
$\frac{\pi}{2}$
$\frac{\pi}{2}+\alpha$
$\frac{\pi}{2}-\alpha$
π
$\pi-\alpha$
α
0, 2π
cos
$\pi+\alpha$
$-\alpha$
3. bölge
Quadrant 3
cos –
sin –
tan +
cot +
$\frac{3\pi}{2}-\alpha$
$\frac{3\pi}{2}+\alpha$
4. bölge
Quadrant 4
cos +
sin –
tan –
cot –
$\frac{3\pi}{2}$

1. Bölge / Quadrant 1

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

i. $\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$

ii. $\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$

iii. $\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$

iv. $\cot\left(\frac{\pi}{2}-\alpha\right) = \tan\alpha$

2. Bölge / Quadrant 2

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$

$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$

$\tan\left(\frac{\pi}{2}+\alpha\right) = -\cot\alpha$

$\cot\left(\frac{\pi}{2}+\alpha\right) = -\tan\alpha$

$\sin(\pi-\alpha) = \sin\alpha$

$\cos(\pi-\alpha) = -\cos\alpha$

$\tan(\pi-\alpha) = -\tan\alpha$

$\cot(\pi-\alpha) = -\cot\alpha$

3. Bölge / Quadrant 3

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{3\pi}{2}-\alpha\right) = -\cos\alpha$	$\sin(\pi+\alpha) = -\sin\alpha$
$\cos\left(\frac{3\pi}{2}-\alpha\right) = -\sin\alpha$	$\cos(\pi+\alpha) = -\cos\alpha$
$\tan\left(\frac{3\pi}{2}-\alpha\right) = \cot\alpha$	$\tan(\pi+\alpha) = \tan\alpha$
$\cot\left(\frac{3\pi}{2}-\alpha\right) = \tan\alpha$	$\cot(\pi+\alpha) = \cot\alpha$

4. Bölge / Quadrant 4

$0 < \alpha < \frac{\pi}{2} \Rightarrow$

$\sin\left(\frac{3\pi}{2}+\alpha\right) = -\cos\alpha$	$\sin(2\pi-\alpha) = -\sin\alpha$
$\cos\left(\frac{3\pi}{2}+\alpha\right) = \sin\alpha$	$\cos(2\pi-\alpha) = \cos\alpha$
$\tan\left(\frac{3\pi}{2}+\alpha\right) = -\cot\alpha$	$\tan(2\pi-\alpha) = -\tan\alpha$
$\cot\left(\frac{3\pi}{2}+\alpha\right) = -\tan\alpha$	$\cot(2\pi-\alpha) = -\cot\alpha$

NEGATİF AÇILARIN TRİGONOMETRİK DEĞERLERİ
TRIGONOMETRIC VALUES of NEGATIVE ANGLES

✓ $\sin(-\alpha) = -\sin\alpha$

✓ $\cos(-\alpha) = \cos\alpha$

✓ $\tan(-\alpha) = -\tan\alpha$

✓ $\cot(-\alpha) = -\cot\alpha$

PEKİŞTİRME / REINFORCE

$\cos 210° = ?$

YANIT / ANSWER : $-\sqrt{3}/2$

ÖRNEK / EXAMPLE 12

$\sin 150° = ?$

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $-\frac{1}{2}$ D) $-\frac{\sqrt{3}}{2}$ E) -1

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\sin 150° = \sin(180° - 30°)$$

$$= \sin 30° = \frac{1}{2}$$

ÖRNEK / EXAMPLE 13

$\tan 840° = ?$

A) -1 B) $-\sqrt{2}$ C) $-\sqrt{3}$ D) $-\frac{\sqrt{3}}{3}$ E) $-\frac{\sqrt{2}}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$840° = 120° + 360°\cdot 2$

$\Rightarrow \tan 840° = \tan 120°$

$\tan 120° = \tan(180° - 60°)$

$\tan 120° = -\tan 60°$

$= -\sqrt{3} \Rightarrow \tan 840° = -\sqrt{3}$

ÖRNEK / EXAMPLE 14

$$\frac{\sin 420° \cdot \cos 190°}{\sin 80° \cdot \tan 225°} = ?$$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) -1 D) $-\frac{1}{2}$ E) $-\frac{\sqrt{3}}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\frac{\sin 420° \cdot \cos 190°}{\sin 80° \cdot \tan 225°} = \frac{\sin 60° \cdot \cos(180° + 10°)}{\sin 80° \cdot (\tan 180° + 45°)}$$

$$= \frac{\sin 60° \cdot (-\cos 10°)}{\sin 80° \cdot \tan 45°}$$

$$= \frac{-\frac{\sqrt{3}}{2}}{1}$$

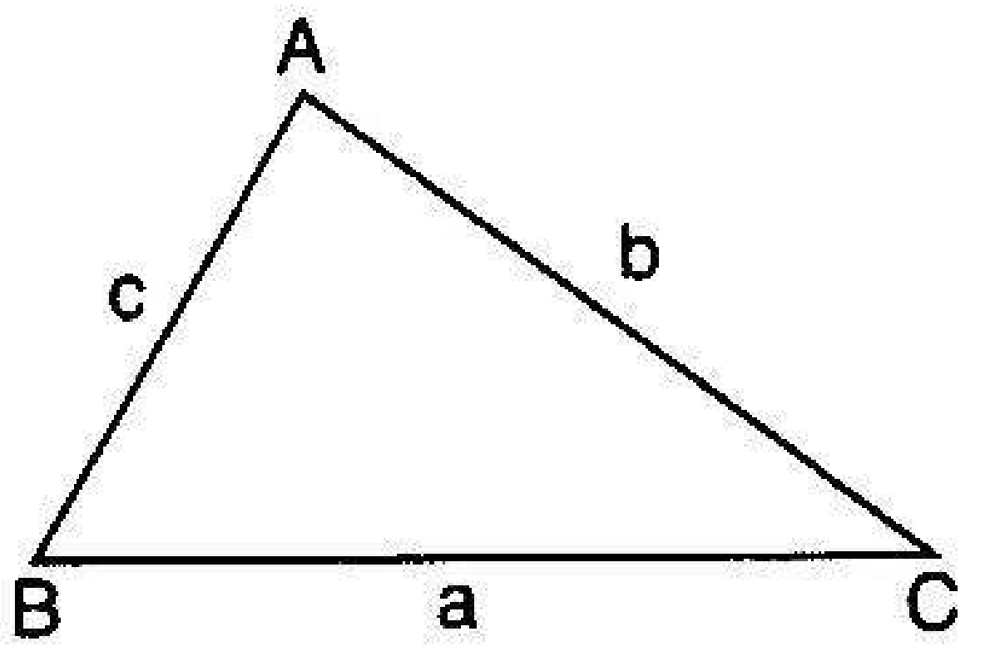
$$= -\frac{\sqrt{3}}{2}$$

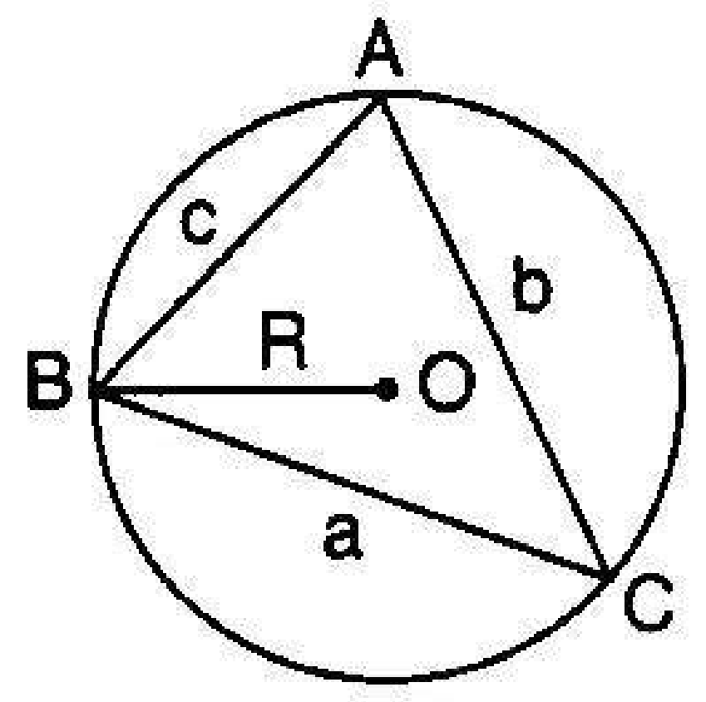
PEKİŞTİRME / REINFORCE

$$\frac{\sin 370°}{\cos 260°} + \tan 225° = ?$$

YANIT / ANSWER : 0

SİNÜS TEOREMİ / THE SINE THEOREM



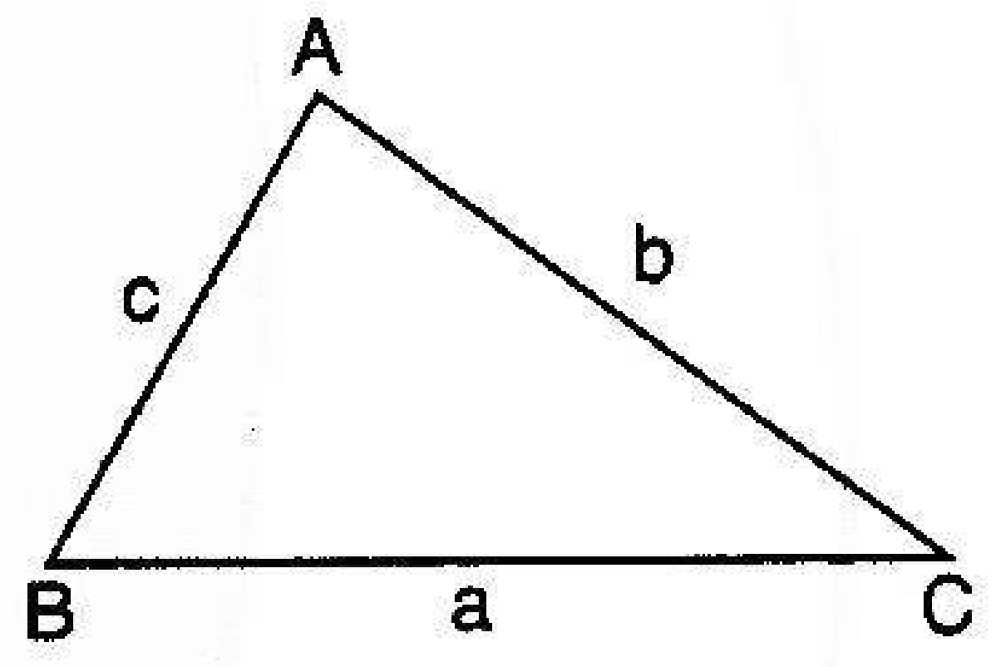


$$\frac{a}{\sin\widehat{A}} = \frac{b}{\sin\widehat{B}} = \frac{c}{\sin\widehat{C}} = 2R$$

R : Çevrel çemberin yarıçapı / *Radius of circumference*

$$A(\overset{\Delta}{ABC}) = \frac{1}{2}\cdot b\cdot c\sin\widehat{A} = \frac{1}{2}\cdot a\cdot c\cdot\sin\widehat{B} = \frac{1}{2}\cdot a\cdot b\cdot\sin\widehat{C}$$

KOSİNÜS TEOREMİ / THE COSINE THEOREM

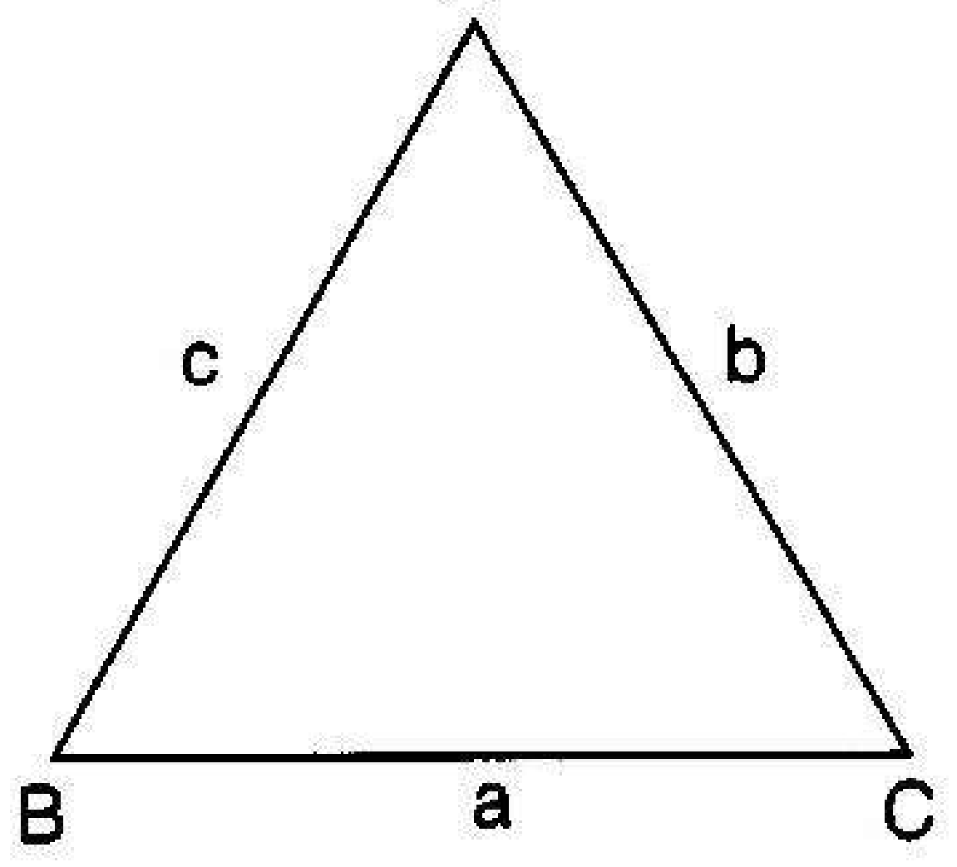


$$a^2 = b^2 + c^2 - 2\cdot b\cdot c\cdot\cos\widehat{A}$$
$$b^2 = a^2 + c^2 - 2\cdot a\cdot c\cdot\cos\widehat{B}$$
$$c^2 = a^2 + b^2 - 2\cdot a\cdot b\cdot\cos\widehat{C}$$

TANJANT TEOREMİ / THE TANGENT THEOREM

|BC| = a, |AC| = b, |AB| = c

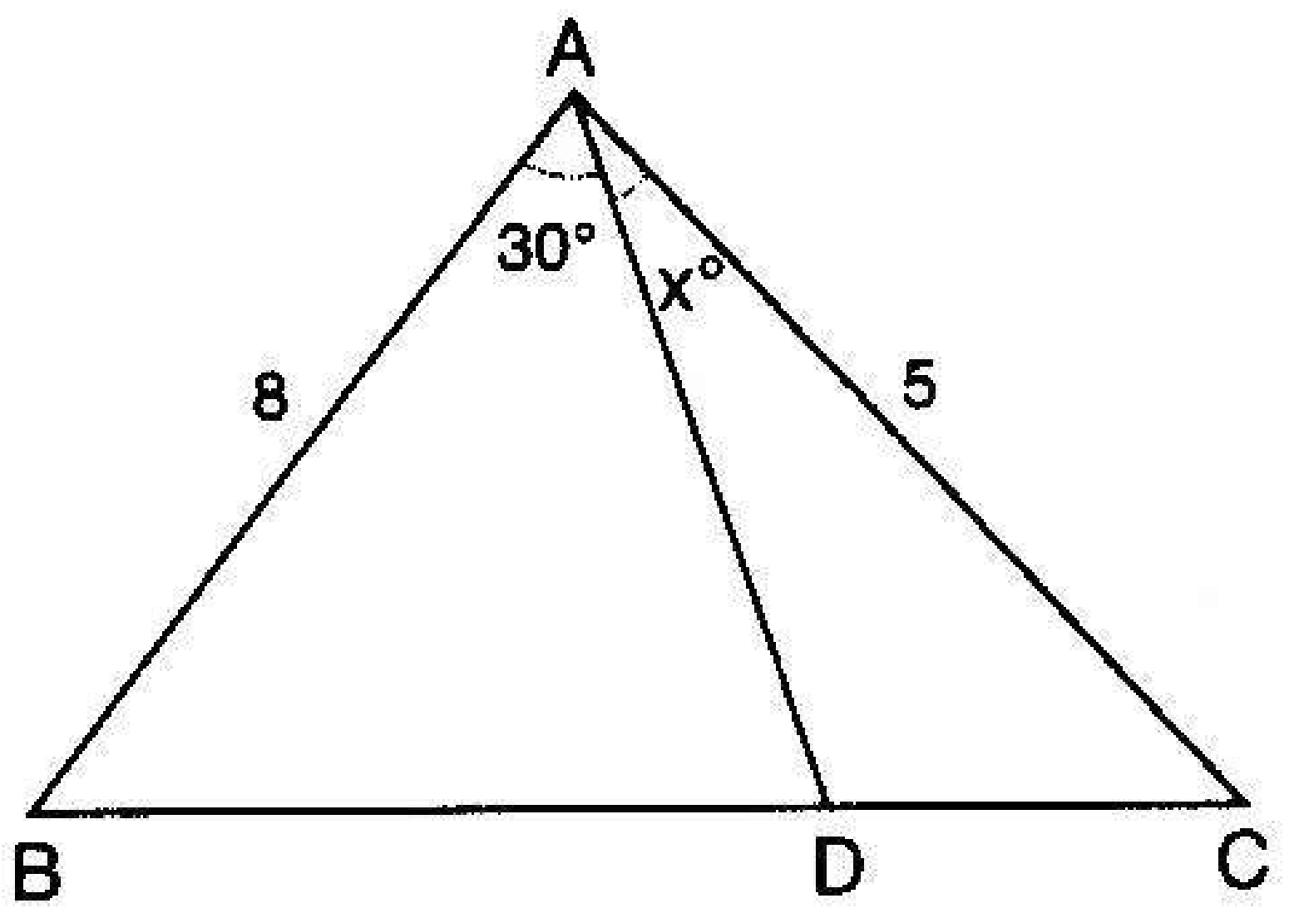
$$\Rightarrow \frac{b+c}{b-c} = \frac{\tan\frac{m(\widehat{B})+m(\widehat{C})}{2}}{\tan\frac{m(\widehat{B})-m(\widehat{C})}{2}}$$



ÖRNEK / EXAMPLE 15

$m(\widehat{BAD}) = 30°$, $|AB| = 8$ cm,
$|AC| = 5$ cm, $|BD| = 2 \cdot |DC|$,
$m(\widehat{DAC}) = x \Rightarrow$ **$\sin x = ?$**

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{2}}{5}$ D) $\frac{\sqrt{21}}{5}$ E) $\frac{2}{5}$



Örnek / Example 15

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$|BD| = 2 \cdot |DC| \Rightarrow A(\overset{\Delta}{ABD}) = 2 \cdot A(\overset{\Delta}{ADC})$$

$$A(\overset{\Delta}{ABD}) = \frac{1}{2} \cdot 8 \cdot |AD| \cdot \sin 30°$$

$$A(\overset{\Delta}{ADC}) = \frac{1}{2} \cdot 5 \cdot |AD| \cdot \sin x$$

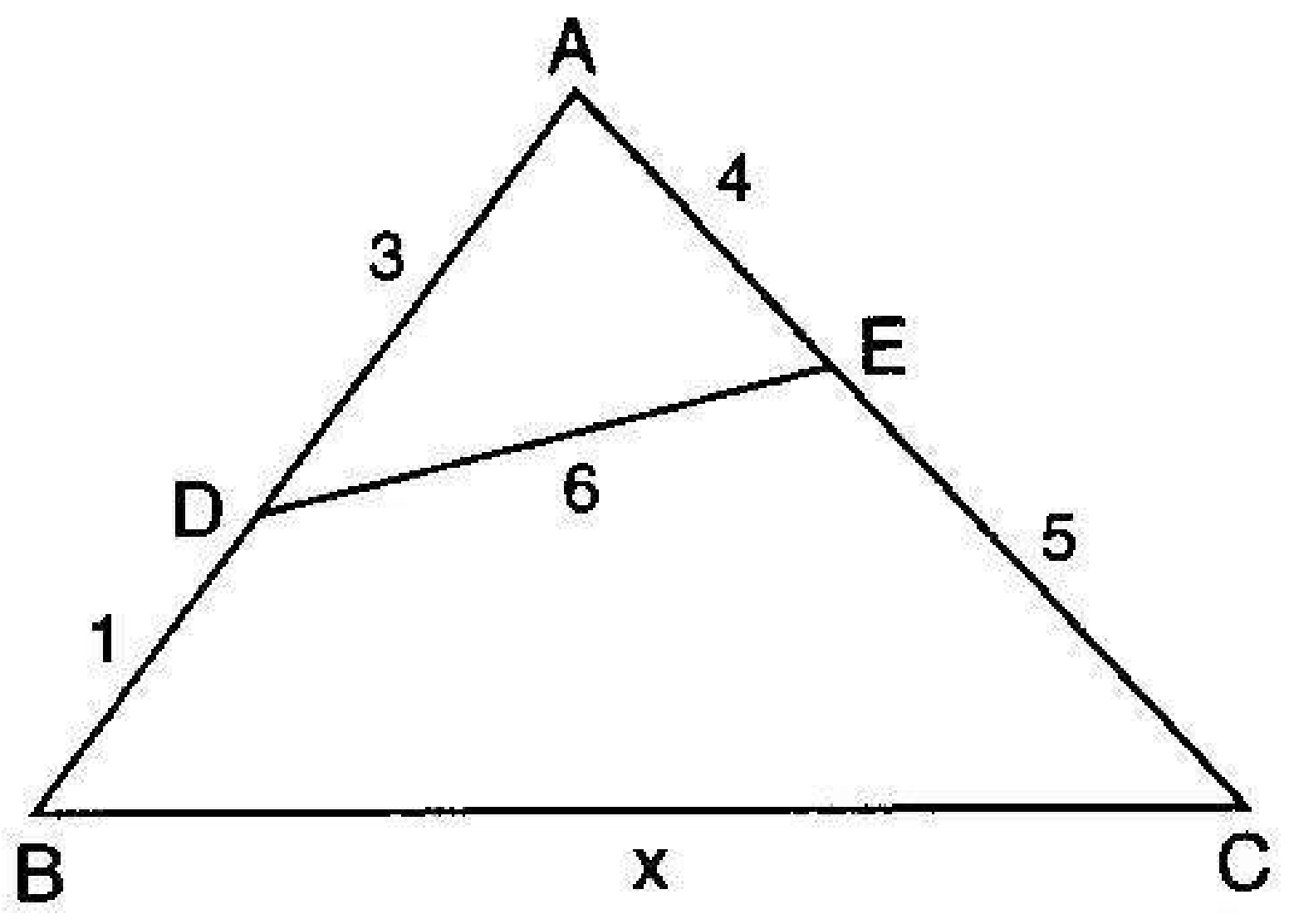
$$\frac{A(\overset{\Delta}{ABD})}{A(\overset{\Delta}{ADC})} = \frac{8 \cdot \sin 30°}{5 \cdot \sin x}$$

$$2 = \frac{4}{5 \cdot \sin x} \Rightarrow \sin x = \frac{2}{5}$$

ÖRNEK / EXAMPLE 16

$|AD| = 3$ cm, $|AE| = 4$ cm,
$|DE| = 6$ cm, $|DB| = 1$ cm,
$|EC| = 5$ cm $\Rightarrow$ **$|BC| = x = ?$ cm**

A) $\frac{\sqrt{91}}{2}$ B) $\frac{3\sqrt{10}}{2}$ C) $2\sqrt{22}$ D) $\sqrt{22}$ E) $\sqrt{130}$



Örnek / Example 16

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\overset{\Delta}{ADE}$: $|DE|^2 = |AD|^2 + |AE|^2 - 2 \cdot |AD| \cdot |AE| \cdot \cos\widehat{A}$

$$6^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos\widehat{A}$$

$$36 = 25 - 24 \cos\widehat{A}$$

$$-11 = 24 \cos\widehat{A} \Rightarrow \cos\widehat{A} = -\frac{11}{24}$$

$\overset{\Delta}{ABC}$: $x^2 = |AB|^2 + |AC|^2 - 2 \cdot |AB| \cdot |AC| \cdot \cos\widehat{A}$

$$= 4^2 + 9^2 - 2 \cdot 4 \cdot 9 \cdot \left(-\frac{11}{24}\right)$$

$$x^2 = 97 + 33 = 130$$

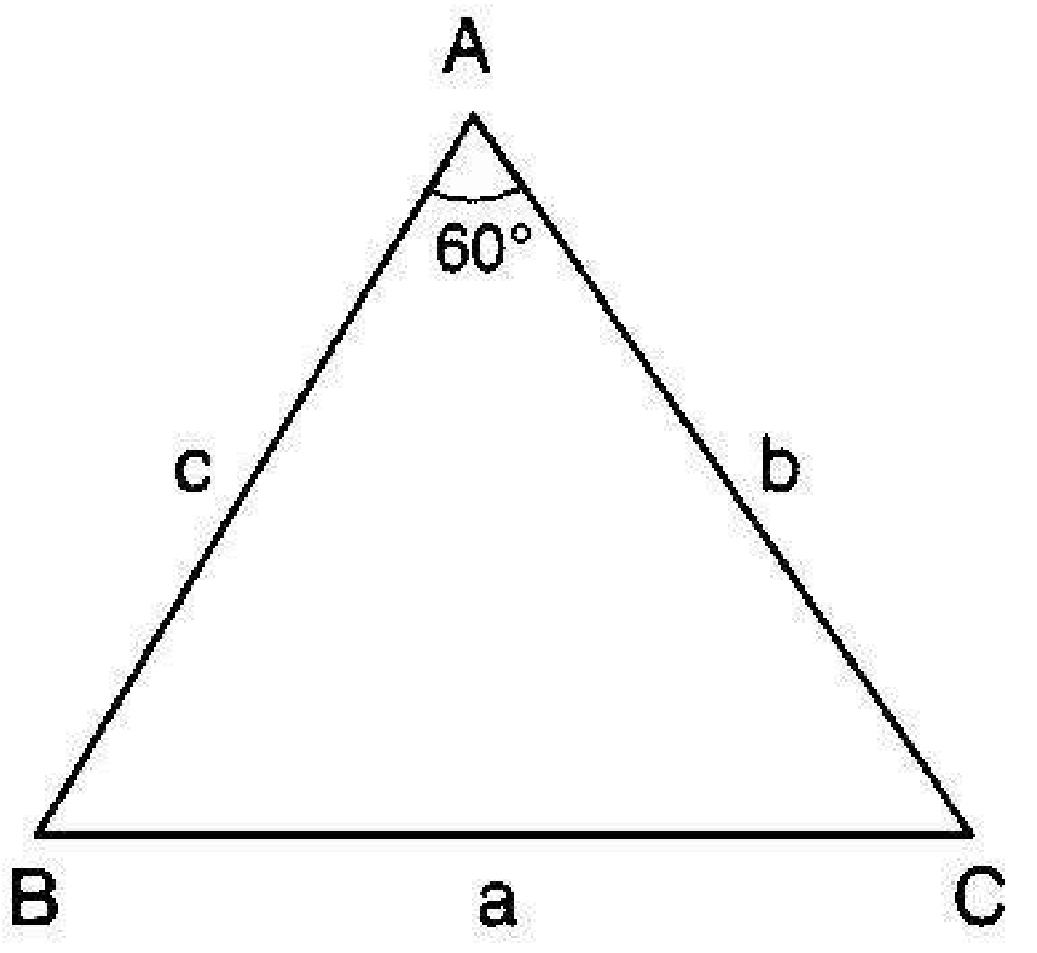
$$x = \sqrt{130}$$

ÖRNEK / EXAMPLE 17

$\dfrac{b+c}{b-c}=\sqrt{3}$,

$m(\widehat{BAC})=60^\circ$

$\Rightarrow$ **$m(\widehat{C})=?$**



A) 15° B) 30° C) 45° D) 60° E) 75°

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{b+c}{b-c}=\frac{\tan\dfrac{m(\widehat{B})+m(\widehat{C})}{2}}{\tan\dfrac{m(\widehat{B})-m(\widehat{C})}{2}}$$

$$m(\widehat{BAC})=60^\circ \Rightarrow m(\widehat{B})+m(\widehat{C})=120^\circ$$

$$\sqrt{3}=\frac{\tan 60^\circ}{\tan\dfrac{m(\widehat{B})-m(\widehat{C})}{2}} \Rightarrow -/\,m(\widehat{B})-m(\widehat{C})=90^\circ$$

$$\underline{m(\widehat{B})+m(\widehat{C})=120^\circ}$$

$$2m(\widehat{C})=30^\circ \Rightarrow m(\widehat{C})=15^\circ$$

ÖRNEK / EXAMPLE 18

ANADOLU ÜNİ. YÖS 2011

$$\frac{\cos 310^\circ\cdot\sin 290^\circ}{\sin 40^\circ\cdot\cos 20^\circ}=?$$

A) 0 B) 1 C) −1 D) $\frac{1}{2}$ E) $\frac{\sqrt{2}}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{\cos 310^\circ\cdot\sin 290^\circ}{\sin 40^\circ\cdot\cos 20^\circ}=\frac{\sin 40^\circ\cdot(-\cos 20^\circ)}{\sin 40^\circ\cdot\cos 20^\circ}=-1$$

ÖRNEK / EXAMPLE 19

ANKARA ÜNİ. YÖS 2011

$$\frac{\cos 15^\circ}{\sin 75^\circ} = ?$$

A) 0 B) -1 C) 1 D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{\cos 15^\circ}{\sin 75^\circ} = \frac{\sin 75^\circ}{\sin 75^\circ} = 1$$

ÖRNEK / EXAMPLE 20

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$\cos(\alpha - 3\pi) + \sin(\alpha - \frac{\pi}{2}) = ?$$

A) $\cos(\alpha)$ B) $\cos(2\alpha)$ C) $\sin(\alpha)$ D) $-2\cos(\alpha)$ E) $-2\sin(\alpha)$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{aligned}\cos(\alpha - 3\pi) + \sin(\alpha - \frac{\pi}{2}) &= \cos(3\pi - \alpha) - \sin(\frac{\pi}{2} - \alpha)\\ &= -\cos\alpha - \cos\alpha\\ &= -2\cos\alpha\end{aligned}$$

ÖRNEK / EXAMPLE 21

MARMARA ÜNİ. YÖS 2015

$x \in (0, \frac{\pi}{2})$

$12\cos x - 5\sin x = 0 \Rightarrow$ **$\sin x = ?$**

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{5}{13}$ D) $\frac{12}{13}$ E) 1

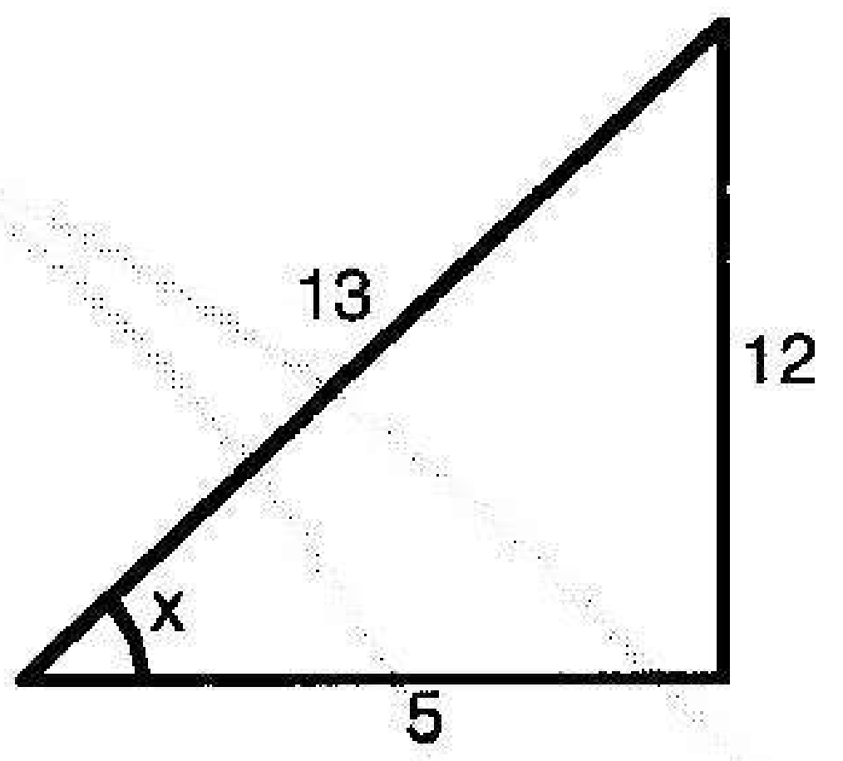
ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$12\cos x - 5\sin x = 0$

$\frac{12\cos x}{\cos x} = \frac{5\sin x}{\cos x}$

$\Rightarrow \frac{12}{5} = \tan x$



$\Rightarrow \sin x = \frac{12}{13}$

ÖRNEK / EXAMPLE 22

ANKARA ÜNİ. YÖS 2016

$$\frac{\mathbf{1+cosecx}}{\mathbf{1+secx}} \cdot \frac{\mathbf{1+cosx}}{\mathbf{1+sinx}} = ?$$

A) sinx B) cosx C) tanx

D) cotx E) sinx·cosx

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{1+\operatorname{cosec}x}{1+\sec x} \cdot \frac{1+\cos x}{1+\sin x} = \frac{1+\frac{1}{\sin x}}{1+\frac{1}{\cos x}} \cdot \frac{1+\cos x}{1+\sin x}$$

$$= \frac{\frac{\sin x+1}{\sin x}}{\frac{\cos x+1}{\cos x}} \cdot \frac{1+\cos x}{1+\sin x}$$

$$= \frac{(\sin x+1)\cdot\cos x}{(\sin x)\cdot(\cos x+1)} \cdot \frac{(1+\cos x)}{(1+\sin x)}$$

$$= \frac{\cos x}{\sin x} = \cot x$$

1. $\frac{5\pi}{4}R = x^\circ \Rightarrow$ **$x = ?$**

A) 135°

B) 175°

C) 180°

D) 210°

E) 225°

2. $\frac{4\pi}{3}R = x^\circ \Rightarrow$ **$x = ?$**

A) 180°

B) 195°

C) 210°

D) 225°

E) 240°

3. $140^\circ = x \cdot \pi \Rightarrow$ **$x = ?$**

A) $\frac{4\pi}{9}$

B) $\frac{5\pi}{9}$

C) $\frac{7\pi}{9}$

D) $\frac{10\pi}{9}$

E) $\frac{4\pi}{9}$

4. $252^\circ = x\pi \Rightarrow$ **$x = ?$**

A) $\frac{7\pi}{5}$

B) $\frac{13\pi}{12}$

C) π

D) $\frac{11\pi}{12}$

E) $\frac{5\pi}{6}$

5. $2578^\circ = x + k \cdot 360^\circ \Rightarrow$ **$x = ?$**

A) 58°

B) 68°

C) 78°

D) 88°

E) 98°

6. $2146^\circ = x + k \cdot 360^\circ \Rightarrow$ **$x = ?$**

A) 314°

B) 330°

C) 346°

D) 352°

E) 356°

7. $-1472° = x + k \cdot 360° \Rightarrow$ **x = ?**

A) 318°

B) 328°

C) 338°

D) 348°

E) 358°

8. $-928° = x + k \cdot 360° \Rightarrow$ **x = ?**

A) 130°

B) 136°

C) 140°

D) 146°

E) 152°

9. $\frac{72\pi}{5} = x + 2k\pi \Rightarrow x = ?$

A) $\frac{\pi}{5}$

B) $\frac{2\pi}{5}$

C) $\frac{3\pi}{5}$

D) $\frac{4\pi}{5}$

E) π

10. $\frac{47\pi}{3} = x + 2k\pi \Rightarrow$ **x = ?**

A) $\frac{11\pi}{6}$

B) $\frac{5\pi}{3}$

C) $\frac{3\pi}{2}$

D) $\frac{4\pi}{3}$

E) $\frac{7\pi}{6}$

11. $\frac{-29\pi}{3} = x + 2k\pi \Rightarrow$ **x = ?**

A) $\frac{\pi}{3}$

B) $\frac{2\pi}{3}$

C) π

D) $\frac{4\pi}{3}$

E) $\frac{5\pi}{3}$

12. $\frac{-34\pi}{5} = x + 2k\pi \Rightarrow x = ?$

A) $\frac{3\pi}{5}$

B) $\frac{4\pi}{5}$

C) π

D) $\frac{6\pi}{5}$

E) $\frac{7\pi}{5}$

YANITLAR / ANSWERS

1. E	**2. E**	**3. C**	**4. A**	**5. A**	**6. C**	**7. B**	**8. E**
9. B	**10. B**	**11. A**	**12. D**				

1. $\frac{D}{180°}=\frac{R}{\pi} \Rightarrow \frac{x}{180°}=\frac{\frac{5\pi}{4}}{\pi}$

$x=180°\cdot\frac{5}{4}$

$x=225°$

Yanıt / The Answer is E

2. $\frac{D}{180°}=\frac{R}{\pi} \Rightarrow \frac{x}{180°}=\frac{\frac{4\pi}{3}}{\pi}$

$x=180°\cdot\frac{4}{3}$

$x=240°$

Yanıt / The Answer is E

3. $\frac{D}{180°}=\frac{R}{\pi} \Rightarrow \frac{140°}{180°}=\frac{x}{\pi}$

$x=\frac{140°\pi}{180°}$

$x=\frac{7\pi}{9}$

Yanıt / The Answer is C

4. $\frac{D}{180°}=\frac{R}{\pi} \Rightarrow \frac{252°}{180°}=\frac{x}{\pi}$

$x=\frac{252°\cdot\pi}{180°}$

$x=\frac{7\pi}{5}$

Yanıt / The Answer is A

5. $$\begin{array}{r|l} 2578 & 360 \\ \cline{2-2} -\,2520 & 7 \\ \hline 58 & \end{array} \Rightarrow 2578°=x+k\cdot 360°$$

$2520°+58°=x+7\cdot 360°$

$x=58°$

Yanıt / The Answer is A

6. $$\begin{array}{r|l} 2146 & 360 \\ \cline{2-2} -\,1800 & 5 \\ \hline 346 & \end{array} \Rightarrow 2146°=x+k\cdot 360°$$

$1800°+346°=x+5\cdot 360°$

$x=346°$

Yanıt / The Answer is C

7.
$$\begin{array}{r|l} -1472 & 360 \\ \underline{-\;-1800} & -5 \\ 328 & \end{array} \Rightarrow \begin{aligned} -1472 &= x + k \cdot 360° \\ -1800 + 328° &= x + (-5) \cdot 360° \\ x &= 328° \end{aligned}$$

Yanıt / The Answer is B

8.
$$\begin{array}{r|l} -928 & 360 \\ \underline{-\;-1080} & -3 \\ 152 & \end{array} \Rightarrow \begin{aligned} -928 &= x + k \cdot 360° \\ -1080 + 252° &= x + (-3) \cdot 360° \\ x &= 152° \end{aligned}$$

Yanıt / The Answer is E

9.
$$\frac{72\pi}{5} = x + 2k\pi$$
$$\frac{70\pi}{5} + \frac{2\pi}{5} = x + 2k\pi$$
$$x = \frac{2\pi}{5}$$

Yanıt / The Answer is B

10.
$$\frac{42\pi}{3} + \frac{5\pi}{3} = x + 2k\pi$$
$$x = \frac{5\pi}{3}$$

Yanıt / The Answer is B

11.
$$-\frac{29\pi}{3} = x + 2k\pi$$
$$-\frac{30\pi}{3} + \frac{\pi}{3} = x + 2k\pi$$
$$x = \frac{\pi}{3}$$

Yanıt / The Answer is A

12.
$$-\frac{34\pi}{5} = x + 2k\pi$$
$$-\frac{40\pi}{5} + \frac{6\pi}{5} = x + 2k\pi$$
$$x = \frac{6\pi}{5}$$

Yanıt / The Answer is D

1. $A = \dfrac{1+\cos x}{2} \Rightarrow$ **? < A < ?**

A) $-1 \leq A \leq 0$

B) $-1 \leq A \leq 1$

C) $0 \leq A \leq 1$

D) $0 < A \leq 1$

E) $-1 < A < 1$

2. $\dfrac{\mathbf{1-cotx}}{\mathbf{1-tanx}} = ?$

A) tanx

B) cotx

C) 1

D) –cotx

E) –tanx

3. $\cos 15° = a \Rightarrow \dfrac{\mathbf{sin^2 15°}}{\mathbf{1-cos15°}} = ?$

A) a

B) a – 1

C) a + 1

D) 2a – 1

E) –2a

4. $\tan a = \dfrac{1}{2}$,

$\dfrac{\cot a - \cot b}{\cot a \cdot \cot b} = 1 \Rightarrow$ **tanb = ?**

A) $\dfrac{1}{3}$

B) $\dfrac{1}{2}$

C) 1

D) $\dfrac{4}{3}$

E) $\dfrac{3}{2}$

5. $\mathbf{\cos^4 x - \sin^4 x = ?}$

A) $1 - \cos^2 x$

B) $\cos^2 x - 1$

C) $1 - \sin^2 x$

D) $1 - 2\sin^2 x$

E) $1 - \sin^2 x$

6. $x \in \left(0, \dfrac{\pi}{2}\right)$

$\Rightarrow \sqrt{\mathbf{1-sinx}} \cdot \sqrt{\mathbf{1+sinx}} = ?$

A) cosx

B) 1

C) sinx

D) cosx + 1

E) sinx + 1

7. $\dfrac{\sin^2x}{1+\cos x}+\cos x = ?$

A) 1 + 2cosx

B) 2sinx

C) 2cosx

D) 1

E) –1

8. $\dfrac{1-\sin^2x}{\cos^2x}+\tan x\cdot\cot x = ?$

A) 0

B) 1

C) sinx

D) cosx

E) 2

9. $\dfrac{1}{\cos^2x}-\dfrac{1}{\cot^2x} = ?$

A) tanx

B) cotx

C) sinx

D) cosx

E) 1

10. $\dfrac{1+\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x} = ?$

A) secx

B) cosecx

C) sinx

D) cosx

E) 2cosecx

11. $\left(\dfrac{\cos x}{1-\sin^2x}-\dfrac{\sin x}{1-\cos^2x}\right):\left(\dfrac{\sin x-\cos x}{\sin x}\right) = ?$

A) secx

B) cosecx

C) –secx

D) –cosecx

E) cosx

12. $\cot x+\dfrac{\sin x}{1+\cos x} = ?$

A) sinx

B) cosx

C) tanx

D) cosecx

E) secx

YANITLAR / ANSWERS

1. C 2. D 3. C 4. E 5. D 6. A 7. D 8. E
9. E 10. E 11. A 12. D

1. $A = \dfrac{1+\cos x}{2}$

$2A = 1+\cos x$

$2A-1 = \cos x$

$-1 \le \cos x \le 1 \Rightarrow -1 \le 2A-1 \le 1$

$0 \le A \le 1$

Yanıt / The Answer is C

---

2. $$\frac{1-\cot x}{1-\tan x} = \frac{1-\dfrac{\cos x}{\sin x}}{1-\dfrac{\sin x}{\cos x}}$$

$$= \frac{\dfrac{\sin x-\cos x}{\sin x}}{\dfrac{\cos x-\sin x}{\cos x}}$$

$$= -\cot x$$

Yanıt / The Answer is D

---

3. $\sin^2 15° = 1-\cos^2 15°$

$$\frac{\sin^2 15°}{1-\cos^2 15°} = \frac{1-\cos^2 15°}{1-\cos 15°}$$

$$= \frac{(1-\cos 15°)\cdot(1+\cos 15°)}{1-\cos 15°}$$

$$= 1+\cos 15°$$

$$= 1+a$$

Yanıt / The Answer is C

---

4. $\tan a = \dfrac{1}{2} \Rightarrow \cot a = 2$

$$\frac{\cot a-\cot b}{\cot a\cdot\cot b} = 1$$

$$\frac{2-\cot b}{2\cdot\cot b} = 1$$

$$2 = 3\cdot\cot b$$

$$\cot b = \frac{2}{3} \Rightarrow \tan b = \frac{3}{2}$$

Yanıt / The Answer is E

---

5. $\cos^4 x-\sin^4 x = (\cos^2 x-\sin^2 x)\cdot(\cos^2 x+\sin^2 x)$

$= (\cos^2 x-\sin^2 x)\cdot 1$

$= 1-\sin^2 x-\sin^2 x$

$= 1-2\sin^2 x$

Yanıt / The Answer is D

---

6. $\sqrt{1-\sin x}\cdot\sqrt{1+\sin x} = \sqrt{1-\sin^2 x}$

$= \sqrt{\cos^2 x}$

$= |\cos x|$

$x \in \left(0, \dfrac{\pi}{2}\right) \Rightarrow |\cos x| = \cos x$

Yanıt / The Answer is A

7. $\frac{\sin^2 x}{1+\cos x}+\cos x=\frac{1-\cos^2 x}{1+\cos x}+\cos x$

$=1-\cos x+\cos x$

$=1$

Yanıt / The Answer is D

8. $\frac{1-\sin^2 x}{\cos^2 x}+\tan x\cdot\cot x=\frac{\cos^2 x}{\cos^2 x}+1$

$=1+1=2$

Yanıt / The Answer is E

9. $\frac{1}{\cos^2 x}-\frac{1}{\cot^2 x}=\frac{1}{\cos^2 x}-\frac{1}{\frac{\cos^2 x}{\sin^2 x}}$

$=\frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}$

$=\frac{1-\sin^2 x}{\cos^2 x}$

$=\frac{\cos^2 x}{\cos^2 x}=1$

Yanıt / The Answer is E

10. $\frac{1+\cos x}{\underset{(1+\cos x)}{\sin x}}+\frac{\sin x}{\underset{(\sin x)}{1+\cos x}}=\frac{(1+\cos x)^2+\sin^2 x}{(1+\cos x)\cdot\sin x}$

$=\frac{1+2\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)\cdot\sin x}$

$=\frac{2+2\cos x}{(1+\cos x)\cdot\sin x}$

$=\frac{2}{\sin x}$

$=2\text{cosec}x$

Yanıt / The Answer is E

11. $\left(\frac{\cos x}{1-\sin^2 x}-\frac{\sin x}{1-\cos^2 x}\right):\left(\frac{\sin x-\cos x}{\sin x}\right)$

$=\left(\frac{\cos x}{\cos^2 x}-\frac{\sin x}{\sin^2 x}\right):\left(\frac{\sin x-\cos x}{\sin x}\right)$

$=\left(\frac{1}{\cos x}-\frac{1}{\sin x}\right)\cdot\left(\frac{\sin x}{\sin x-\cos x}\right)$

$=\frac{\sin x-\cos x}{\sin x\cdot\cos x}\cdot\frac{\sin x}{\sin x-\cos x}$

$=\frac{1}{\cos x}=\sec x$

Yanıt / The Answer is A

12. $\cot x+\frac{\sin x}{1+\cos x}=\frac{\cos x}{\sin x}+\frac{\sin x}{1+\cos x}$

$=\frac{\cos(1+\cos x)+\sin^2 x}{\sin x\cdot(1+\cos x)}$

$=\frac{\cos x+\cos^2 x+\sin^2 x}{\sin x\cdot(1+\cos x)}$

$=\frac{1+\cos x}{\sin x\cdot(1+\cos x)}$

$=\frac{1}{\sin x}=\text{cosec}x$

Yanıt / The Answer is D

1. $0 < x < \frac{\pi}{2}$, $\sin x = 0{,}6 \Rightarrow$ **$\tan x = ?$**

A) $\frac{5}{3}$

B) $\frac{4}{3}$

C) $\frac{3}{4}$

D) $\frac{3}{5}$

E) $\frac{1}{2}$

2. **$\cos\frac{11\pi}{3} = ?$**

A) $\frac{1}{2}$

B) $\frac{3}{5}$

C) $\frac{3}{4}$

D) $\frac{4}{5}$

E) $\frac{9}{10}$

3. **$\tan 210° = ?$**

A) $\sqrt{2}$

B) $\sqrt{3}$

C) $\frac{1}{\sqrt{2}}$

D) $\frac{\sqrt{3}}{3}$

E) $\frac{1}{2}$

4. **$\frac{\sin 300°}{1-\cos 240°} = ?$**

A) $-\sqrt{3}$

B) $-\frac{\sqrt{3}}{2}$

C) $-\frac{\sqrt{3}}{3}$

D) $-\frac{\sqrt{2}}{2}$

E) -1

5. $\frac{\pi}{2} < x < \pi$, $\tan x = -\frac{4}{3} \Rightarrow$ **$\cos x = ?$**

A) $-\frac{3}{5}$

B) $-\frac{4}{5}$

C) $\frac{3}{5}$

D) $\frac{4}{5}$

E) $\frac{2}{5}$

6. **$(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = ?$**

A) -3

B) -2

C) 0

D) 2

E) 3

7. $\dfrac{\cos 37°}{\sin 127°} = ?$

A) $-\cos 37°$

B) -1

C) 1

D) $\cos 37°$

E) $\tan 37°$

8. $0 < x < 2\pi$

$2\sin^2 x - 5 \cdot \sin x + 2 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {60° , 150°}

B) {45° , 135°}

C) {60° , 120°}

D) {30° , 150°}

E) {30° , 120°}

9. $3\sin x - 4\cos y = A \Rightarrow$ **max(A) = ?**

A) -1

B) 0

C) 2

D) 3

E) 7

10. $\dfrac{\sin 30° + \cos 30°}{\tan 45° + \cot 30°} = ?$

A) 1

B) $\dfrac{1}{2}$

C) $\dfrac{\sqrt{3}}{2}$

D) $\dfrac{\sqrt{2}}{2}$

E) $\dfrac{1+\sqrt{3}}{2}$

11. $\dfrac{\cos 150° - \sin 150°}{\cos(-330°) - \sin(-750°)} = ?$

A) $-\dfrac{\sqrt{3}}{3}$

B) -1

C) $\dfrac{\sqrt{3}}{3}$

D) 1

E) 3

12. $\dfrac{\pi}{2} < x < \pi$

$\sin x = \dfrac{3}{5} \Rightarrow$ **$\tan x - \cos x = ?$**

A) $-\dfrac{1}{10}$ B) $-\dfrac{1}{20}$ C) 0

D) $\dfrac{1}{10}$ E) $\dfrac{1}{20}$

YANITLAR / ANSWERS

1. C	2. A	3. D	4. C	5. A	6. D	7. C	8. D
9. E	10. B	11. B	12. E				

1. $\sin x = 0{,}6$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

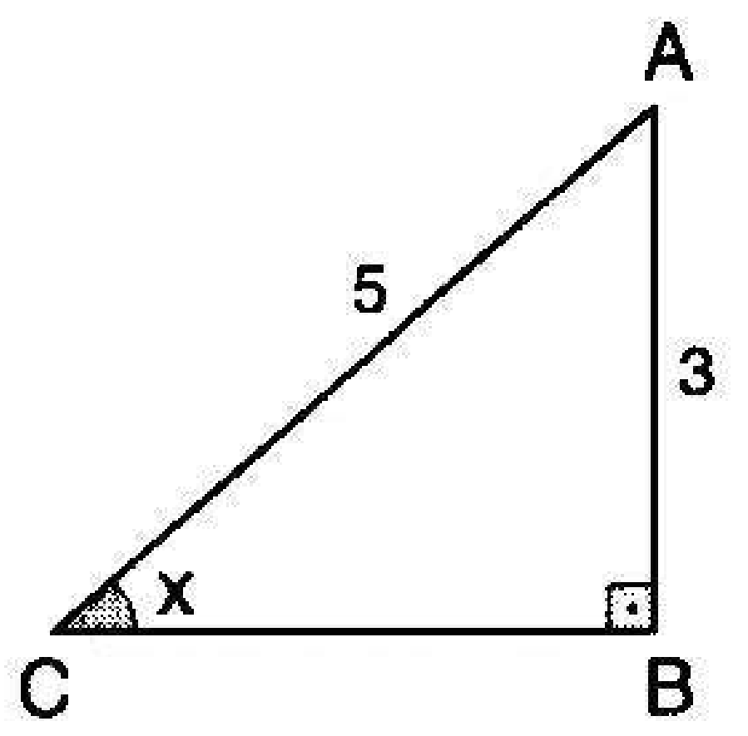
$$\sin x = \frac{|AB|}{|AC|}$$

$$|AC|^2 = |AB|^2 + |BC|^2$$

$$5^2 = 3^2 + |BC|^2$$

$$|BC| = 4$$

$$\Rightarrow \tan x = \frac{|AB|}{|BC|} = \frac{3}{4}$$



Yanıt / The Answer is C

2. $\frac{11\pi}{3} = x + 2k\pi$

$$\frac{6\pi}{3} + \frac{5\pi}{3} = x + 2k\pi \Rightarrow x = \frac{5\pi}{3}$$

$$\cos\frac{11\pi}{3} = \cos\frac{5\pi}{3}$$

$$= \cos\left(2\pi - \frac{\pi}{3}\right)$$

$$= \cos\frac{\pi}{3} = \frac{1}{2}$$

Yanıt / The Answer is A

3. $\tan 210° = \tan(180° + 30°)$

$$= \tan 30°$$

$$= \frac{\sqrt{3}}{3}$$

Yanıt / The Answer is D

4. $$\frac{\sin 300°}{1 - \cos 240°} = \frac{\sin(360° - 60°)}{1 - \cos(180° + 60°)}$$

$$= \frac{-\sin 60°}{1 - (-\cos 60°)}$$

$$= \frac{-\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}}$$

$$= -\frac{\sqrt{3}}{3}$$

Yanıt / The Answer is C

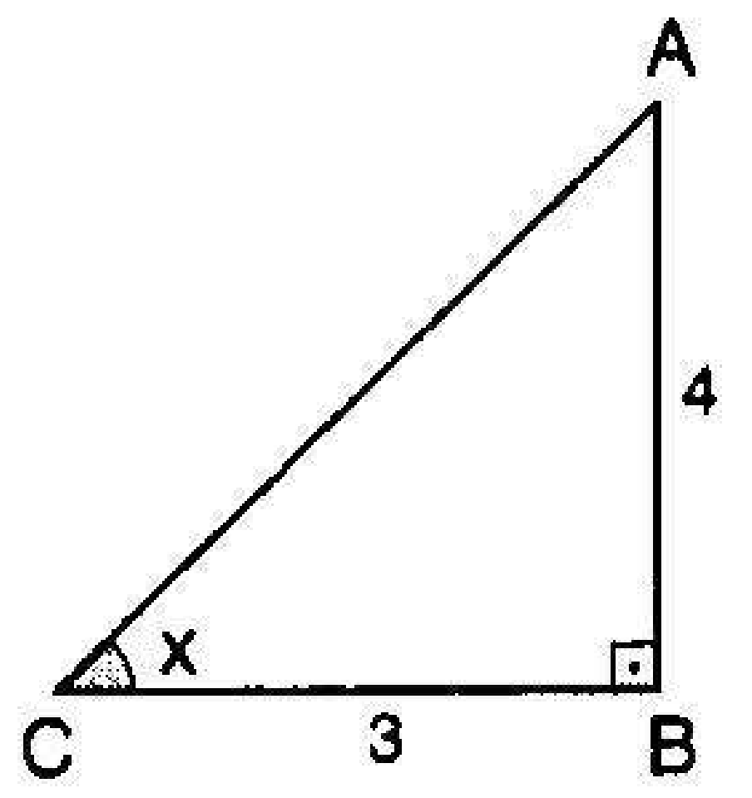
5. $\tan x = -\frac{4}{3}$

$$|AC|^2 = |AB|^2 + |BC|^2$$

$$|AC|^2 = 4^2 + 3^2$$

$$|AC| = 5$$

$$\cos x = -\frac{3}{5}$$



Yanıt / The Answer is A

6. $(\sin x - \cos x)^2 + (\sin x + \cos x)^2$

$$= \sin^2 x + \cos^2 x - 2\sin x \cdot \cos x + \sin^2 x + \cos^2 x + 2\sin x \cdot \cos x$$

$$= 1 - 2\sin x \cdot \cos x + 1 + 2\sin x \cdot \cos x$$

$$= 2$$

Yanıt / The Answer is D

7. $\frac{\cos 37°}{\sin 127°}=\frac{\cos 37°}{\cos(90°+37°)}$

$=\frac{\cos 37°}{\cos 37°}$

$=1$

Yanıt / The Answer is C

8. $2\sin^2 x-5\cdot\sin x+2=0$

$(2\sin x-1)\cdot(\sin x-2)=0$

$2\sin x-1=0$ veya (or) $\sin x-2=0$

$\sin x=\frac{1}{2}$ $\qquad$ $\sin x=2\Rightarrow$ ÇK(SS) $=\varnothing$

$x=30°,\quad x=150°$

ÇK(SS) $=\{30°,150°\}$

Yanıt / The Answer is D

9. $-1\le\sin x\le 1\ \Rightarrow\ -3\le 3\sin x\le 3$

$-1\le\cos y\le 1\ \Rightarrow\ \underline{-4\le -4\cos y\le 4}$

$-7\le 3\sin x-4\cos y\le 7$

$-7\le A\le 7\ \Rightarrow\ \max(A)=7$

Yanıt / The Answer is E

10. $\frac{\sin 30°+\cos 30°}{\tan 45°+\cot 30°}=\frac{\frac{1}{2}+\frac{\sqrt{3}}{2}}{1+\sqrt{3}}$

$=\frac{\frac{1+\sqrt{3}}{2}}{1+\sqrt{3}}$

$=\frac{1}{2}$

Yanıt / The Answer is B

11. $\frac{\cos 150°-\sin 150°}{\cos(-330°)-\sin(-750°)}$

$=\frac{\cos(180°-30°)-\sin(180°-30°)}{\cos(360°-30°)+\sin 30°}$

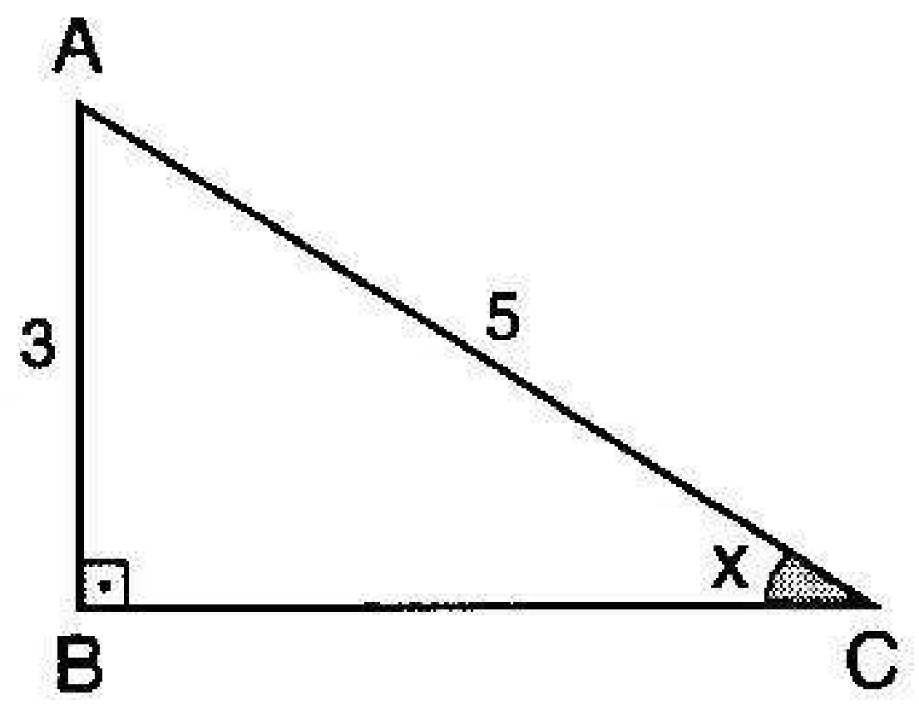
$=\frac{-\cos 30°-\sin 30°}{\cos 30°+\sin 30°}$

$=-1$

Yanıt / The Answer is B

12. $\frac{\pi}{2}<x<\pi$

$\sin x=\frac{3}{5}\ \Rightarrow$



$|AC|^2=|AB|^2+|BC|^2$

$25=9+|BC|^2\ \Rightarrow\ |BC|=4$

$\frac{\pi}{2}<x<\pi\ \Rightarrow\ \tan x=-\frac{3}{4}\ ,\ \cos x=-\frac{4}{5}$

$\tan x-\cos x=-\frac{3}{4}-\left(-\frac{4}{5}\right)$

$=-\frac{3}{4}+\frac{4}{5}=\frac{1}{20}$

Yanıt / The Answer is E

1. $120° = k \cdot \pi \Rightarrow k = ?$

Yanıt / The Answer is: $\frac{2\pi}{3}$

2. $\left.\begin{array}{l} \alpha = \frac{3\pi}{4} \\ \alpha = x° \end{array}\right\} \Rightarrow x = ?$

Yanıt / The Answer is: 135°

3. $\frac{1-\cos^2 x}{\sec^2 x - 1} = ?$

Yanıt / The Answer is: $\cos^2 x$

4. $\frac{1}{1+\tan^2 x} + \frac{1}{1+\cot^2 x} = ?$

Yanıt / The Answer is: 1

5. $\frac{\cos 300° + \sin 120°}{\tan 45° + \tan 60°} = ?$

Yanıt / The Answer is: $\frac{1}{2}$

6. $\frac{\cos 210° - \sin 420°}{\sin(-240°) + \cos(-30°)} = ?$

Yanıt / The Answer is: -1

7. $\frac{3\pi}{2} < \alpha < 2\pi$

$\sin\alpha = -\frac{5}{13} \Rightarrow \boldsymbol{\cos\alpha = ?}$

Yanıt / The Answer is: $\boldsymbol{\frac{12}{13}}$

---

8. $\boldsymbol{\cos(180° - x) = ?}$

Yanıt / The Answer is: **$-\cos x$**

---

9. $\frac{\boldsymbol{\sin x}}{\boldsymbol{\mathrm{cosec}\, x}} + \frac{\boldsymbol{\cos x}}{\boldsymbol{\sec x}} = ?$

Yanıt / The Answer is: **1**

---

10. $1356° = x + k \cdot 360° \Rightarrow \boldsymbol{x = ?}$

Yanıt / The Answer is: **276°**

---

11. $-935° = x + k \cdot 360° \Rightarrow \boldsymbol{x = ?}$

Yanıt / The Answer is: **145°**

---

12. $\frac{45\pi}{4} = x + k \cdot 2\pi \Rightarrow \boldsymbol{x = ?}$

Yanıt / The Answer is: $\boldsymbol{\frac{5\pi}{4}}$

13. $\frac{-78\pi}{7} = x + k \cdot 2\pi \quad \Rightarrow \quad \mathbf{x = ?}$

Yanıt / The Answer is: $\mathbf{\frac{6\pi}{7}}$

14. $\left.\begin{array}{l} a = \sin 50° \\ b = \cos 130° \\ c = \tan 200° \\ d = \cot 300° \end{array}\right\} \Rightarrow \mathbf{a, b, c, d = ?, ?, ?, ?}$

Yanıt / The Answer is: **+ , – , + , –**

15. $\left.\begin{array}{l} a = \cos 20° \\ b = \cos 95° \\ c = \cos 155° \end{array}\right\} \Rightarrow \mathbf{? < ? < ?}$

Yanıt / The Answer is: **c < b < a**

16. $\dfrac{\cos(\pi - x) + \sin\left(\dfrac{\pi}{2} - x\right)}{\tan\left(\dfrac{3\pi}{2} - x\right)} = ?$

Yanıt / The Answer is: **0**

17. $m \in Z,$

$m = 5 - 2\sin^2\alpha \;\Rightarrow\; \mathbf{\sum m = ?}$

Yanıt / The Answer is: **12**

18. **cos420° + sin(–300°) + tan765° = ?**

Yanıt / The Answer is: $\mathbf{\frac{3+\sqrt{3}}{2}}$

19. $\dfrac{\cos^2 x}{1+\sin x} + \sin x = ?$

Yanıt / The Answer is: **1**

20. $\tan x + \cot x = 4 \Rightarrow$ $\mathbf{\tan^2 x + \cot^2 x = ?}$

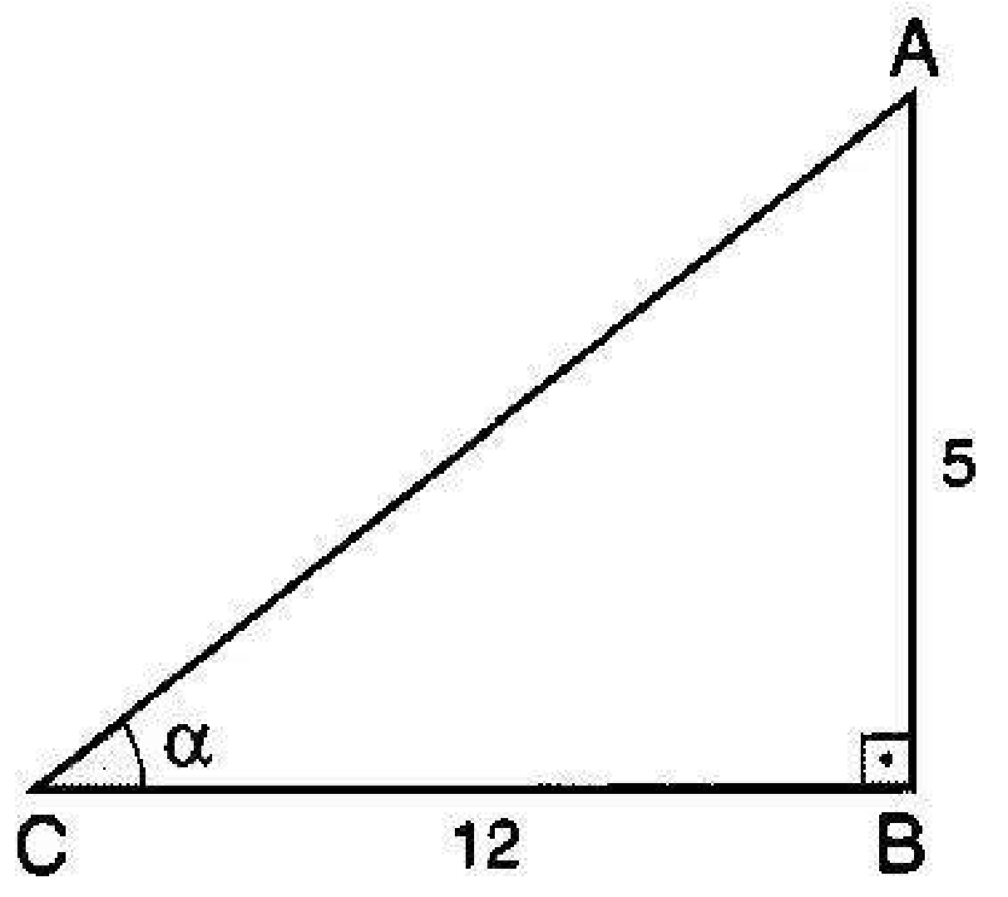
Yanıt / The Answer is: **14**

21. $0 < x < \dfrac{\pi}{2}$

$\dfrac{3\sin x - \cos x}{2\sin x + 5\cos x} = \dfrac{3}{4} \Rightarrow$ **tanx = ?**

Yanıt / The Answer is: $\dfrac{19}{6}$

22.



$-\sin\alpha - \cos(180° - \alpha) + \tan(90° + \alpha) = ?$

Yanıt / The Answer is: $-\dfrac{121}{65}$

23. |BC| = 4 cm,

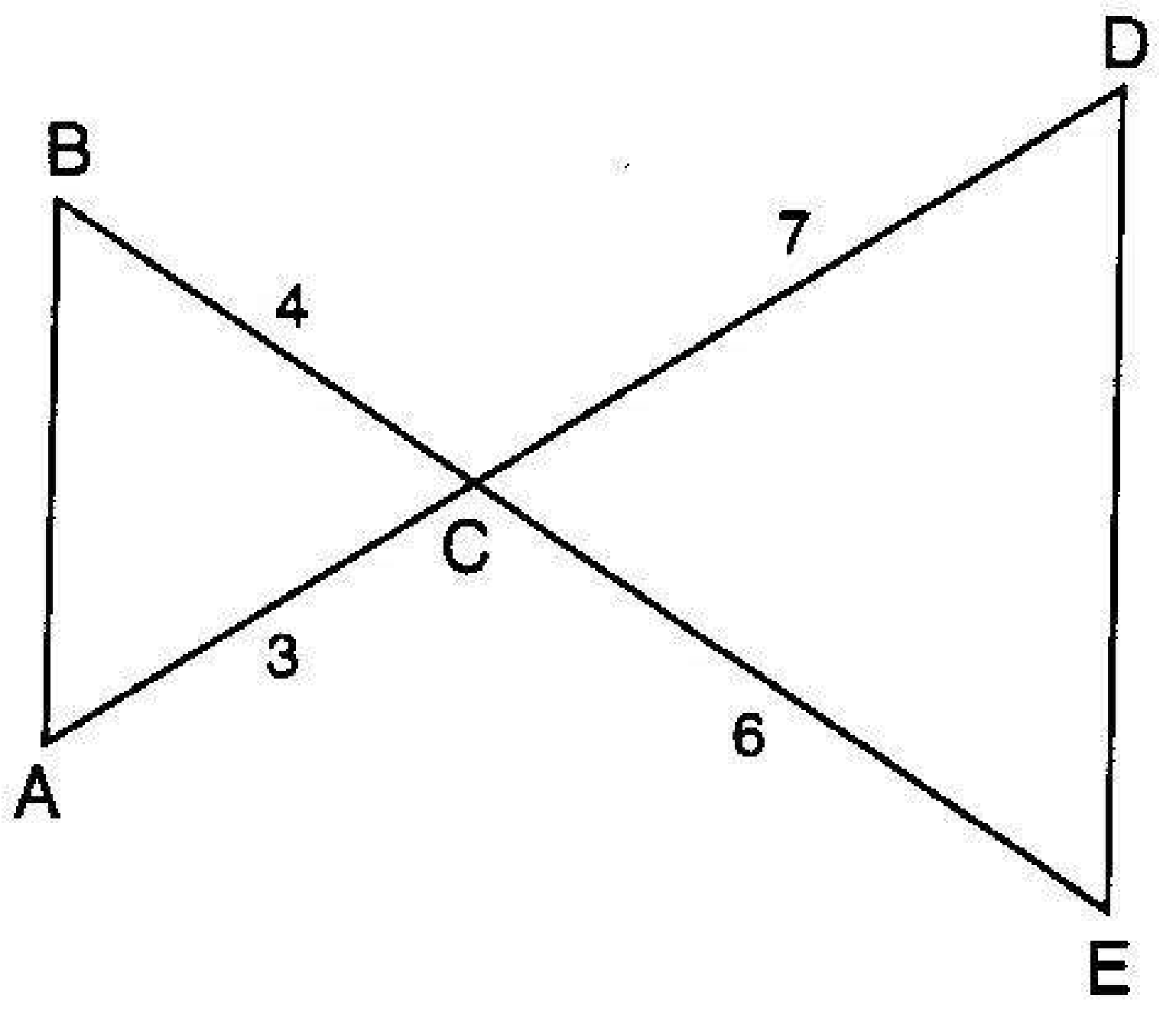
|AC| = 3 cm,

|CD| = 7 cm,

|CE| = 6 cm,

$A(\overset{\Delta}{ABC}) = 4\ cm^2$.

$A(\overset{\Delta}{CDE}) = ?\ cm^2$



Yanıt / The Answer is: **14**

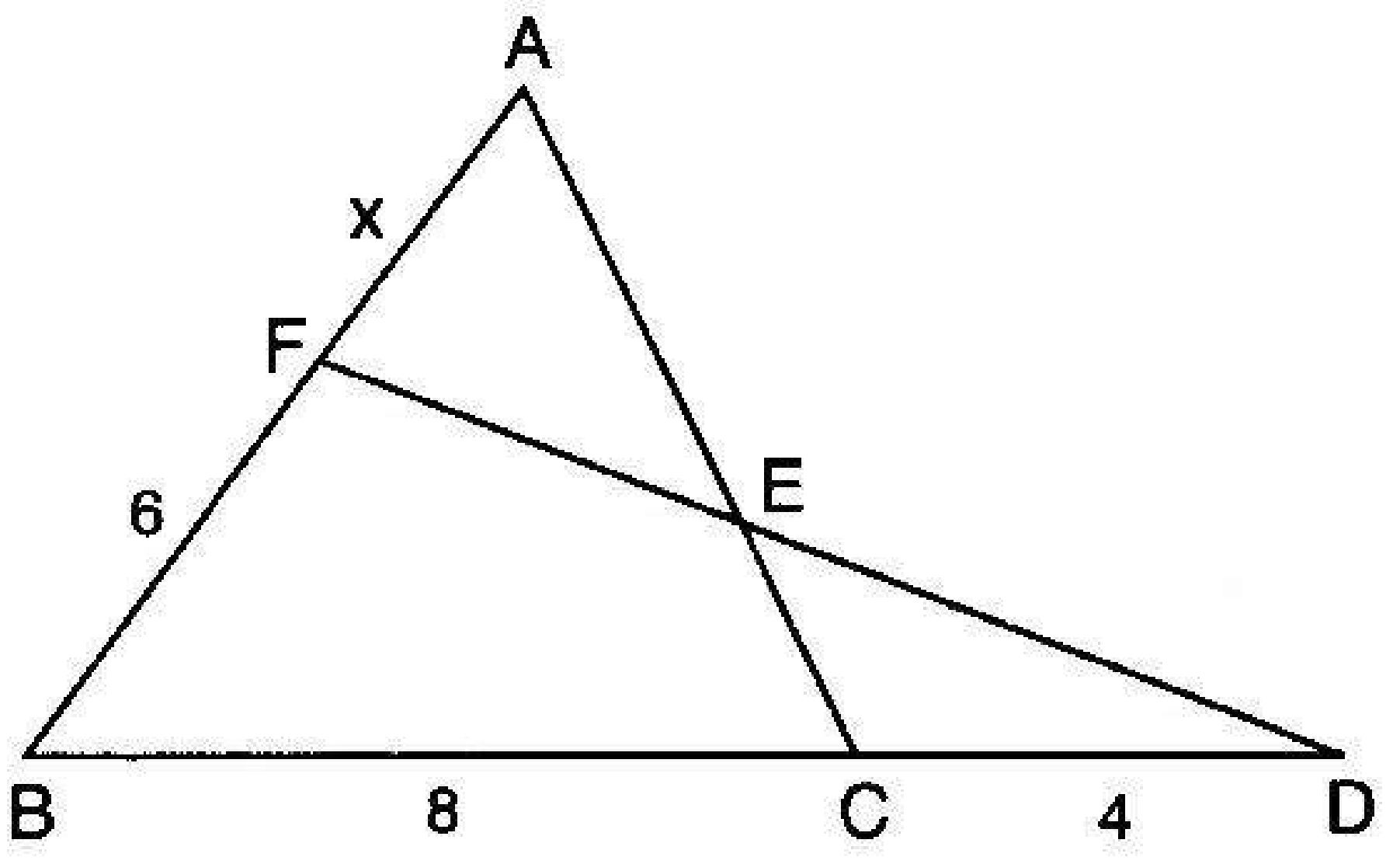
24. |FB| = 6 cm,

|BC| = 8 cm,

|CD| = 4 cm,

$A(\overset{\Delta}{AFE}) = A(\overset{\Delta}{ECD})$.

|AF| = x = ? cm



Yanıt / The Answer is: **3**

1. $50° = xR \Rightarrow$ **x = ?**

A) $\frac{5\pi}{18}$

B) $\frac{2\pi}{9}$

C) $\frac{\pi}{6}$

D) $\frac{\pi}{9}$

E) $\frac{\pi}{18}$

2. $\frac{3\pi}{5} \cdot R = x° \Rightarrow$ **x = ?**

A) 84

B) 96

C) 108

D) 120

E) 132

3. $1280° = x + 360° \cdot k \Rightarrow$ **x = ?**

A) 160°

B) 170°

C) 180°

D) 190°

E) 200°

4. $-1370° = x + 360° \cdot k \Rightarrow$ **x = ?**

A) 25°

B) 40°

C) 60°

D) 70°

E) 95°

5. $\frac{47\pi}{3} = x + 2k\pi \Rightarrow$ **x = ?**

A) $\frac{\pi}{3}$

B) $\frac{2\pi}{3}$

C) π

D) $\frac{4\pi}{3}$

E) $\frac{5\pi}{3}$

6. $-\frac{51\pi}{4} = x + 2k\pi \Rightarrow$ **x = ?**

A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) $\frac{3\pi}{4}$

D) π

E) $\frac{5\pi}{4}$

7. $$\left.\begin{array}{l} a=\sin 100^\circ \\ b=\sin 125^\circ \\ c=\sin 145^\circ \end{array}\right\} \Rightarrow ?<?<?$$

A) $a<b<c$

B) $b<c<a$

C) $a<c<b$

D) $c<b<a$

E) $b<a<c$

8. $\sqrt{\cos^2 x}-\sqrt{1-\sin^2 x}=?$

A) 0

B) 1

C) 2

D) 3

E) 4

9. $\dfrac{\sin x}{\tan x}+\dfrac{\sin^2 x}{1+\cos x}=?$

A) –1

B) 0

C) 1

D) sinx

E) tanx

10. $\dfrac{\sin^2 x-\cos^2 x}{\sin x+\cos x}=\dfrac{1}{2} \Rightarrow \sin x\cdot\cos x=?$

A) 1

B) $\frac{1}{4}$

C) $\frac{2}{7}$

D) $\frac{3}{8}$

E) $\frac{1}{2}$

11. $\dfrac{\sin^3 x+\sin x\cdot\cos^2 x}{\cos x}=?$

A) 1

B) sinx

C) cosx

D) tanx

E) cotx

12. $(1+\sin x)\cdot\left(1-\dfrac{1}{\sin x}\right)\cdot\tan x=?$

A) cosx

B) –cosx

C) sinx

D) –cotx

E) tanx

YANITLAR / ANSWERS

1. A	2. C	3. E	4. D	5. E	6. E	7. D	8. A
9. C	10. D	11. D	12. B				

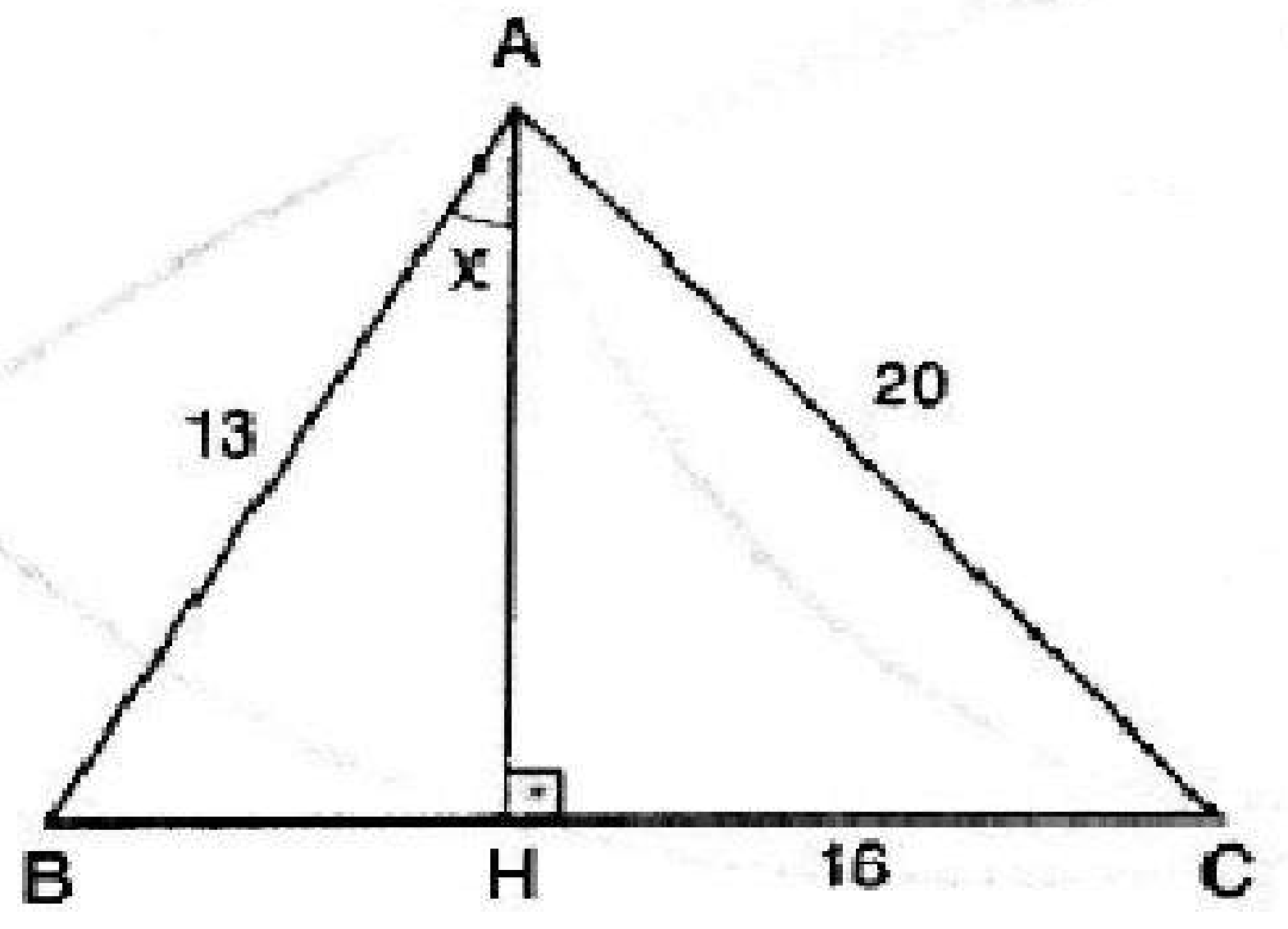
1. $\frac{\pi}{2} < x < \pi, \quad \sin x = \frac{2}{3} \Rightarrow \tan x = ?$

A) $\frac{3\sqrt{5}}{5}$
B) $\frac{2\sqrt{5}}{5}$
C) $\frac{\sqrt{5}}{5}$
D) $-\frac{\sqrt{5}}{5}$
E) $-\frac{2\sqrt{5}}{5}$

2. $0 < x < \frac{\pi}{2}, \quad \tan x = \frac{1}{2} \Rightarrow \sin^2 x + 2 \cdot \cos x \cdot \sin x = ?$

A) $\frac{1}{5}$
B) $\frac{2}{5}$
C) $\frac{3}{5}$
D) $\frac{4}{5}$
E) 1

3. $\sin x = ?$



A) $\frac{5}{13}$
B) $\frac{8}{13}$
C) $\frac{9}{13}$
D) $\frac{10}{13}$
E) $\frac{12}{13}$

4. $\pi < x < \frac{3\pi}{2}$

$16^{\sin x} = 8^{\cos x} \Rightarrow \sin x = ?$

A) $-\frac{3}{5}$
B) $-\frac{4}{5}$
C) $\frac{3}{5}$
D) $\frac{4}{5}$
E) 1

5. $\frac{\pi}{2} < x < \pi,$

$\tan x = -3 \Rightarrow 4\sin x + 2\cos x = ?$

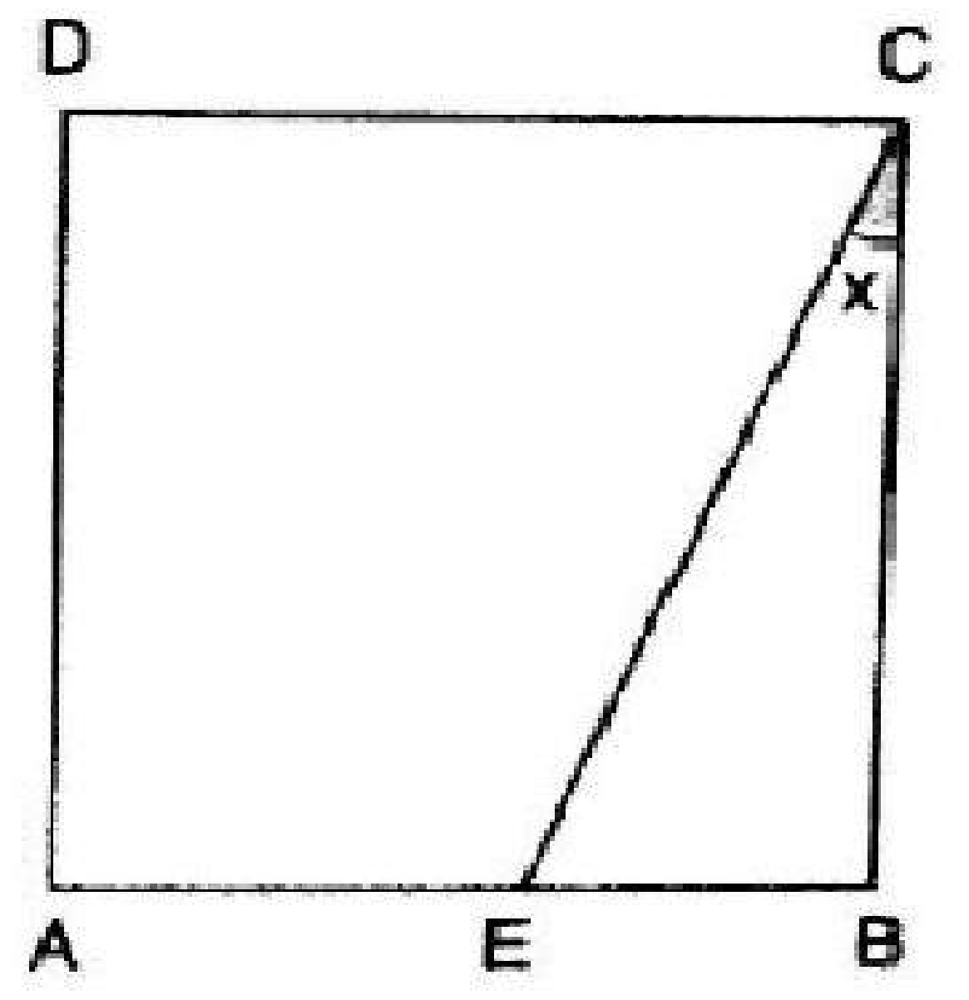
A) $\sqrt{7}$
B) $2\sqrt{2}$
C) 3
D) $\sqrt{10}$
E) $\sqrt{11}$

6. ABCD bir karedir.

ABCD is a square.

$3|EB| = 2|AE|$

$\Rightarrow \cot x = ?$



A) 1
B) $\frac{3}{2}$
C) 2
D) $\frac{5}{2}$
E) 3

7. **$\cos 240° = ?$**

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $-\frac{1}{2}$

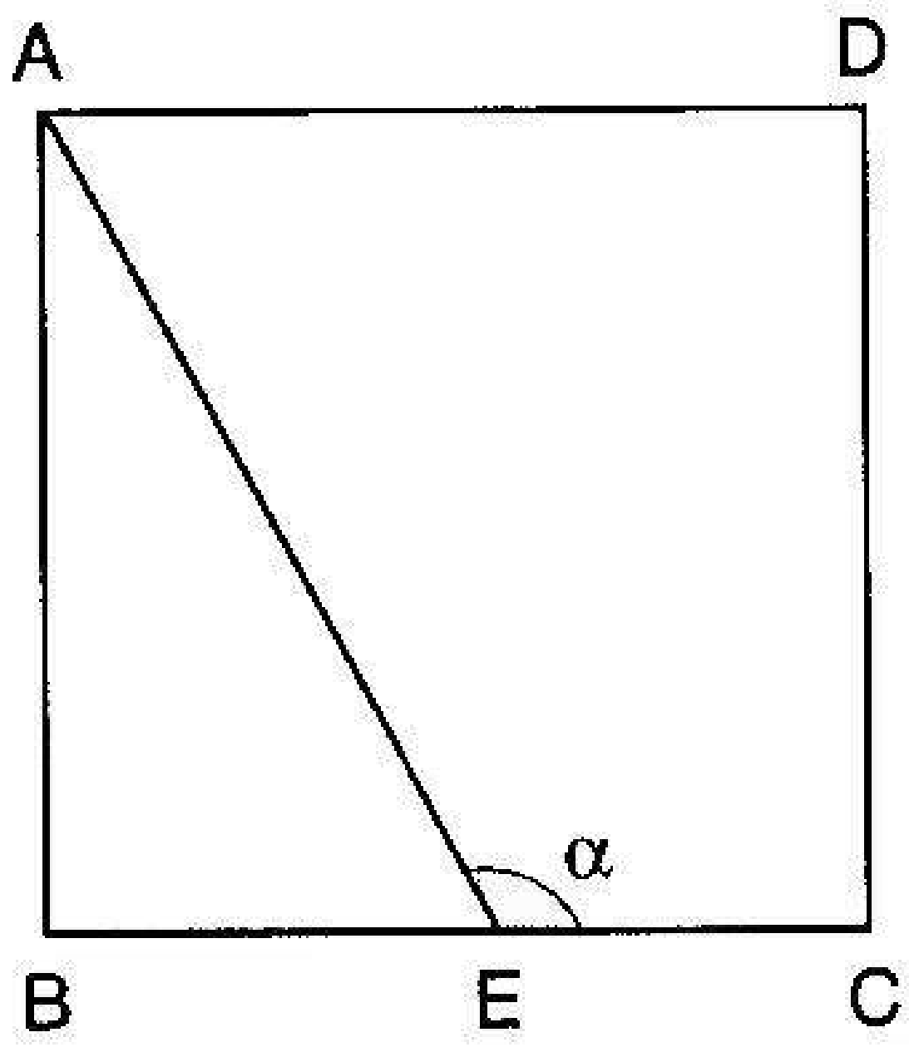
D) $-\frac{\sqrt{3}}{2}$ E) -1

8. $$\frac{\sin\left(\frac{3\pi}{2}+x\right)-\cos(\pi+x)}{\cos\left(\frac{3\pi}{2}-x\right)+\sin(-x)}=?$$

A) –tanx B) 0 C) 1

D) cotx E) tanx

9. ABCD bir karedir.
ABCD is a square.
$3|BE| = 4|EC|$
$\Rightarrow$ **$\cot\alpha = ?$**



A) $\frac{3}{7}$ B) $\frac{4}{7}$ C) $-\frac{3}{7}$

D) $-\frac{4}{7}$ E) $-\frac{5}{7}$

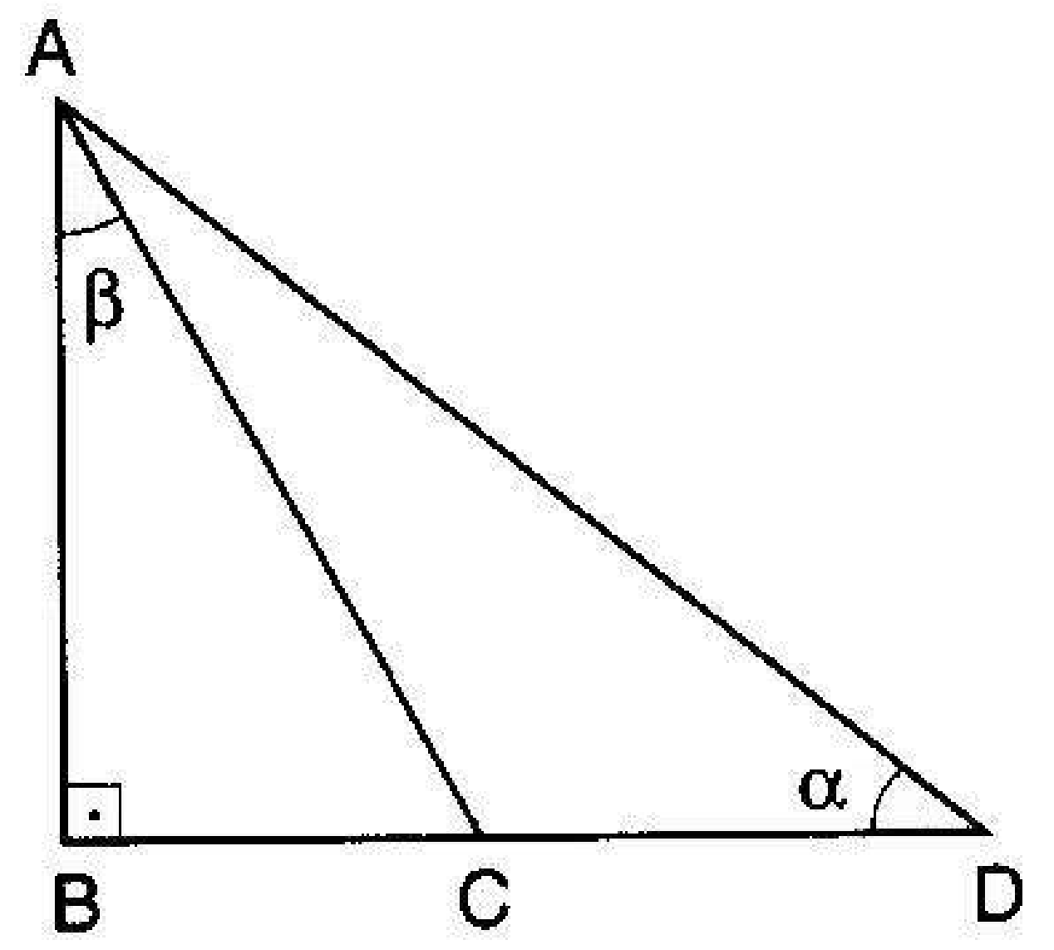
10. $0 < x < \frac{\pi}{2}$, $\tan x = \frac{5}{12}$

$\Rightarrow$ **$\cos(\pi - x) + \cos\left(\frac{\pi}{2}+x\right) = ?$**

A) $-\frac{7}{13}$ B) $-\frac{12}{13}$ C) -1

D) $-\frac{15}{13}$ E) $-\frac{17}{13}$

11. $\tan\alpha = \frac{2}{5}$,
$\cot\beta = 2$,
$|CD| = 8$.
$|BC| = ?$



A) 1 B) 2 C) 3

D) 4 E) 5

12. $0 < x < \pi$,
$\sin^2 x + \sin x - 2 = 0 \Rightarrow$ **$x = ?$**

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$

D) $\frac{5\pi}{2}$ E) 2π

YANITLAR / ANSWERS							
1. E	2. E	3. A	4. A	5. D	6. D	7. C	8. B
9. D	10. E	11. B	12. A				

1. $\sin 50° = m$

$\Rightarrow$ **$\cot 680° + \tan 230° - 2\cdot\sin(-50°) = ?$**

A) m

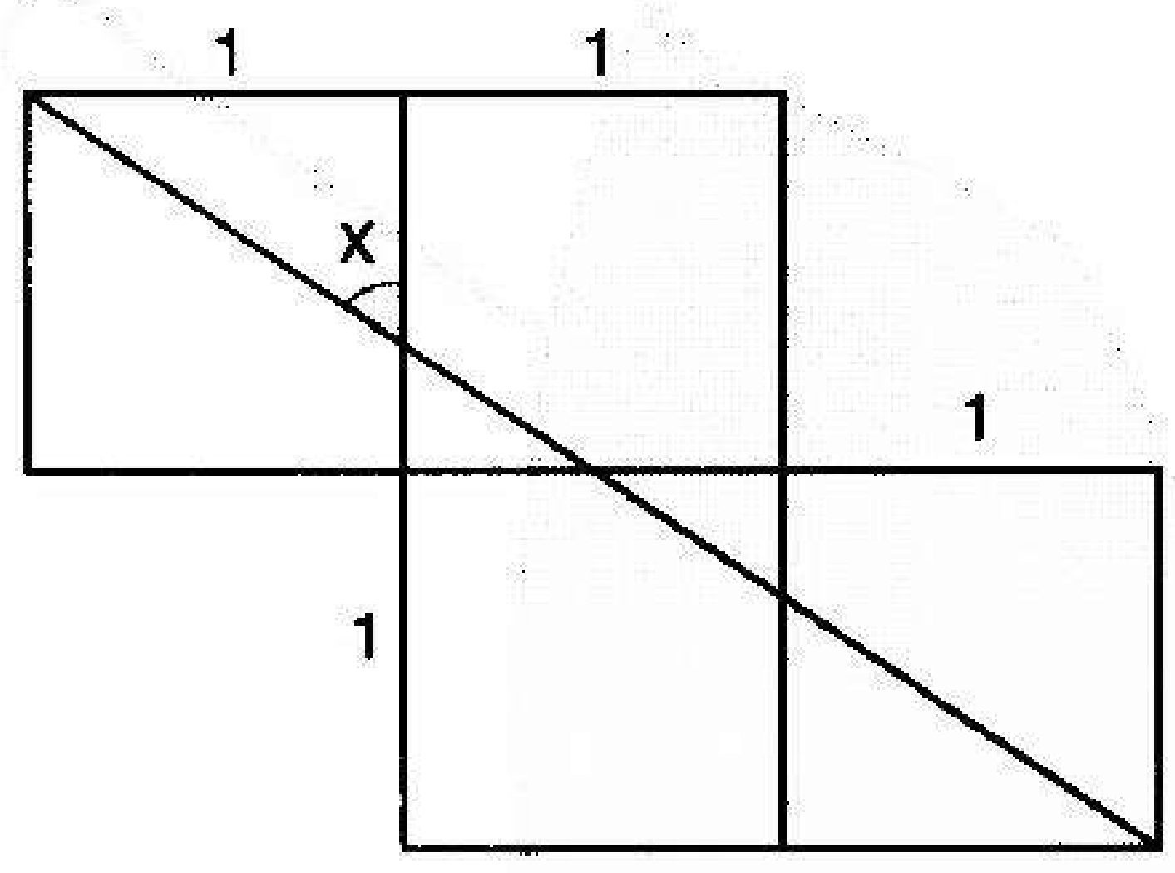
B) 2m

C) 3m

D) 4m

E) 9m

2. **$\sin x\cdot\cot x = ?$**



A) $\frac{2\sqrt{13}}{13}$

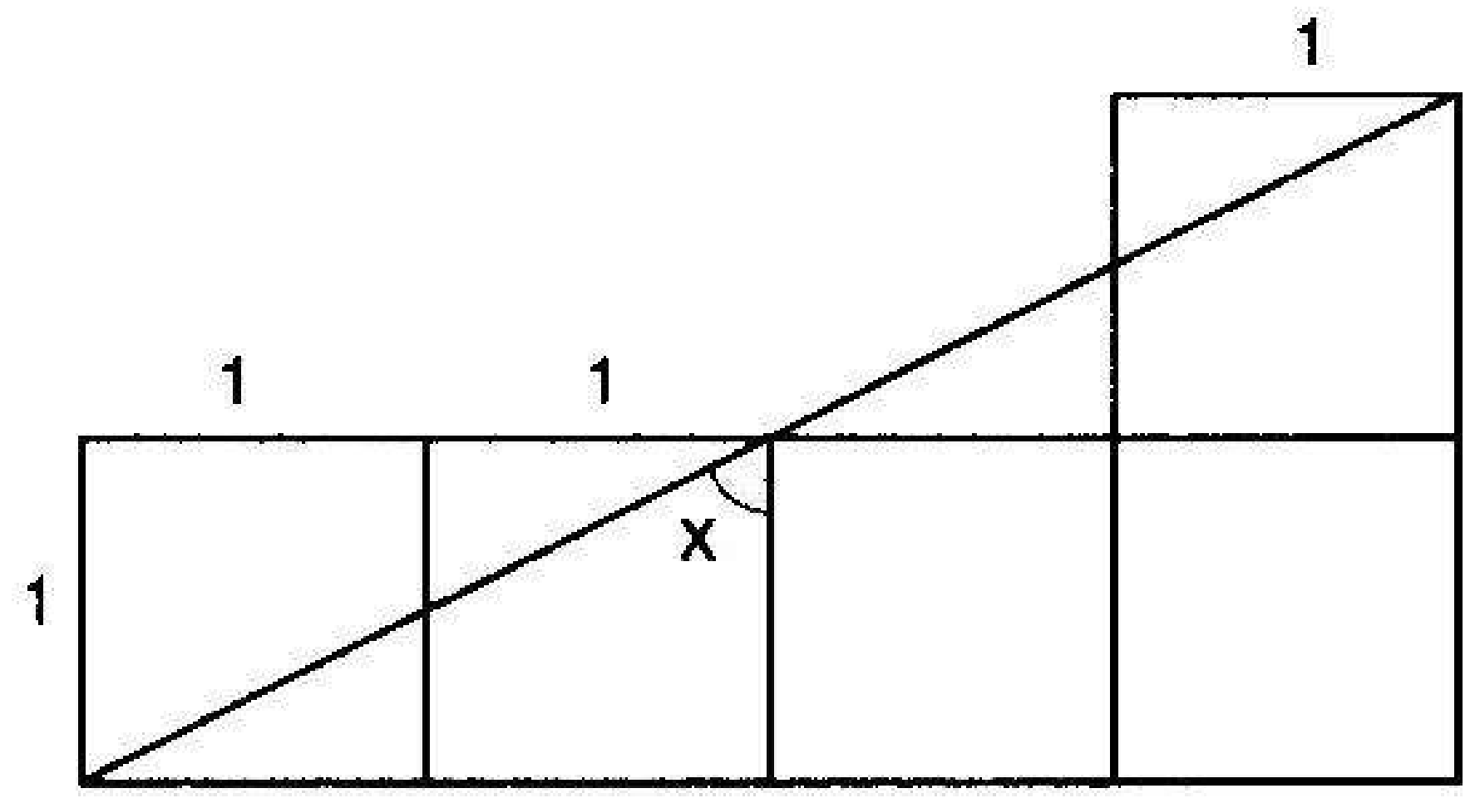
B) $\frac{\sqrt{13}}{2}$

C) $\frac{\sqrt{5}}{3}$

D) $\frac{\sqrt{2}}{5}$

E) $\frac{1}{2}$

3. **$\tan x = ?$**



A) 2

B) $\frac{3}{2}$

C) 1

D) $\frac{1}{2}$

E) $\frac{1}{3}$

4.

$$\left.\begin{aligned} a &= \sin 70° \\ b &= \cos 40° \\ c &= \sin 125° \\ d &= \cos(-70°) \end{aligned}\right\} \Rightarrow ? < ? < ? < ?$$

A) a < b < c < d

B) d < b < c < a

C) d < c < b < a

D) c < d < b < a

E) c < d < a < b

5.

$$\left.\begin{aligned} a &= \tan 62° \\ b &= \sin 110° \\ c &= \cos 15° \\ d &= \cot 30° \end{aligned}\right\} \Rightarrow ? < ? < ? < ?$$

A) b < c < d < a

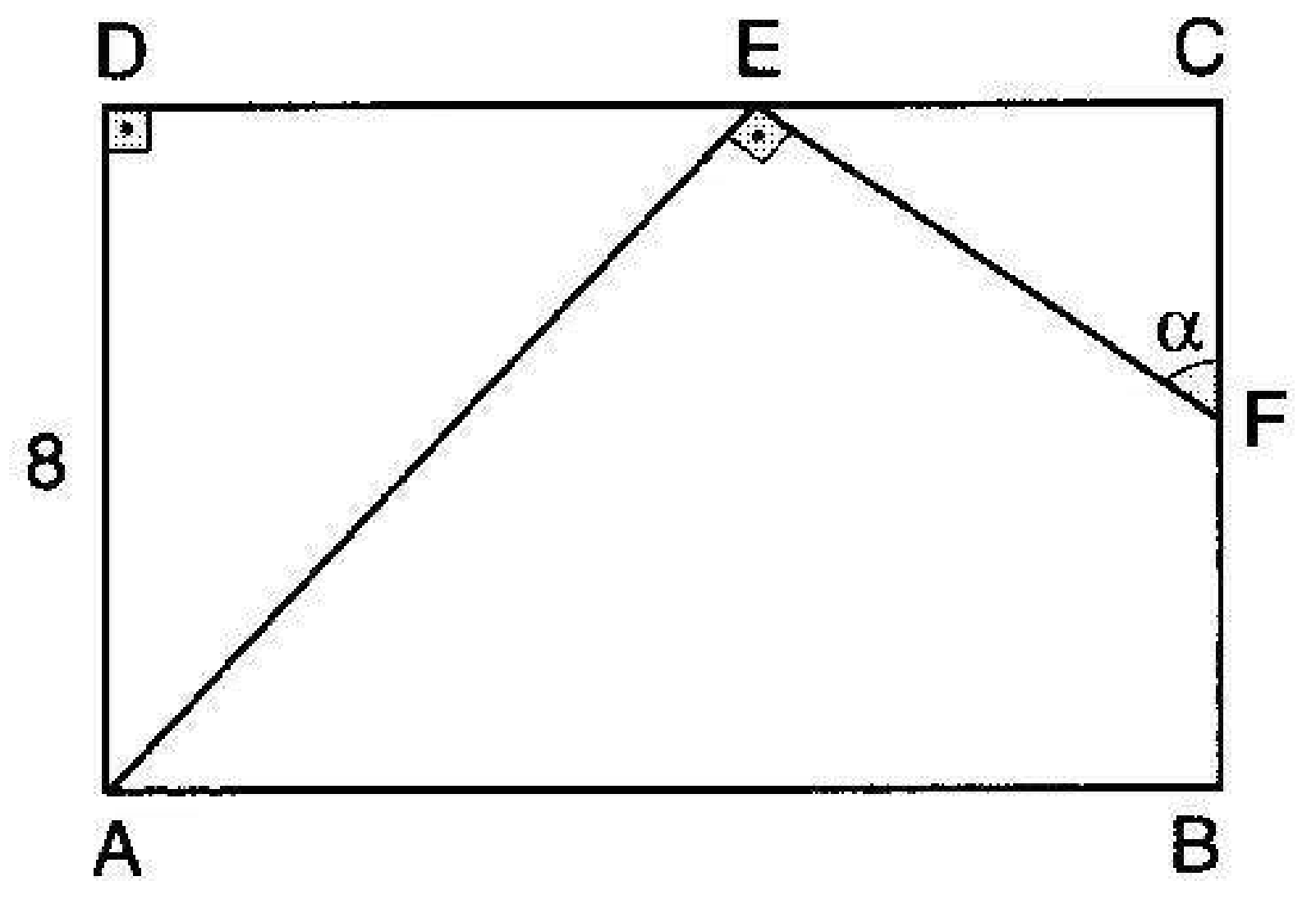
B) b < c < a < d

C) d < a < c < b

D) b < a < c < d

E) d < b < c < a

6. ABCD bir dikdörtgendir.
ABCD is a rectangle.
$m(\widehat{EFC}) = \alpha$,
$m(\widehat{AEF}) = 90°$,
$|AD| = 8$ cm,
$|DE| = 15$ cm.
$\sin\alpha = ?$



A) $\frac{8}{17}$

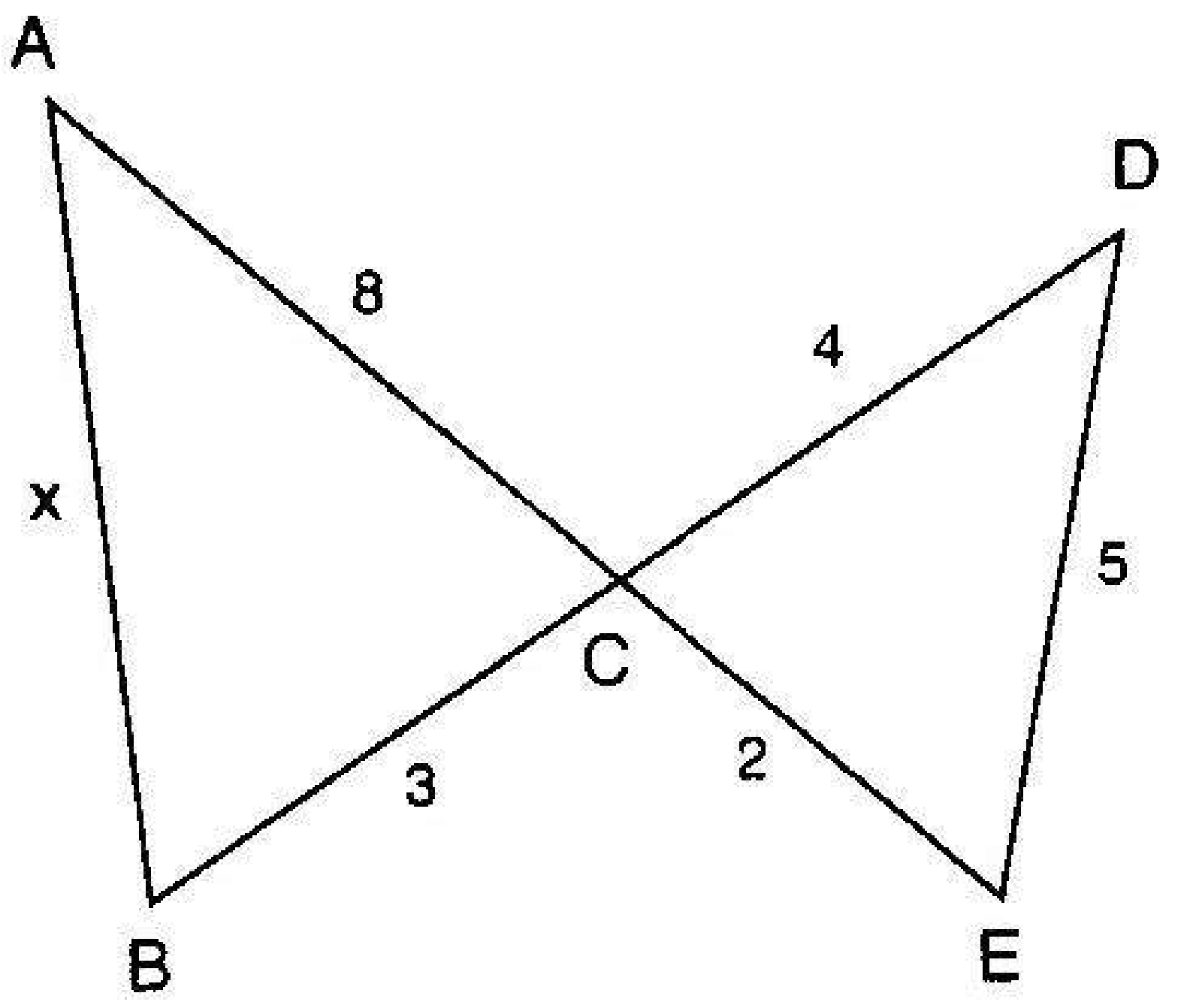
B) $\frac{15}{17}$

C) $\frac{5}{13}$

D) $\frac{3}{5}$

E) $\frac{4}{5}$

7. **x = ?**


A
D
8
4
x
5
C
3
2
B
E

A) $\sqrt{22}$

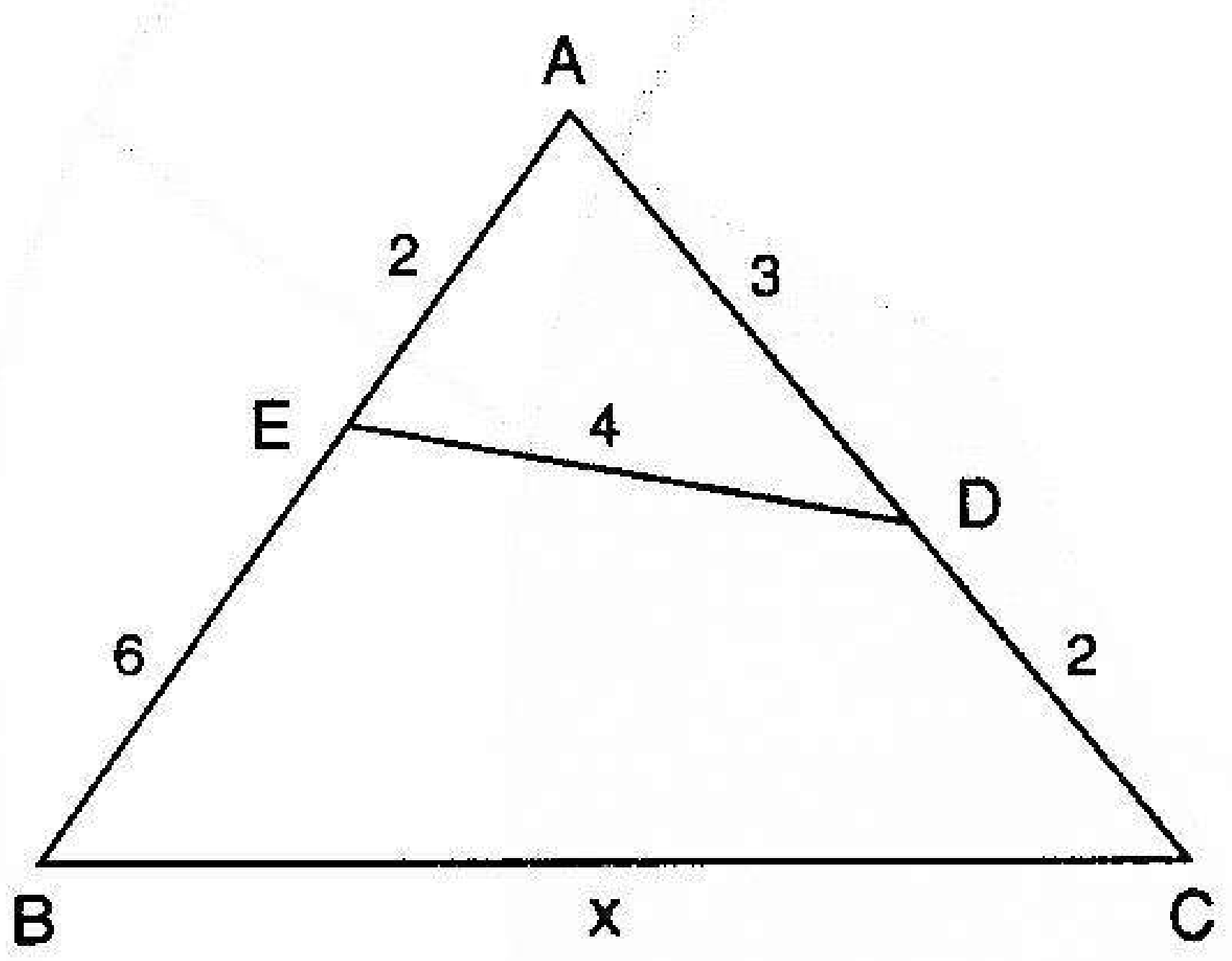
B) $2\sqrt{22}$

C) $2\sqrt{26}$

D) $2\sqrt{29}$

E) $2\sqrt{34}$

8. **x = ?**


A
2
3
E
4
D
6
2
B
x
C

A) $\sqrt{103}$

B) $\sqrt{105}$

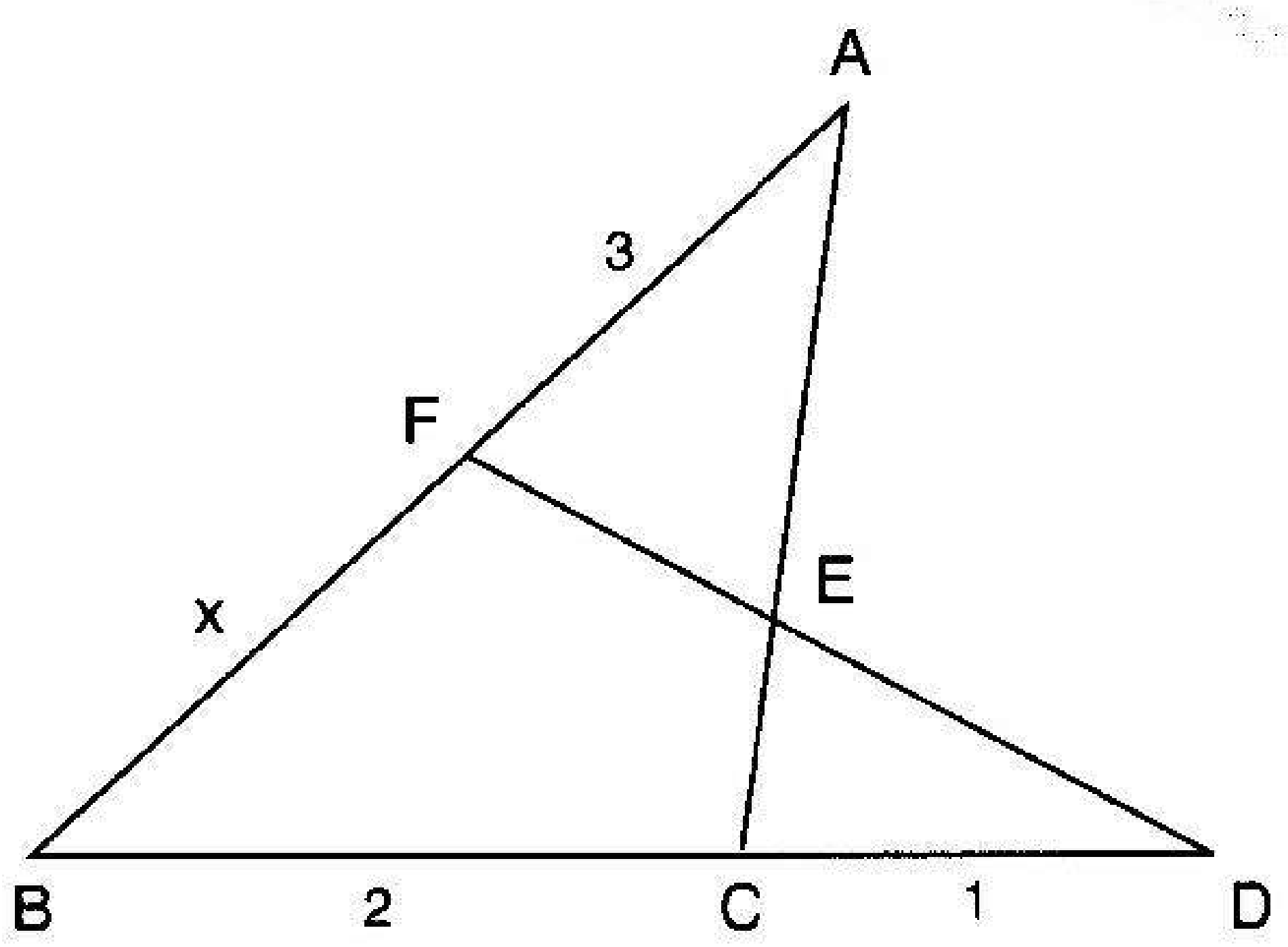
C) $\sqrt{107}$

D) $\sqrt{109}$

E) $\sqrt{113}$

9. $A(\overset{\Delta}{AFE}) = A(\overset{\Delta}{ECD})$

⇒ **x = ?**


A
3
F
x
E
B
2
C
1
D

A) 3

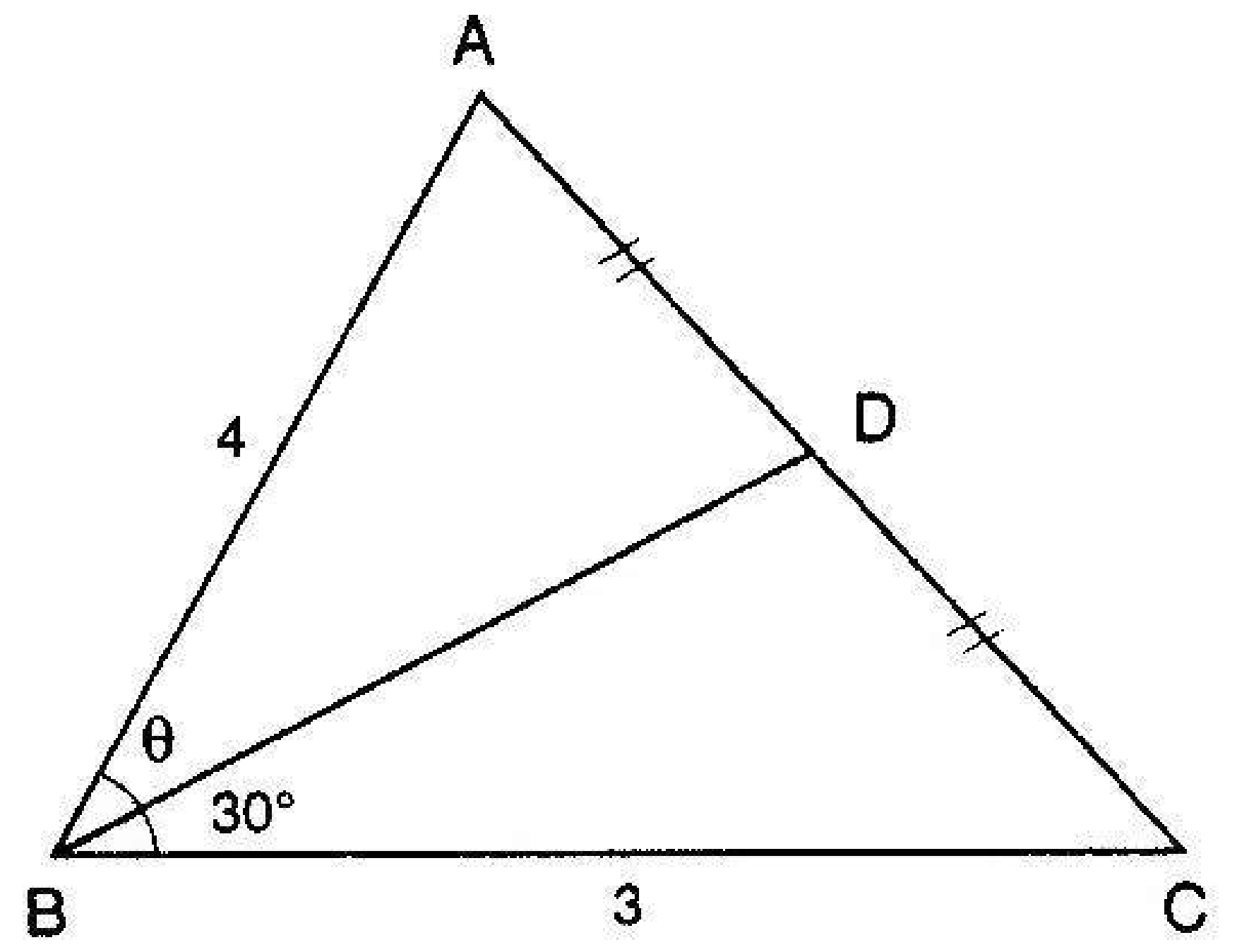
B) 4

C) 5

D) 6

E) 8

10. **sinθ = ?**



A) $\frac{1}{8}$

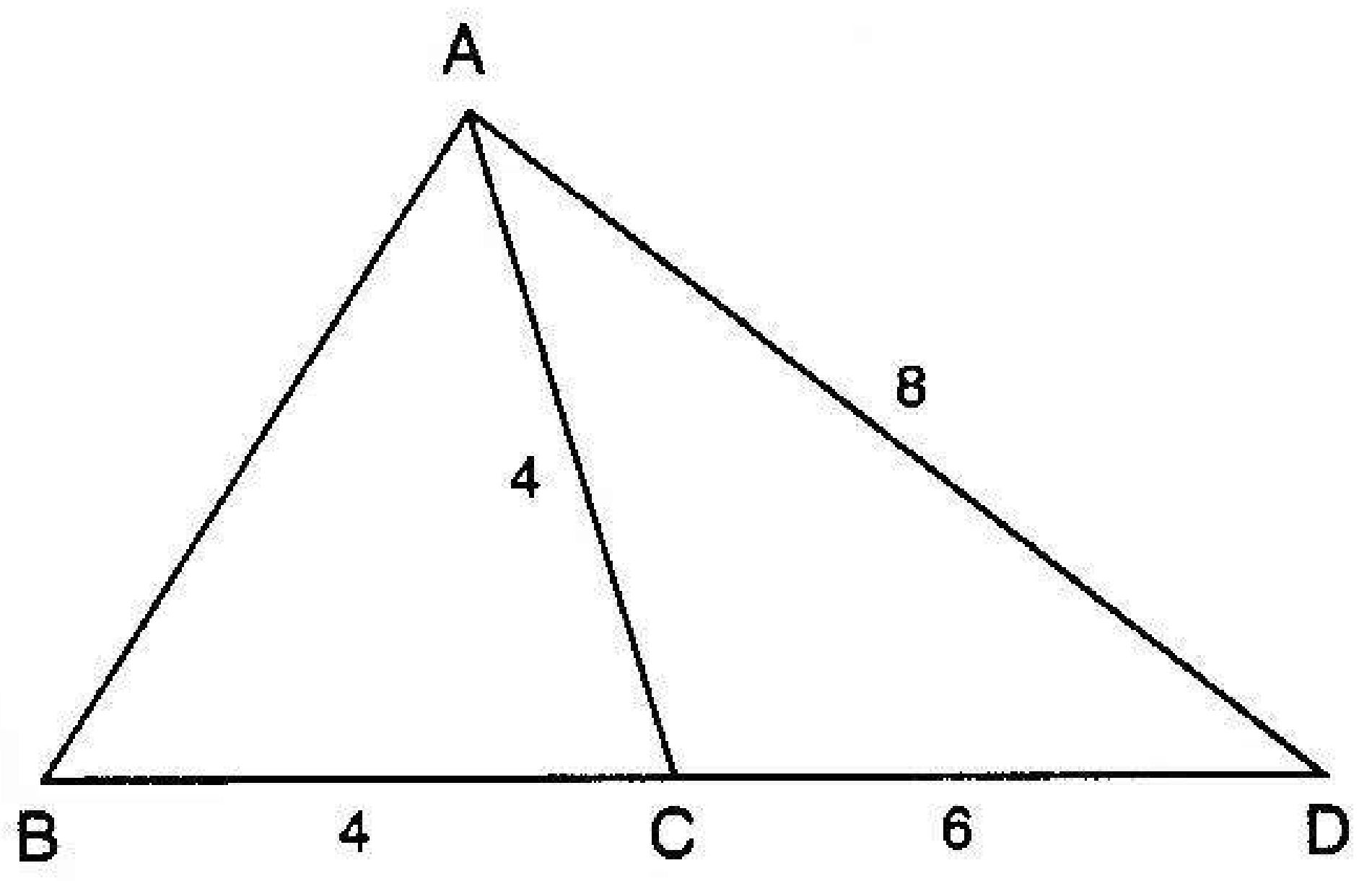
B) $\frac{1}{4}$

C) $\frac{3}{8}$

D) $\frac{1}{2}$

E) $\frac{5}{8}$

11. **$\sin(\widehat{B}) = ?$**



A) $\frac{\sqrt{6}}{4}$

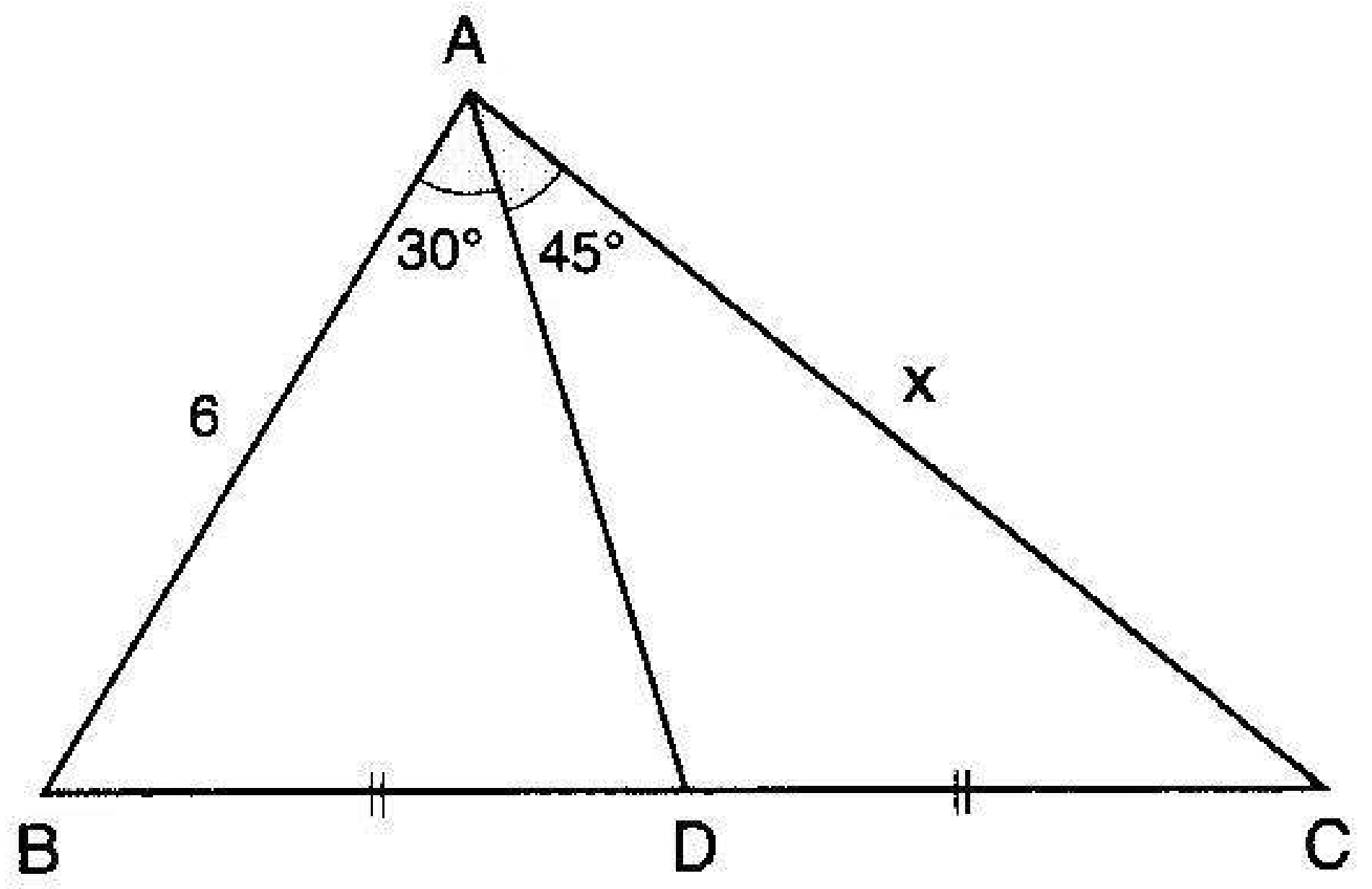
B) $\frac{\sqrt{7}}{4}$

C) $\frac{\sqrt{10}}{4}$

D) $\frac{\sqrt{11}}{4}$

E) $\frac{\sqrt{15}}{4}$

12. **x = ?**


A
30°
45°
6
x
B
D
C

A) $2\sqrt{2}$

B) $3\sqrt{2}$

C) $4\sqrt{2}$

D) $2\sqrt{3}$

E) $3\sqrt{3}$

YANITLAR / ANSWERS

1. B	**2. A**	**3. A**	**4. B**	**5. A**	**6. A**	**7. B**	**8. D**
9. D	**10. C**	**11. C**	**12. B**				

TRİGONOMETRİ II
TRIGONOMETRY II

BÖLÜM / CHAPTER 2

TOPLAM VE FARK FORMÜLLERİ / SUM and DIFFERENCE FORMULAS

1. $\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$
2. $\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$
3. $\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$
4. $\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$
5. $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$
6. $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$
7. $\cot(a+b) = \dfrac{\cot a \cdot \cot b - 1}{\cot b + \cot a}$
8. $\cot(a-b) = \dfrac{\cot a \cdot \cot b + 1}{\cot b - \cot a}$

NOT / NOTE

$f(x) = a\sin x + b\cos x \Rightarrow$

$\max f(x) = \sqrt{a^2 + b^2}$

$\min f(x) = -\sqrt{a^2 + b^2}$

ÖRNEK / EXAMPLE 1

$\sin 105° = ?$

A) $\dfrac{\sqrt{6}+\sqrt{3}}{4}$ B) $\dfrac{\sqrt{6}+\sqrt{2}}{2}$ C) $\dfrac{\sqrt{6}+1}{2}$

D) $\dfrac{\sqrt{6}+\sqrt{3}}{2}$ E) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

PEKİŞTİRME / REINFORCE

$\cos 75° = ?$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\sin 105° = \sin(60° + 45°)$

$\sin(60° + 45°) = \sin 60° \cdot \cos 45° + \cos 60° \cdot \sin 45°$

$$\sin 105° = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\sin 105° = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

YANIT / ANSWER : $(\sqrt{6} - \sqrt{2}) / 4$

ÖRNEK / EXAMPLE 2

$x+y=\frac{\pi}{6} \Rightarrow (\cos x+\sin y)^2+(\cos y+\sin x)^2=?$

A) $\frac{1}{2}$ B) 1 C) $\frac{\sqrt{3}}{2}$ D) 3 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{aligned}(\cos x+\sin y)^2+(\cos y+\sin x)^2 &= \cos^2x+2\cos x\cdot\sin y+\sin^2y+\cos^2y+2\cos y\cdot\sin x+\sin^2x\\ &= 1+1+2\sin(x+y)\\ &= 2+2\sin(x+y)\end{aligned}$$

$$x+y=\frac{\pi}{6} \Rightarrow 2+2\sin\frac{\pi}{6}=2+2\cdot\frac{1}{2}=3$$

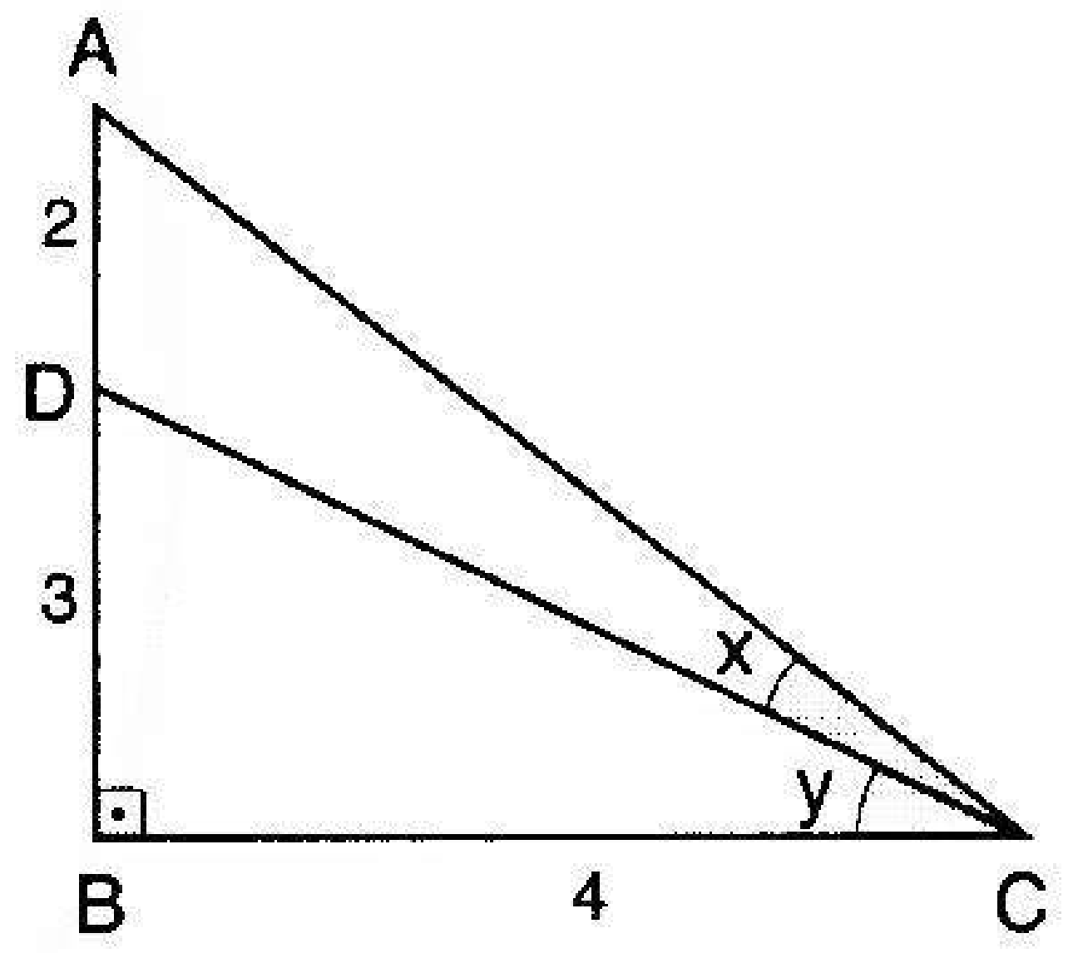
ÖRNEK / EXAMPLE 3

|AD| = 2 br,

|DB| = 3 br,

|BC| = 4 br.

⇒ **tanx = ?**



A) $\frac{18}{31}$ B) $\frac{15}{31}$ C) $\frac{12}{31}$ D) $\frac{10}{31}$ E) $\frac{8}{31}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\cdot\tan y}=\frac{\tan x+\frac{3}{4}}{1-\tan x\cdot\frac{3}{4}}$$

$$\frac{5}{4}=\frac{\frac{4\tan x+3}{4}}{\frac{4-3\tan x}{4}}$$

$$\frac{5}{4}=\frac{4\tan x+3}{4-3\tan x} \Rightarrow 16\tan x+12=20-15\tan x$$

$$31\tan x=8$$

$$\tan x=\frac{8}{31}$$

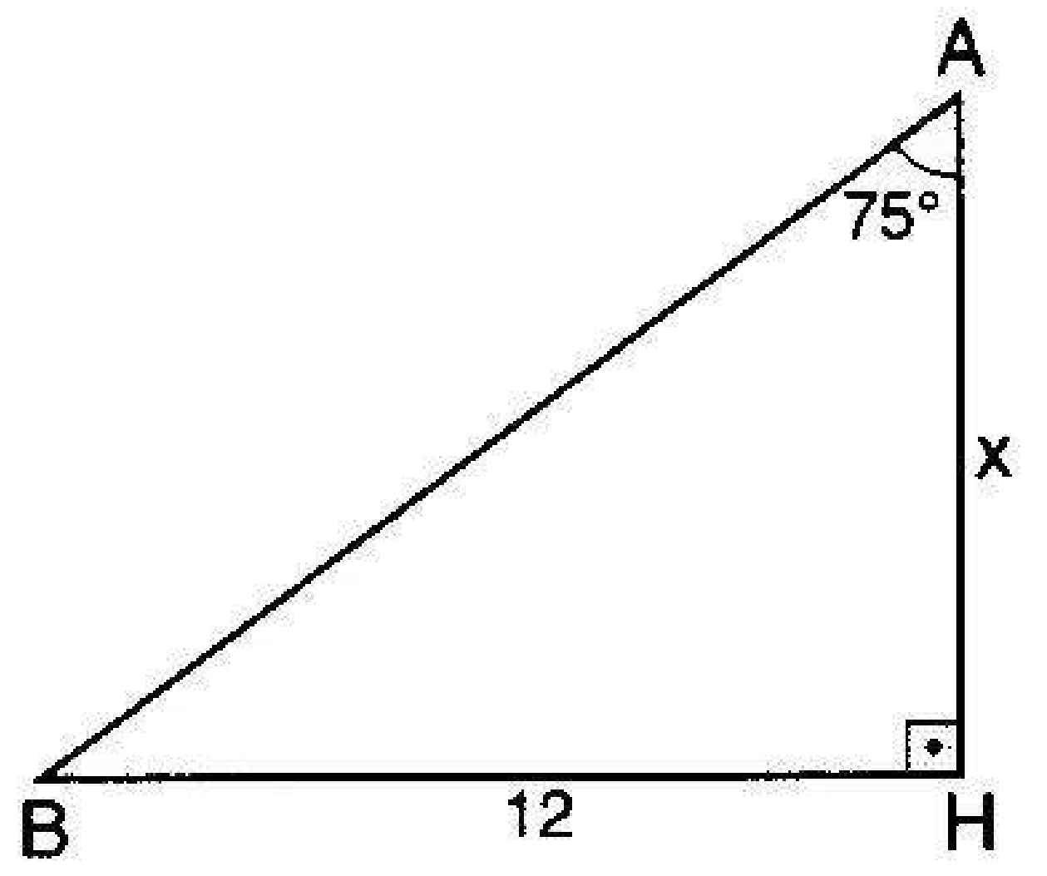
ÖRNEK / EXAMPLE 4

[AH] ⊥ [BH],

|BH| = 12 cm,

$m(\widehat{BAH}) = 75°$.

|AH| = x = ? cm



A) $12 \cdot (2-\sqrt{3})$ B) $6 \cdot (1+\sqrt{3})$ C) $4 \cdot (2-\sqrt{3})$

D) 6 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\tan 75° = \tan(45°+30°)$$

$$= \frac{\tan 45° + \tan 30°}{1-\tan 45° \cdot \tan 30°} = \frac{3+\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{(3+\sqrt{3})\cdot(3+\sqrt{3})}{(3+\sqrt{3})\cdot(3-\sqrt{3})}$$

$$= \frac{12+6\sqrt{3}}{6} = 2+\sqrt{3}$$

$$\Rightarrow x = \frac{12}{2+\sqrt{3}} = 12\cdot(2-\sqrt{3})$$

ÖRNEK / EXAMPLE 5

$\sin(\alpha-45°) = 3\cdot\cos(\alpha-45°) \Rightarrow$ **$\tan\alpha = ?$**

A) -2 B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) $\frac{\sqrt{2}}{3}$ E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\sin(\alpha-45°) = 3\cdot\cos(\alpha-45°)$$

$$\tan(\alpha-45°) = 3$$

$$\frac{\tan\alpha - \tan 45°}{1+\tan\alpha \cdot \tan 45°} = 3$$

$$\frac{\tan\alpha - 1}{1+\tan\alpha \cdot 1} = 3$$

$$3+3\tan\alpha = \tan\alpha - 1$$

$$\tan\alpha = -2$$

Düşünün / Think about it

$k > 2, \ \alpha = \frac{2\pi}{k}$

$\Rightarrow 1+\cos^2\alpha + \ldots + \cos^2((k-1)\alpha) = ?$

Yanıt / Answer

k / 2

ÖZET / SUMMARY

✓ $\sin 2a = 2\sin a \cdot \cos a$

✓ $\cos 2a = \cos^2 a - \sin^2 a$
$= 2\cos^2 a - 1$
$= 1 - 2\sin^2 a$

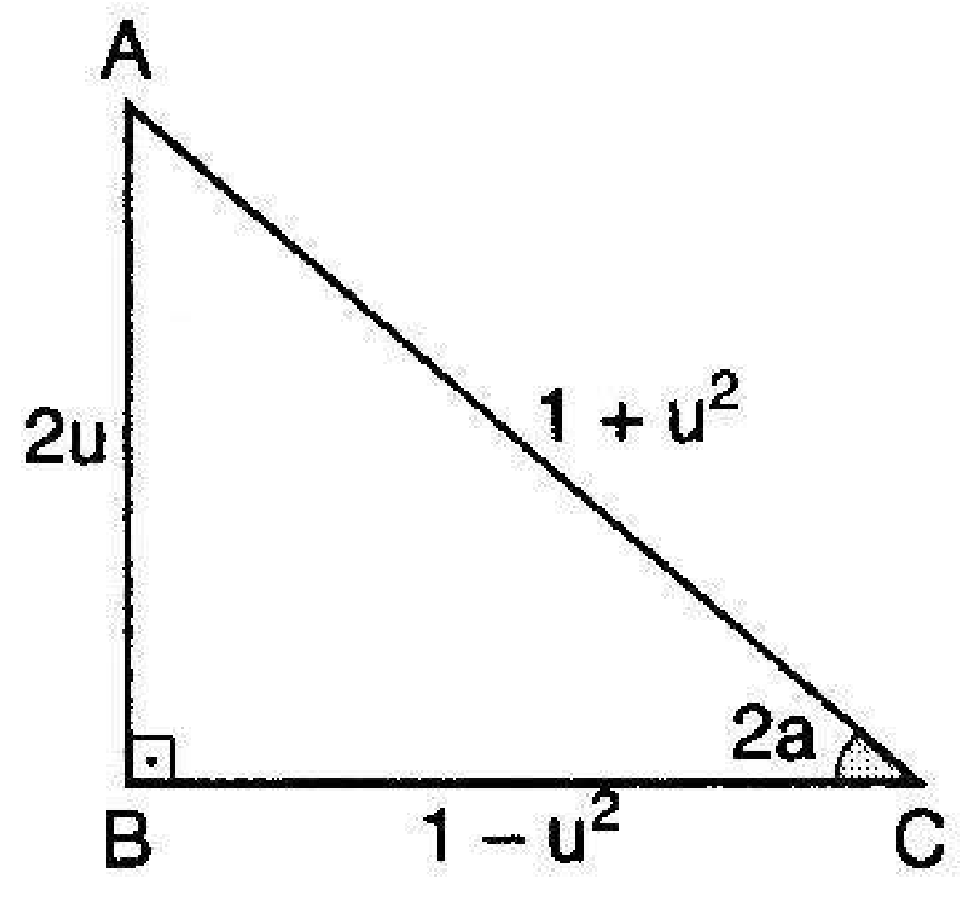
✓ $\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$

✓ $\cot 2a = \dfrac{\cot^2 a - 1}{2\cot a}$

KANIT / PROOF

$$\tan 2a = \frac{2\tan a}{1-\tan^2 a}$$

$$\tan a = u \Rightarrow \tan 2a = \frac{2u}{1-u^2}$$



$$\Rightarrow \sin 2a = \frac{2\tan a}{1+\tan^2 a}$$

$$\cos 2a = \frac{1-\tan^2 a}{1+\tan^2 a}$$

YARIM AÇI FORMÜLLERİ / HALF ANGLE FORMULAS

i. 1. toplam fark formulünde a = b alınırsa,

If we take a = b in the sum and difference formula – 1, we get

$\sin(a + a) = \sin a \cdot \cos a + \cos a \cdot \sin a$

$\sin 2a = 2\sin a \cdot \cos a$ elde edilir.

ii. 3. toplam fark formulünde a = b alınırsa,

$\cos(a + a) = \cos a \cdot \cos a - \sin a \cdot \sin a = \cos^2 a - \sin^2 a$ elde edilir.

$\cos^2 a = 1 - \sin^2 a$ olduğundan, $\cos 2a = 1 - \sin^2 a - \sin^2 a = 1 - 2\sin^2 a$ elde edilir veya

$\sin^2 a = 1 - \cos^2 a$ olduğundan, $\cos 2a = \cos^2 a - (1 - \cos^2 a) = 2\cos^2 a - 1$ elde edilir.

If we take a = b in the sum and difference formula – 3, we get

$\cos(a + a) = \cos a \cdot \cos a - \sin a \cdot \sin a = \cos^2 a - \sin^2 a$.

Since $\cos^2 a = 1 - \sin^2 a$ we get $\cos 2a = 1 - \sin^2 a - \sin^2 a = 1 - 2\sin^2 a$ or

since $\sin^2 a = 1 - \cos^2 a$, we get $\cos 2a = \cos^2 a - (1 - \cos^2 a) = 2\cos^2 a - 1$.

iii. 5. toplam fark formulünde a = b alınırsa,

If we take a = b in the sum and difference formula – 5, we get

$$\tan(a+a) = \frac{\tan a + \tan a}{1-\tan a \cdot \tan a} = \frac{2\tan a}{1-\tan^2 a}$$

iv. 7. toplam fark formulünde a = b alınırsa,

If we take a = b in the sum and difference formula – 7, we get

$$\cot(a+a) = \frac{\cot a \cdot \cot a - 1}{\cot a + \cot a} = \frac{\cot^2 a - 1}{2\cot a}$$

ÖRNEK / EXAMPLE 6

$\cos x = \dfrac{1}{4} \Rightarrow$ **$\cos 2x = ?$**

A) $-\dfrac{8}{9}$ B) $-\dfrac{7}{8}$ C) $\dfrac{7}{9}$ D) $-\dfrac{8}{11}$ E) $-\dfrac{6}{7}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$\cos 2x = 2\cdot\cos^2 x - 1$ [Yarım açı formülü / *Half – open formula*]

$= 2\cdot\left(\dfrac{1}{4}\right)^2 - 1$ [cosx yerine 1/4 yaz. / *Substitute 1/4 for cosx.*]

$= 2\cdot\dfrac{1}{16} - 1$ [İndirge. / *Reduce.*]

$= \dfrac{2}{16} - 1$ [Sadeleştir. / *Simplify.*]

$= \dfrac{1}{8} - 1 = -\dfrac{7}{8}$ [Sonuç / *Conclusion*]

ÖRNEK / EXAMPLE 7

$\sin x - \cos x = \frac{5}{13} \Rightarrow$ **$\sin 2x = ?$**

A) $\frac{251}{169}$ B) $\frac{48}{169}$ C) $\frac{108}{169}$ D) $\frac{144}{169}$ E) $\frac{145}{169}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\sin x - \cos x = \frac{5}{13}$ [Verilen eşitlik / *Given equality*]

$(\sin x - \cos x)^2 = \left(\frac{5}{13}\right)^2$ [Her iki tarafın karesini al. / *Square both sides.*]

$\sin^2 x - 2\sin x \cdot \cos x + \cos^2 x = \frac{25}{169}$ [Tam kare / *Perfect square*]

$\underbrace{\sin^2 x + \cos^2 x}_{1} - 2\sin x \cdot \cos x = \frac{25}{169}$ [Trigonometrik özdeşlik / *Trigonometric identity*]

$1 - \sin 2x = \frac{25}{169}$ [Yarım açı formülü / *Half – open formula*]

$1 - \frac{25}{169} = \sin 2x$ [Benzer terimleri topla. / *Combine like terms.*]

$\frac{144}{169} = \sin 2x$ [Sonuç / *Conclusion*]

ÖRNEK / EXAMPLE 8

$\sin 10° = m \Rightarrow$ **$\sin 70° = ?$**

A) $2m$ B) $m^2 - 1$ C) $2m^2 - 1$

D) $1 - m^2$ E) $1 - 2m^2$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{aligned} \sin 10° = m \Rightarrow \sin 70° &= \cos 20 \\ &= \cos(2 \cdot 10°) \\ &= 1 - 2 \cdot \sin^2 10° \\ &= 1 - 2m^2 \end{aligned}$$

PEKİŞTİRME / REINFORCE

$\sin x = t \Rightarrow \sin 2x = ?$

YANIT / ANSWER : $2t\sqrt{1 - t^2}$

ÖRNEK / EXAMPLE 9

$\cos 20° \cdot \cos 40° \cdot \cos 80° = ?$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{12}$ E) $\frac{1}{16}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{aligned}
\cos 20° \cdot \cos 40° \cdot \cos 80° &= \frac{2\cdot\sin 20°\cdot\cos 20°\cdot\cos 40°\cdot\cos 80°}{2\cdot\sin 20°}\\
&= \frac{\sin 40°\cdot\cos 40°\cdot\cos 80°}{2\cdot\sin 20°}\\
&= \frac{2\cdot\sin 40°\cdot\cos 40°\cdot\cos 80°}{2\cdot 2\cdot\sin 20°}\\
&= \frac{\sin 80°\cdot\cos 80°}{4\cdot\sin 20°}\\
&= \frac{2\sin 80°\cdot\cos 80°}{2\cdot 4\cdot\sin 20°}\\
&= \frac{\sin 160°}{8\cdot\sin 20°}\\
&= \frac{1}{8}
\end{aligned}$$

ÖRNEK / EXAMPLE 10

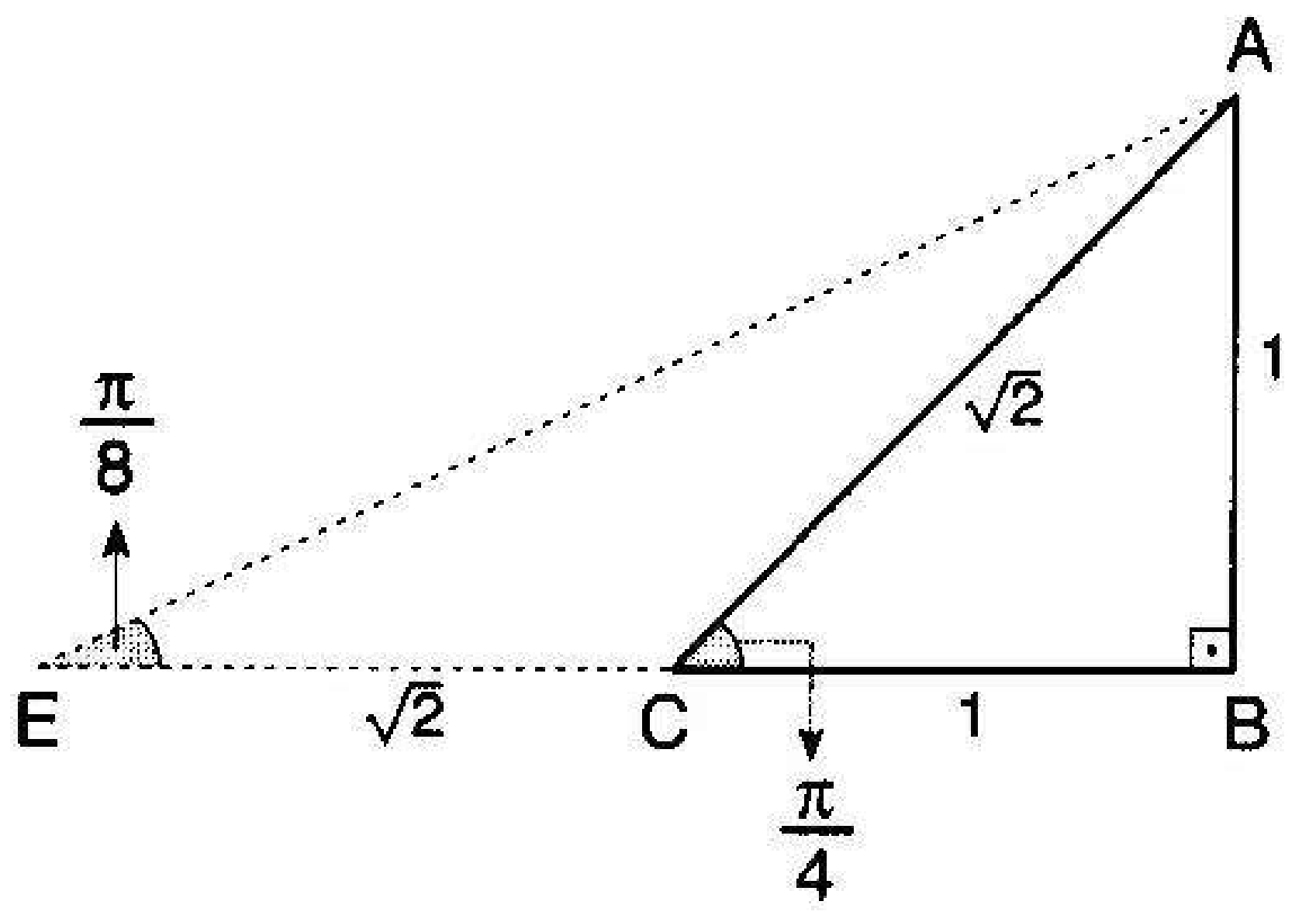
$\tan\frac{\pi}{8} = ?$

A) $\sqrt{2}+1$ B) $\sqrt{2}$ C) $\sqrt{2}-1$

D) $\sqrt{2}-2$ E) $2-\sqrt{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**



$$\tan\frac{\pi}{4} = 1 \Rightarrow \tan\frac{\pi}{8} = \frac{1}{\sqrt{2}+1}$$

$$= \sqrt{2}-1$$

DÖNÜŞÜM FORMÜLLERİ / CONVERSION FORMULAS

I. $\sin a + \sin b = 2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right)$

II. $\sin a - \sin b = 2 \cdot \sin\left(\frac{a-b}{2}\right) \cdot \cos\left(\frac{a+b}{2}\right)$

III. $\cos a + \cos b = 2 \cdot \cos\left(\frac{a-b}{2}\right) \cdot \cos\left(\frac{a+b}{2}\right)$

IV. $\cos a - \cos b = -2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \sin\left(\frac{a-b}{2}\right)$

V. $\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cdot \cos b}$

VI. $\tan a - \tan b = \frac{\sin(a-b)}{\cos a \cdot \cos b}$

VII. $\cot a + \cot b = \frac{\sin(a+b)}{\sin a \cdot \sin b}$

VIII. $\cot a - \cot b = \frac{-\sin(a-b)}{\sin a \cdot \sin b}$

ÖRNEK / EXAMPLE 11

$\cos 75° + \cos 15° = ?$

A) $\frac{\sqrt{3}}{2}$ B) 1 C) $\frac{\sqrt{5}}{2}$ D) $\frac{\sqrt{6}}{2}$ E) $\sqrt{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\cos a + \cos b = 2 \cdot \cos\left(\frac{a-b}{2}\right) \cdot \cos\left(\frac{a+b}{2}\right)$$

$$\cos 75° + \cos 15° = 2 \cdot \cos\frac{75°+15°}{2} \cdot \cos\frac{75°-15°}{2}$$

$$= 2 \cdot \cos 45° \cdot \cos 30°$$

$$= 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{6}}{2}$$

PEKİŞTİRME / REINFORCE

$\sin 75° + \sin 15° = ?$

YANIT / ANSWER : $\sqrt{6}/2$

PEKİŞTİRME / REINFORCE

$$\frac{\cos 15° + \sin 15°}{\cos 15° - \sin 15°} = ?$$

YANIT / ANSWER : $-\sqrt{3}$

PEKİŞTİRME / REINFORCE

$$2x = \frac{\pi}{2} \Rightarrow \frac{\cos 3x + \cos 5x}{\sin 3x - \sin 5x} = ?$$

YANIT / ANSWER : -1

ÖRNEK / EXAMPLE 12

$$\frac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°} = ?$$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2 D) $\sqrt{5}$ E) $\sqrt{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°} = \frac{2 \cdot \cos \frac{75° + 15°}{2} \cdot \cos \frac{75° - 15°}{2}}{2 \cdot \sin \frac{75° - 15°}{2} \cdot \cos \frac{75° + 15°}{2}}$$

$$= \frac{2 \cdot \cos 45° \cdot \cos 30°}{2 \cdot \sin 30° \cdot \cos 45°}$$

$$= \cot 30°$$

$$= \sqrt{3}$$

ÖRNEK / EXAMPLE 13

$$a = \frac{\pi}{24} \Rightarrow \frac{\cos 5a + \cos 7a}{\sin 5a + \sin 7a} = ?$$

A) $-\sqrt{2}$ B) -1 C) 1 D) $\sqrt{2}$ E) $\sqrt{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{\cos 5a + \cos 7a}{\sin 5a + \sin 7a} = \frac{2 \cdot \cos \frac{5a + 7a}{2} \cdot \cos \frac{7a - 5a}{2}}{2 \cdot \sin \frac{5a + 7a}{2} \cdot \cos \frac{7a - 5a}{2}}$$

$$= \frac{2 \cdot \cos 6a \cdot \cos a}{2 \cdot \sin 6a \cdot \cos a}$$

$$= \cot 6a$$

$$a = \frac{\pi}{24} \Rightarrow \cot 6a = \cot \frac{\pi}{4} = 1$$

ÖRNEK / EXAMPLE 14

$$\frac{\cos 5°+\cos 20°+\cos 35°}{\sin 5°+\sin 20°+\sin 35°}=?$$

A) sin20° B) cos20° C) tan20° D) cot20° E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

Çözüm / *Solution* 1:

$$\frac{\cos 5°+\cos 20°+\cos 35°}{\sin 5°+\sin 20°+\sin 35°}$$

$$=\frac{\cos 5°+\cos 35°+\cos 20°}{\sin 5°+\sin 35°+\sin 20°}$$

$$=\frac{2\cdot\cos 15°\cdot\cos 20°+\cos 20°}{2\cdot\sin 20°\cdot\cos 15°+\sin 20°}$$

$$=\frac{\cos 20°(2\cos 15°+1)}{\sin 20°(2\cos 15°+1)}$$

$$=\cot 20°$$

Çözüm / *Solution* 2:

$$\frac{5+35}{2}=20 \Rightarrow$$

$$\frac{\cos 5°+\cos 20°+\cos 35°}{\sin 5°+\sin 20°+\sin 35°}=\cot 20°$$

ÖRNEK / EXAMPLE 15

$$3a=\pi \Rightarrow \frac{\sin a+\sin 2a+\sin 3a}{1+\cos a+\cos 2a}=?$$

A) $-\sqrt{3}$ B) 0 C) $\sqrt{3}$ D) $\frac{1}{2}$ E) $\frac{1}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\sin a + \sin 3a = 2\cdot\sin 2a\cdot\cos a$

$\cos 2a + 1 = \cos 2a + \cos 0° = 2\cdot\cos^2 a$

$$\Rightarrow \frac{\sin a+\sin 2a+\sin 3a}{1+\cos a+\cos 2a}=\frac{2\sin 2a\cdot\cos a+\sin 2a}{2\cos^2 a+\cos a}$$

$$=\frac{\sin 2a(2\cos a+1)}{\cos a(2\cos a+1)}$$

$$=\frac{\sin 2a}{\cos a}=2\sin a=\sqrt{3}$$

NOT / NOTE

$$\frac{x+y}{2}=z \Rightarrow$$

$$\frac{\sin x+\sin z+\sin y}{\cos x+\cos z+\cos y}=\tan z$$

TERS DöNÜŞÜM FORMÜLLERİ / INVERSE CONVERSION FORMULAS

I. $\sin a \cdot \sin b = -\frac{1}{2}\,[\cos(a+b)-\cos(a-b)]$

II. $\cos a \cdot \cos b = \frac{1}{2}\,[\cos(a+b)+\cos(a-b)]$

III. $\sin a \cdot \cos b = \frac{1}{2}\,[\sin(a+b)+\sin(a-b)]$

IV. $\cos a \cdot \sin b = \frac{1}{2}\,[\sin(a+b)-\sin(a-b)]$

ÖRNEK / EXAMPLE 16

$\cos\frac{5\pi}{12} \cdot \cos\frac{\pi}{12} = ?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{2}}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\cos\frac{5\pi}{12}\cdot\cos\frac{\pi}{12} = \frac{1}{2}\left[\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{\pi}{2}+\cos\frac{\pi}{3}\right]$$

$$= \frac{1}{2}\left[0+\frac{1}{2}\right] = \frac{1}{4}$$

ÖRNEK / EXAMPLE 17

$2\cdot\cos 40° \cdot \cos 80° + \cos 140° = ?$

A) 1 B) $\frac{1}{2}$ C) $-\frac{1}{2}$ D) $-\frac{\sqrt{3}}{2}$ E) -1

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$2\cdot\cos 40°\cdot\cos 80° + \cos 140° = 2\cdot\frac{1}{2}[\cos(40°+80°)+\cos(40°-80°)]+\cos 140°$$

$$= \cos 120° + \cos 40° - \cos 40°$$

$$= \cos 120° = -\frac{1}{2}$$

TERS TRİGONOMETRİK FONKSİYONLAR
INVERSE TRIGONOMETRIC FUNCTIONS

1. $y = \arcsin x = \sin^{-1} x \Leftrightarrow x = \sin y$

   $x \in [-1,1]$ ve (and) $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. $y = \arccos x = \cos^{-1} x \Rightarrow x = \cos y$

   $x \in [-1,1]$ ve (and) $y \in [0, \pi]$

3. $y = \arctan x = \tan^{-1} x \Leftrightarrow x = \tan y$

   $x \in R$ ve (and) $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4. $y = \text{arccot}\, x = \cot^{-1} x \Rightarrow x = \cot y$

   $x \in R$ ve (and) $y \in [0, \pi]$

ÖRNEK / EXAMPLE 18

$\arcsin \frac{1}{2} = ?$

A) $\frac{\pi}{12}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{5}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$\arcsin \frac{1}{2} = \theta$

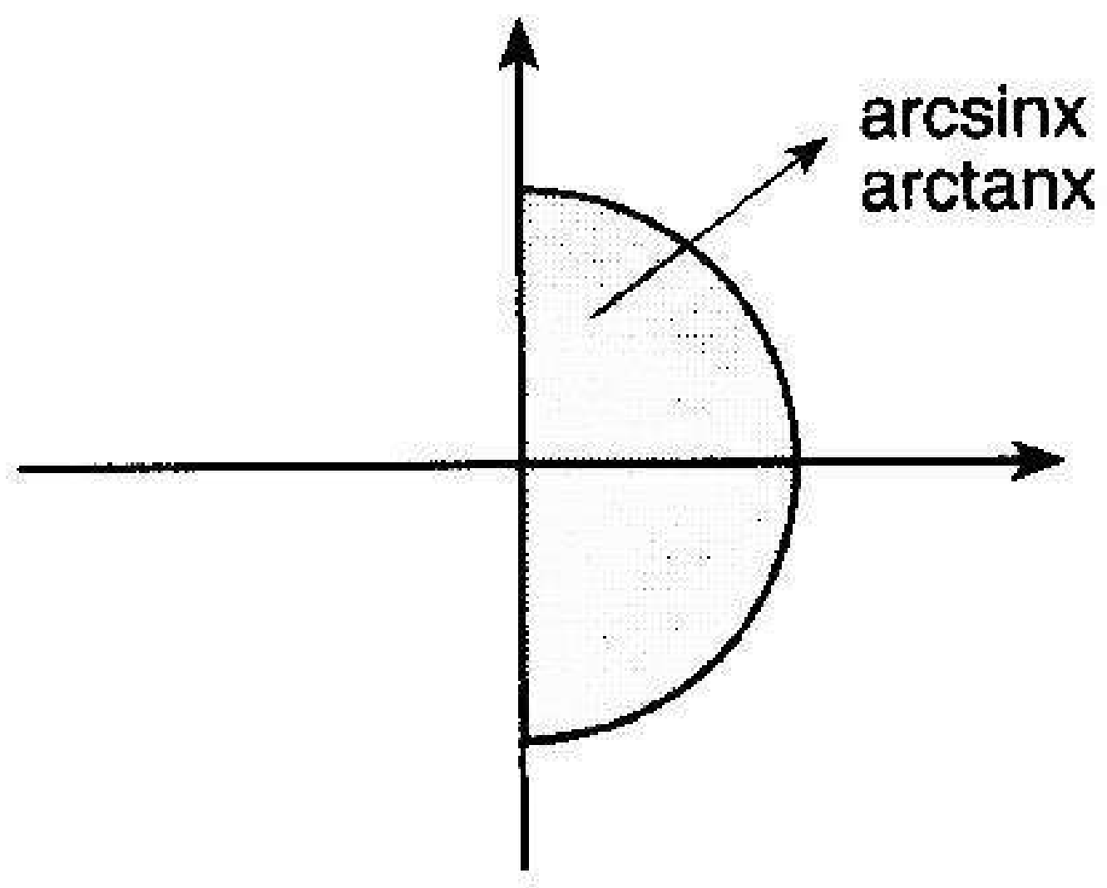
$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

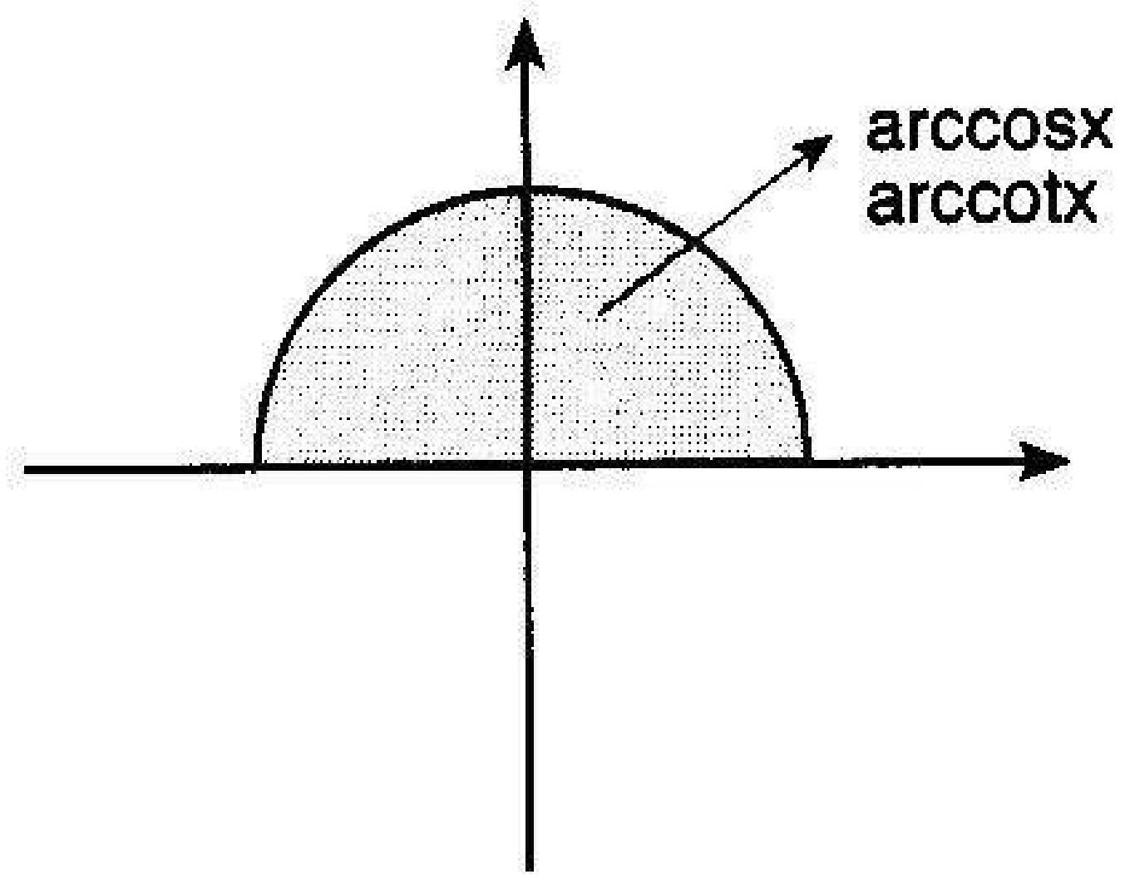
NOT / NOTE

✓ I. $\arcsin(\sin x) = x$
II. $\arccos(\cos x) = x$
III. $\arctan(\tan x) = x$
IV. $\text{arccot}(\cot x) = x$

✓ I. $\sin(\arcsin x) = x$
II. $\cos(\arccos x) = x$
III. $\tan(\arctan x) = x$
IV. $\cot(\text{arccot}\, x) = x$

NOT / NOTE





NOT / NOTE

Fonksiyon *Function*	Tanım kümesi *Domain*	Görüntü kümesi *Range*
arcsinx	[–1, 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
arccosx	[–1, 1]	$[0, \pi]$
arctanx	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
arccotx	R	$[0, \pi]$

PEKİŞTİRME / REINFORCE

cot(arccot2) = ?

YANIT / ANSWER : 2

ÖRNEK / EXAMPLE 19

$\sin\left(\arctan\frac{1}{3}\right) = ?$

A) $\frac{\sqrt{10}}{3}$ B) $\frac{\sqrt{10}}{5}$ C) $\frac{\sqrt{10}}{6}$ D) $\frac{\sqrt{10}}{8}$ E) $\frac{\sqrt{10}}{10}$

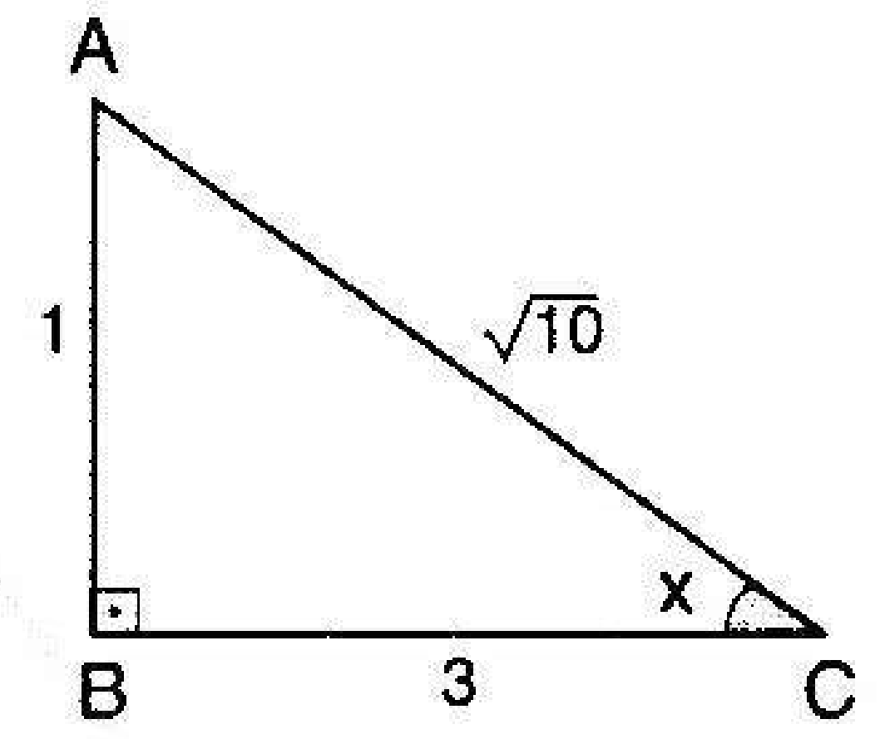
ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\arctan\frac{1}{3} = x \Rightarrow \tan x = \frac{1}{3}$

$\sin\left(\arctan\frac{1}{3}\right) = \sin x$

$\tan x = \frac{1}{3} \Rightarrow \sin x = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$



ÖRNEK / EXAMPLE 20

sin(2arctan2) = ?

A) $\frac{1}{6}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) $\frac{4}{7}$

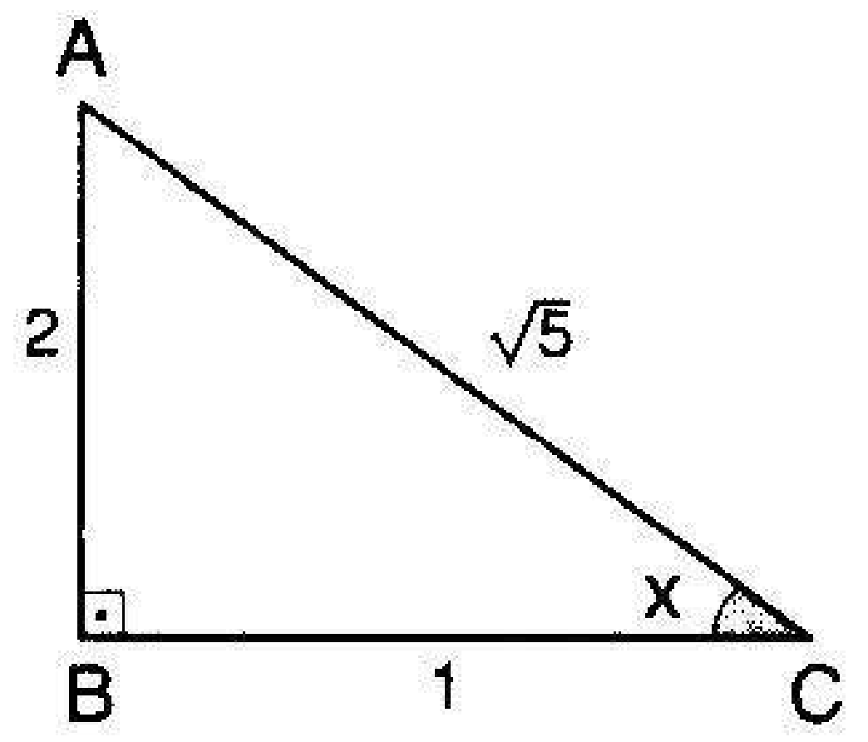
ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\arctan 2 = x \Rightarrow \tan x = 2$

$\tan x = 2 \Rightarrow \sin x = \frac{2}{\sqrt{5}}, \quad \cos x = \frac{1}{\sqrt{5}}$

$$\begin{aligned}\sin(2\arctan 2) &= \sin(2x)\\ &= 2\cdot\sin x\cdot\cos x\\ &= 2\cdot\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\\ &= \frac{4}{5}\end{aligned}$$

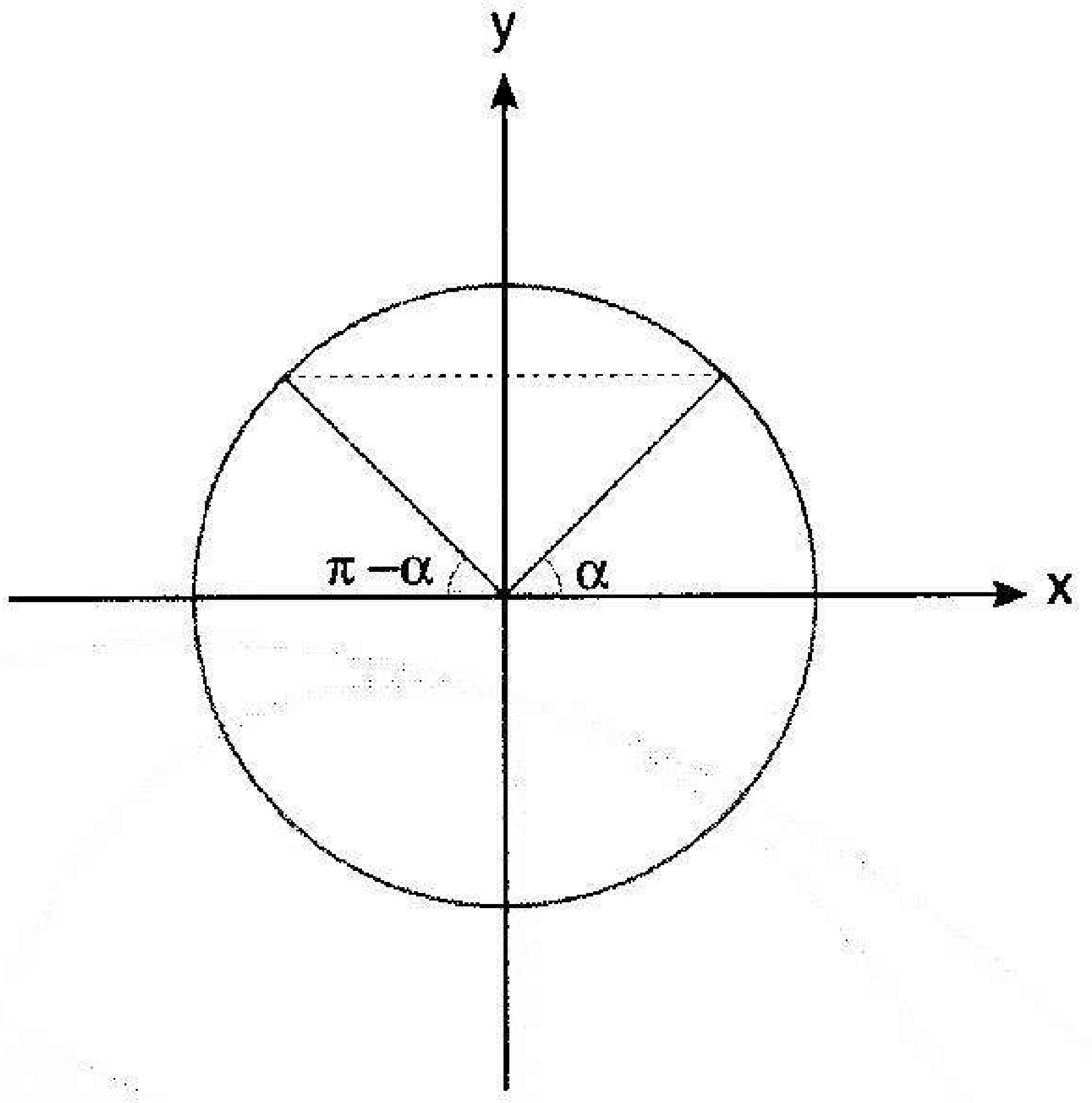


TRİGONOMETRİK DENKLEMLER / TRIGONOMETRIC EQUATIONS

1. sinx = sinα Denkleminin Çözümü / Solution of the Equation sinx = sinα

$$\left.\begin{aligned} \sin x &= \sin\alpha \\ \sin x &= \sin(\pi-\alpha) \end{aligned}\right\} \Rightarrow x_1 = \alpha \ , \ x_2 = \pi - \alpha$$

$$\sin x = \sin\alpha \Rightarrow \begin{cases} x_1 = \alpha + 2k\pi & , \ k \in Z \\ x_2 = (\pi-\alpha) + 2k\pi \ , & k \in Z \end{cases}$$



ÖRNEK / EXAMPLE 21

$\sin x = \dfrac{\sqrt{3}}{2} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x: x = \dfrac{\pi}{3} + 2k\pi \ \vee \ x = \dfrac{2\pi}{3} + 2k\pi\right\}$

B) $\left\{x: x = \dfrac{\pi}{3} + 2k\pi \ \vee \ x = \dfrac{5\pi}{6} + 2k\pi\right\}$

C) $\left\{x: x = \dfrac{\pi}{6} + 2k\pi \ \vee \ x = \dfrac{5\pi}{6} + 2k\pi\right\}$

D) $\left\{x: x = \dfrac{\pi}{6} + 2k\pi \ \vee \ x = \dfrac{2\pi}{3} + 2k\pi\right\}$

E) $\left\{x: x = \dfrac{\pi}{3} + 2k\pi \ \vee \ x = \dfrac{4\pi}{3} + 2k\pi\right\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\sin x = \frac{\sqrt{3}}{2} \Rightarrow \sin x = \sin\frac{\pi}{3}$$

$$\left.\begin{aligned} \sin x &= \sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} &= \sin\frac{2\pi}{3} \end{aligned}\right\} \Rightarrow x_1 = \frac{\pi}{3} + 2k\pi \ , \ x_2 = \frac{2\pi}{3} + 2k\pi$$

$$ÇK(SS) = \left\{x: x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$$

NOT / NOTE

$k \in Z$

✓ $\sin x = 1$

$\Rightarrow x = \dfrac{\pi}{2} + 2k\pi$

✓ $\sin x = -1$

$\Rightarrow x = \dfrac{3\pi}{2} + 2k\pi$

✓ $\sin x = 0 \Rightarrow x = k\pi$

NOT / NOTE

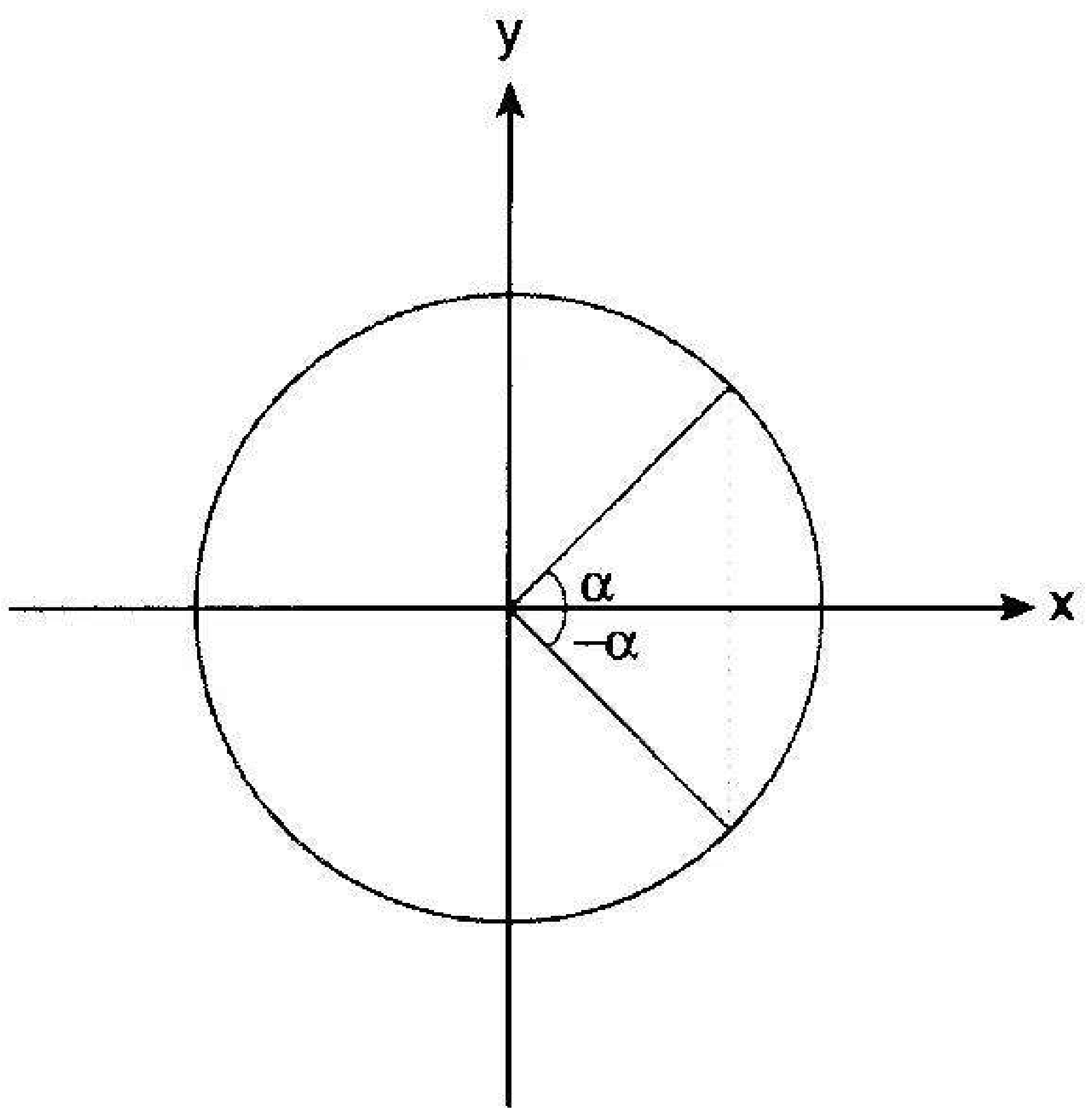
$k \in Z$

- ✓ $\cos x = 1 \Rightarrow x = 2k\pi$
- ✓ $\cos x = -1 \Rightarrow x = \pi + 2k\pi$
- ✓ $\cos x = 0$

  $\Rightarrow x = \frac{\pi}{2} + k\pi$

2. cosx = cosα Denkleminin Çözümü / Solution of the Equation cosx = cosα

$$\left.\begin{array}{l}\cos x = \cos\alpha \\ \cos x = \cos(-\alpha)\end{array}\right\} \Rightarrow x_1 = \alpha \quad , \quad x_2 = -\alpha$$



$$\cos x = \cos\alpha \Rightarrow \begin{cases} x_1 = \alpha + 2k\pi & , \ k \in Z \\ x_2 = -\alpha + 2k\pi & , \ k \in Z \end{cases}$$

ÖRNEK / EXAMPLE 22

$\cos x = \frac{1}{2} \Rightarrow$ ÇK(SS) = ?

A) $\left\{x : x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{2\pi}{3} + 2k\pi\right\}$

B) $\left\{x : x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{4\pi}{3} + 2k\pi\right\}$

C) $\left\{x : x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{5\pi}{3} + 2k\pi\right\}$

D) $\left\{x : x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{11\pi}{6} + 2k\pi\right\}$

E) $\left\{x : x = \frac{\pi}{6} + 2k\pi \ \vee \ x = \frac{5\pi}{6} + 2k\pi\right\}$

ÇÖZÜM / SOLUTION

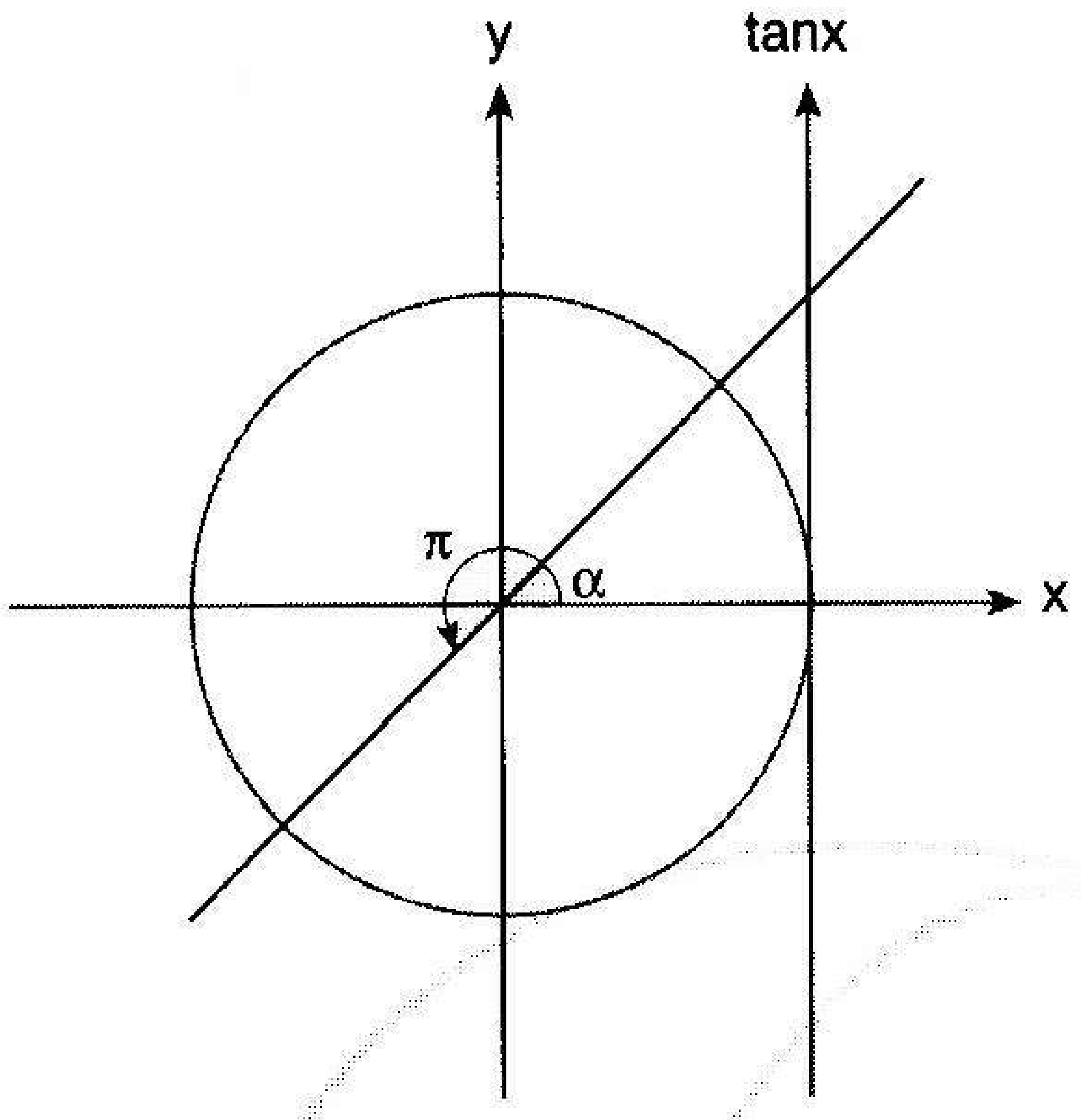
Yanıt / Answer **C**

$$\cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3}$$

$$\left.\begin{array}{l}\cos x = \cos\frac{\pi}{3} \\ \cos\frac{\pi}{3} = \cos\frac{5\pi}{3}\end{array}\right\} \Rightarrow x_1 = \frac{\pi}{3} + 2k\pi \quad , \quad x_2 = \frac{5\pi}{3} + 2k\pi$$

$$\text{ÇK(SS)} = \left\{x : x = \frac{\pi}{3} + 2k\pi \ \vee \ x = \frac{5\pi}{3} + 2k\pi\right\}$$

3. tanx = tanα Denkleminin Çözümü / Solution of the Equation tanx = tanα



$\tan x = \tan\alpha \Rightarrow x = \alpha + k\pi$

ÖRNEK / EXAMPLE 23

$\tan(2x - 60°) = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x: x = \frac{7\pi}{6} + k\pi \ , \ k \in Z\right\}$

B) $\left\{x: x = \frac{7\pi}{12} + \frac{k\pi}{2} \ , \ k \in Z\right\}$

C) $\left\{x: x = \frac{7\pi}{12} + k\pi \ , \ k \in Z\right\}$

D) $\left\{x: x = \frac{7\pi}{24} + \frac{k\pi}{2} \ , \ k \in Z\right\}$

E) $\left\{x: x = \frac{7\pi}{24} + k\pi \ , \ k \in Z\right\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\tan(2x - 60°) = 1$

$\tan(2x - 60°) = \tan 45°$

$2x - 60° = 45° + k\pi$

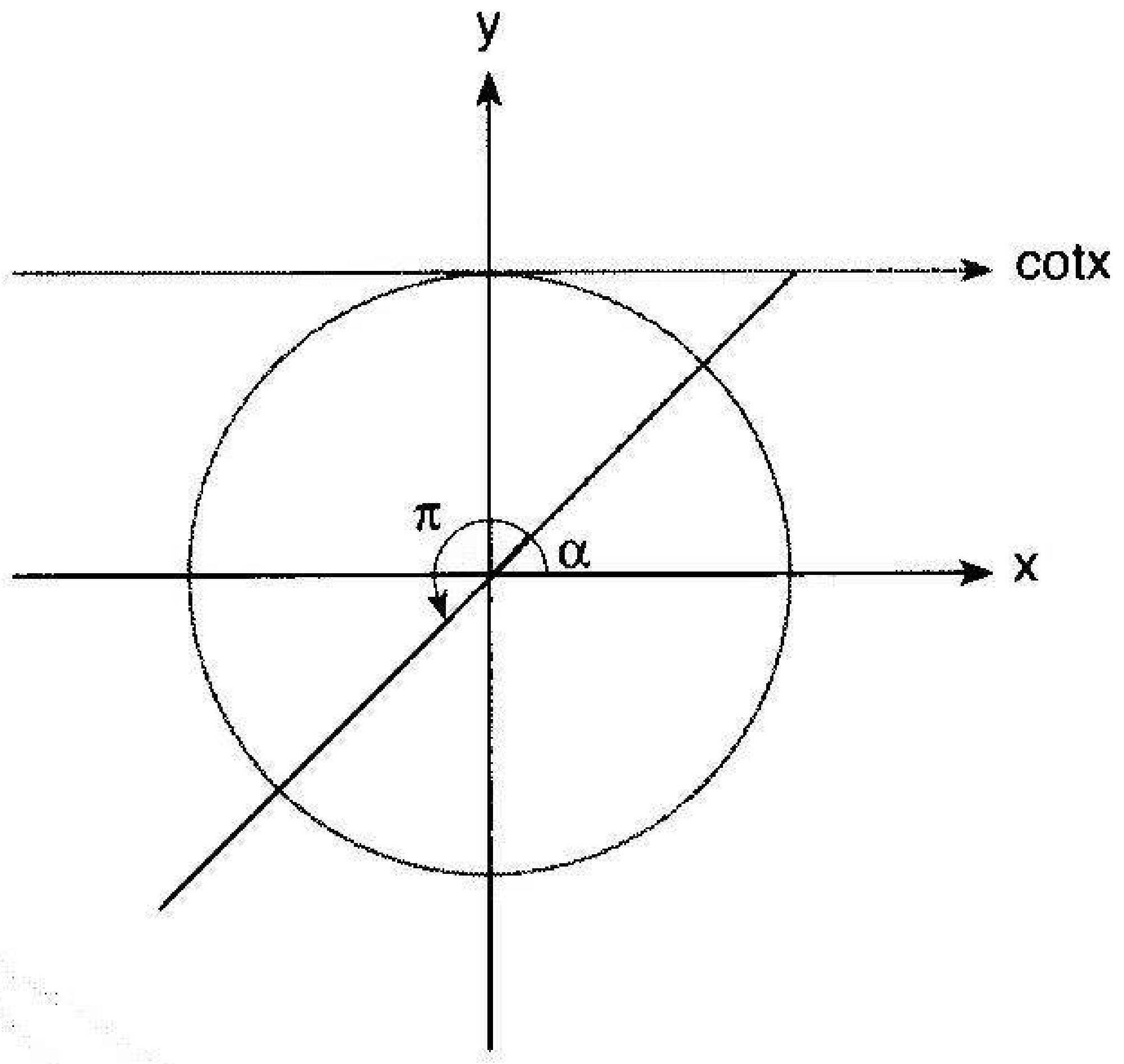
$2x - \frac{\pi}{3} = \frac{\pi}{4} + k\pi$

$2x = \frac{\pi}{3} + \frac{\pi}{4} + k\pi$

$2x = \frac{7\pi}{12} + k\pi$

$x = \frac{7\pi}{24} + \frac{k\pi}{2}$

4. cotx = cotα Denkleminin Çözümü / Solution of the Equation cotx = cotα



$\cot x = \cot\alpha \Rightarrow x = \alpha + k\pi$

ÖRNEK / EXAMPLE 24

$\cot(3x - 10°) = \cot x \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x : x = \frac{\pi}{36} + k\pi \ , \ k \in Z\right\}$

B) $\left\{x : x = \frac{\pi}{36} + \frac{k\pi}{2} \ , \ k \in Z\right\}$

C) $\left\{x : x = \frac{\pi}{18} + k\pi \ , \ k \in Z\right\}$

D) $\left\{x : x = \frac{\pi}{18} + \frac{k\pi}{2} \ , \ k \in Z\right\}$

E) $\left\{x : x = \frac{\pi}{18} + \frac{k\pi}{4} \ , \ k \in Z\right\}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$\cot(3x - 10°) = \cot x$

$3x - 10° = x + 180°\cdot k$

$2x = 10° + 180°\cdot k$

$x = 5° + 90°\cdot k$

$\Rightarrow \text{ÇK(SS)} = \left\{x : x = \frac{\pi}{36} + \frac{k\pi}{2} \ , \ k \in Z\right\}$

ÖRNEK / EXAMPLE 25

YILDIZ TEKNİK ÜNİ. 2012 YÖS

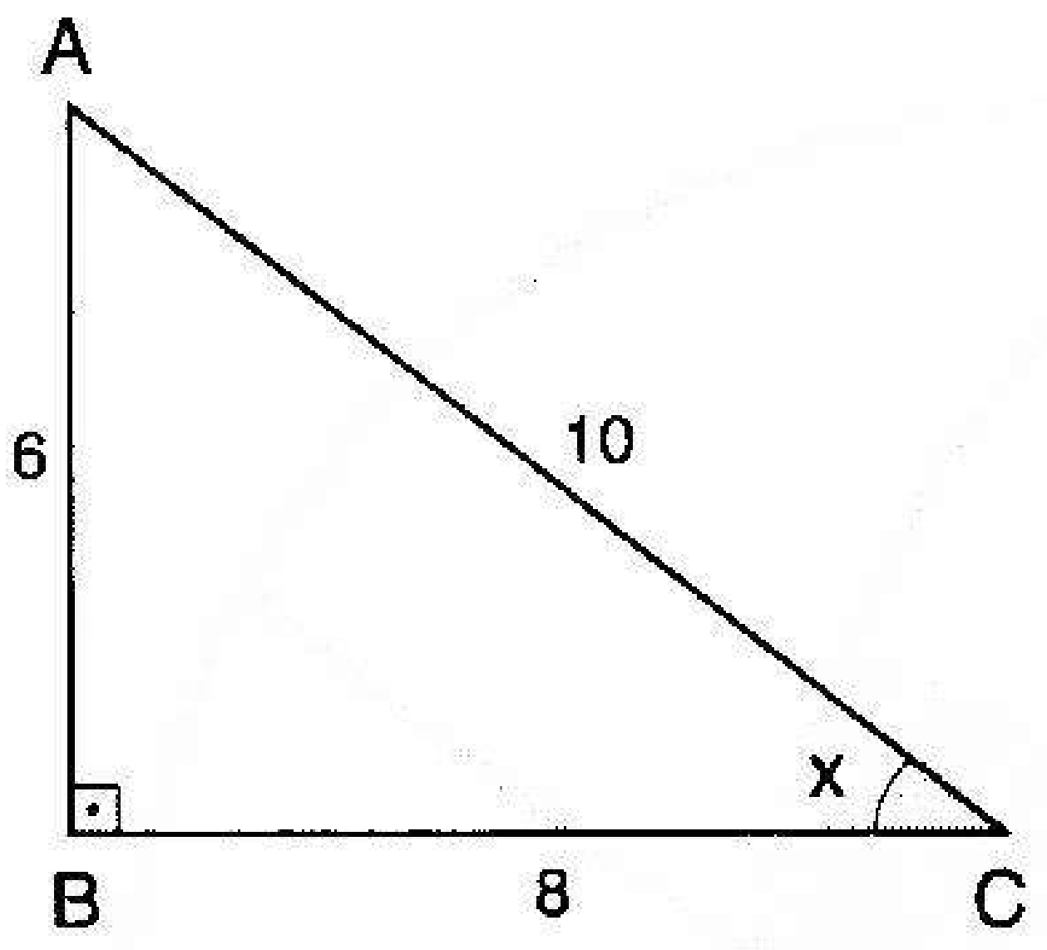
$\sin x = 0{,}6 \Rightarrow \sin 2x = ?$

A) $\frac{25}{24}$ B) $\frac{4}{3}$ C) $\frac{6}{5}$ D) $\frac{3}{4}$ E) $\frac{24}{25}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{aligned}\sin 2x &= 2\sin x\cdot\cos x\\ &= 2\cdot\frac{6}{10}\cdot\frac{8}{10}\\ &= \frac{24}{25}\end{aligned}$$



ÖRNEK / EXAMPLE 26

ANKARA ÜNİ. 2013 YÖS

$\frac{1}{\tan x}\cdot\frac{\cos 2x-1}{\sin 2x} = ?$

A) $-\sin x$ B) $-\cos x$ C) -1 D) $\sin x$ E) $\frac{1}{\cos x}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{aligned}\frac{1}{\tan x}\cdot\frac{\cos 2x-1}{\sin 2x} &= \frac{1}{\frac{\sin x}{\cos x}}\cdot\frac{1-2\sin^2 x-1}{2\sin x\cdot\cos x}\\ &= \frac{\cos x}{\sin x}\cdot\frac{-2\sin^2 x}{2\sin x\cdot\cos x} = -1\end{aligned}$$

ÖRNEK / EXAMPLE 27

İSTANBUL ÜNİ. 2013 YÖS

$\tan(x + 45°) = 2$

$\Rightarrow$ **sinx = ?**

A) $\frac{1}{\sqrt{10}}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{\sqrt{10}}$ E) $\frac{3}{\sqrt{10}}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\tan(x+45°) = \frac{\tan x + \tan 45°}{1 - \tan x \cdot \tan 45°}$$

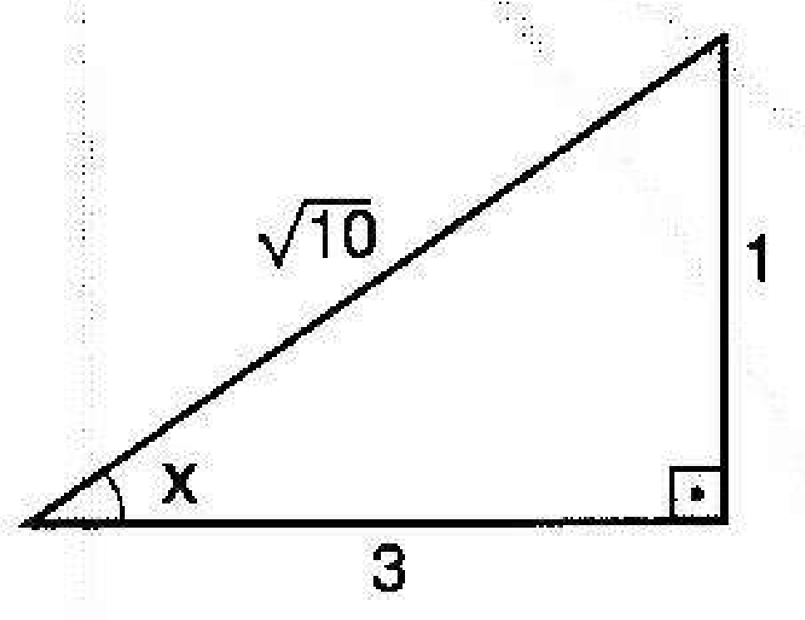
$\tan x = t \Rightarrow$

$t + 1 = 2 - 2t$

$3t = 1$

$t = \frac{1}{3}$

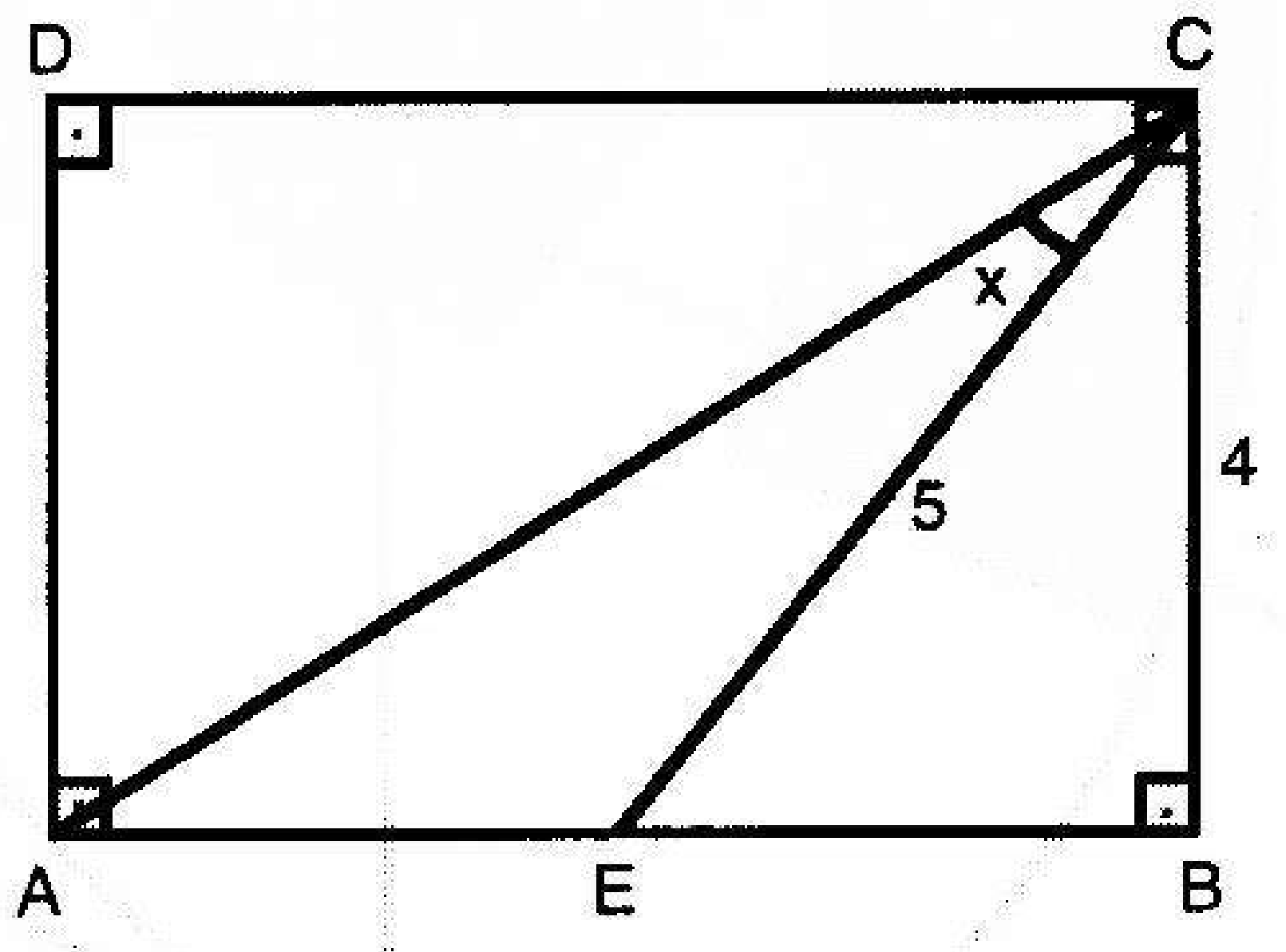
$\tan x = \frac{1}{3}$



$\Rightarrow \sin x = \frac{1}{\sqrt{10}}$

ÖRNEK / EXAMPLE 28

EGE ÜNİV. YÖS 2015



ABCD bir dikdörtgendir. / *ABCD is a rectangle.*

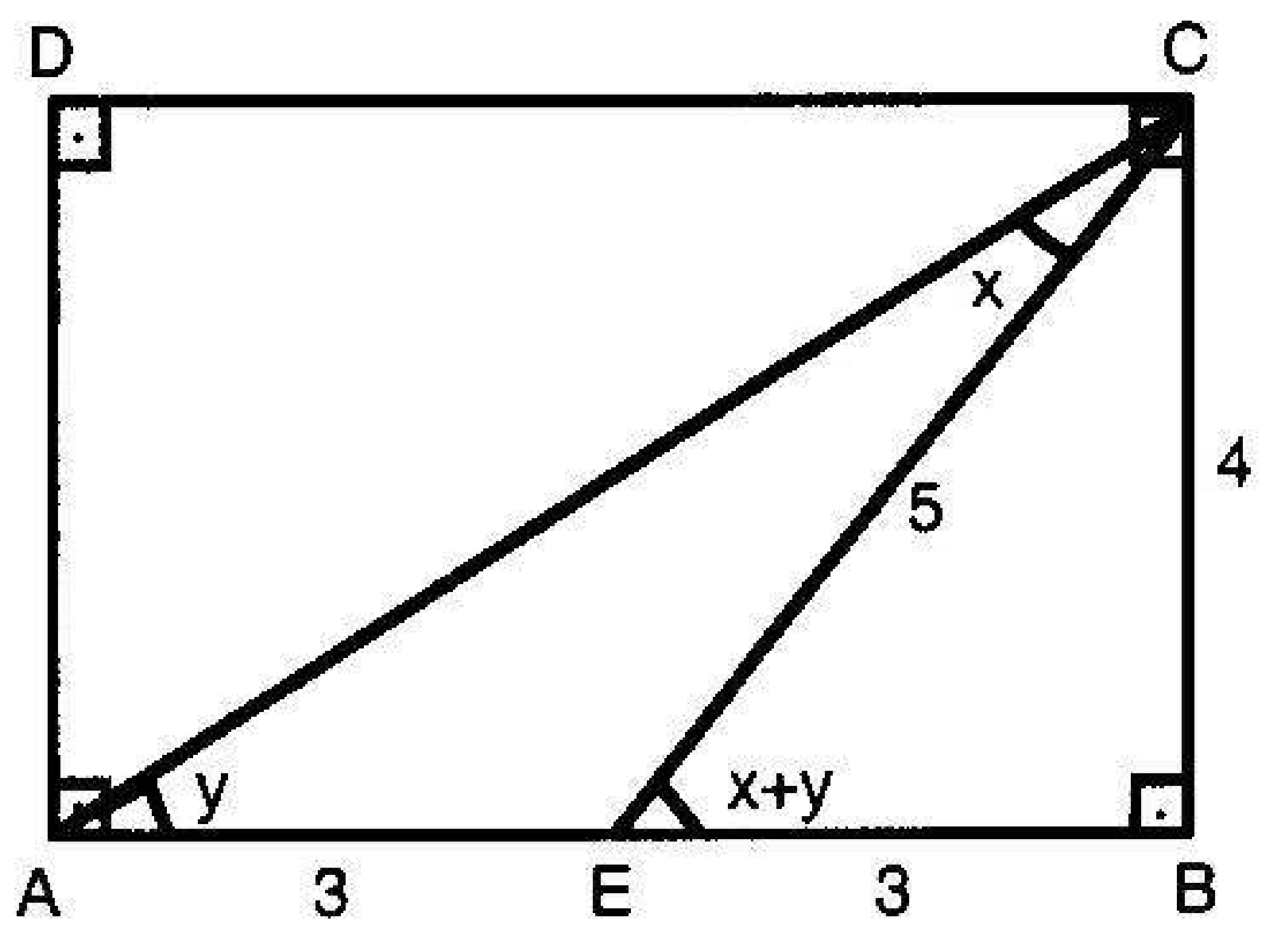
$|BC| = 4$ cm, $|EC| = 5$ cm,

$|AE| = |EB|$ $\Rightarrow$ **tanx = ?**

A) $\frac{6}{17}$ B) $\frac{5}{17}$ C) $\frac{3}{17}$ D) $\frac{2}{17}$ E) $\frac{1}{17}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**



$|EB|^2 + 4^2 = 5^2 \Rightarrow |EB| = 3$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\frac{4}{3} = \frac{\tan x + \frac{4}{6}}{1 - \tan x \cdot \frac{4}{6}} \Rightarrow \tan x = \frac{6}{17}$$

ÖRNEK / EXAMPLE 29

KARADENİZ TEKNİK ÜNİV. 2016 YÖS

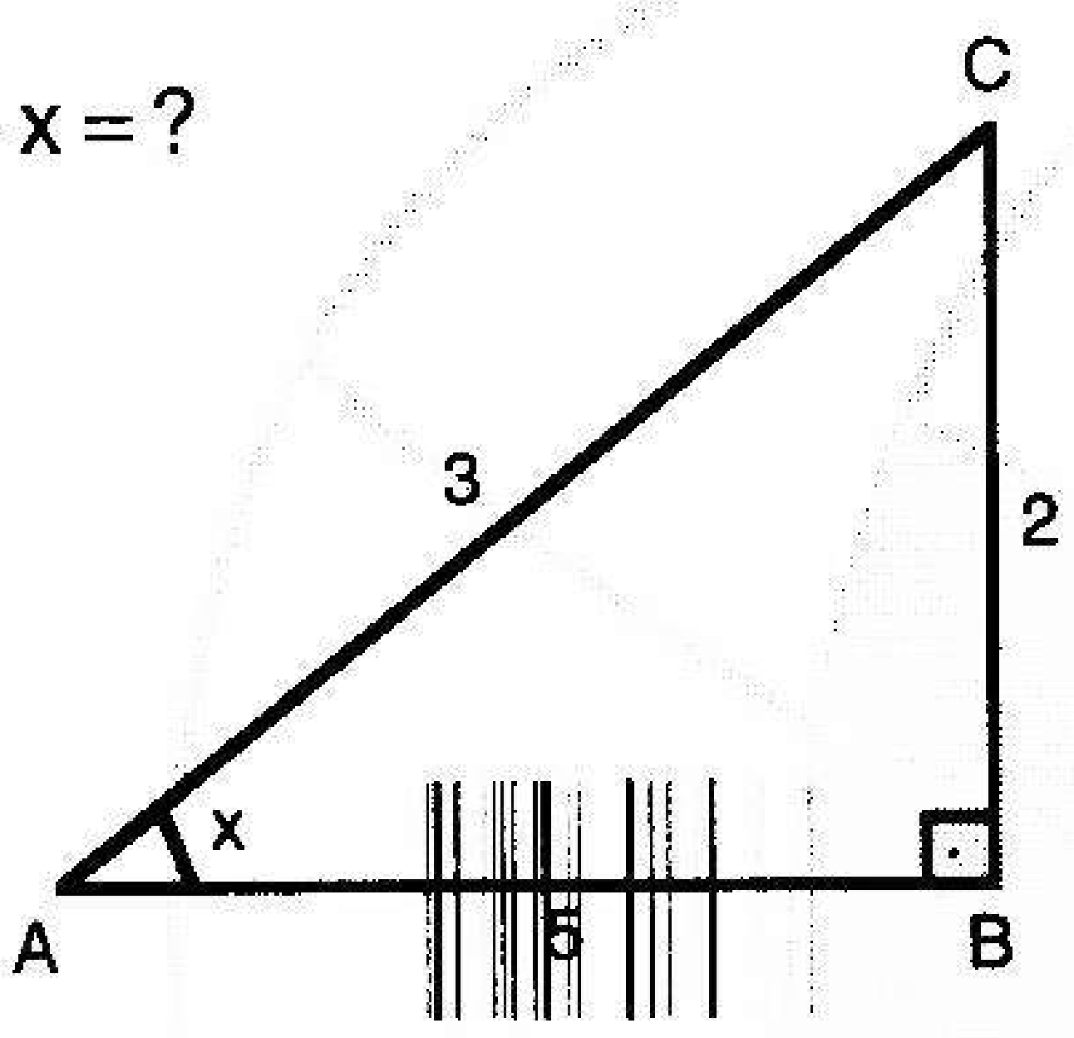
$\cos\left(\arcsin\dfrac{2}{3}\right)=?$

A) $\dfrac{1}{3}$ B) $\dfrac{2}{3}$ C) $\dfrac{\sqrt{2}}{3}$ D) $\dfrac{\sqrt{5}}{3}$ E) $\dfrac{\sqrt{5}}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\underbrace{\cos\left(\arcsin\dfrac{2}{3}\right)}_{x} \Rightarrow \cos x=?$



$\arcsin\dfrac{2}{3}=x$

$\sin x=\dfrac{2}{3}$

$3^2=2^2+|AB|^2$

$|AB|^2=9-4=5$

$\cos x=\dfrac{\sqrt{5}}{3}$

ÖRNEK / EXAMPLE 30

MARMARA ÜNİV. YÖS 2016

$\left(\sin\dfrac{\pi}{12}-\cos\dfrac{\pi}{12}\right)^2=?$

A) -1 B) 0 C) $\dfrac{1}{2}$ D) $\dfrac{\sqrt{2}}{2}$ E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)^2$$

$$=\sin^2\frac{\pi}{12}-2\sin\frac{\pi}{12}\cdot\cos\frac{\pi}{12}+\cos^2\frac{\pi}{12}$$

$$=1-\sin\frac{\pi}{6}=1-\frac{1}{2}=\frac{1}{2}$$

ÖRNEK / EXAMPLE 31 SÜLEYMAN DEMİREL ÜNİV. YÖS 2016

$\arccos(\log_5 3x) = \frac{\pi}{3} \Rightarrow x = ?$

A) $\sqrt{5}$ B) $\frac{1}{2}$ C) 1 D) $\frac{\sqrt{5}}{3}$ E) $\frac{5}{3}$

ÇÖZÜM / SOLUTION Yanıt / Answer D

$$\arccos(\log_5 3x) = \frac{\pi}{3}$$

$$\cos\frac{\pi}{3} = \log_5 3x$$

$$\frac{1}{2} = \log_5 3x$$

$$3x = 5^{\frac{1}{2}}$$

$$3x = \sqrt{5} \Rightarrow x = \frac{\sqrt{5}}{3}$$

ÖRNEK / EXAMPLE 32 ANADOLU ÜNİV. YÖS 2016

$0 < x < 90°$

$\sin(2x + 20°) = \cos 10° \Rightarrow \sum x = ?$

A) 50° B) 60° C) 70° D) 80° E) 90°

ÇÖZÜM / SOLUTION Yanıt / Answer C

$\sin(2x + 20°) = \cos 10°$

$\sin(2x + 20°) = \sin 80°$

I. $2x + 20° = k \cdot 360° + 80, \quad (0 < x < 90°)$

$k = 0 \Rightarrow 2x + 20° = 80°$

$2x = 60° \Rightarrow x = 30°$

II. $2x + 20° = (180° - 80°) + k \cdot 360°$

$k = 0 \Rightarrow 2x + 20° = 100°$

$2x = 80° \Rightarrow x = 40°$

$\sum x = 70°$

1. **$\tan 15° = ?$**

A) $1 + \sqrt{3}$

B) $2 - \sqrt{3}$

C) $1 - \sqrt{3}$

D) $2 + \sqrt{3}$

E) $\frac{2-\sqrt{3}}{2}$

2. **$\cot 105° = ?$**

A) $2 + \sqrt{3}$

B) $2 - \sqrt{3}$

C) $-(2 - \sqrt{3})$

D) $\sqrt{3} + 1$

E) $-\sqrt{3} + 1$

3. **$\cos 165° = ?$**

A) $\frac{-\sqrt{6}+\sqrt{2}}{2}$

B) $\frac{-\sqrt{6}-\sqrt{2}}{4}$

C) $\frac{\sqrt{6}-\sqrt{2}}{4}$

D) $\frac{\sqrt{6}+\sqrt{2}}{4}$

E) $\frac{\sqrt{3}-\sqrt{2}}{2}$

4. $\frac{\pi}{2} < x < \frac{3\pi}{4}$, $\tan x = -3 \Rightarrow$ **$\cos 2x = ?$**

A) $-\frac{2}{5}$

B) $-\frac{3}{5}$

C) $-\frac{4}{5}$

D) $-\frac{9}{10}$

E) $-\frac{14}{15}$

5. $0 < x < \frac{\pi}{2}$, $\sin x = 0{,}8 \Rightarrow$ **$\cos 2x - \sin 2x = ?$**

A) $-\frac{31}{25}$

B) $-\frac{27}{25}$

C) -1

D) $-\frac{17}{25}$

E) $-\frac{12}{15}$

6. ABCD bir dikdörtgendir.

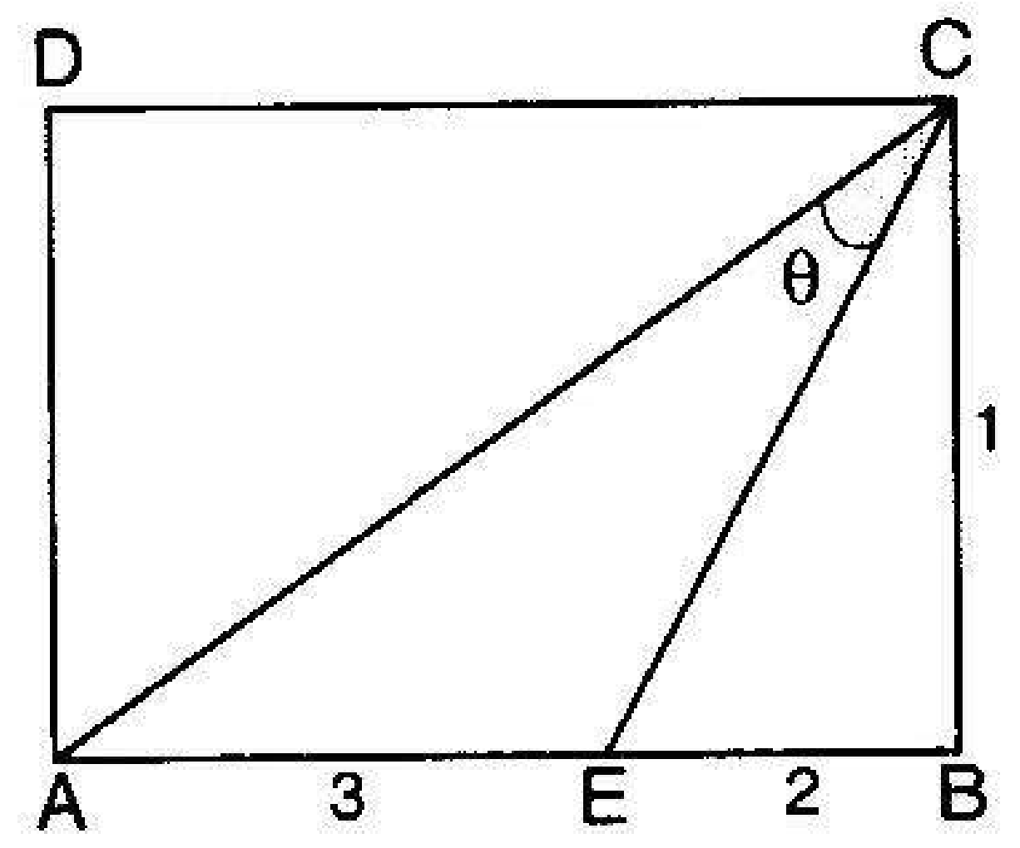
ABCD is a rectangle.

$|AE| = 3$ cm,

$|EB| = 2$ cm,

$|BC| = 1$ cm.

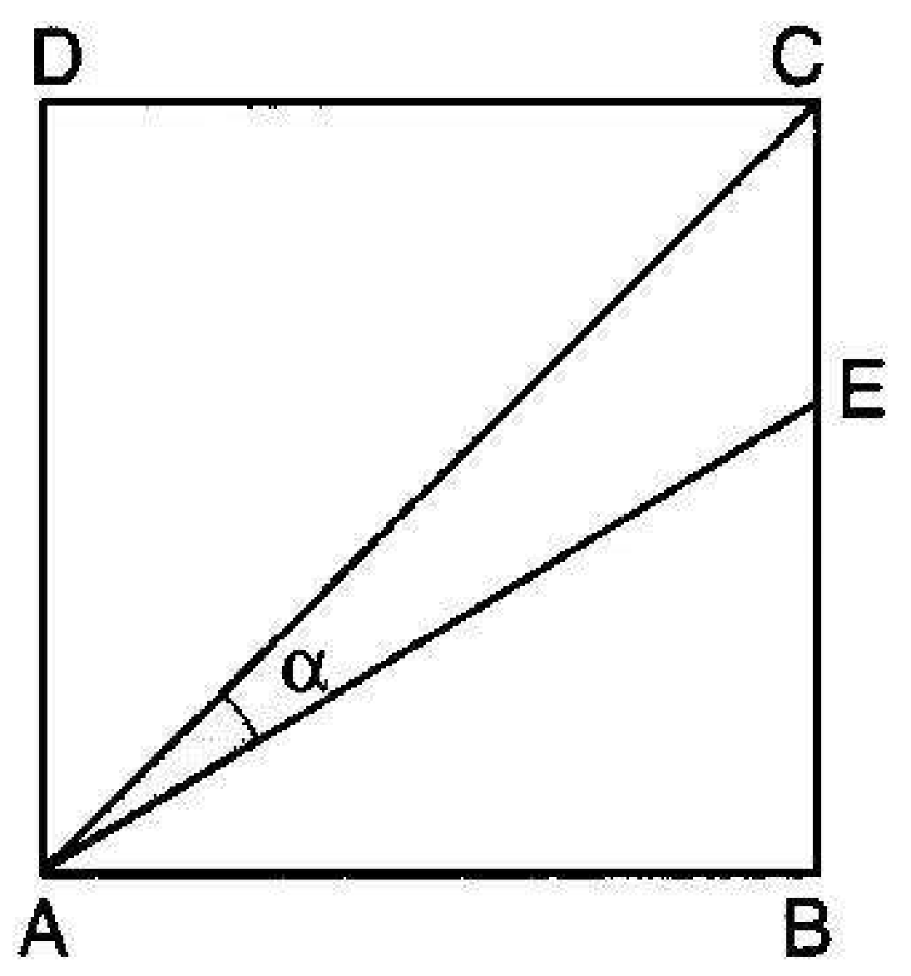
$\tan\theta = ?$



A) $\frac{1}{11}$ B) $\frac{2}{11}$ C) $\frac{3}{11}$

D) $\frac{4}{11}$ E) $\frac{5}{11}$

7. ABCD bir karedir.
ABCD is a square.
$2\cdot|CE| = |EB|$.
$\tan\alpha$ = ?



A) $\frac{5}{9}$

B) $\frac{1}{5}$

C) $\frac{9}{5}$

D) 5

E) $\frac{3}{5}$

8. $\cos 40° = 2m \Rightarrow$ **$\sin 12{,}5°\cdot\cos 12{,}5°\cdot\sin 65°$ = ?**

A) 8m

B) 4m

C) 2m

D) $\frac{m}{2}$

E) $\frac{m}{4}$

9. $A = \sin\frac{\pi}{12}\cdot\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{6} \Rightarrow$ **A^2 = ?**

A) $\frac{1}{64}$

B) $\frac{1}{32}$

C) $\frac{4}{3}$

D) $\frac{3}{64}$

E) 1

10. $0 < x < 90°$,

$$\frac{1+\cos 2x}{\sin 2x} = \sqrt{3} \Rightarrow \mathbf{x = ?}$$

A) 15°

B) 30°

C) 45°

D) 60°

E) 75°

11. $\frac{1}{\cos 80°} - \frac{\sqrt{3}}{\sin 80°} = A \Rightarrow$ **A = ?**

A) 4

B) 3

C) 2

D) 1

E) $\frac{1}{4}$

12. **$\sin^4 a - \cos^4 a$ = ?**

A) cos2a

B) sin2a

C) 1

D) –sin2a

E) –cos2a

YANITLAR / ANSWERS

1. B	2. C	3. B	4. C	5. A	6. C	7. B	8. D
9. D	10. B	11. A	12. E				

1.
$$\tan 15° = \tan(45°-30°)$$
$$= \frac{\tan 45° - \tan 30°}{1+\tan 45°\cdot\tan 30°}$$
$$= \frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$$
$$= \frac{3-\sqrt{3}}{3+\sqrt{3}}$$
$$= \frac{\left(3-\sqrt{3}\right)^2}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)} = \frac{12-6\sqrt{3}}{6}$$
$$= 2-\sqrt{3}$$

Yanıt / The Answer is B

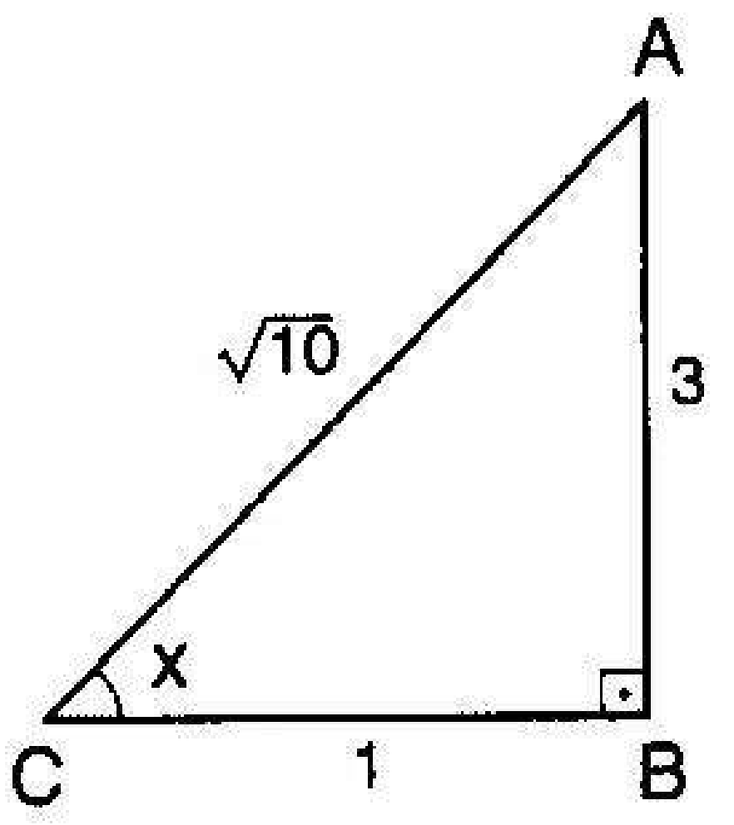
2.
$$\cot 105° = \cot(60°+45°)$$
$$= \frac{\cot 60°\cdot\cot 45° - 1}{\cot 60° + \cot 45°}$$
$$= \frac{\sqrt{3}-3}{\sqrt{3}+3}$$
$$= \frac{\left(\sqrt{3}-3\right)^2}{\left(\sqrt{3}+3\right)\cdot\left(\sqrt{3}-3\right)}$$
$$= \frac{12-6\sqrt{3}}{3-9}$$
$$= -(2-\sqrt{3})$$

Yanıt / The Answer is C

3.
$$\cos 165° = \cos(180° - 15°)$$
$$= -\cos 15°$$
$$= -\cos(45° - 30°)$$
$$= -[\cos 45°\cdot\cos 30° + \sin 45°\cdot\sin 30°]$$
$$= -\left[\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right] = \frac{-\sqrt{6}-\sqrt{2}}{4}$$

Yanıt / The Answer is B

4.
$$\tan x = -3 \Rightarrow \cos x = \frac{1}{\sqrt{10}}$$
$$\cos 2x = 2\cos^2 x - 1$$
$$= 2\cdot\left(\frac{1}{\sqrt{10}}\right)^2 - 1$$
$$= 2\cdot\frac{1}{10} - 1$$
$$= \frac{2}{10} - 1$$
$$= -\frac{8}{10}$$
$$= -\frac{4}{5}$$



$|AC|^2 = |AB|^2 + |BC|^2$

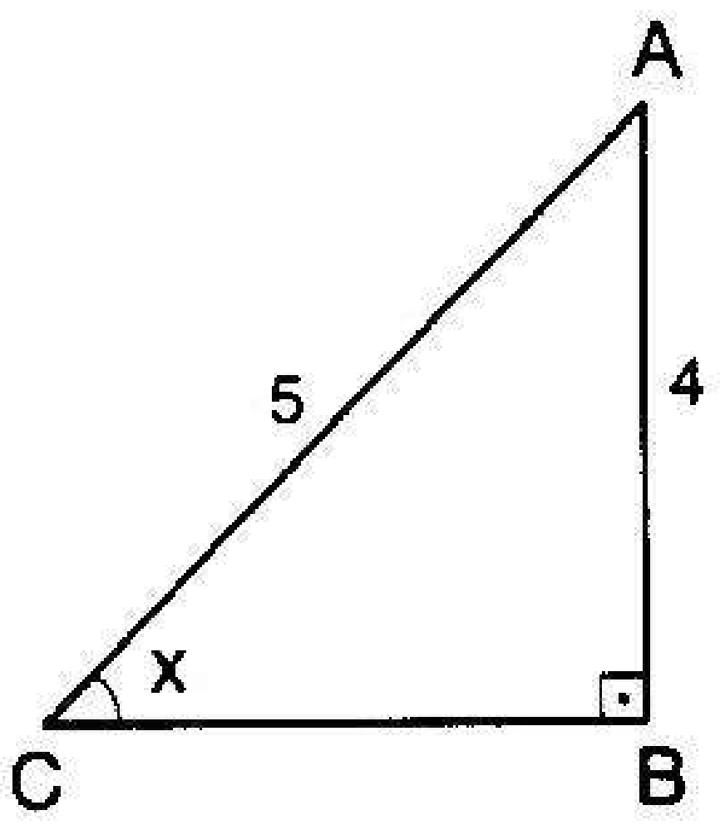
$|AC|^2 = 3^2 + 1^2$

$|AC| = 10$

$|AC| = \sqrt{10}$

Yanıt / The Answer is C

5.
$$\sin x = \frac{8}{10} = \frac{4}{5}$$
$$\cos x = \frac{3}{5}$$
$$\cos 2x - \sin 2x$$
$$= 2\cos^2 x - 1 - 2\cdot\sin x\cdot\cos x$$
$$= 2\cdot\left(\frac{3}{5}\right)^2 - 1 - 2\cdot\frac{4}{5}\cdot\frac{3}{5}$$
$$= \frac{18}{25} - \frac{25}{25} - \frac{24}{25} = -\frac{31}{25}$$



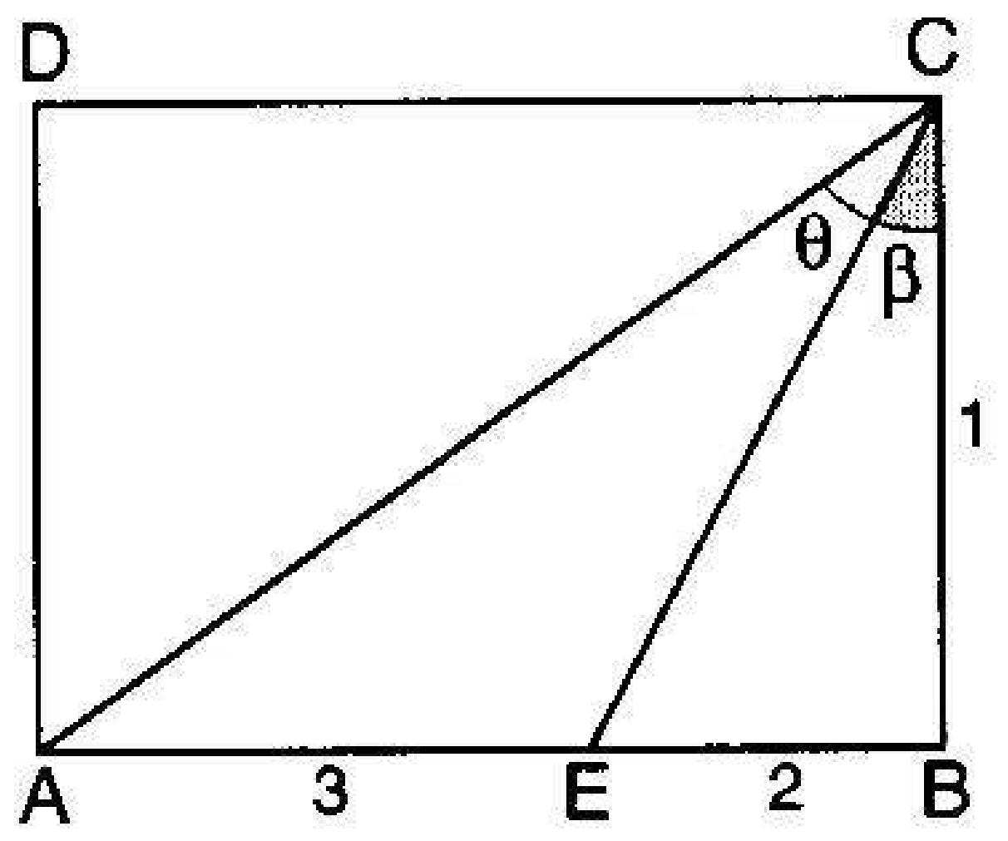
$|AC|^2 = |AB|^2 + |BC|^2$

$5^2 = 4^2 + |BC|^2$

$|BC| = 3$

Yanıt / The Answer is A

6.
$$\tan(\theta+\beta) = 5$$
$$\frac{\tan\theta + \tan\beta}{1-\tan\theta\cdot\tan\beta} = 5$$
$$\frac{\tan\theta + 2}{1-2\tan\theta} = 5$$
$$\tan\theta + 2 = 5 - 10\tan\theta$$
$$11\tan\theta = 3$$
$$\tan\theta = \frac{3}{11}$$



Yanıt / The Answer is C

7. $2 \cdot |CE| = |EB| \Rightarrow$

$|CE| = k\ ,\ |EB| = 2k$

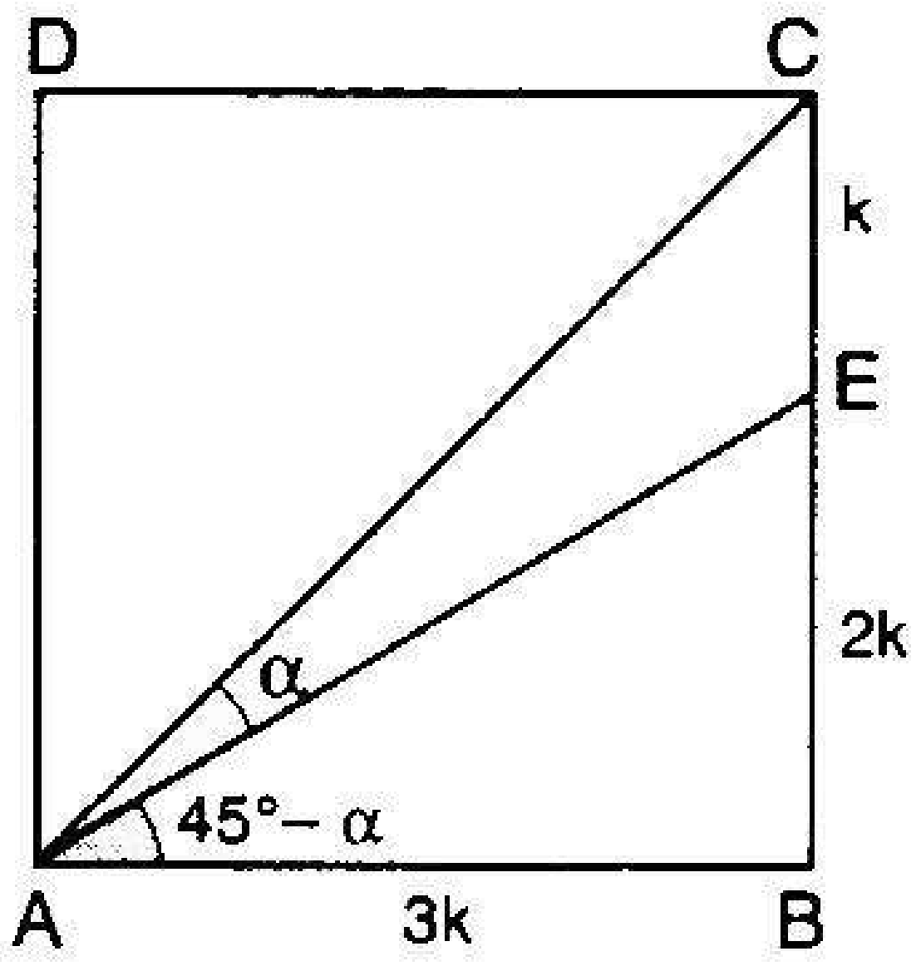
$\tan(\alpha + (45° - \alpha)) = \tan 45° = 1$

$$\frac{\tan\alpha + \tan(45° - \alpha)}{1 - \tan\alpha \cdot \tan(45° - \alpha)} = 1$$

$$\frac{\tan\alpha + \frac{2}{3}}{1 - \tan\alpha \cdot \frac{2}{3}} = 1$$

$$\frac{5}{3}\tan\alpha = \frac{1}{3}$$

$$\tan\alpha = \frac{1}{5}$$



Yanıt / The Answer is B

8. $\sin 12{,}5° \cdot \cos 12{,}5° \cdot \sin 65° = \dfrac{\sin 25° \cdot \sin 65°}{2}$

$$\sin 25° = \cos 65° \Rightarrow \frac{\sin 25° \cdot \sin 65°}{2} = \frac{\cos 65° \cdot \sin 65°}{2}$$

$$= \frac{\sin 130°}{4}$$

$\sin 130° = \sin(90° + 40°) = \cos 40°$

$$\Rightarrow \frac{\sin 130°}{4} = \frac{2m}{4} = \frac{m}{2}$$

Yanıt / The Answer is D

9. $A = \sin\dfrac{\pi}{12} \cdot \cos\dfrac{\pi}{12} \cdot \cos\dfrac{\pi}{6}$

$$= \frac{2 \cdot \sin\frac{\pi}{12} \cdot \cos\frac{\pi}{12} \cdot \cos\frac{\pi}{6}}{2}$$

$$= \frac{\sin\frac{\pi}{6} \cdot \cos\frac{\pi}{6}}{2} = \frac{\sin\frac{\pi}{3}}{4}$$

$$A^2 = \frac{\left(\sin\frac{\pi}{3}\right)^2}{16} = \frac{3}{64}$$

Yanıt / The Answer is D

10. $\dfrac{1 + \cos 2x}{\sin 2x} = \sqrt{3}$

$$\frac{1 + 2\cos^2 x - 1}{2 \cdot \sin x \cdot \cos x} = \sqrt{3}$$

$$\frac{\cos x}{\sin x} = \sqrt{3}$$

$0 < x < 90° \Rightarrow x = 30°$

Yanıt / The Answer is B

11. $\dfrac{1}{\cos 80°} - \dfrac{\sqrt{3}}{\sin 80°} = \dfrac{\sin 80° - \tan 60° \cdot \cos 80°}{\sin 80° \cdot \cos 80°}$

$$= \frac{\sin 80° - \frac{\sin 60°}{\cos 60°} \cdot \cos 80°}{\sin 80° \cdot \cos 80°}$$

$$= \frac{\sin 80° \cdot \cos 60° - \sin 60° \cdot \cos 80°}{\cos 60° \cdot \sin 80° \cdot \cos 80°}$$

$$= \frac{2 \cdot \sin 20°}{\frac{1}{2} \cdot 2 \cdot \sin 80° \cdot \cos 80°}$$

$$= \frac{2 \cdot \sin 20°}{\frac{1}{2} \cdot \sin 160°} = 4$$

Yanıt / The Answer is A

12. $\sin^4 a - \cos^4 a = -(\cos^4 a - \sin^4 a)$

$= -(\cos^2 a - \sin^2 a) \cdot (\cos^2 a + \sin^2 a)$

$= -(\cos 2a) \cdot 1$

$= -\cos 2a$

Yanıt / The Answer is E

1. $\dfrac{\sin 77^\circ + \cos 47^\circ}{\sin 73^\circ} = ?$

A) $2\sqrt{3}\sin 17^\circ$
B) $\sqrt{3}$
C) $2\sqrt{3}$
D) $2\sqrt{2}\cos 17^\circ$
E) $\tan 17^\circ$

2. $\cos 105^\circ - \cos 15^\circ = ?$

A) $\dfrac{\sqrt{6}}{2}$
B) $-\dfrac{\sqrt{6}}{2}$
C) 0
D) 1
E) $\dfrac{\sqrt{3}}{2}$

3. $\dfrac{\sin 75^\circ + \cos 5^\circ}{\cos 75^\circ - \sin 5^\circ} = ?$

A) $\tan 10^\circ$
B) 1
C) $\cot 10^\circ$
D) $\cot 5^\circ$
E) $\tan 5^\circ$

4. $\tan 15^\circ + \cot 15^\circ = ?$

A) $\dfrac{\sqrt{3}}{2}$
B) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$
C) $\dfrac{\sqrt{3}+1}{2}$
D) 2
E) 4

5. $\dfrac{\cos 37^\circ + \sin 37^\circ}{\cos 37^\circ - \sin 37^\circ} = ?$

A) $\sin 8^\circ$
B) $\cos 8^\circ$
C) $\tan 8^\circ$
D) $\cot 8^\circ$
E) $\sec 8^\circ$

6. $x = \dfrac{\pi}{17} \Rightarrow \dfrac{\cos 14x + \cos 12x}{\cos 3x + \cos 5x} = ?$

A) -2
B) $-\dfrac{3}{2}$
C) -1
D) 0
E) 1

7. $\dfrac{\sin 5a + \sin 4a + \sin 3a}{\sin 9a + \sin 8a + \sin 7a} = ?$

A) $\sin 4a$

B) $\sin 8a$

C) $\tan 2a$

D) $\dfrac{\sec 4a}{2}$

E) $\dfrac{\sec 8a}{2}$

8. $16 \cdot \sin 10° \cdot \sin 30° \cdot \sin 50° \cdot \sin 70° = ?$

A) 8

B) 4

C) 2

D) 1

E) $\dfrac{1}{2}$

9. $\cos 106° \cdot \sin 14° + \sin 30° \cdot \cos 2° = ?$

A) $-\dfrac{1}{4}$

B) $-\dfrac{1}{2}$

C) 0

D) $\dfrac{1}{2}$

E) $\dfrac{\sqrt{3}}{4}$

10. $\arccos 5x = \dfrac{\pi}{3} \Rightarrow x = ?$

A) $\dfrac{\sqrt{3}}{10}$

B) $\dfrac{\sqrt{2}}{10}$

C) $\dfrac{1}{10}$

D) $\dfrac{\sqrt{2}}{5}$

E) $\dfrac{\sqrt{3}}{5}$

11. $\sin\left(2\arcsin\dfrac{4}{5}\right) = ?$

A) $\dfrac{12}{25}$

B) $\dfrac{16}{25}$

C) $\dfrac{18}{25}$

D) $\dfrac{24}{25}$

E) 1

12. $x \in [0, \pi]$,
$\cos 2x + \cos x = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\dfrac{\pi}{2}, \dfrac{5\pi}{6}\right\}$ B) $\left\{\dfrac{\pi}{4}, \dfrac{\pi}{2}\right\}$ C) $\left\{\dfrac{\pi}{3}, \pi\right\}$

D) $\left\{\dfrac{\pi}{3}, \dfrac{3\pi}{4}\right\}$ E) $\left\{\dfrac{3\pi}{4}, \pi\right\}$

YANITLAR / ANSWERS

1. B 2. B 3. D 4. E 5. D 6. C 7. D 8. D
9. E 10. C 11. D 12. C

1.
$$\frac{\sin 77°+\cos 47°}{\sin 73°}=\frac{\sin 77°+\sin 43°}{\cos 17°}$$

$$=\frac{2\cdot\sin\left(\frac{77°+43°}{2}\right)\cdot\cos\left(\frac{77°-43°}{2}\right)}{\cos 17°}$$

$$=\frac{2\cdot\sin 60°\cdot\cos 17°}{\cos 17°}$$

$$=2\cdot\frac{\sqrt{3}}{2}=\sqrt{3}$$

Yanıt / The Answer is B

2.
$$\cos 105°-\cos 15°=-2\sin\frac{105°+15°}{2}\cdot\sin\frac{105°-15°}{2}$$

$$=-2\sin 60°\cdot\sin 45°$$

$$=-2\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$=-\frac{\sqrt{6}}{2}$$

Yanıt / The Answer is B

3.
$$\frac{\sin 75°+\cos 5°}{\cos 75°-\sin 5°}=\frac{\sin 75°+\sin 85°}{\cos 75°-\cos 85°}$$

$$=\frac{2\cdot\sin\frac{75°+85°}{2}\cdot\cos\frac{85°-75°}{2}}{-2\cdot\sin\frac{75°+85°}{2}\cdot\sin\frac{75°-85°}{2}}$$

$$=\frac{2\cdot\sin 80°\cdot\cos 5°}{2\cdot\sin 80°\cdot\sin 5°}$$

$$=\cot 5°$$

Yanıt / The Answer is D

4.
$$\tan 15°+\cot 15°=\tan 15°+\tan 75°$$

$$=\frac{\sin(15°+75°)}{\cos 15°\cdot\cos 75°}$$

$$=\frac{\sin 90°}{\sin 75°\cdot\cos 75°}$$

$$=\frac{2\cdot\sin 90°}{2\cdot\sin 75°\cdot\cos 75°}$$

$$=\frac{2}{\sin 150°}=\frac{2}{\frac{1}{2}}=4$$

Yanıt / The Answer is E

5.
$$\frac{\cos 37°+\sin 37°}{\cos 37°-\sin 37°}=\frac{\cos 37°+\cos 53°}{\cos 37°-\cos 53°}$$

$$=\frac{2\cdot\cos\frac{37°+53°}{2}\cdot\cos\frac{53°-37°}{2}}{-2\cdot\sin\frac{37°+53°}{2}\cdot\sin\frac{37°-53°}{2}}$$

$$=\frac{2\cdot\cos 45°\cdot\cos 8°}{2\cdot\sin 45°\cdot\sin 8°}=\cot 8°$$

Yanıt / The Answer is D

6.
$$\frac{\cos 14x+\cos 12x}{\cos 3x+\cos 5x}=\frac{2\cdot\cos\frac{14x+12x}{2}\cdot\cos\frac{14x-12x}{2}}{2\cdot\cos\frac{3x+5x}{2}\cdot\cos\frac{5x-3x}{2}}$$

$$=\frac{2\cdot\cos 13x\cdot\cos x}{2\cdot\cos 4x\cdot\cos x}$$

$$=\frac{\cos 13x}{\cos 4x}$$

$$13x+4x=\pi\ \Rightarrow\ \frac{\cos 13x}{\cos 4x}=-1$$

Yanıt / The Answer is C

7. $$\frac{\sin 5a+\sin 4a+\sin 3a}{\sin 9a+\sin 8a+\sin 7a}$$

$$=\frac{2\cdot\sin\frac{5a+3a}{2}\cdot\cos\frac{5a-3a}{2}+\sin 4a}{2\cdot\sin\frac{9a+7a}{2}\cdot\cos\frac{9a-7a}{2}+\sin 8a}$$

$$=\frac{2\cdot\sin 4a\cdot\cos a+\sin 4a}{2\cdot\sin 8a\cdot\cos a+\sin 8a}$$

$$=\frac{\sin 4a(2\cos a+1)}{\sin 8a(2\cos a+1)}$$

$$=\frac{\sin 4a}{2\cdot\sin 4a\cdot\cos 4a}=\frac{1}{2\cos 4a}=\frac{1}{2}\sec 4a$$

Yanıt / The Answer is D

8. $$16\cdot\sin 10°\cdot\sin 30°\cdot\sin 50°\cdot\sin 70°$$

$$=16\cdot\sin 10°\cdot\frac{1}{2}\cdot\sin 50°\cdot\sin 70°$$

$$=8\cdot\left[-\frac{1}{2}(\cos 80°-\cos 60°)\right]\cdot\sin 50°$$

$$=(-4\cdot\cos 80°+2)\cdot\sin 50°$$

$$=-4\cdot\cos 80°\cdot\sin 50°+2\cdot\sin 50°$$

$$=-4\cdot\left[\frac{1}{2}(\sin 130°-\sin 30°)\right]+2\cdot\sin 50°$$

$$=-2\cdot\sin 130°-(-2)\cdot\left(\frac{1}{2}\right)+2\cdot\sin 50°$$

$$=1$$

Yanıt / The Answer is D

9. $$\cos 106°\cdot\sin 14°+\sin 30°\cdot\cos 2°$$

$$=\frac{1}{2}\cdot(\sin 120°-\sin 92°)+\frac{1}{2}\cdot\cos 2°$$

$$=\frac{1}{2}\left(\frac{\sqrt{3}}{2}-\cos 2°\right)+\frac{1}{2}\cos 2°$$

$$=\frac{\sqrt{3}}{4}-\frac{1}{2}\cos 2°+\frac{1}{2}\cos 2°$$

$$=\frac{\sqrt{3}}{4}$$

Yanıt / The Answer is E

10. $$\arccos 5x=\frac{\pi}{3}\Rightarrow\cos\frac{\pi}{3}=5x$$

$$\frac{1}{2}=5x$$

$$x=\frac{1}{10}$$

Yanıt / The Answer is C

11. $$\arcsin\frac{4}{5}=x\Rightarrow\sin x=\frac{4}{5}$$

$$\sin x=\frac{4}{5}\Rightarrow\cos x=\frac{3}{5}$$

$$\sin\left(2\arcsin\frac{4}{5}\right)=\sin 2x$$

$$\sin 2x=2\cdot\sin x\cdot\cos x$$

$$=2\cdot\frac{4}{5}\cdot\frac{3}{5}$$

$$=\frac{24}{25}$$

Yanıt / The Answer is D

12. $$\cos 2x+\cos x=0$$

$$2\cos^2 x-1+\cos x=0$$

$$(2\cos x-1)\cdot(\cos x+1)=0$$

$$\cos x=\frac{1}{2}\ ,\quad \cos x=-1$$

$$x\in[0,\pi]\Rightarrow x_1=\frac{\pi}{3}\ ,\quad x_2=\pi$$

$$\text{ÇK(SS)}=\left\{\frac{\pi}{3},\pi\right\}$$

Yanıt / The Answer is C

1. $\cos 105° = ?$

Yanıt / The Answer is: $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

---

2. $\sin 72° \cdot \cos 18° + \sin 18° \cdot \cos 72° = ?$

Yanıt / The Answer is: **1**

---

3. $x + y = \dfrac{\pi}{6}$

$\sin x + \cos y = A, \ \ \cos x + \sin y = B \Rightarrow A^2 + B^2 = ?$

Yanıt / The Answer is: **3**

---

4. $\dfrac{\cos 3a}{\sin a} + \dfrac{\sin 3a}{\cos a} = ?$

Yanıt / The Answer is: **2cot 2a**

---

5. $A = \sin 10° + \tan 40° \cdot \cos 10° \Rightarrow A = ?$

Yanıt / The Answer is: **1**

---

6. $\tan x = \dfrac{3}{5} \Rightarrow \sin 2x = ?$

Yanıt / The Answer is: $\dfrac{15}{17}$

7. $2\sin 5° \cdot \cos 5° \cdot \cos 10° = a \Rightarrow \mathbf{\cos 40° = ?}$

Yanıt / The Answer is: $\mathbf{1 - 8a^2}$

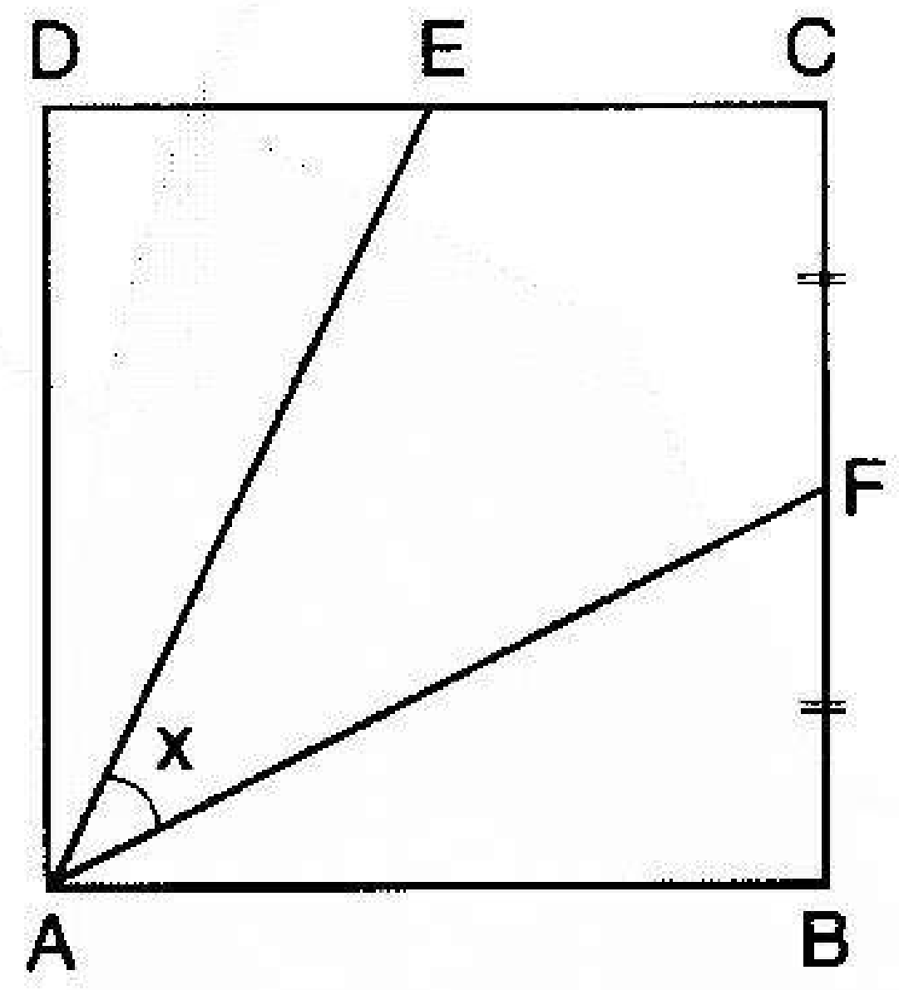
8. ABCD bir karedir.

ABCD is a square.

$|BF| = |FC|$,

$|EC| = 2|DE|$

$\Rightarrow \mathbf{m(\widehat{EAF}) = x = ?}$



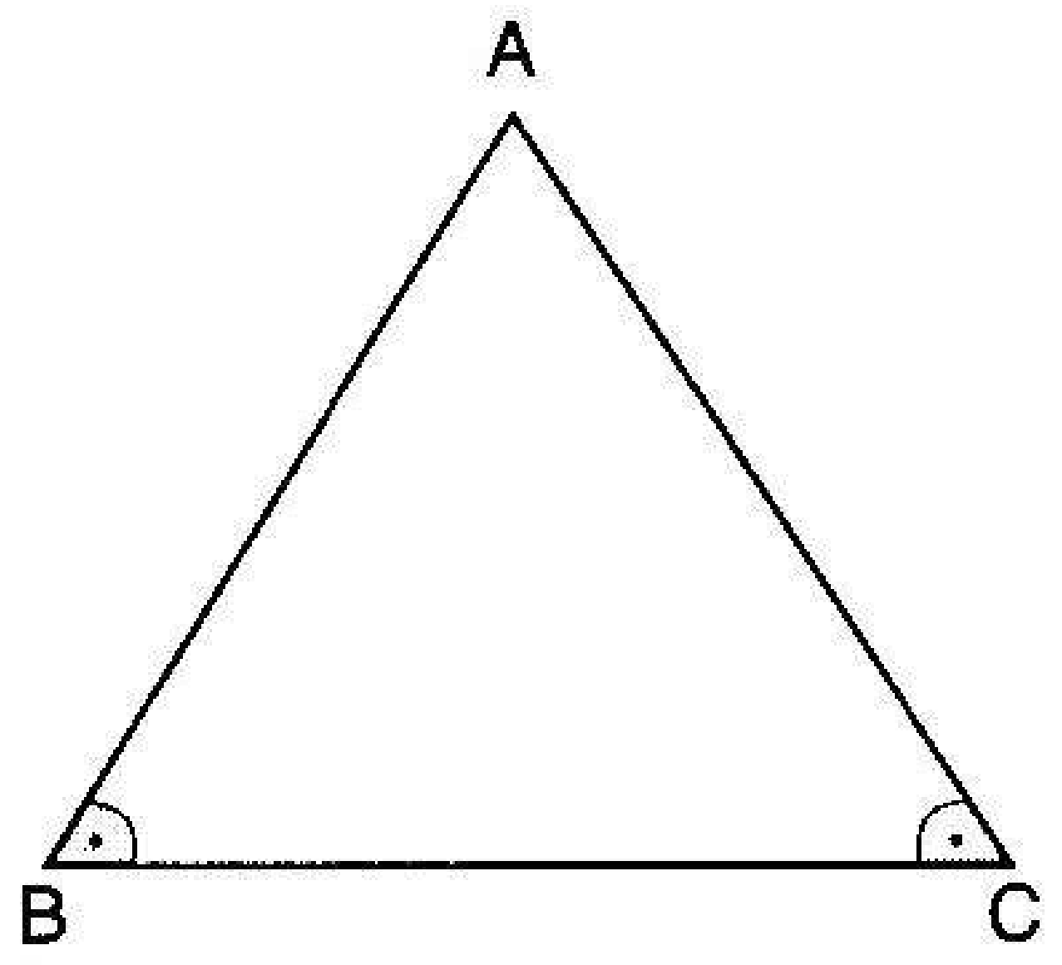
Yanıt / The Answer is: $\mathbf{\frac{\pi}{4}}$

9. ABC üçgeni ikizkenardır.

ABC is an isosceles triangle.

$m(\widehat{B}) = m(\widehat{C})$,

$\tan A = \frac{5}{12} \Rightarrow \mathbf{\cos \widehat{C} = ?}$



Yanıt / The Answer is: $\mathbf{\frac{1}{\sqrt{26}}}$

10. $\mathbf{\frac{\cos 40°}{\sin 10°} - \frac{\sin 40°}{\cos 10°} = ?}$

Yanıt / The Answer is: **4cos20°**

11. $\left.\begin{array}{l} a = \cos 12° \cdot \cos 18° \\ b = \cos 78° \cdot \cos 72° \end{array}\right\} \Rightarrow \mathbf{a - b = ?}$

Yanıt / The Answer is: $\mathbf{\frac{\sqrt{3}}{2}}$

12. $\cos 16° = a \Rightarrow \mathbf{\sin 58° = ?}$

Yanıt / The Answer is: $\mathbf{2a^2 - 1}$

13. $x + 2y = \frac{\pi}{2}$

$\Rightarrow \sin(2x + 2y) - \sin 2y = ?$

Yanıt / The Answer is: **0**

14. $\sin 2a = \frac{3}{5} \quad \Rightarrow \quad \tan a = ?$

Yanıt / The Answer is: $\frac{1}{3}$

15. $\frac{\cos 4x - \cos 8x}{\cos 8x - \cos 4x} = ?$

Yanıt / The Answer is: **–1**

16. $\cos 40° + \cos 80° = ?$

Yanıt / The Answer is: **cos20°**

17. $\sin 75° + \sin 15° = ?$

Yanıt / The Answer is: $\frac{\sqrt{6}}{2}$

18. $16x = \pi$

$\Rightarrow \frac{\sin 12x - \sin 8x}{\cos 16x + \cos 4x} = ?$

Yanıt / The Answer is: **1**

19. $\dfrac{\sin 2x+\sin 3x+\sin 4x}{\cos 2x+\cos 3x+\cos 4x}=?$

Yanıt / The Answer is: **tan3x**

---

20. $\cos\left(\arcsin\left(\dfrac{1}{2}\right)\right)=?$

Yanıt / The Answer is: $\dfrac{\sqrt{3}}{2}$

---

21. $\arccos x = 12° \Rightarrow \sin 66° = ?$

Yanıt / The Answer is: $2x^2-1$

---

22. $\arctan\dfrac{1}{3}+\arctan\dfrac{1}{2}=?$

Yanıt / The Answer is: $\dfrac{\pi}{4}$

---

23. $\arctan\dfrac{1}{3}+\arctan 3=?$

Yanıt / The Answer is: **90°**

---

24. $3\cos x + 4\sin x = A \Rightarrow \max(A) = ?$

Yanıt / The Answer is: **5**

1. **$\sin 28° \cdot \cos 32° + \cos 28° \cdot \sin 32° = ?$**

A) 0

B) $\frac{1}{2}$

C) $\frac{\sqrt{2}}{2}$

D) $\frac{\sqrt{3}}{2}$

E) 1

2. **$\cos 40° \cdot \cos 10° + \cos 50° \cdot \cos 80° = ?$**

A) 1

B) $\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{2}}{2}$

D) $\frac{1}{2}$

E) 0

3. **$\tan 75° = ?$**

A) $1 + \sqrt{3}$

B) $2 + \sqrt{3}$

C) $3 + \sqrt{3}$

D) $4 + \sqrt{3}$

E) $5 + \sqrt{3}$

4. $\left.\begin{array}{l}\tan x = 2\\ \cot y = 3\end{array}\right\} \Rightarrow$ **$\tan(x+y) = ?$**

A) -7

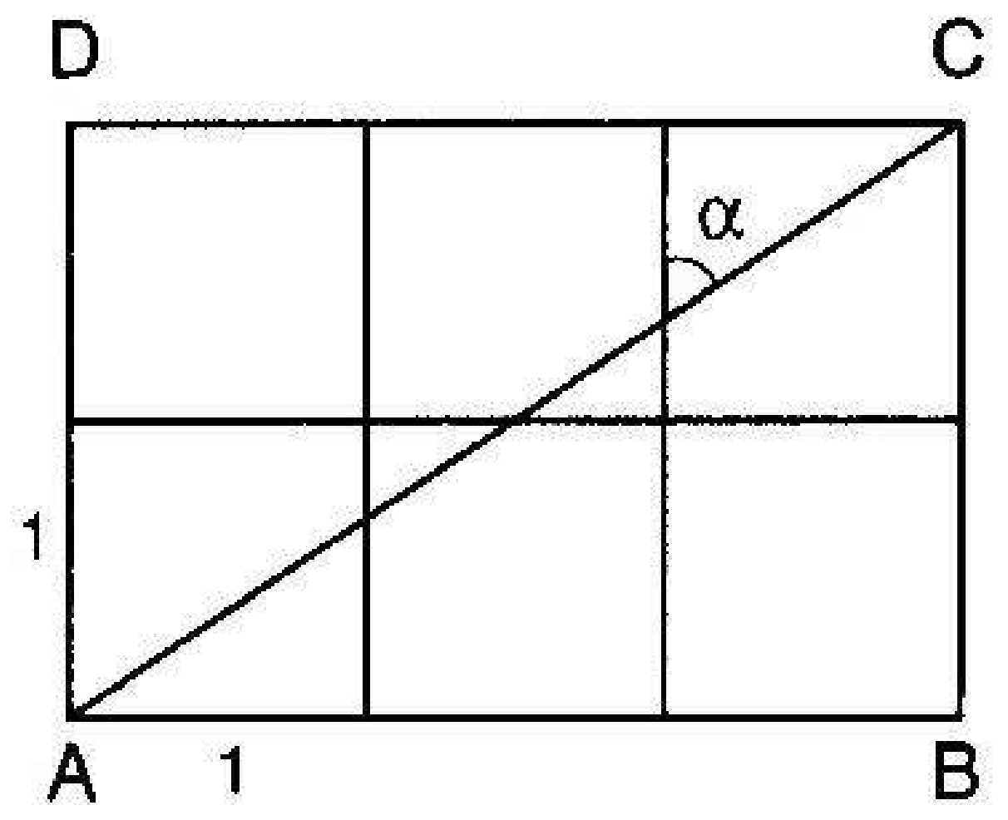
B) $-\frac{5}{3}$

C) -1

D) $\frac{5}{3}$

E) 7

5. **$\sin\alpha + \cos\alpha = ?$**



A) $\frac{\sqrt{13}}{13}$

B) $\frac{\sqrt{13}}{5}$

C) $\frac{5\sqrt{13}}{13}$

D) $\frac{3\sqrt{13}}{13}$

E) $\frac{4\sqrt{13}}{13}$

6. $$\frac{\sin 20° + \sqrt{3} \cdot \cos 20°}{\cos 10°} = ?$$

A) $\frac{1}{2}$

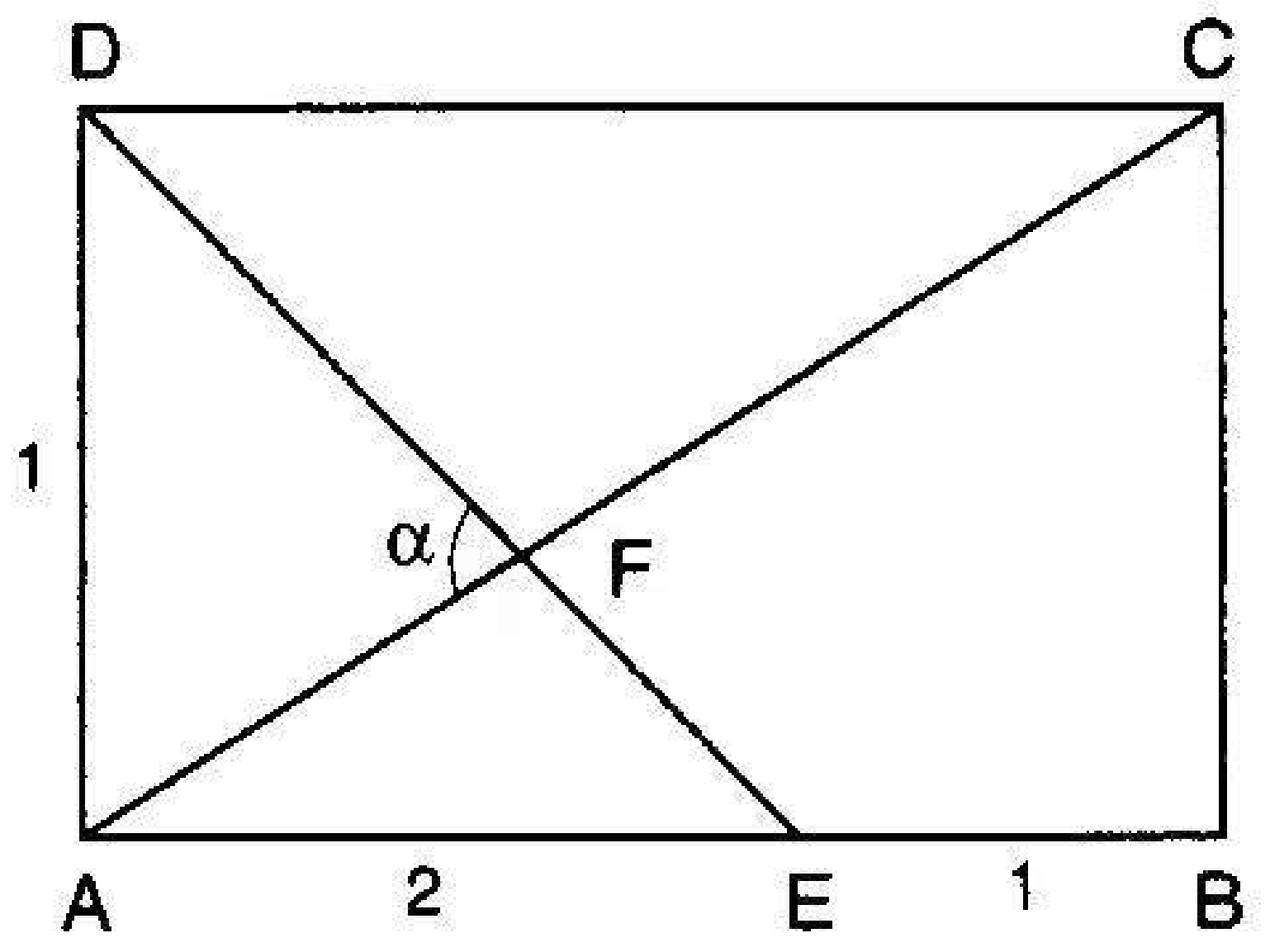
B) $\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{3}}{2}$

D) 1

E) 2

7.



ABCD bir dikdörtgendir. / ABCD is a rectangle.

$|AD| = 1$, $|EB| = 1$, $|AE| = 2$ $\Rightarrow$ **$\tan\alpha = ?$**

A) $-\frac{1}{3}$

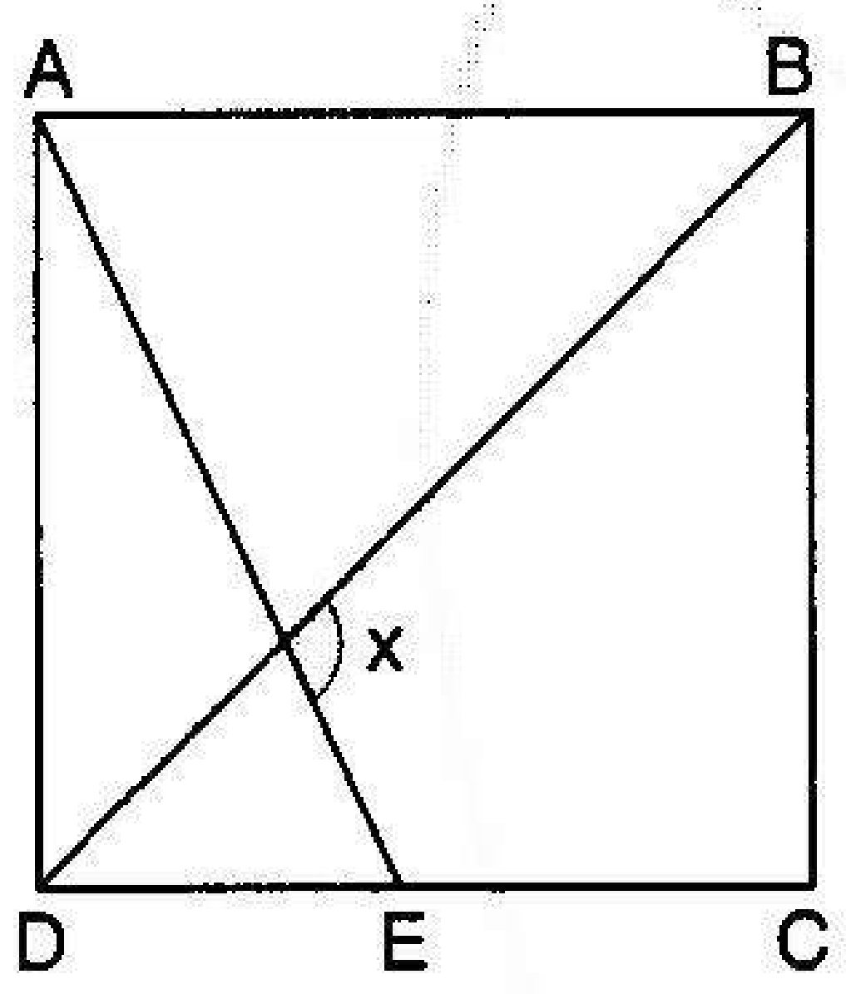
B) $-\frac{1}{6}$

C) 1

D) $\frac{1}{6}$

E) $\frac{1}{3}$

8.



ABCD bir karedir. / ABCD is a square.

$3\cdot|DC| = 4\cdot|DE| \Rightarrow$ **$\sin x = ?$**

A) $\frac{7\sqrt{2}}{5}$

B) $\frac{2\sqrt{2}}{5}$

C) $\frac{\sqrt{2}}{10}$

D) $\frac{\sqrt{3}}{15}$

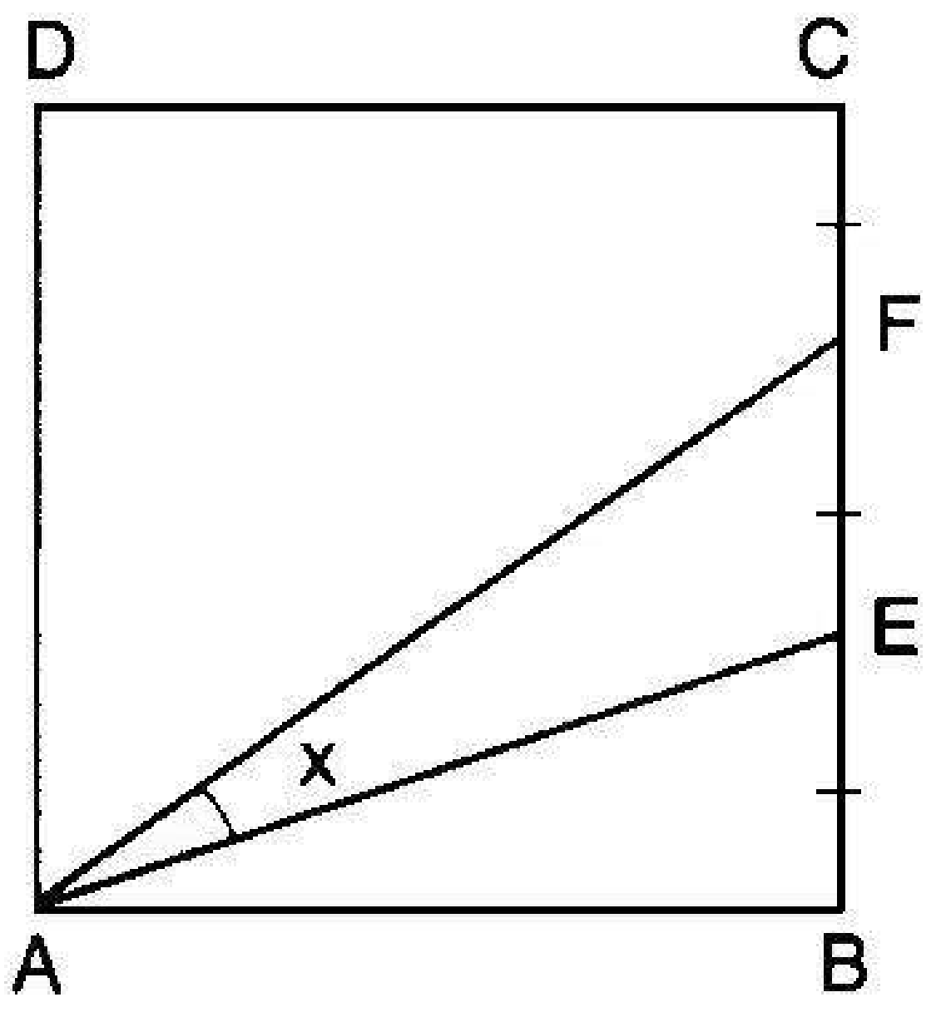
E) $\frac{7\sqrt{2}}{10}$

9. ABCD bir karedir.

ABCD is a square.

$|CF| = |FE| = |EB|$

$\Rightarrow$ **$\tan x = ?$**



A) $\frac{1}{7}$

B) $\frac{1}{8}$

C) $\frac{1}{9}$

D) $\frac{1}{11}$

E) $\frac{3}{11}$

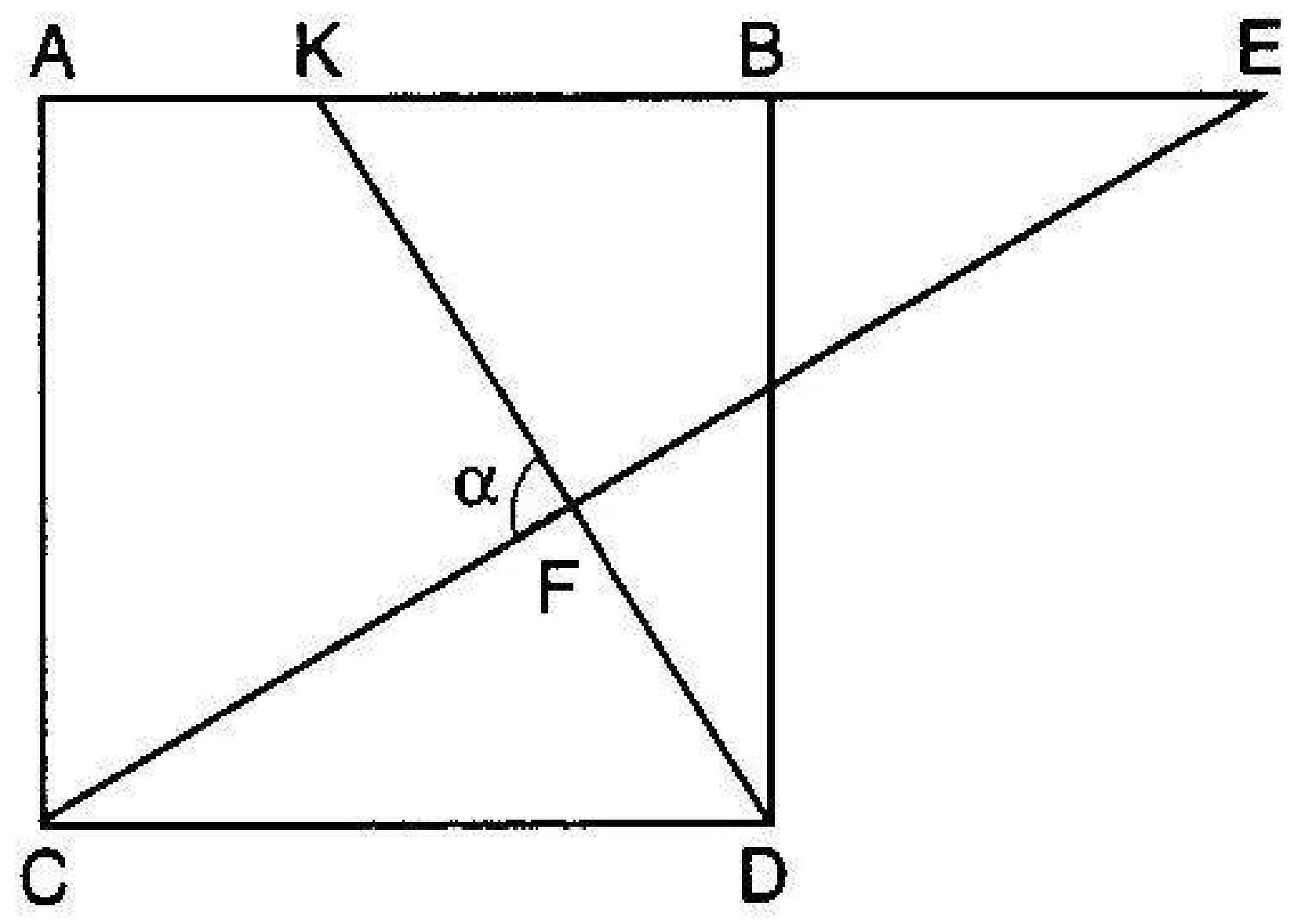
10 ABCD bir karedir.

ABCD is a square.

$2|KB| = 6|KA| = 3|BE|$

$m(\widehat{KFD}) = \alpha$

$\Rightarrow$ **$\cot\alpha = ?$**



A) 18

B) 16

C) 1

D) $\frac{1}{16}$

E) $\frac{1}{18}$

11. $\frac{\tan^2x-\tan^2y}{1-\tan^2x\cdot\tan^2y} = ?$

A) $\tan(x-y)$

B) 1

C) -1

D) $\tan(x+y)$

E) $\tan(x-y)\cdot\tan(x+y)$

12. $A = 4\sin x + 8\cos x$ $\Rightarrow$ **$\max(A) = ?$**

A) $\sqrt{5}$

B) $2\sqrt{5}$

C) $3\sqrt{5}$

D) $4\sqrt{5}$

E) 10

YANITLAR / ANSWERS

1. D 2. B 3. B 4. E 5. C 6. E 7. C 8. E

9. E 10. E 11. E 12. D

1. $\sin x - \cos x = \frac{3}{4} \Rightarrow$ **sin2x = ?**

A) $-\frac{7}{32}$

B) $-\frac{7}{16}$

C) $\frac{7}{16}$

D) $\frac{7}{32}$

E) $\frac{9}{16}$

2. $\sin^2 \frac{\pi}{8} = ?$

A) $\frac{2-\sqrt{2}}{4}$

B) $\frac{\sqrt{2}}{4}$

C) $\frac{\sqrt{2}}{2}$

D) $\sqrt{2}-1$

E) $\frac{2\sqrt{2}+1}{2}$

3. $\frac{\sin^4 x + 2\cos^2 x - 1}{\cos^4 x} = ?$

A) cos2x

B) –sin2x

C) 1

D) –cos2x

E) sin2x

4. $\cot x = 3 \Rightarrow$ **sin2x = ?**

A) $\frac{1}{5}$

B) $\frac{2}{5}$

C) $\frac{3}{5}$

D) $\frac{4}{5}$

E) 1

5. $0 < x < \frac{\pi}{2}$, $\cos 2x = -\frac{7}{9} \Rightarrow$ **tanx = ?**

A) $-2\sqrt{2}$

B) $-\sqrt{2}$

C) 1

D) $\sqrt{2}$

E) $2\sqrt{2}$

6. $\sin 20° = a \Rightarrow$ **cos40°·cos50°·cos80° = ?**

A) $\frac{a}{4}$

B) $\frac{a}{2}$

C) a

D) 2a

E) 4a

7. $\dfrac{\sin 21°}{\sin 7°} - \dfrac{\cos 21°}{\cos 7°} = ?$

A) $\dfrac{1}{8}$

B) $\dfrac{1}{4}$

C) $\dfrac{1}{2}$

D) 1

E) 2

---

8. $\cos^2 \dfrac{\pi}{12} + \sin^2 \dfrac{5\pi}{12} = ?$

A) $\dfrac{2+\sqrt{3}}{2}$

B) $\dfrac{3+\sqrt{3}}{3}$

C) $\dfrac{2+\sqrt{2}}{2}$

D) $\dfrac{3+\sqrt{2}}{3}$

E) $\dfrac{\sqrt{2}+\sqrt{3}}{2}$

---

9. $\sin 14° = a \Rightarrow \sin 62° = ?$

A) $1 - a$

B) $1 + a$

C) a^2

D) $1 - 2a^2$

E) $1 + 2a^2$

---

10. $\dfrac{1-\cos 2x}{\sin x} = ?$

A) $\cot x$

B) $\tan x$

C) $-\tan x$

D) $\sin x$

E) $2 \cdot \sin x$

---

11. $\dfrac{1-\cos 32°}{\sin^2 16°} = ?$

A) 2

B) 1

C) 0

D) –1

E) –2

---

12. $\tan \dfrac{x}{2} = \dfrac{4}{3} \Rightarrow \tan \dfrac{3x}{4} = ?$

A) $\dfrac{5}{2}$

B) $\dfrac{7}{2}$

C) $\dfrac{9}{2}$

D) $\dfrac{11}{2}$

E) $\dfrac{13}{2}$

YANITLAR / ANSWERS

1. C	2. A	3. C	4. C	5. E	6. A	7. E	8. A
9. D	10. E	11. A	12. D				

1. $x = \frac{\pi}{30} \Rightarrow \frac{\cos 14x + \cos 6x}{\cos 4x} = ?$

A) $\frac{1}{2}$

B) $-\frac{1}{2}$

C) 1

D) -1

E) $\frac{\sqrt{3}}{4}$

---

2. $a = \frac{\pi}{15} \Rightarrow \frac{\cos 7a + \cos 5a}{\cos a \cdot \cos 9a} = ?$

A) -3

B) -2

C) 0

D) 1

E) 2

---

3. $x = \frac{\pi}{12} \Rightarrow \frac{\sin 3x + \sin 11x}{\sin 5x \cdot \cos 4x} = ?$

A) -2

B) -1

C) 1

D) $\frac{1}{2}$

E) 2

---

4. $\frac{\sin 82° + \sin 22°}{\sin 68° - \sin 8°} = ?$

A) $-\sqrt{3}$

B) $-\frac{\sqrt{3}}{3}$

C) $\frac{\sqrt{3}}{3}$

D) $\frac{\sqrt{3}}{2}$

E) $\sqrt{3}$

---

5. $x + y = \frac{\pi}{3} \Rightarrow \frac{\sin x - \sin y}{\cos y - \cos x} = ?$

A) $\sqrt{3}$

B) $\sqrt{2}$

C) $-\sqrt{3}$

D) $-\frac{\sqrt{3}}{2}$

E) -1

---

6. $\frac{\sin 75° - \cos 75°}{\sin 75° + \cos 75°} = ?$

A) $\frac{\sqrt{2}}{2}$

B) $\frac{\sqrt{6}}{2}$

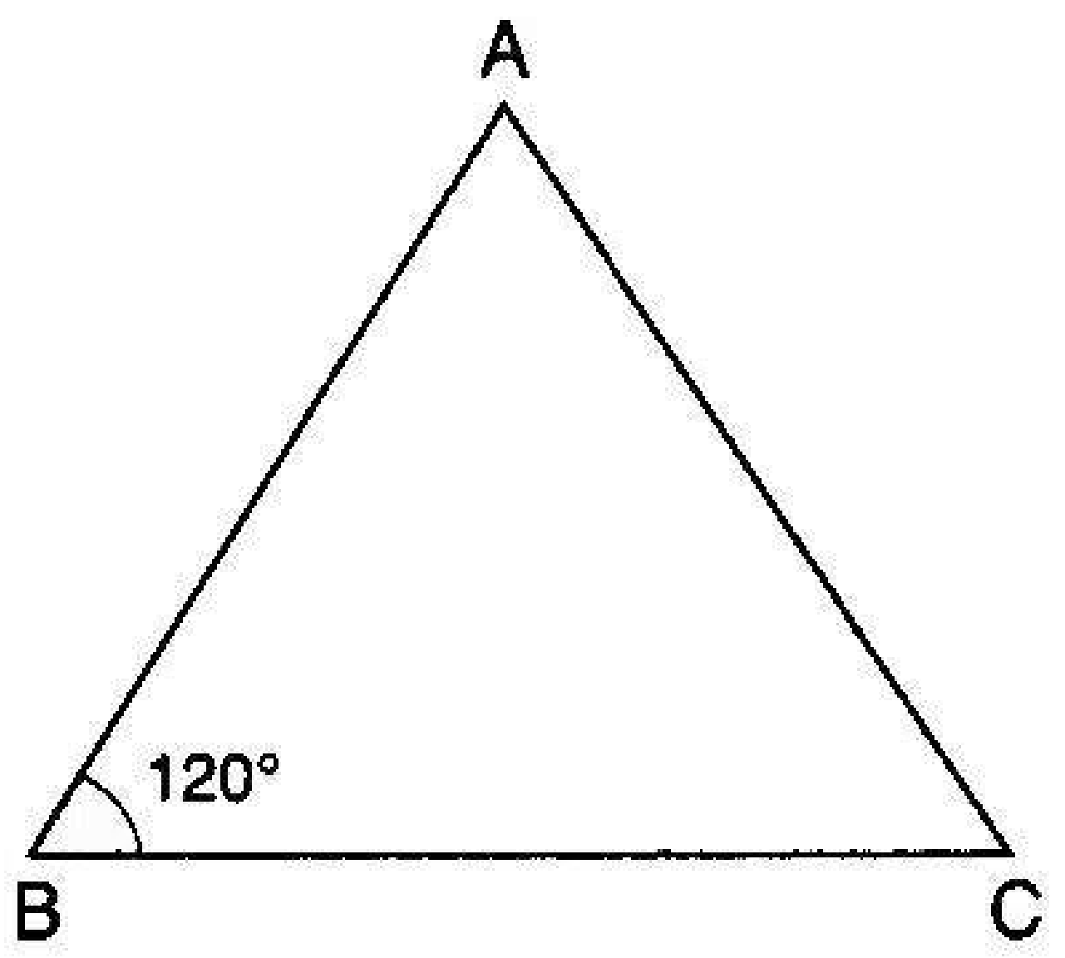
C) $\frac{1}{2}$

D) $\frac{\sqrt{3}}{2}$

E) $\frac{\sqrt{3}}{3}$

7. $m(\widehat{B}) = 120°$

$\Rightarrow \dfrac{\sin \widehat{A} + \sin \widehat{C}}{\cos \widehat{A} + \cos \widehat{C}} = ?$



A) $\dfrac{\sqrt{3}}{2}$ B) $\dfrac{\sqrt{3}}{4}$ C) $\dfrac{\sqrt{3}}{3}$

D) $\sqrt{3}$ E) $\dfrac{1}{2}$

8. $\dfrac{\sin x + \sin 2x + \sin 3x}{1 + \cos x + \cos 2x} = ?$

A) sinx

B) 2sinx

C) cosx

D) 2cosx

E) sin2x

9. $\dfrac{\sin 42° + \sin 50° + \sin 58°}{\cos 42° + \cos 50° + \cos 58°} = ?$

A) tan40°

B) cot40°

C) cot38°

D) tan42°

E) tan58°

10. $\dfrac{\sin 10x - \sin 6x - \sin 2x}{\sin 9x - \sin 7x - \sin x} = ?$

A) 2sinx

B) 2tan4x

C) cot3x

D) 2cosx

E) sin2x

11. $\cos 190° \cdot \cos 70° \cdot \cos 50° = ?$

A) $\dfrac{\sqrt{3}}{8}$

B) $\dfrac{\sqrt{3}}{4}$

C) $-\dfrac{\sqrt{3}}{2}$

D) $-\dfrac{\sqrt{3}}{4}$

E) $-\dfrac{\sqrt{3}}{8}$

12. $\sin 10° \cdot \sin 30° \cdot \sin 50° \cdot \sin 70° = ?$

A) $\dfrac{1}{4}$

B) $\dfrac{1}{8}$

C) $\dfrac{1}{16}$

D) $\dfrac{1}{32}$

E) $\dfrac{1}{64}$

YANITLAR / ANSWERS

1. C	2. B	3. E	4. E	5. A	6. E	7. C	8. B
9. B	10. D	11. E	12. C				

1. **$\arccos\left(-\frac{1}{2}\right) = ?$**

A) $\frac{\pi}{6}$

B) $\frac{\pi}{3}$

C) π

D) $\frac{2\pi}{3}$

E) $\frac{5\pi}{6}$

2. **$\tan\left(\arcsin\frac{4}{5}\right) = ?$**

A) $\frac{1}{3}$

B) $\frac{2}{3}$

C) 1

D) $\frac{4}{3}$

E) $\frac{5}{3}$

3. **$\sin\left(\arctan\frac{1}{3} + \arctan\frac{\sqrt{6}}{2}\right) = ?$**

A) $\frac{2+\sqrt{6}}{10}$

B) $\frac{1+2\sqrt{6}}{10}$

C) $\frac{2+3\sqrt{6}}{10}$

D) $\frac{1+4\sqrt{6}}{10}$

E) $\frac{3+\sqrt{6}}{10}$

4. **$\cos\left(2\arcsin\frac{2}{3}\right) = ?$**

A) $\frac{1}{7}$

B) $\frac{1}{8}$

C) $\frac{1}{9}$

D) $\frac{1}{10}$

E) $\frac{1}{12}$

5. **$\arctan\frac{1}{2} + \arctan\frac{1}{3} = ?$**

A) $\frac{\pi}{2}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{5}$

E) $\frac{\pi}{6}$

6. **$\arctan(-\sqrt{3}) + \arccos\left(-\frac{1}{2}\right) = ?$**

A) $\frac{\pi}{3}$

B) $\frac{2\pi}{3}$

C) $\frac{\pi}{4}$

D) $\frac{3\pi}{4}$

E) π

7. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) + \text{arccot}\,1 = ?$

A) $-\frac{\pi}{12}$

B) $-\frac{\pi}{6}$

C) $-\frac{\pi}{5}$

D) $-\frac{\pi}{4}$

E) $-\frac{\pi}{3}$

8. $\sin(\arctan -2) = ?$

A) $-\frac{3\sqrt{5}}{5}$

B) $-\frac{2\sqrt{5}}{5}$

C) $-\frac{\sqrt{5}}{5}$

D) $\frac{\sqrt{5}}{5}$

E) $\frac{2\sqrt{5}}{5}$

9. $\sin\left(\text{arccot} -\frac{1}{3}\right) = ?$

A) $\frac{3\sqrt{10}}{10}$

B) $-\frac{2\sqrt{10}}{10}$

C) $-\frac{\sqrt{10}}{10}$

D) $\frac{\sqrt{10}}{10}$

E) $\frac{2\sqrt{10}}{10}$

10. $\tan\left(\arcsin\frac{\sqrt{5}}{5}\right) = ?$

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{5}$

E) $\frac{1}{6}$

11. $\arccos x + \arcsin x = ?$

A) $\frac{\pi}{6}$

B) $\frac{\pi}{5}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{3}$

E) $\frac{\pi}{2}$

12. $\arcsin\left(\cos\left(-\frac{\pi}{6}\right)\right) = ?$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{5}$

D) $\frac{\pi}{6}$ E) $\frac{\pi}{8}$

YANITLAR / ANSWERS

1. D 2. D 3. C 4. C 5. C 6. A 7. A 8. B
9. A 10. A 11. E 12. A

1. $\sin x = \frac{1}{2} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{\pi}{3} + 2k\pi \lor x = \frac{2\pi}{3} + 2k\pi\right\}$

B) $\left\{x \mid x = \frac{\pi}{6} + 2k\pi \lor x = \frac{5\pi}{6} + 2k\pi\right\}$

C) $\left\{x \mid x = \frac{\pi}{4} + 2k\pi \lor x = \frac{3\pi}{4} + 2k\pi\right\}$

D) $\left\{x \mid x = \frac{3\pi}{5} + 2k\pi \lor x = \frac{2\pi}{5} + 2k\pi\right\}$

E) $\left\{x \mid x = \frac{\pi}{2} + 2k\pi \lor x = \frac{3\pi}{2} + 2k\pi\right\}$

2. $\cos x = -1 \Rightarrow$ **ÇK(SS) = ?**

A) $\{x \mid x = \pi + 2k\pi\}$

B) $\left\{x \mid x = \frac{\pi}{2} + 2k\pi\right\}$

C) $\left\{x \mid x = \frac{3\pi}{2} + 2k\pi\right\}$

D) $\left\{x \mid x = \frac{11\pi}{6} + 2k\pi\right\}$

E) $\{x \mid x = 2k\pi\}$

3. $\sin 4x = \sin\left(x + \frac{\pi}{4}\right) \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{\pi}{3} + \frac{2k\pi}{4} \lor x = \frac{\pi}{4} + \frac{2k\pi}{3}\right\}$

B) $\left\{x \mid x = \frac{\pi}{6} + \frac{2k\pi}{5} \lor x = \frac{3\pi}{2} + 2k\pi\right\}$

C) $\left\{x \mid x = \frac{\pi}{2} + 2k\pi \lor x = \frac{3\pi}{4} + 2k\pi\right\}$

D) $\left\{x \mid x = \frac{\pi}{6} + \frac{2k\pi}{6} \lor x = \frac{2\pi}{5} + \frac{2k\pi}{5}\right\}$

E) $\left\{x \mid x = \frac{\pi}{12} + \frac{2k\pi}{3} \lor x = \frac{3\pi}{20} + \frac{2k\pi}{5}\right\}$

4. $2\cos^2 x + 5\cos x - 3 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{\pi}{6} + 2k\pi \lor x = \frac{5\pi}{6} + 2k\pi\right\}$

B) $\left\{x \mid x = \frac{\pi}{4} + 2k\pi \lor x = \frac{3\pi}{4} + 2k\pi\right\}$

C) $\left\{x \mid x = \frac{\pi}{3} + 2k\pi \lor x = \frac{5\pi}{3} + 2k\pi\right\}$

D) $\left\{x \mid x = \frac{3\pi}{7} + 2k\pi \lor x = \frac{4\pi}{7} + 2k\pi\right\}$

E) $\left\{x \mid x = \frac{\pi}{2} + 2k\pi \lor x = \pi + 2k\pi\right\}$

5. $\cos 3x = \cos\frac{4\pi}{5} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{5\pi}{12} + \frac{2k\pi}{4} \lor x = \frac{4\pi}{3} + \frac{2k\pi}{5}\right\}$

B) $\left\{x \mid x = \frac{4\pi}{15} + \frac{2k\pi}{3} \lor x = \frac{2\pi}{5} + \frac{2k\pi}{3}\right\}$

C) $\left\{x \mid x = \frac{3\pi}{8} + \frac{2k\pi}{3} \lor x = \frac{\pi}{6} + \frac{2k\pi}{3}\right\}$

D) $\left\{x \mid x = \frac{\pi}{3} + \frac{k\pi}{4} \lor x = \frac{2\pi}{3} + \frac{k\pi}{4}\right\}$

E) $\left\{x \mid x = \frac{3\pi}{4} + \frac{k\pi}{5} \lor x = \frac{\pi}{3} + \frac{k\pi}{5}\right\}$

6. $x \in [0, \pi)$,

$2\sin x + \cos 2x = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$

B) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

C) $\left\{\frac{\pi}{2}, \frac{3\pi}{4}\right\}$

D) $\left\{0, \frac{\pi}{2}\right\}$

E) $\left\{\frac{\pi}{2}, \frac{3\pi}{4}\right\}$

7. $x \in [0, \pi]$,

$\sin^3 x - \frac{1}{4}\sin x = 0 \Rightarrow \boldsymbol{\sum x = ?}$

A) 0

B) $\frac{\pi}{2}$

C) π

D) $\frac{3\pi}{2}$

E) 2π

8. $x \in [0, \pi]$,

$\sin 2x - \cos x = 0 \Rightarrow \boldsymbol{\sum x = ?}$

A) 0

B) $\frac{\pi}{3}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{2}$

E) $\frac{3\pi}{2}$

9. $x \in [0, \pi]$,

$\cos^2 x + 3\cos x - 4 = 0 \Rightarrow \boldsymbol{\sum x = ?}$

A) 0

B) $\frac{\pi}{6}$

C) $\frac{\pi}{2}$

D) $\frac{3\pi}{4}$

E) π

10. $\sin 2x = \cos\left(x + \frac{\pi}{3}\right) \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{\pi}{3} + \frac{2k\pi}{3} \vee x = \frac{2\pi}{3} + \frac{2k\pi}{3}\right\}$

B) $\left\{x \mid x = \frac{4\pi}{3} + \frac{2k\pi}{3} \vee x = \frac{\pi}{2} + k\pi\right\}$

C) $\left\{x \mid x = \frac{\pi}{8} + \frac{2k\pi}{4} \vee x = \frac{\pi}{3} + 2k\pi\right\}$

D) $\left\{x \mid x = \frac{\pi}{12} + \frac{2k\pi}{5} \vee x = \frac{2\pi}{3} + 2k\pi\right\}$

E) $\left\{x \mid x = \frac{\pi}{18} + \frac{2k\pi}{3} \vee x = \frac{5\pi}{6} + 2k\pi\right\}$

11. $x \in [0, \pi]$,

$\cos 2x - 3\sin x + 1 = 0 \Rightarrow \boldsymbol{\sum x = ?}$

A) $\frac{\pi}{3}$

B) $\frac{\pi}{6}$

C) $\frac{\pi}{2}$

D) $\frac{5\pi}{6}$

E) π

12. $x \in [0, \pi)$,

$\sin\left(x + \frac{\pi}{3}\right) = \sin\left(2x - \frac{\pi}{3}\right) \Rightarrow \boldsymbol{\sum x = ?}$

A) $\frac{4\pi}{3}$ B) $\frac{3\pi}{2}$ C) $\frac{7\pi}{4}$

D) $\frac{11\pi}{3}$ E) π

YANITLAR / ANSWERS

1. B	**2. A**	**3. E**	**4. C**	**5. B**	**6. D**	**7. E**	**8. E**
9. A	**10. E**	**11. E**	**12. E**				

KARMAŞIK SAYILAR
COMPLEX NUMBERS

BÖLÜM 3
CHAPTER 3

İkinci dereceden bir denklemde $\Delta < 0$ iken çözüm kümesi boş küme, yani; bu denklemi sağlayan reel kök yoktur. Bu nedenle, $\Delta < 0$ durumunda denklemin çözüm kümesini bulmak için reel sayılardan daha geniş bir kümeye gerek vardır.

We said that the solution set is empty set when $\Delta < 0$ in quadratic equation i.e, we have not got real root of this equation. In situation $\Delta < 0$, there are need to set more larger than real numbers for the solution set.

NOT / NOTE

$N^{+} \subset N \subset Z \subset Q \subset R \subset C$

Tanım: Sanal sayı birimi $i = \sqrt{-1}$ ile tanımlanır.
Definition: *The imaginary unit i is defined by $i = \sqrt{-1}$.*

Tanım: $a, b \in R$ olmak üzere, $z = a + bi$ şeklinde yazılan sayılara *karmaşık sayılar,* bu sayıların oluşturduğu kümeye de *karmaşık sayılar kümesi* denir ve C ile gösterilir.
Definition: *A complex numbers is a number of the form a + bi where a and b are real numbers. The set of complex numbers is consist of complex numbers and this set is denoted by C.*

Tanım: $z = a + bi$ karmaşık sayısında, a sayısına karmaşık sayının *reel kısmı* denir ve $Re(z) = a$ ile gösterilir.
Definition: *In complex number $z = a + bi$, the real number a is called the real part of the complex number and that is denoted by $Re(z) = a$.*

Tanım: $z = a + bi$ karmaşık sayısında, b sayısına karmaşık sayının *sanal (imajiner) kısmı* denir ve $Im(z)$ ile gösterilir.
Definition: *In complex number $z = a + bi$, the real number b is called the imaginary part of the complex number and that is denoted by $Im(z) = b$.*

ÖRNEK / EXAMPLE 1

$A = \sqrt{-4} \cdot \sqrt{-9} + \sqrt{-2} \cdot \sqrt{-8} \Rightarrow$ **A = ?**

A) –2 B) –4 C) –6 D) –8 E) –10

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\sqrt{-4} = 2\sqrt{-1} = 2i$

$\sqrt{-9} = 3\sqrt{-1} = 3i$

$\sqrt{-2} = \sqrt{-1} \cdot \sqrt{2} = \sqrt{2}i$

$\sqrt{-8} = 2\sqrt{2} \cdot \sqrt{-1} = 2\sqrt{2}i$

$A = (2i)\cdot(3i) + (\sqrt{2}i)\cdot(2\sqrt{2}i) = 6i^2 + 4i^2 = 10i^2 = -10$

PEKİŞTİRME / REINFORCE

$\sqrt{-36} + \sqrt{-16} = ?$

YANIT / ANSWER : 10i

PEKİŞTİRME / REINFORCE

$2x^2 - 3x + 2 = 0 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : $(3 + \sqrt{7}\, i) / 4$

NOT / NOTE

$i^m \cdot i^n = i^{m+n}$

ÖRNEK / EXAMPLE 2

$x^2 - 2x + 10 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {± 2i}
B) {1 ± 3i}
C) {2 ± 3i}
D) {± 5i}
E) {2 ± 5i}

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$x^2 - 2x + 10 = 0$$

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-2)^2 - 4\cdot 1\cdot 10 \\ &= -36 < 0 \end{aligned}$$

$$x_{1,2} = \frac{-(-2) \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$$

i' nin TAMSAYI KUVVETLERİ / INTEGER POWERS of i

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$i^5 = i$
$i^6 = -1$

$$i^n = \begin{cases} 1, & n \equiv 0 \pmod 4 \\ i, & n \equiv 1 \pmod 4 \\ -1, & n \equiv 2 \pmod 4 \\ -i, & n \equiv 3 \pmod 4 \end{cases}$$

ÖRNEK / EXAMPLE 3

$i^{101} + i^{202} + i^{303} + i^{404} = ?$

A) –1 B) 1 C) –i D) i E) 0

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\begin{array}{r|l} 101 & \underline{4} \\ -100 & 25 \\ \hline \end{array}$ ①
$\begin{array}{r|l} 202 & \underline{4} \\ - & \\ \hline \end{array}$ ②
$\begin{array}{r|l} 303 & \underline{4} \\ - & \\ \hline \end{array}$ ③
$\begin{array}{r|l} 404 & \underline{4} \\ - & \\ \hline \end{array}$ ④

$$\begin{aligned} i^{101} + i^{202} + i^{303} + i^{404} &= i^1 + i^2 + i^3 + i^4 \\ &= i - 1 - i + 1 = 0 \end{aligned}$$

▼ **I.** $(i + 1)^2 = i^2 + 2i + 1 = -1 + 2i + 1 = 2i$

II. $(i - 1)^2 = i^2 - 2i + 1 = -1 - 2i + 1 = -2i$

ÖRNEK / EXAMPLE 4

$(1 + i)^{28} = ?$

A) $2^{13}i$ B) 2^{14} C) $2^{14}i$ D) -2^{14} E) $-2^{14}i$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{aligned} (1 + i)^{28} &= ((i + 1)^2)^{14} \\ &= (2i)^{14} \\ &= 2^{14} \cdot i^2 \\ &= -2^{14} \end{aligned}$$

$\begin{array}{r|l} 14 & \underline{4} \\ - & 3 \\ \hline \end{array}$ ②

PEKİŞTİRME / REINFORCE

$(i - 1)^{16} = ?$

YANIT / ANSWER : 2^8

İKİ KARMAŞIK SAYININ EŞİTLİĞİ / EQUALITY of TWO COMPLEX NUMBERS

$z_1 = a + bi$
$z_2 = c + di,$
$z_1 = z_2 \Rightarrow a = c, \; b = d$

ÖRNEK / EXAMPLE 5

$3x + 5 + yi = 8 + (2x - y)i \Rightarrow$ **$x - y = ?$**

A) 0 B) 1 C) 2 D) 3 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$3x + 5 + yi = 8 + (2x - y)i$$
$$\Rightarrow \; 3x + 5 = 8 \quad , \quad y = 2x - y$$
$$3x = 3 \qquad 2y = 2x$$
$$x = 1 \qquad y = x = 1$$
$$\Rightarrow \; x - y = 1 - 1 = 0$$

KARMAŞIK SAYILARIN EŞLENİĞİ / CONJUGATE COMPLEX NUMBERS

$z = a + bi$ karmaşık sayısının eşleniği $\overline{z} = a - bi$ dir.

The conjugate of $z = a + bi$ is denoted by $\overline{z}$ and $\overline{z} = a - bi$.

Eşlenik Özellikleri / Properties of Conjugates

1. $\overline{\left(\overline{z}\right)} = z$

2. $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

3. $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

4. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}, \; (z_2 \neq 0)$

5. $z = a + bi \Rightarrow z \cdot \overline{z} = a^2 + b^2$

ÖRNEK / EXAMPLE 6

$z = \frac{2-i}{3+i} \Rightarrow \text{Im}(\bar{z}) = ?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$z = \frac{2-i}{\underset{(3-i)}{3+i}} = \frac{6-2i-3i+i^2}{9-i^2}$$

$$= \frac{5-5i}{10}$$

$$= \frac{5}{10} - \frac{5}{10}i$$

$$= \frac{1}{2} - \frac{1}{2}i \Rightarrow \bar{z} = \frac{1}{2} + \frac{1}{2}i$$

$$\Rightarrow \text{Im}(\bar{z}) = \frac{1}{2}$$

PEKİŞTİRME / REINFORCE

$z = \frac{1-i}{3+i} \Rightarrow \text{Re}(z) = ?$

YANIT / ANSWER : 1 / 5

ÖRNEK / EXAMPLE 7

$2z + 1 = \bar{z} \cdot i + 9 - 7i \Rightarrow z = ?$

A) 4 – i B) 3 + 4i C) 3 – 2i D) 5 + i E) 5 + 2i

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$2z + 1 = \bar{z} \cdot i + 9 - 7i$

$2(a + bi) + 1 = (a - bi)\cdot i + 9 - 7i$

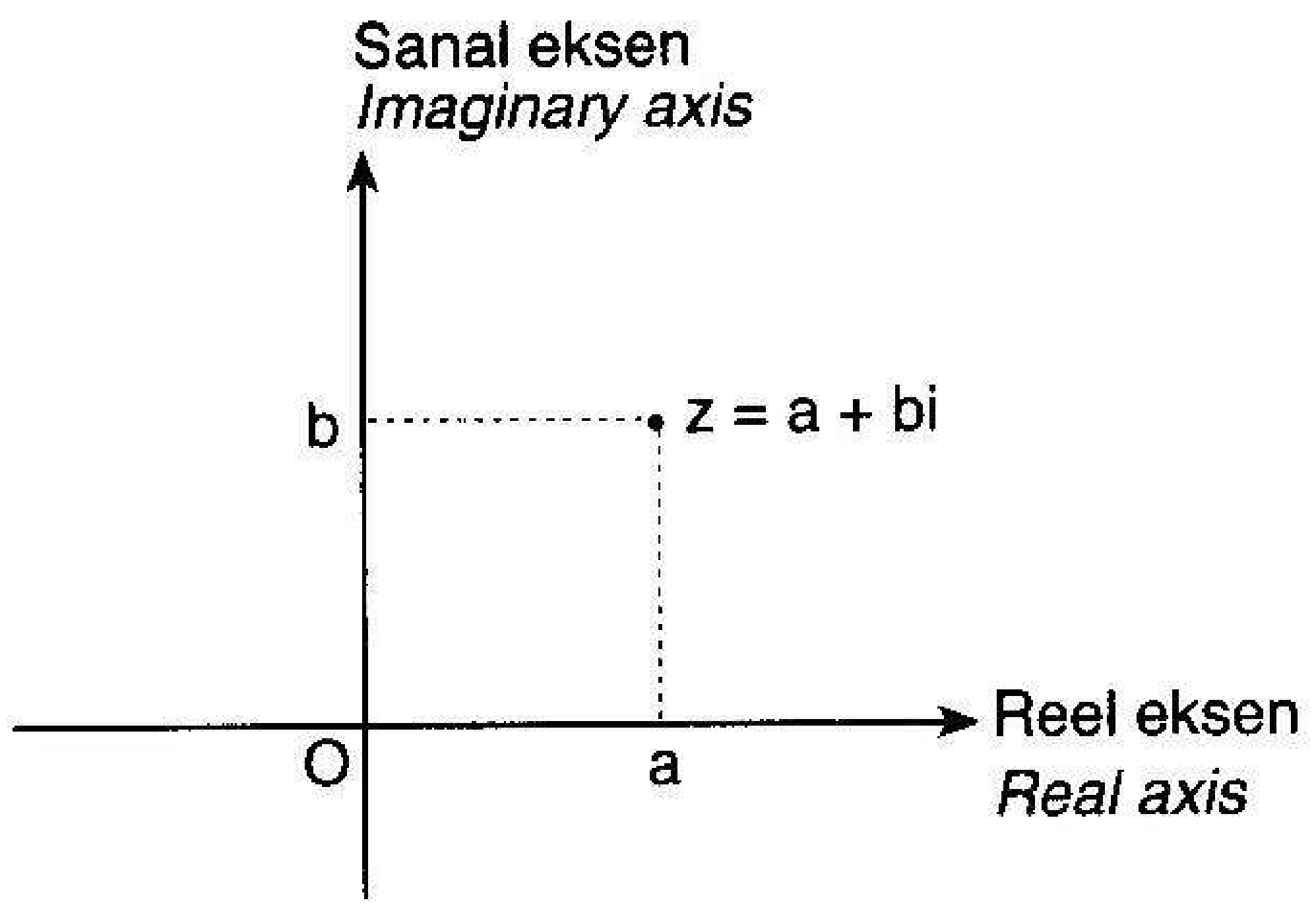
$2a + 2bi + 1 = ai + b + 9 - 7i$

$(2a + 1) + 2bi = (a - 7)i + b + 9$

$$\begin{array}{rcl} 2a + 1 = b + 9 & \Rightarrow & 2a - b = 8 \\ 2b = a - 7 & \Rightarrow & \underline{+\ a - 2b = 7} \\ & & 3b = -6 \\ & & b = -2\ ,\ a = 3 \end{array}$$

$\Rightarrow z = 3 - 2i$

KARMAŞIK SAYILARIN ANALİTİK DÜZLEMDE GÖSTERİLMESİ
INTERPRETATION of COMPLEX NUMBERS on ANALYTIC PLANE

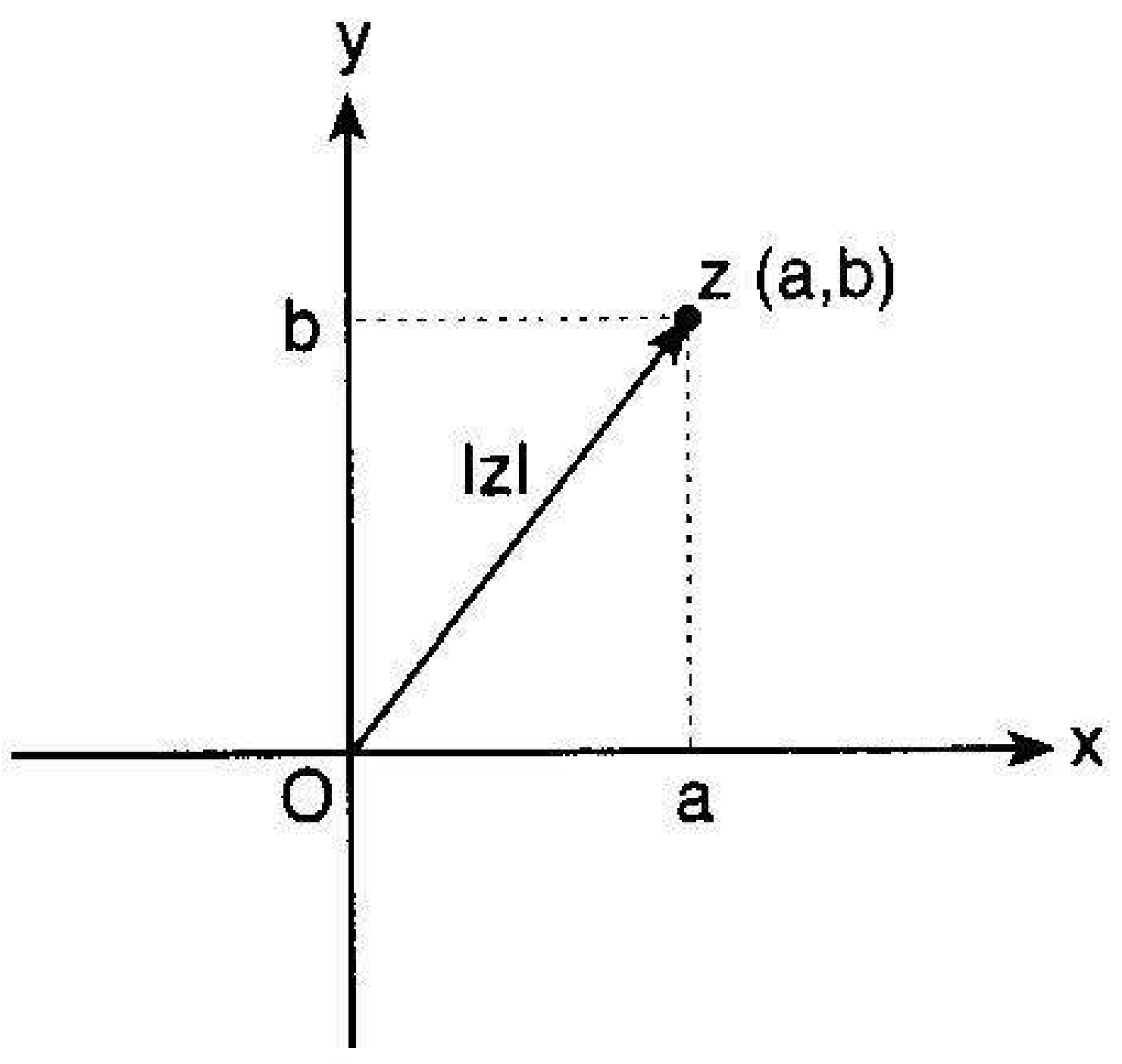


BİR KARMAŞIK SAYININ MUTLAK DEĞERİ (MODÜLÜ)
ABSOLUTE VALUE (MODULUS) of a COMPLEX NUMBER

Bir karmaşık sayının orijine olan uzaklığına o karmaşık sayının *mutlak değeri (modülü)* denir. |z| biçiminde gösterilir.

The distance from a complex number z to the origin is called the absolute value or modulus of z. It is shown by |z|.

$$|z| = \sqrt{a^2 + b^2}$$



Modülün Özellikleri / Properties of Modulus

1. $z \cdot \bar{z} = |z|^2$
2. $|z| = |\bar{z}| = |-z| = |-\bar{z}|$
3. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \quad (z_2 \neq 0)$
4. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
5. $|z^n| = |z|^n$
6. $\left|\frac{1}{z}\right| = \frac{1}{|z|}$

ÖRNEK / EXAMPLE 8

$z = \dfrac{(2+i)^2 \cdot (3-4i)}{3+i} \Rightarrow |z| = ?$

A) $\dfrac{3\sqrt{10}}{2}$ B) $\dfrac{5\sqrt{10}}{2}$ C) $\dfrac{7\sqrt{10}}{2}$

D) $\dfrac{9\sqrt{10}}{2}$ E) $\dfrac{11\sqrt{10}}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$z = \frac{(2+i)^2 \cdot (3-4i)}{3+i} \Rightarrow |z| = \frac{(\sqrt{2^2+1^2}\,)^2\,(\sqrt{3^2+4^2}\,)}{\sqrt{3^2+1^2}}$$

$$= \frac{5 \cdot 5}{\sqrt{10}} = \frac{25\sqrt{10}}{10} = \frac{5\sqrt{10}}{2}$$

KARMAŞIK SAYILARDA DÖRT İŞLEM / FOUR OPERATIONS in the COMPLEX NUMBERS

$z_1 = a + bi$, $z_2 = c + di$

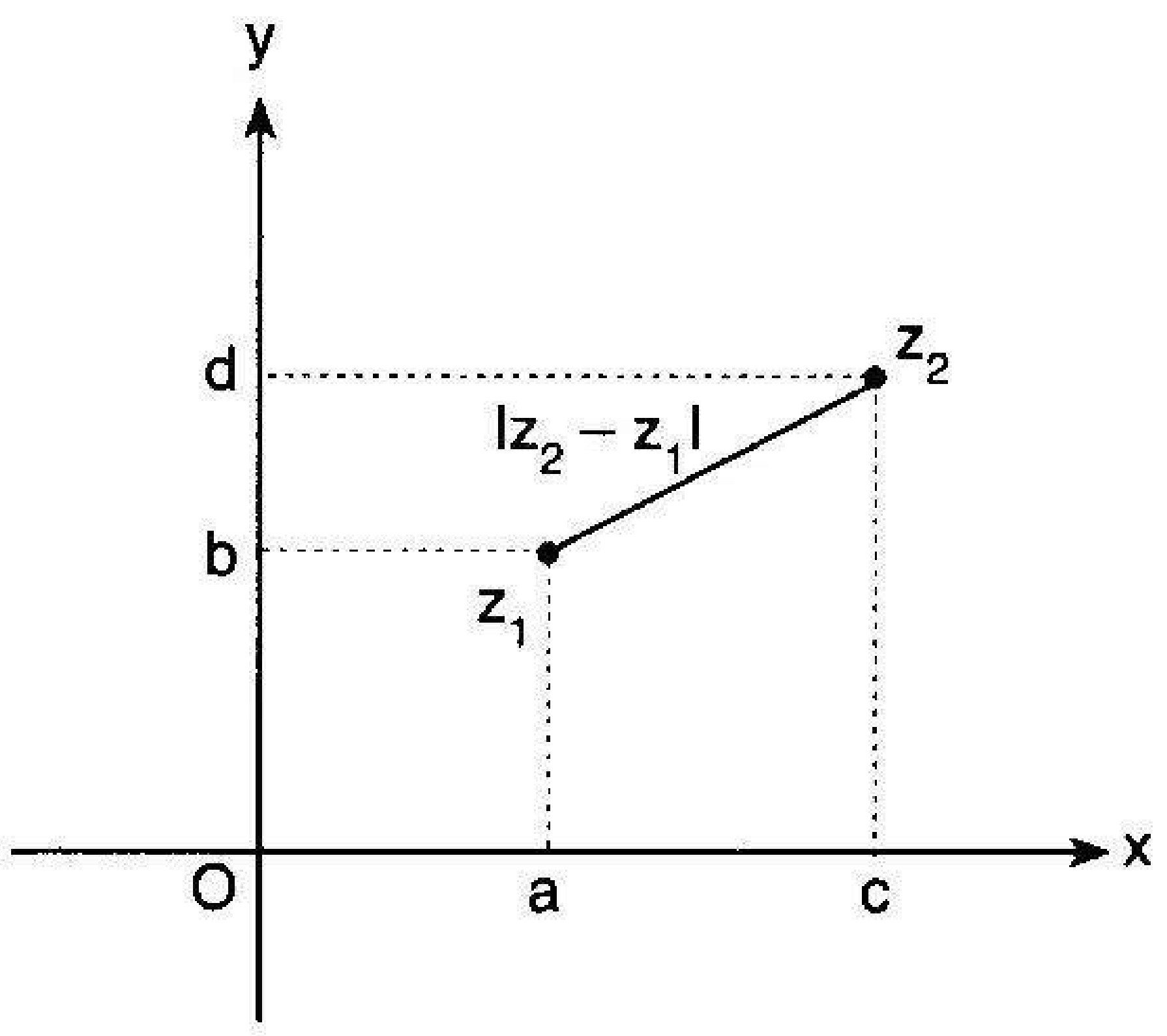
1. $z_1 + z_2 = (a + c) + (b + d)i$

2. $z_1 - z_2 = (a - c) + (b - d)i$

3. $z_1 \cdot z_2 = (a + bi)\cdot(c + di)$

4. $\dfrac{z_1}{z_2} = \dfrac{a+bi}{c+di}$

KARMAŞIK DÜZLEMDE İKİ NOKTA ARASINDAKİ UZAKLIK
THE DISTANCE BETWEEN the TWO POINTS on the COMPLEX PLANE



$z_1 = a + bi$, $z_2 = c + di$

$|z_2 - z_1| = \sqrt{(d-b)^2 + (c-a)^2}$

Düşünün / Think about it

$x, y \in R$,

$(x-7)^2 + (y+24)^2 = 196$

$\Rightarrow \min(x^2 + y^2) = ?$

Yanıt / Answer

100

ÖRNEK / EXAMPLE 9

$x \in R^+$,

$z_1 = x + 4i,\ z_2 = 3 - i$

$|z_1 - z_2| = \sqrt{34} \Rightarrow$ **x = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$|z_1 - z_2| = \sqrt{34}$$

$$(x-3)^2 + (4-(-1))^2 = 34$$

$$(x-3)^2 + 25 = 34$$

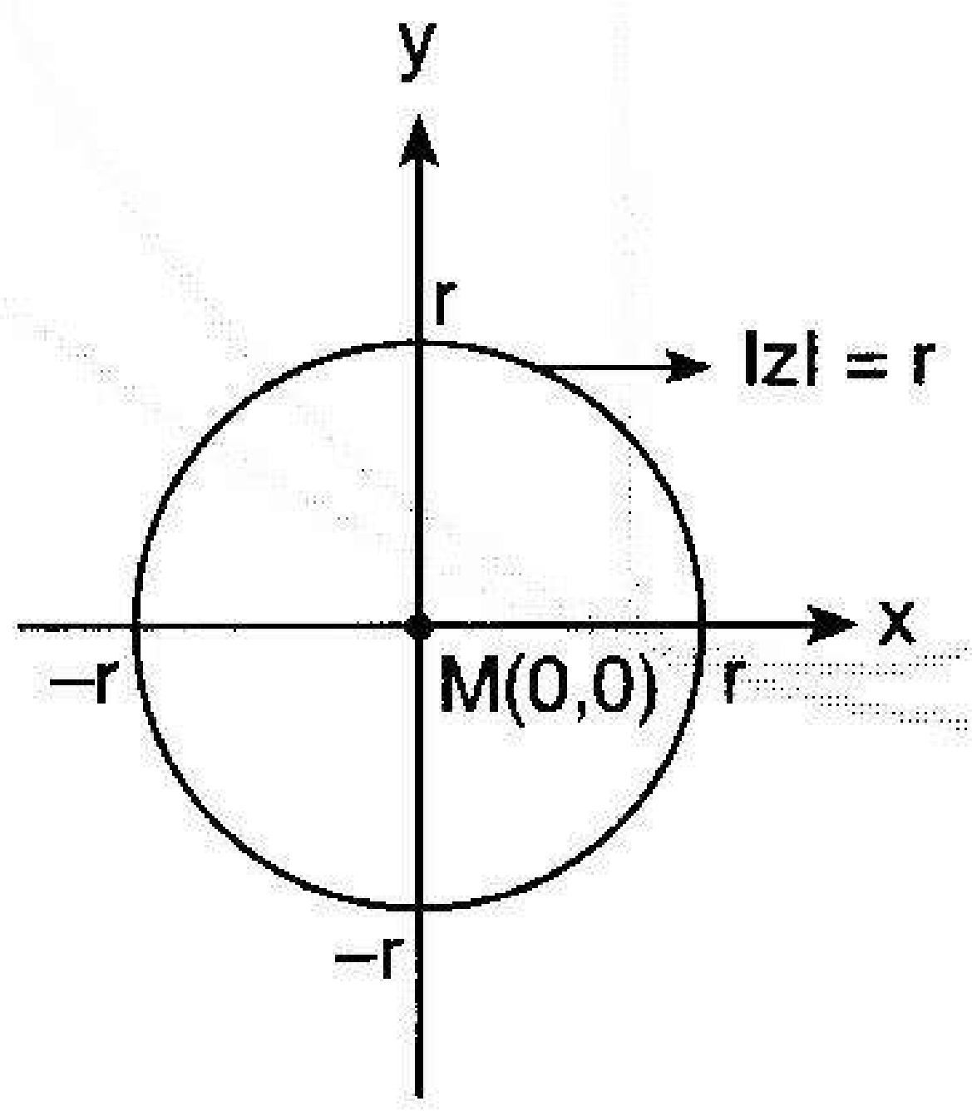
$$(x-3)^2 = 9$$

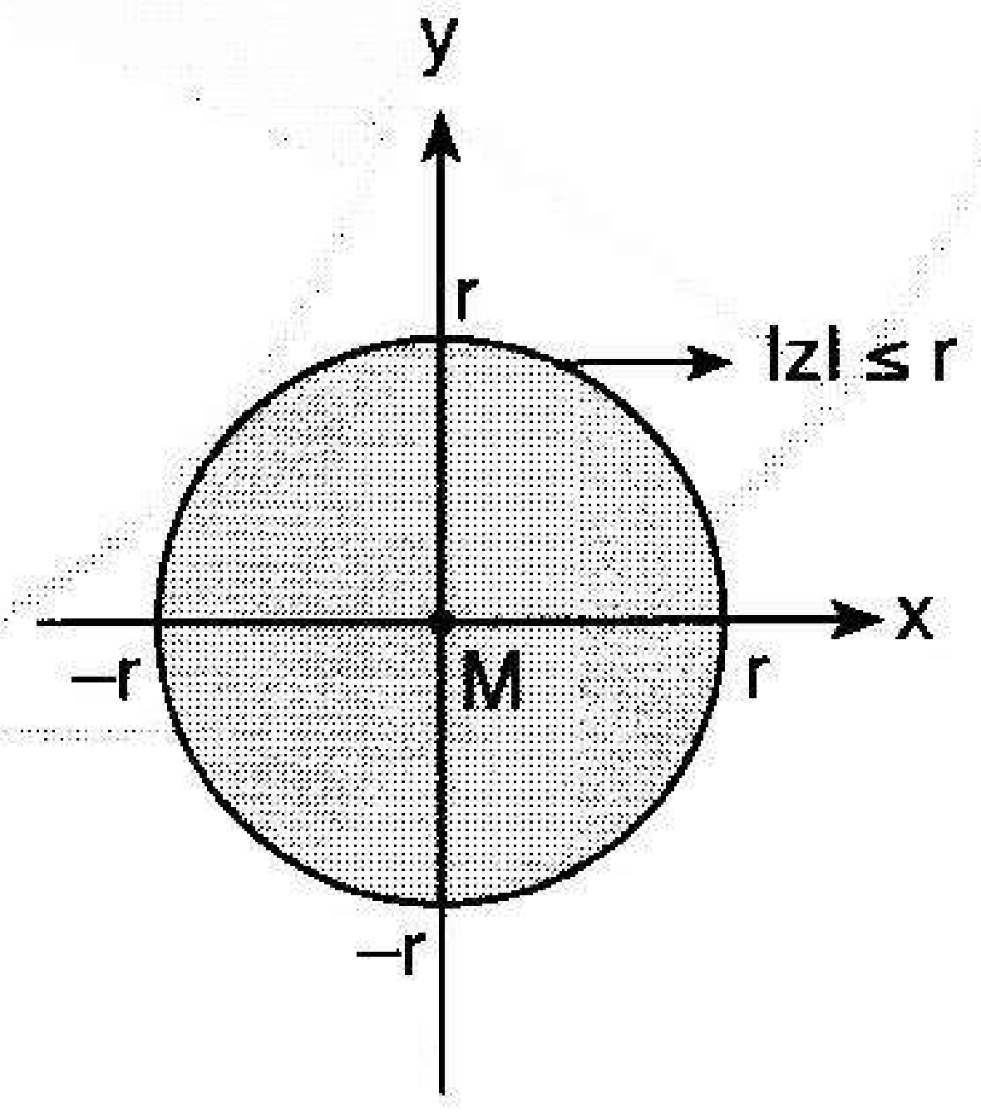
$$x - 3 = 3 \quad , \quad x - 3 = -3$$

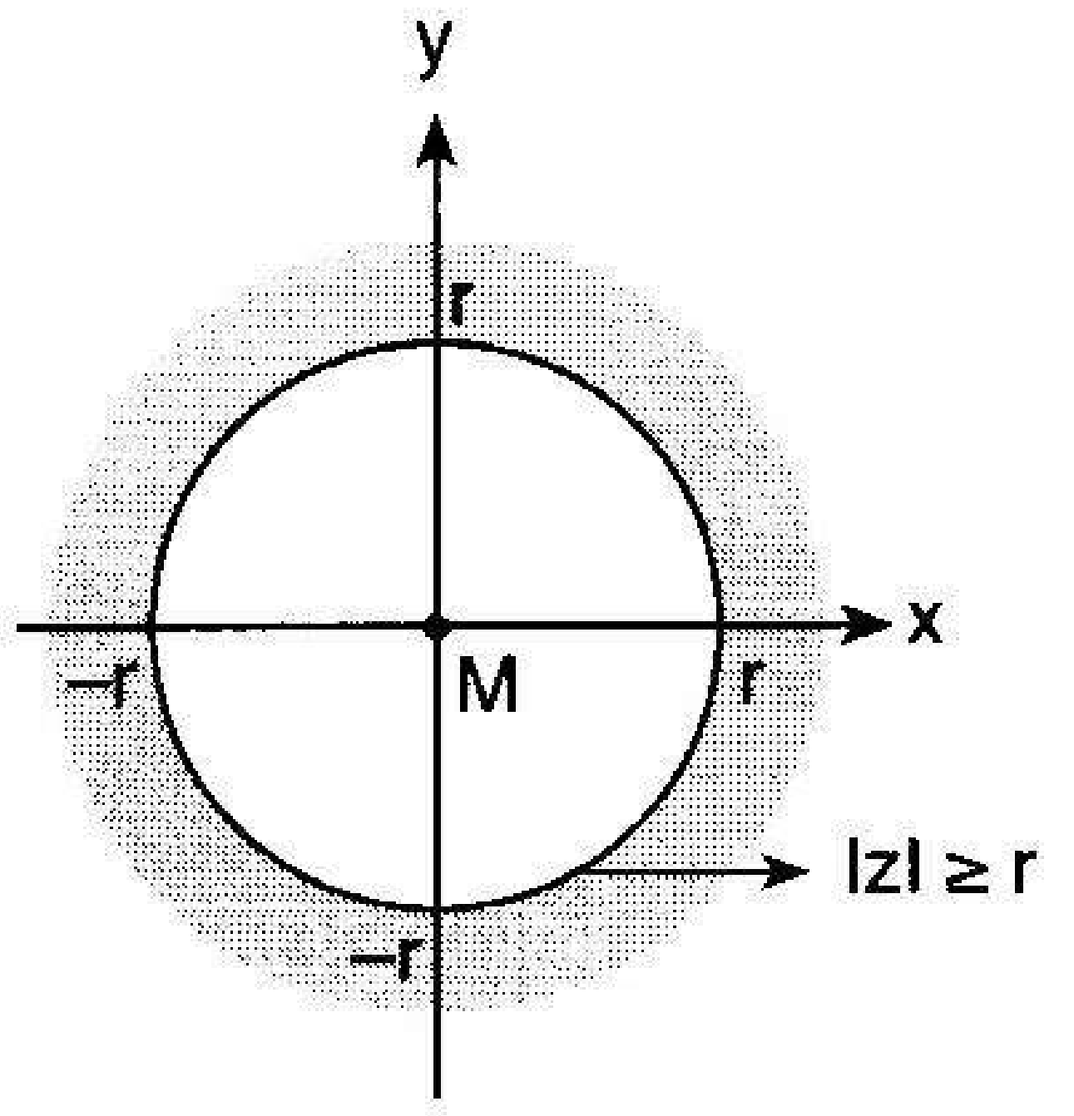
$$x = 6 \qquad x = 0$$

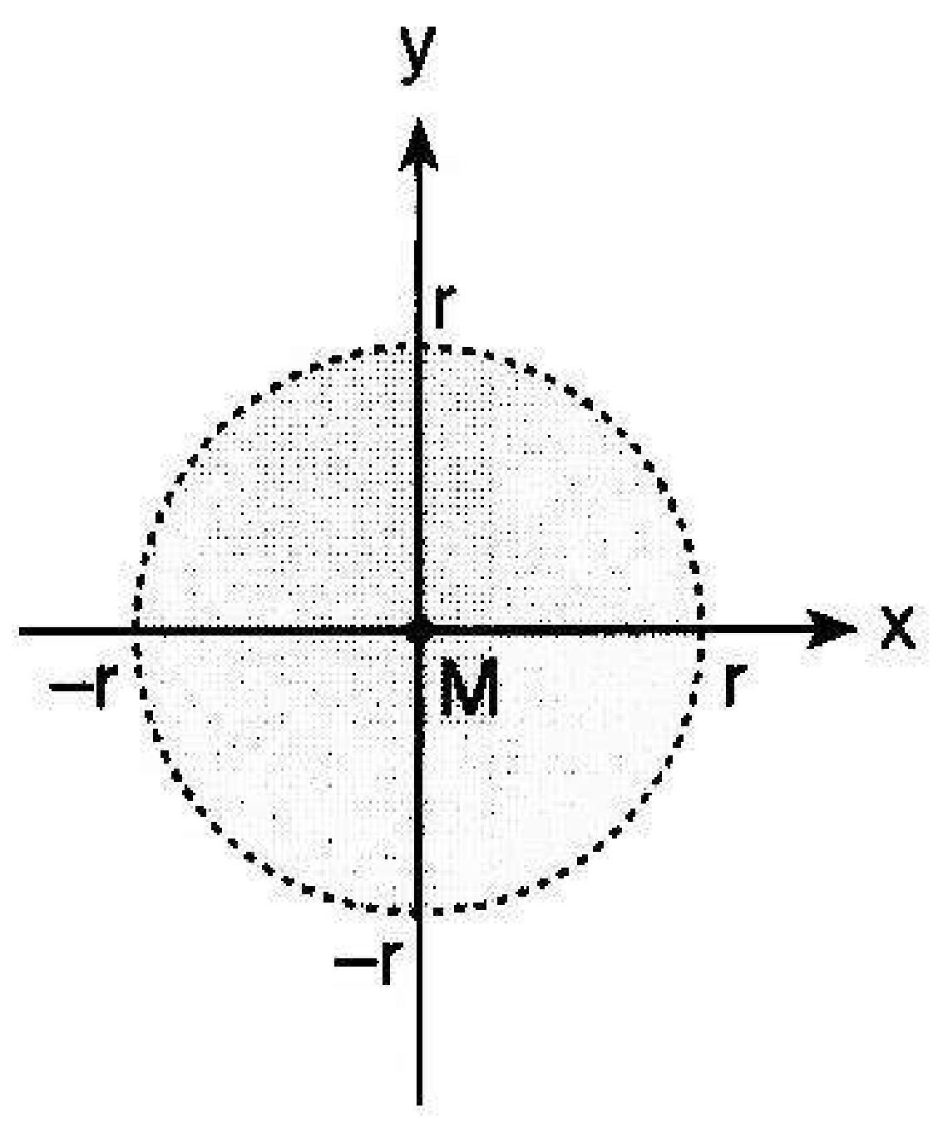
$x \in R^+ \Rightarrow x = 6$

Sonuçlar / Conclusions

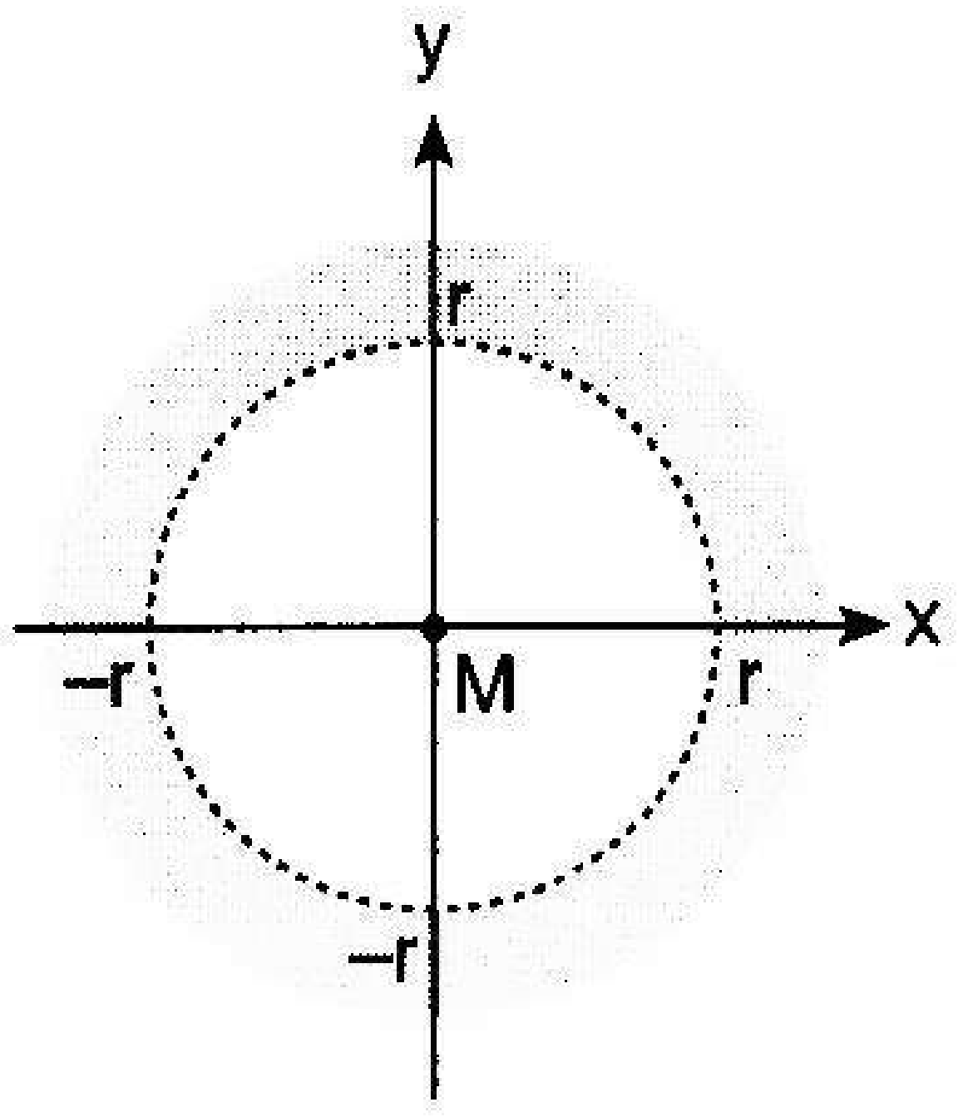

y
r
IzI = r
x
–r
M(0,0)
r
–r


y
r
IzI ≤ r
x
–r
M
r
–r

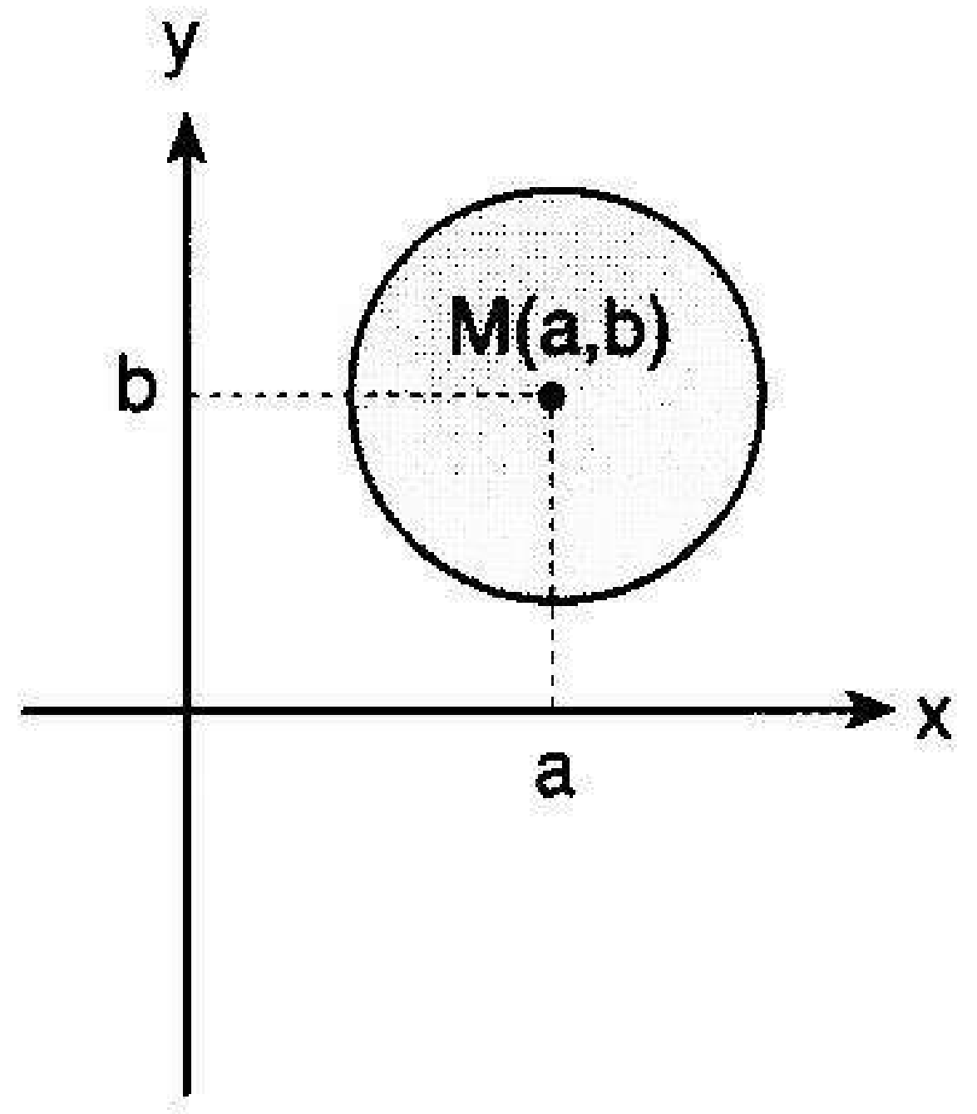

y
r
x
–r
M
r
IzI ≥ r
–r



IzI < r



IzI > r



Iz – (a+bi)I ≤ r

ÖRNEK / EXAMPLE 10

Taralı bölge / *The shaded region* = ?

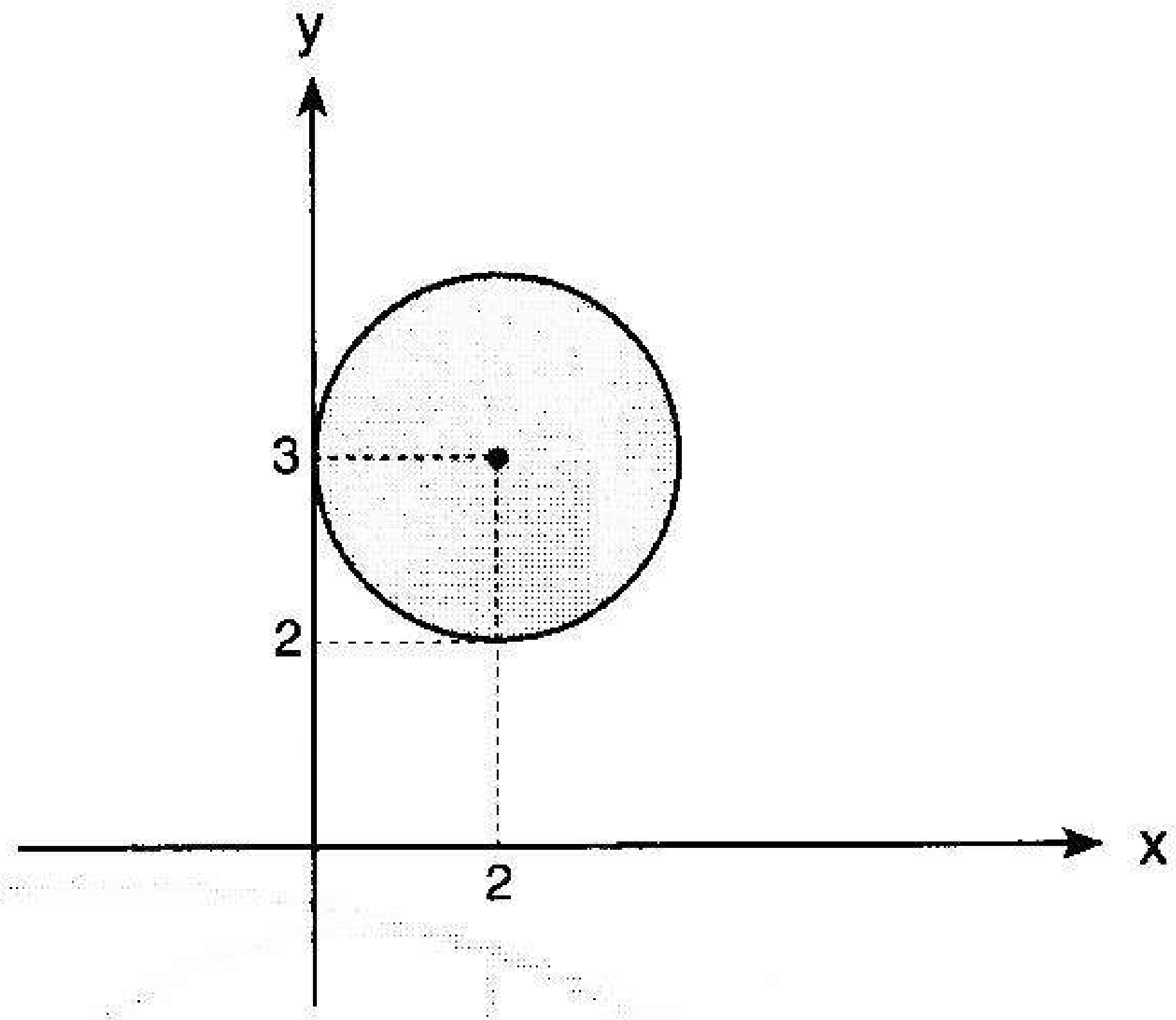
A) $|z - (2 + 3i)| \leq 1$

B) $|z - (2 + 3i)| > 1$

C) $|z - (2 + 3i)| < 1$

D) $|z - (2 + 3i)| \geq 1$

E) $|z + (2 + 3i)| < 1$



ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

M(2, 3) , r = 1

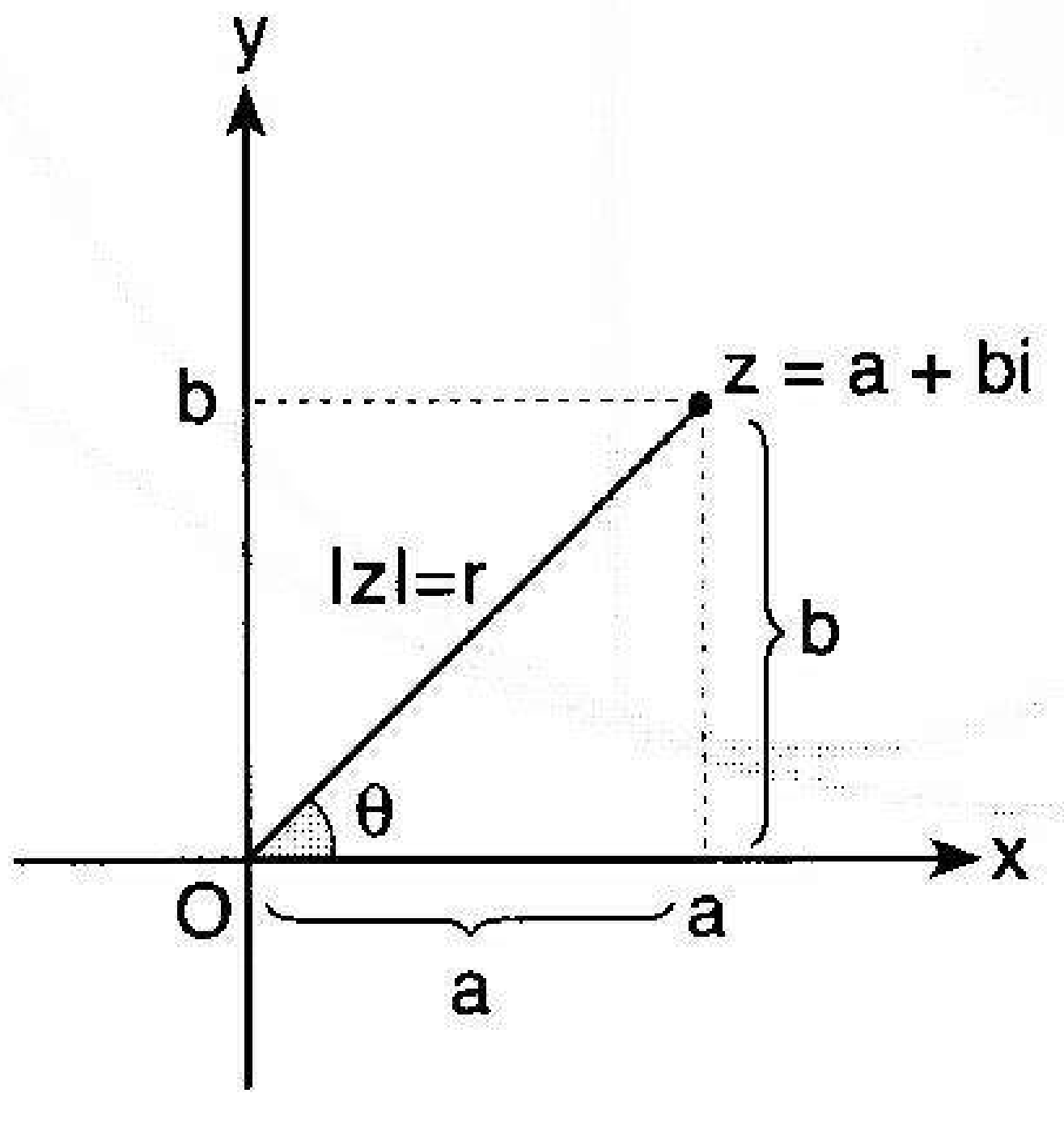
⇒ taralı bölge / *the shaded region* = $|z - (2 + 3i)| \leq 1$

KARMAŞIK SAYILARIN KUTUPSAL FORMDA YAZILIŞI
POLAR (Trigonometric) FORM of COMPLEX NUMBERS

$z = a + bi$,

$\cos\theta = \frac{a}{|z|} \Rightarrow a = |z| \cdot \cos\theta$

$\sin\theta = \frac{b}{|z|} \Rightarrow b = |z| \cdot \sin\theta$



Bu değerler z = a + bi ifadesinde yerine yazılırsa aşağıdaki denklemler bulunur.

Substituting these values in z = a + bi we get

$z = |z| \cdot \cos\theta + i\,|z| \cdot \sin\theta$

$z = |z|\,(\cos\theta + i\sin\theta) = r\,\text{cis}\theta$

Karmaşık sayıların bu şekilde yazılışına *kutupsal (trigonometrik) biçimde yazılış* denir. Ox eksenli ile pozitif yönde yapılan θ gerçel (reel) sayısına *z nin argümenti* denir ve arg(z) ile gösterilir.

This form is called the polar or trigonometric form of a complex number. The angle θ is called the argument of z, shown by arg(z).

NOT / NOTE

$\text{Arg}\, z = \theta \Rightarrow$

✓ $\text{Arg}\, \bar{z} = -\theta$

✓ $\text{Arg}\, z^n = n\theta$

✓ $\text{Arg}\, \frac{1}{z} = -\theta$

ÖRNEK / EXAMPLE 11

$z = 2 - 2i \Rightarrow$ **$z = r \cdot cis\theta = ?$**

A) $2\sqrt{2} \cdot cis\frac{\pi}{4}$

B) $2\sqrt{2} \cdot cis\frac{3\pi}{4}$

C) $2\sqrt{2} \cdot cis\frac{5\pi}{4}$

D) $2\sqrt{2} \cdot cis\frac{7\pi}{4}$

E) $2\sqrt{2} \cdot cis\frac{7\pi}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

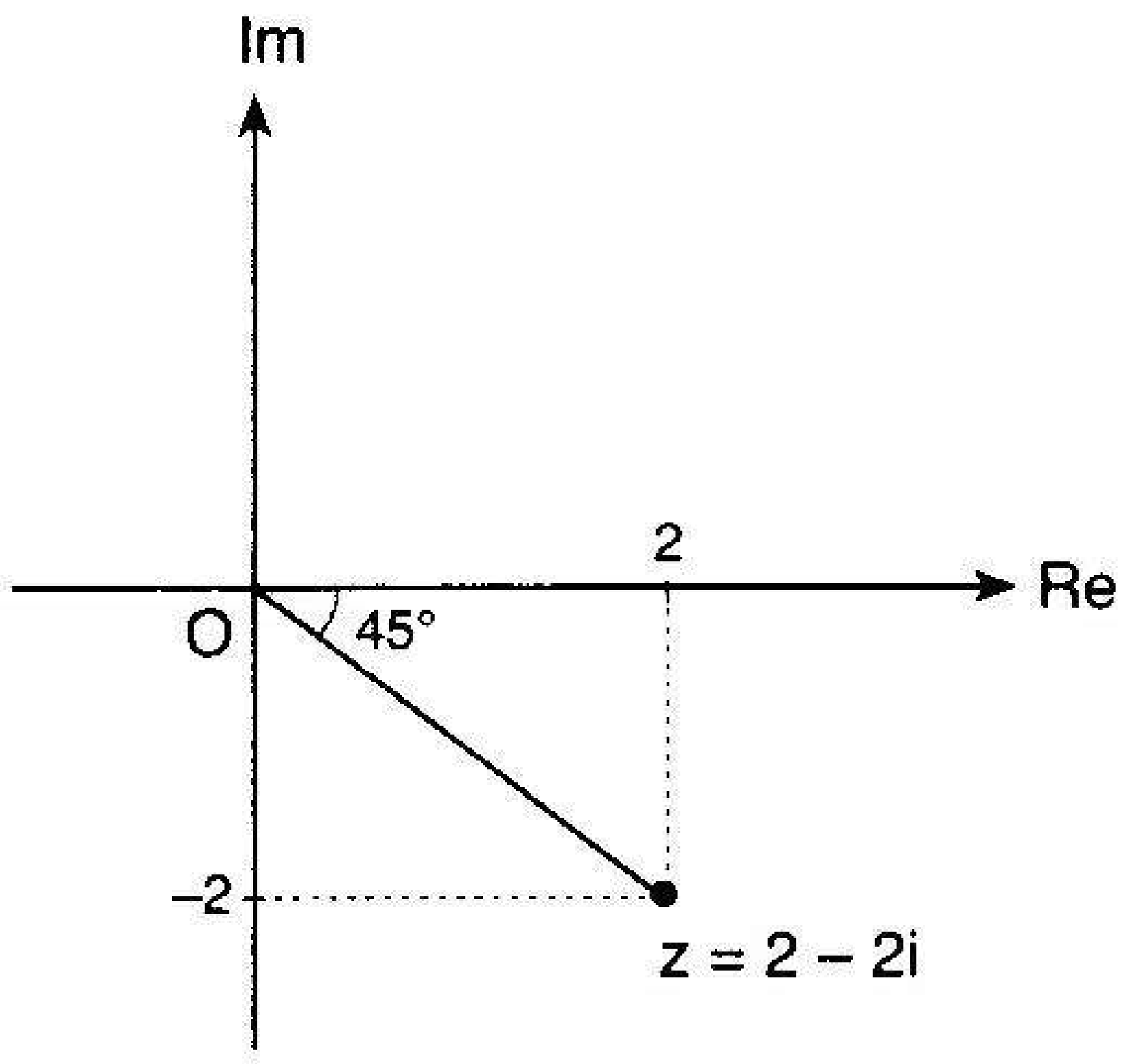
$z = 2 - 2i \Rightarrow Re(z) = 2,\ Im(z) = -2$

$$\tan\theta = \frac{Im(z)}{Re(z)} = \frac{-2}{2} = -1 \Rightarrow \theta = 360° - 45° = 315°$$

$$|z| = r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$$

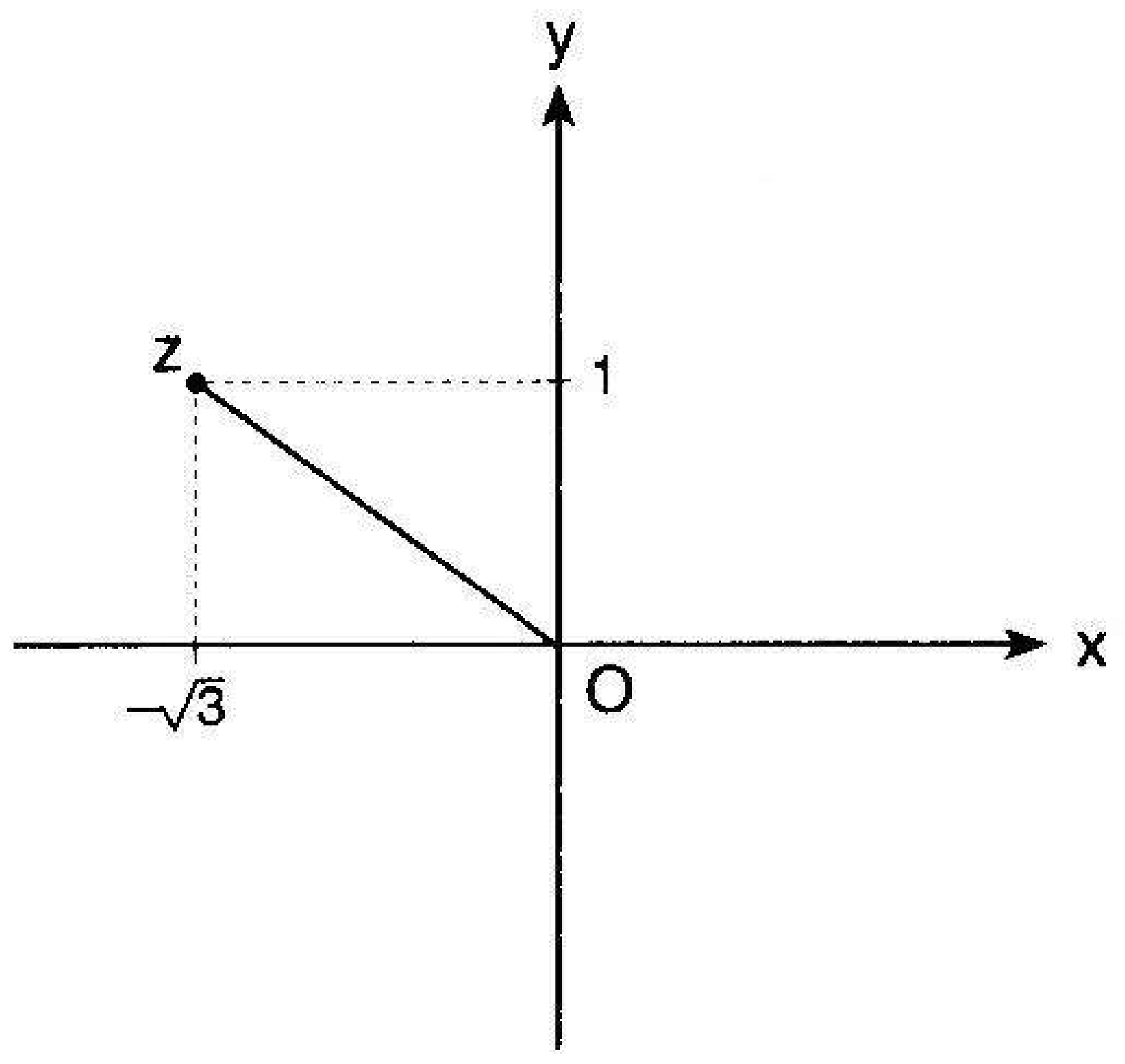
$$z = 2\sqrt{2}\, cis 315°$$

$$= 2\sqrt{2}\, cis\frac{7\pi}{4}$$



ÖRNEK / EXAMPLE 12

z = ?



A) $2 cis\frac{\pi}{6}$

B) $2 cis\frac{5\pi}{6}$

C) $2 cis\frac{11\pi}{6}$

D) $\sqrt{2}\, cis\frac{5\pi}{6}$

E) $\sqrt{2}\, cis\frac{11\pi}{6}$

ÇÖZÜM / SOLUTION

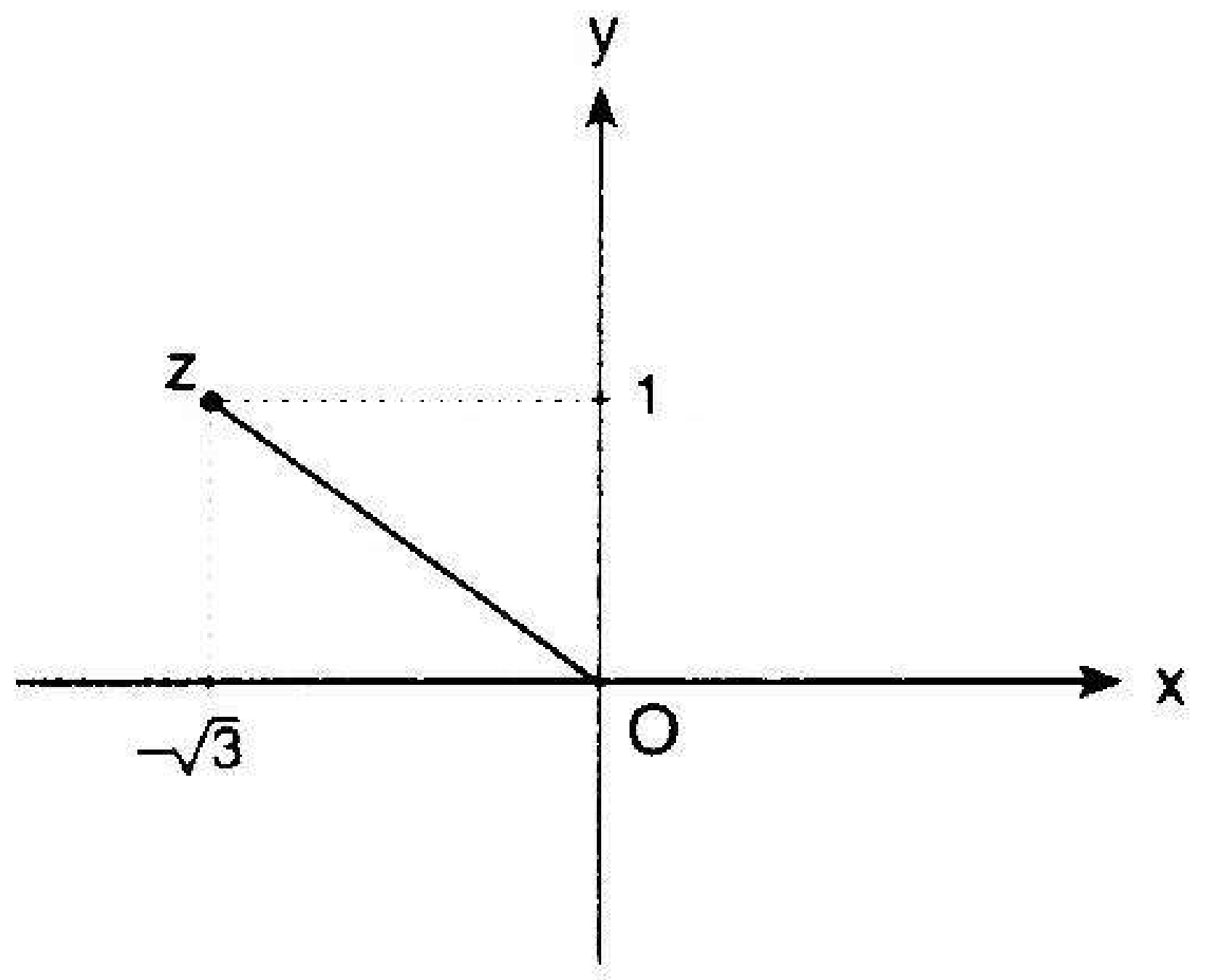
Yanıt / Answer **B**

$$\tan\theta = \frac{Im(z)}{Re(z)} = \frac{-1}{\sqrt{3}}$$

$$\Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$|z| = r = \sqrt{1^2 + (-\sqrt{3})^2} = 2$$

$$z = 2 cis\frac{5\pi}{6}$$



Kutupsal Gösterimin Özellikleri / Properties of Polar Form

$z_1 = r_1 \text{ cis}\theta_1$, $z_2 = r_1 \text{ cis}\theta_2$

1. $z_1 \cdot z_2 = r_1 \cdot r_2 \text{ cis } (\theta_1 + \theta_2)$

2. $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \text{ cis } (\theta_1 - \theta_2)$

3. $z_1^n = r_1^n \text{ cis}(\theta \cdot n)$ [De Moivre]

ÖRNEK / EXAMPLE 13

$z_1 \cdot z_2 = ?$

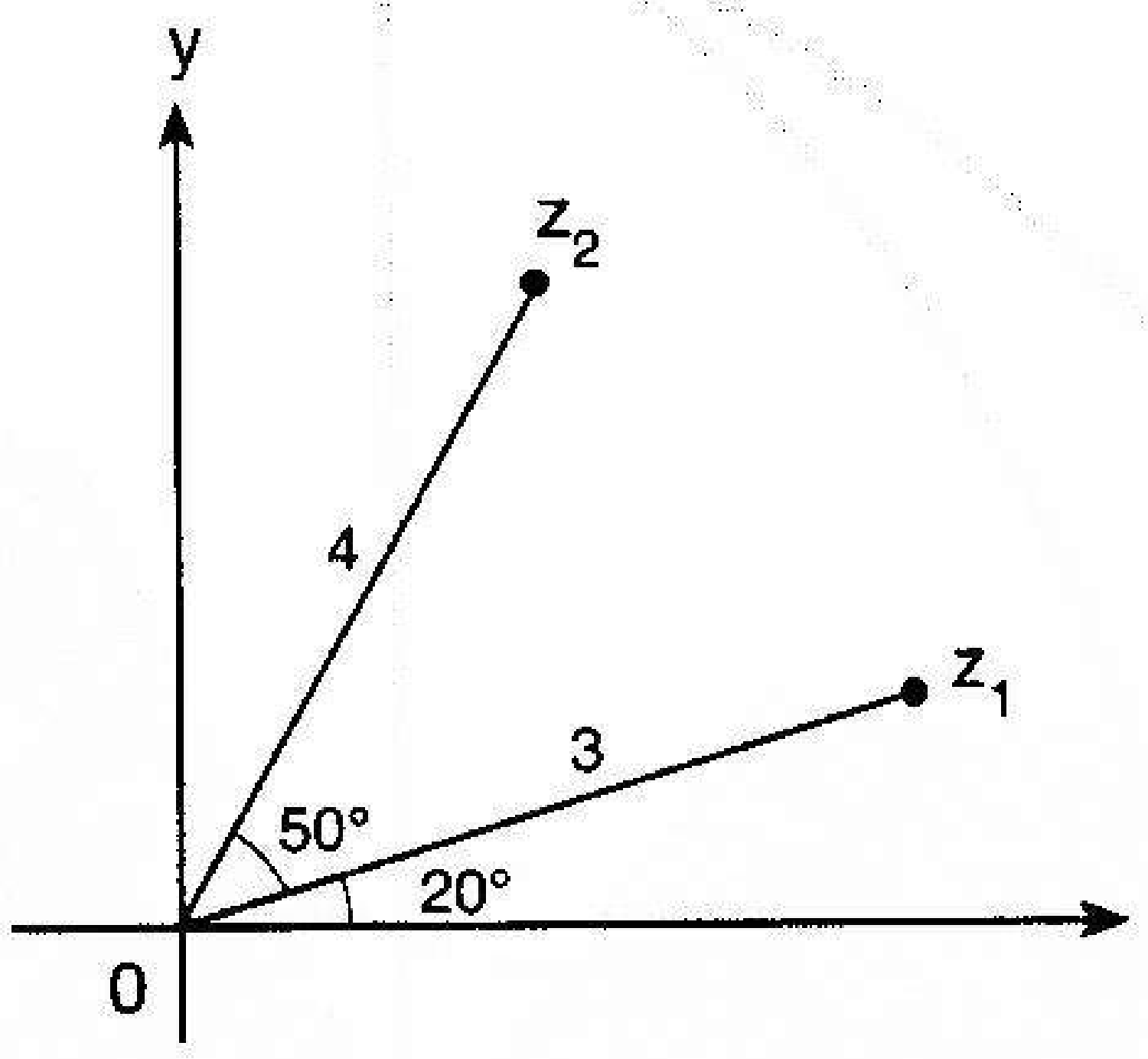
A) –6

B) 6

C) 6i

D) 12

E) 12i



ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$z_1 = 3 \cdot \text{cis}20°$

$z_2 = 4 \cdot \text{cis}70°$

$\Rightarrow z_1 \cdot z_2 = 3 \cdot 4 \text{ cis}(20° + 70°)$

$= 12 \cdot [\cos 90° + i\sin 90°] = 12i$

BİR KARMAŞIK SAYININ KÖKLERİ / ROOTS of a COMPLEX NUMBER

z bir karmaşık sayı ise / *if z is a complex number,*

$z = r\,(\cos(\theta + 2k\pi) + i\,\sin(\theta + 2k\pi))$.

$$w_k = \sqrt[n]{z} = r^{\frac{1}{n}}\left(\cos\frac{1}{n}(\theta + 2k\pi) + i\sin\frac{1}{n}(\theta + 2k\pi)\right)$$

($k \in \{0, 1, 2, 3, \ldots\ldots, k-1\}$) ifadesine z nin n. *kuvvetten kökleri* denir.

the expression above is the n^{th} roots of z.

✓ z karmaşık sayısının kare köklerinden biri w_0 ise ikinci kök $w_1 = -w_0$ dır.

If one of the square roots of z is w_0, then other root is $w_1 = -w_0$

✓ $z = a \pm bi$ karmaşık sayısının kare kökleri aşağıdaki formül ile bulunabilir.

The square roots of $z = a + bi$ can be found also by the following formula.

$$w_{0,1} = \pm\left(\sqrt{\frac{|z|+a}{2}} \pm \sqrt{\frac{|z|-a}{2}}\; i\right)$$

ÖRNEK / EXAMPLE 14

$z = 3 - 4i \Rightarrow w_1 = ?$

A) $i - 2$ B) $1 - i$ C) $-i$ D) $3 - i$ E) $i - 3$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$z = 3 - 4i \Rightarrow$

$a = 3,$

$b = -4,$

$$|z| = \sqrt{3^2 + (-4)^2} = 5$$

$$w_{0,1} = \pm\left(\sqrt{\frac{5+3}{2}} \pm \sqrt{\frac{5-3}{2}}\,i\right)$$

$$= \pm(2 \pm i)$$

ÖRNEK / EXAMPLE 15

YILDIZ TEKNİK ÜNİV. YÖS 2011

$i^2 = -1$

$$\frac{3}{\sqrt{2} - i} + \frac{2}{i^{17}} = ?$$

A) $2\sqrt{2} + i$ B) $2\sqrt{2} - i$ C) $\sqrt{2} - i$

D) $\sqrt{2} + i$ E) $\sqrt{2} + 2i$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{array}{r|l} 17 & \underline{4} \\ -\,16 & 4 \\ \hline \textcircled{1} & \end{array}$$

$$\underset{(\sqrt{2}+i)}{\frac{3}{\sqrt{2} - i}} + \underset{(i)}{\frac{2}{i}} = \frac{3(\sqrt{2} + i)}{2+1} + \frac{2i}{-1}$$

$$= \sqrt{2} + i - 2i$$

$$= \sqrt{2} - i$$

ÖRNEK / EXAMPLE 16

YILDIZ TEKNİK ÜNİ. YÖS 2013

$i = \sqrt{-1}$

$(1 + 3i)^{15} = a - bi$

$\Rightarrow \mathbf{(1 - 3i)^{15} = ?}$

A) $a + bi$
B) $2abi$
C) $a - bi$
D) $(a + bi)^2$
E) $-a - bi$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$i = \sqrt{-1}$

$(1 + 3i)^{15} \cdot (1 - 3i)^{15} = 10^{15}$

$10^{15} \in Q$

$(a - bi)\ (a + bi) \in Q$

$\Rightarrow (1 - 3i)^{15} = a + bi$

ÖRNEK / EXAMPLE 17

MARMARA ÜNİ. YÖS 2015

$z_1 = \sqrt{7} - \sqrt{2}i,\quad z_2 = 3 - 4i \quad \Rightarrow \mathbf{|z_1 \cdot z_2| = ?}$

A) 10
B) 14
C) 15
D) 18
E) 20

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{array}{rl} & z_1 = \sqrt{7} - \sqrt{2}i \\ \times & z_2 = 3 - 4i \\ \hline \end{array}$$

$$z_1 \cdot z_2 = 3\sqrt{7} - 4\sqrt{7}i - 3\sqrt{2}i + 4\sqrt{2}i^2$$

$$= (3\sqrt{7} - 4\sqrt{2}) + (-4\sqrt{7} - 3\sqrt{2})i$$

$$|z_1 \cdot z_2| = \sqrt{(3\sqrt{7} - 4\sqrt{2})^2 + (4\sqrt{7} + 3\sqrt{2})^2}$$

$$= \sqrt{63 + 32 - 24\sqrt{14} + 112 + 18 + 24\sqrt{14}}$$

$$= \sqrt{95 + 130} = \sqrt{225} = 15$$

ÖRNEK / EXAMPLE 18

SÜLEYMAN DEMİREL ÜNİV. YÖS 2016

$$\left.\begin{array}{l} z_1 = 3\,\text{Cis}\,20^\circ \\ z_2 = 6\,\text{Cis}\,70^\circ \end{array}\right\} \Rightarrow \quad \mathbf{z_1 \cdot z_2 = ?}$$

A) –18i B) –9i C) i D) 9i E) 18i

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{array}{rl} & z_1 = 3\text{cis}20^\circ \\ \text{x} & \underline{z_2 = 6\text{cis}70^\circ} \end{array}$$

$$\begin{aligned} z_1 \cdot z_2 &= 3 \cdot 6 \cdot \text{cis}(20^\circ + 70^\circ) \\ &= 18(\cos 90^\circ + i\sin 90^\circ) \\ &= 18(0 + i \cdot 1) = 18i \end{aligned}$$

ÖRNEK / EXAMPLE 19

ANKARA ÜNİ. YÖS 2016

$i^2 = -1$

$z - 1 = iz - 5i \Rightarrow$ **z = ?**

A) –2 – 3i B) 3 – 2i C) 2 – 3i D) 3 + 2i E) 2 + 3i

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$z - iz = 1 - 5i$

$z(1 - i) = 1 - 5i$

$$\Rightarrow z = \frac{1 - 5i}{\underset{(1+i)}{1 - i}} = \frac{1 - 5i + i + 5}{2}$$

$z = 3 - 2i$

1. $x^2 + 9 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-i, i\}$

B) $\{-2i, 2i\}$

C) $\{-3i, 3i\}$

D) $\{-4i, 4i\}$

E) $\{2i, 3i\}$

2. $P(x) = 3x^{50} - 2x^{47} + x^{36} \Rightarrow$ **P(i) = ?**

A) $2 + i$

B) $1 + i$

C) $-1 + i$

D) $-2 + 2i$

E) $-3 + 3i$

3. $(z - i)\cdot(1 + i) = 4 + 6i \Rightarrow$ **z = ?**

A) $4 + i$

B) $5 + 2i$

C) $3 + 4i$

D) $2 - i$

E) $4 - 3i$

4. $z + \dfrac{1-i}{3+2i} = 4 + i \Rightarrow$ **Re(z) = ?**

A) $\dfrac{41}{13}$

B) $\dfrac{47}{13}$

C) $\dfrac{51}{13}$

D) $\dfrac{53}{13}$

E) $\dfrac{55}{13}$

5. $z = -2 + 2\sqrt{3}i \Rightarrow$ **z = r·cisθ = ?**

A) $4\text{cis}\dfrac{\pi}{6}$

B) $4\text{cis}\dfrac{\pi}{3}$

C) $4\text{cis}\dfrac{\pi}{2}$

D) $4\text{cis}\dfrac{2\pi}{3}$

E) $4\text{cis}\dfrac{5\pi}{6}$

6. $z = 3\text{cis}\dfrac{5\pi}{6} \Rightarrow$ **Re(z) = ?**

A) $-\dfrac{3}{2}$

B) $\dfrac{3}{2}$

C) $-\dfrac{3\sqrt{3}}{2}$

D) $\dfrac{3\sqrt{3}}{2}$

E) 3

7. $z_1 = 18\text{cis } 230°$

$z_2 = 3\text{cis } 50° \Rightarrow \text{Re}\left(\dfrac{z_1}{z_2}\right) = ?$

A) –6

B) –3

C) 2

D) 3

E) 6

8. $z = \sqrt{2}\text{ cis}18° \Rightarrow \text{Re}(z^{10}) = ?$

A) –2

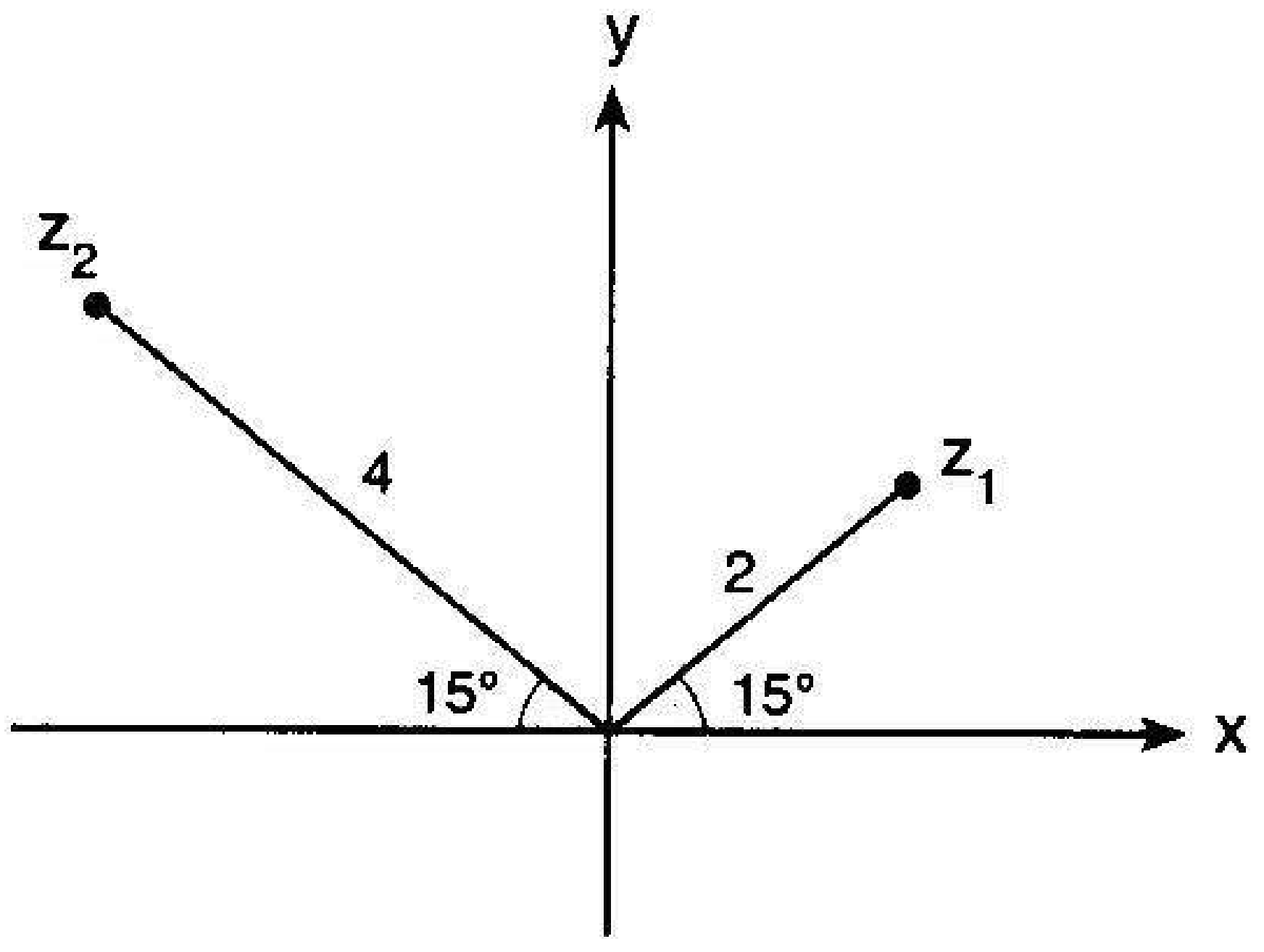
B) –4

C) –8

D) –16

E) –32

9. $\text{Re}\left(\dfrac{z_2}{z_1}\right) = ?$



A) $-\sqrt{3}$

B) –1

C) 1

D) $\sqrt{3}$

E) 2

10. $z_1 = 2 - 2i$

$z_2 = 3 + 3\sqrt{3}i \Rightarrow \text{Arg}\left(\dfrac{z_1}{z_2^2}\right) = ?$

A) 180°

B) 185°

C) 190°

D) 195°

E) 200°

11. $z = \dfrac{1}{2-3i} \Rightarrow \text{im}(z^2) = ?$

A) $\dfrac{12}{169}$

B) $\dfrac{3}{13}$

C) $\dfrac{5}{169}$

D) $-\dfrac{5}{169}$

E) $-\dfrac{12}{169}$

12. $z = -1 + \sqrt{3}i \Rightarrow w_1 = ?$

A) $\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{6}}{2}i$

B) $\dfrac{\sqrt{3}}{3} + \dfrac{\sqrt{6}}{2}i$

C) $\dfrac{\sqrt{3}}{3} + \dfrac{\sqrt{7}}{2}i$

D) $\sqrt{3} + \dfrac{\sqrt{6}}{2}i$

E) $1 + \dfrac{\sqrt{6}}{2}i$

YANITLAR / ANSWERS							
1. C	2. D	3. B	4. C	5. D	6. C	7. A	8. E
9. A	10. D	11. A	12. A				

1. $x^2 + 9 = 0$

$x^2 = -9$

$x = \mp\sqrt{-9}$

$x = \mp 3i$

Yanıt / The Answer is C

2. $P(i) = 3i^{50} - 2i^{47} + i^{36}$

$= 3i^2 - 2i^3 + i^0$

$= -3 + 2i + 1$

$= -2 + 2i$

Yanıt / The Answer is D

3. $(z - i)(1 + i) = 4 + 6i$

$z = a + bi \Rightarrow (a + bi - i)(1 + i) = 4 + 6i$

$a + ai + bi - b - i + 1 = 4 + 6i$

$(a - b + 1) + (a + b - 1)i = 4 + 6i$

$a - b + 1 = 4 \Rightarrow \quad a - b = 3$

$a + b - 1 = 6 \Rightarrow \underline{+\, a + b = 7}$

$2a = 10$

$a = 5 \Rightarrow a - b = 3$

$5 - b = 3$

$5 - 3 = b$

$2 = b$

$\Rightarrow z = 5 + 2i$

Yanıt / The Answer is B

4. $z + \dfrac{1-i}{3+2i} = 4 + i$

$z = 4 + i - \dfrac{1-i}{3+2i}$

$= \dfrac{12 + 8i + 3i - 2 - 1 + i}{3+2i}$

$= \dfrac{9+12i}{3+2i}$

$= \dfrac{(9+12i)(3-2i)}{13} = \dfrac{51 - 18i + 36i}{13}$

$= \dfrac{51}{13} + \dfrac{18}{13}i \Rightarrow \mathrm{Re}(z) = \dfrac{51}{13}$

Yanıt / The Answer is C

5. $\tan\theta = \dfrac{2\sqrt{3}}{-2} = -\sqrt{3} \Rightarrow \theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

$|z| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$

$\Rightarrow z = 4\mathrm{cis}\dfrac{2\pi}{3}$

Yanıt / The Answer is D

6. $z = 3\mathrm{cis}\dfrac{5\pi}{6}$

$= 3\left[\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right]$

$= 3\left[-\dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}\right]$

$= \dfrac{-3\sqrt{3}}{2} + \dfrac{3}{2}i$

$\Rightarrow \mathrm{Re}(z) = \dfrac{-3\sqrt{3}}{2}$

Yanıt / The Answer is C

7. $\frac{z_1}{z_2} = \frac{18}{3}\,\text{cis}\,(230° - 50°)$

$= 6\,(\cos 180° + i \sin 180°)$

$= -6$

$\Rightarrow \text{Re}\,(\frac{z_1}{z_2}) = -6$

Yanıt / The Answer is A

8. $z = \sqrt{2}\,\text{cis}18° \Rightarrow z^{10} = (\sqrt{2})^{10}\,\text{cis}(18° \cdot 10)$

$= 32\,\text{cis}180°$

$\Rightarrow \text{Re}(z) = 32\cos 180° = -32$

Yanıt / The Answer is E

9. $z_1 = 2\,\text{cis}15°$

$z_2 = 4\,\text{cis}(180° - 15°) = 4\,\text{cis}165°$

$\frac{z_2}{z_1} = \frac{4}{2}\,\text{cis}(165° - 15°) = 2\text{cis}150°$

$\text{Re}\left(\frac{z_2}{z_1}\right) = 2\cos 150° = 2 \cdot \left(\frac{-\sqrt{3}}{2}\right) = -\sqrt{3}$

Yanıt / The Answer is A

10. $z_1 = 2 - 2i \Rightarrow \tan\theta_1 = \frac{-2}{2} = -1$

$\theta_1 = 360° - 45° = 315°$

$z_2 = 3 + 3\sqrt{3}\,i \Rightarrow \tan\theta_2 = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\Rightarrow \theta_2 = 60°$

$\text{Arg}\left(\frac{z_1}{z_2^2}\right) = 315° - 2 \cdot 60° = 195°$

Yanıt / The Answer is D

11. $z = \underset{(2+3i)}{\frac{1}{2-3i}} = \frac{2+3i}{13}$

$z^2 = \frac{(2+3i)^2}{169} = \frac{4+12i-9}{169}$

$\Rightarrow \text{Im}(z) = \frac{12}{169}$

Yanıt / The Answer is A

12. $z = -1 + \sqrt{3}i \Rightarrow a = -1,\ b = \sqrt{3},\ |z| = 2$

$w_{0,1} = \pm\left(\sqrt{\frac{2-1}{2}} \pm \sqrt{\frac{2+1}{2}}\,i\right)$

$= \pm\left(\frac{\sqrt{2}}{2} \pm \frac{\sqrt{6}}{2}\,i\right)$

Yanıt / The Answer is A

1. $A=\sqrt{-169}+\sqrt{-16}$

$B=\sqrt{-25}-\sqrt{-36}$

$\Rightarrow$ **$A \cdot B = ?$**

A) –17

B) 17i

C) 17

D) –17i

E) 15i

2. **$i^{1987}+i^{1985}=?$**

A) –i

B) –1

C) 0

D) 1

E) i

3. $z=\dfrac{1+i}{1-i} \Rightarrow$ **$\mathrm{Re}(z)=?$**

A) –1

B) 0

C) 1

D) 2

E) 3

4. $x>y$,

$z=(x-y)^2+6i$,

$w=16+(x+y)i$,

$z=w \Rightarrow$ **$x \cdot y = ?$**

A) –1

B) 1

C) 2

D) 3

E) 5

5. $z_1=x+2i$,

$z_2=y-(3+x)\,i$,

$z_1+z_2=6+12i$

$\Rightarrow$ **$x-y=?$**

A) –12

B) –16

C) –24

D) –32

E) –36

6. $z+\bar{z}\cdot 2=9-8i \Rightarrow$ **$|z|=?$**

A) $\sqrt{65}$

B) $\sqrt{67}$

C) $\sqrt{69}$

D) $\sqrt{71}$

E) $\sqrt{73}$

7. $z_1 = 4 - 3i$

$z_2 = 1 - xi,$

$|z_1 - z_2| = 5$

$\Rightarrow \sum x = ?$

A) -1

B) 1

C) 3

D) 6

E) 8

8. $z = 2 - 2\sqrt{3}i \Rightarrow z = r\,\text{cis}\theta = ?$

A) $4\,\text{cis}\frac{5\pi}{6}$

B) $2\,\text{cis}\frac{\pi}{6}$

C) $4\,\text{cis}\frac{5\pi}{3}$

D) $2\,\text{cis}\frac{5\pi}{6}$

E) $4\,\text{cis}\frac{\pi}{6}$

9. $z = -2 - 2i \Rightarrow z^{16} = ?$

A) 2^{24}

B) -2^{24}

C) $2^{24}i$

D) $-2^{24}i$

E) 0

10. $z_1 = -2 + 2i$

$z_2 = 3 - \sqrt{3}i$

$\Rightarrow z_1 \cdot z_2 = ?$

A) $4\sqrt{6}\,\text{cis}\frac{\pi}{12}$

B) $4\sqrt{3}\,\text{cis}\frac{7\pi}{12}$

C) $4\sqrt{6}\,\text{cis}\frac{7\pi}{12}$

D) $4\sqrt{6}\,\text{cis}\frac{7\pi}{6}$

E) $2\sqrt{6}\,\text{cis}\frac{7\pi}{12}$

11. $\text{Re}\left(\frac{z_2}{z_1}\right) = ?$

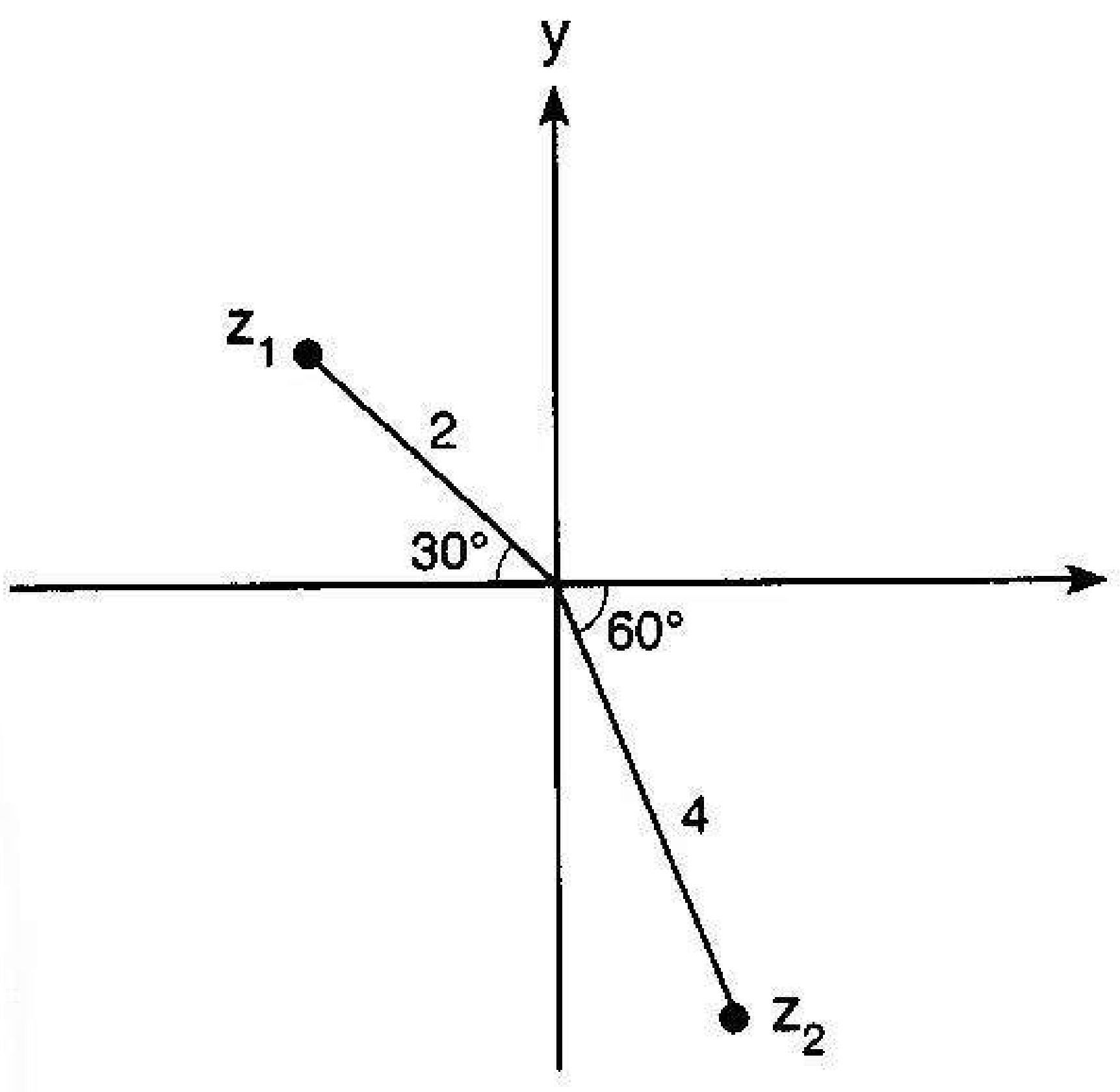
A) $\sqrt{3}$

B) $-\sqrt{3}$

C) $2\sqrt{3}$

D) $-2\sqrt{3}$

E) $4\sqrt{3}$



12. $|z_1 - z_2| = ?$

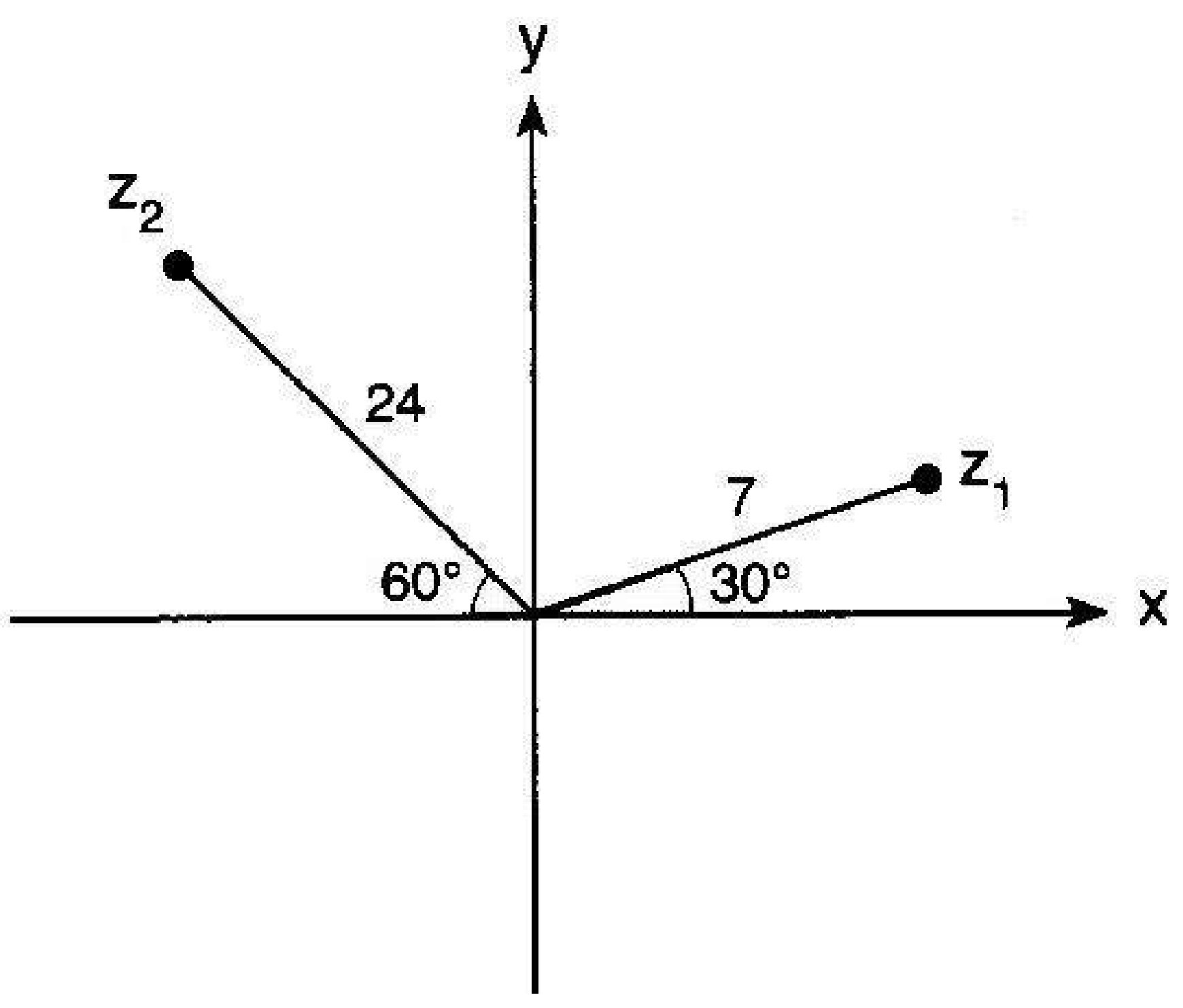
A) 12

B) 15

C) 25

D) 33

E) 36



YANITLAR / ANSWERS

1. C 2. C 3. B 4. E 5. D 6. E 7. D 8. C

9. A 10. C 11. B 12. C

1. $A = 13i + 4i = 17i$

$B = 5i - 6i = -i$

$\Rightarrow A \cdot B = -17i^2 = 17$

Yanıt / The Answer is C

2.
$$\begin{array}{r|l} 1987 & \underline{\ 4\ } \\ \hline ③ & \end{array} \qquad \begin{array}{r|l} 1985 & \underline{\ 4\ } \\ \hline ① & \end{array}$$

$i^{1987} + i^{1985} = i^3 + i = -i + i = 0$

Yanıt / The Answer is C

3. $z = \frac{1+i}{\underset{(1+i)}{1-i}} = \frac{2i}{1-i^2} = \frac{2i}{2} = i$

$\Rightarrow \text{Re}(z) = 0$

Yanıt / The Answer is B

4. $(x-y)^2 = 16 \Rightarrow x - y = 4$

$$\begin{array}{r} x - y = 4 \\ +\ x + y = 6 \\ \hline 2x = 10 \\ x = 5 \\ y = 1 \end{array}$$

$\Rightarrow x \cdot y = 5 \cdot 1 = 5$

Yanıt / The Answer is E

5. $z_1 + z_2 = x + y + (2 - 3 - x)\,i = 6 + 12i$

$\Rightarrow x + y = 6$

$-1 - x = 12$

$x = -13 \Rightarrow y = 19$

$x - y = -13 - 19 = -32$

Yanıt / The Answer is D

6. $z = a + bi \Rightarrow \bar{z} = a - bi$

$a + bi + 2(a - bi) = 9 - 8i$

$3a - bi = 9 - 8i$

$\Rightarrow 3a = 9 \quad , \quad -b = -8$

$a = 3 \quad , \quad b = 8$

$|z| = \sqrt{a^2 + b^2} = \sqrt{3^2 + 8^2} = \sqrt{73}$

Yanıt / The Answer is E

7. $z_1 - z_2 = 3 - (3 - x)i \Rightarrow |z_1 - z_2| = \sqrt{3^2 + (3 - x)^2} = 5$

$9 + (3 - x)^2 = 25$

$(3 - x)^2 = 4^2 \Rightarrow \quad 3 - x = 4 \quad , \quad 3 - x = -4$

$x = -1 \quad , \quad x = 7$

$\Rightarrow \sum x = -1 + 7 = 6$

Yanıt / The Answer is D

8. $|z| = r = \sqrt{2^2 + (2\sqrt{3})^2} = 4$

$\tan\theta = \dfrac{-2\sqrt{3}}{2} = -\sqrt{3} \Rightarrow \theta = 2\pi - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$

$\Rightarrow z = 4\,\text{cis}\dfrac{5\pi}{3}$

Yanıt / The Answer is C

9. $|z| = \sqrt{4 + 4} = \sqrt{8}$

$\tan\theta = \dfrac{-2}{-2} = 1 \Rightarrow \theta = \pi + \dfrac{\pi}{4} = \dfrac{5\pi}{4}$

$z = \sqrt{8}\,\text{cis}\dfrac{5\pi}{4} \Rightarrow z^{16} = (\sqrt{8})^{16}\,\text{cis}\left(\dfrac{5\pi}{4} \cdot 16\right)$

$= 2^{24}\,\text{cis}\,20\pi$

$= 2^{24}\,\text{cis}0° = 2^{24}$

Yanıt / The Answer is A

10. $z_1: \quad |z_1| = \sqrt{2^2 + 2^2} = \sqrt{8}$

$\tan\theta_1 = \dfrac{2}{-2} = -1 \Rightarrow \theta_1 = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

$z_1 = 2\sqrt{2}\ \text{cis}\dfrac{3\pi}{4}$

$z_2: \quad |z_2| = \sqrt{3^2 + (\sqrt{3})^2} = 2\sqrt{3}$

$\tan\theta_2 = \dfrac{-\sqrt{3}}{3} = \dfrac{-1}{\sqrt{3}} \Rightarrow \theta_2 = 2\pi - \dfrac{\pi}{6} = \dfrac{11\pi}{6}$

$z_2 = 2\sqrt{3}\ \text{cis}\dfrac{11\pi}{6}$

$z_1 \cdot z_2 = 2\sqrt{2} \cdot 2\sqrt{3} \cdot \text{cis}\left(\dfrac{3\pi}{4} + \dfrac{11\pi}{6}\right)$

$= 4\sqrt{6}\ \text{cis}\dfrac{31\pi}{12} = 4\sqrt{6}\ \text{cis}\ \dfrac{7\pi}{12}$

Yanıt / The Answer is C

11. $z_1 = 2\text{cis}(180° - 30°) = 2\text{cis}150°$

$z_2 = 4\text{cis}(360° - 60°) = 4\text{cis}300°$

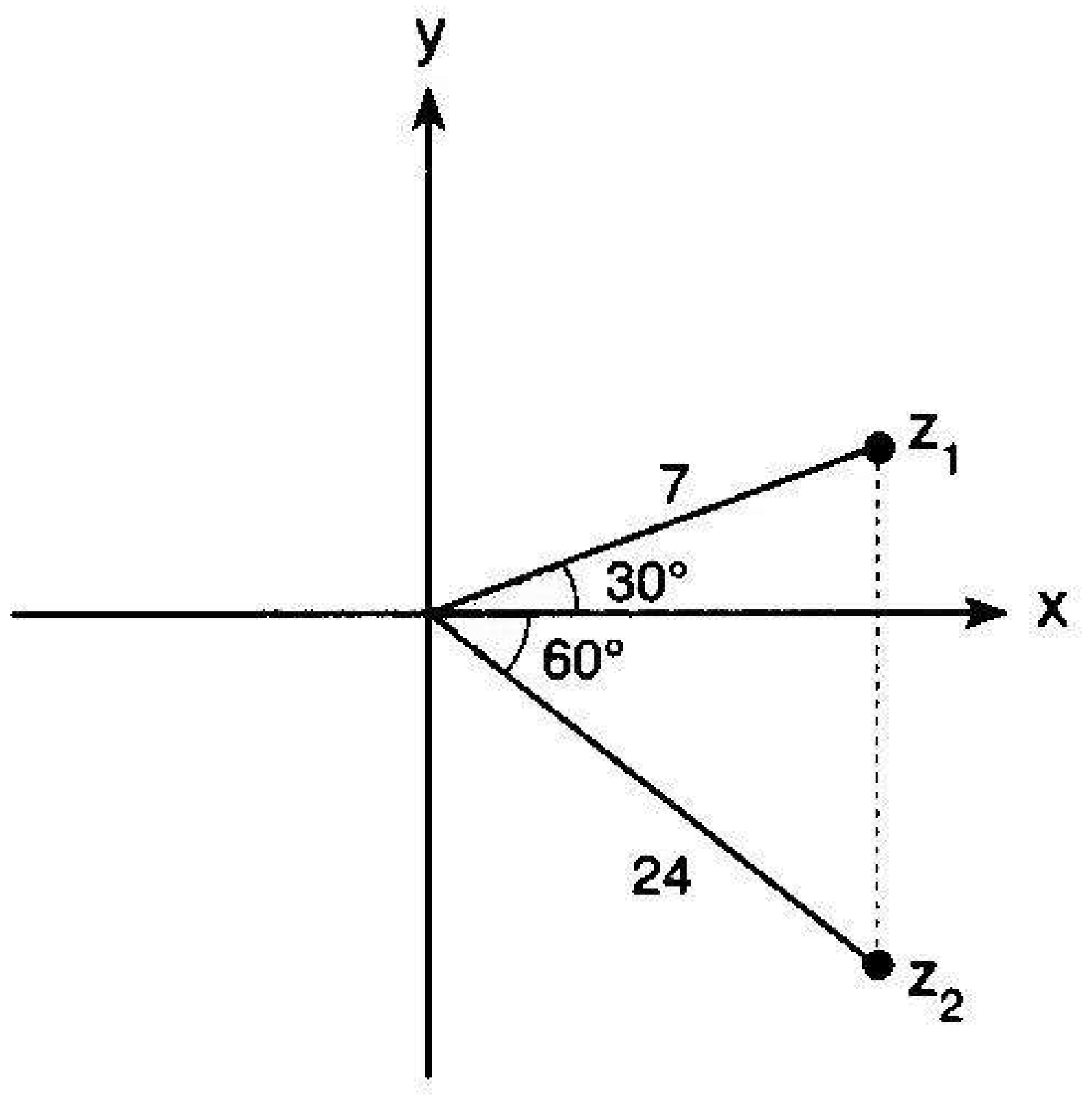
$\dfrac{z_2}{z_1} = \dfrac{4}{2}\text{cis}(300° - 150°) = 2\text{cis}\ 150°$

$\Rightarrow \text{Re}\left(\dfrac{z_2}{z_1}\right) = 2\cos 150° = 2 \cdot \dfrac{-\sqrt{3}}{2} = -\sqrt{3}$

Yanıt / The Answer is B

12. $|z_1 - z_2|^2 = 7^2 + 24^2$

$\Rightarrow |z_1 - z_2| = 25$



Yanıt / The Answer is C

1. $\dfrac{\sqrt{-16}+\sqrt{-4}}{\sqrt{-9}}=?$

Yanıt / The Answer is: **2**

2. $P(x) = x^{121} + 2x^{98} - x^{72} \Rightarrow$ **$P(i) = ?$**

Yanıt / The Answer is: **$i - 3$**

3. $z = -3 + \sqrt{7}i \Rightarrow$ **$|z| = ?$**

Yanıt / The Answer is: **4**

4. $z + 4i = iz - 8 \Rightarrow$ **$z = ?$**

Yanıt / The Answer is: **$-6i - 2$**

5. $z = \dfrac{1}{4+i} - \dfrac{1}{4-i} \Rightarrow$ **$\mathrm{Re}(z) = ?$**

Yanıt / The Answer is: **0**

6. $z = -3 + 3i \Rightarrow$ **$z = r \cdot \mathrm{cis}\theta = ?$**

Yanıt / The Answer is: **$3\sqrt{2}\ \mathrm{cis}\ \dfrac{3\pi}{4}$**

7. $z = 1 + \sqrt{3}i \Rightarrow \mathbf{z = r \cdot cis\theta = ?}$

Yanıt / The Answer is: $\mathbf{2 cis \frac{\pi}{3}}$

---

8. $z = 12 \text{ cis } \frac{\pi}{2} \Rightarrow \mathbf{Im(z) = ?}$

Yanıt / The Answer is: **12**

---

9. $z = 2 - 2i \Rightarrow \mathbf{z = r \cdot cis\alpha = ?}$

Yanıt / The Answer is: $\mathbf{2\sqrt{2} \text{ cis } \frac{7\pi}{4}}$

---

10. $\left.\begin{array}{l} z_1 = 3 - 2i \\ z_2 = -3 - 10i \end{array}\right\} \Rightarrow \mathbf{|z_1 - z_2| = ?}$

Yanıt / The Answer is: **10**

---

11. $z = -8i \Rightarrow \mathbf{z = r \cdot cis\theta = ?}$

Yanıt / The Answer is: $\mathbf{8 cis \frac{3\pi}{2}}$

---

12. $z = 3\sqrt{2} + 3\sqrt{6}i \Rightarrow \mathbf{z = r \cdot cis\theta = ?}$

Yanıt / The Answer is: $\mathbf{6\sqrt{2} \text{ cis } \frac{\pi}{3}}$

1. $x^2 + mx + n = 0$,

ÇK(SS) = $\{2 \pm 5i\}$ $\Rightarrow$ **m + n = ?**

A) 21

B) 24

C) 25

D) 27

E) 30

2. $P(x) = 3x^{21} + x^{16} - 2x^{11}$ $\Rightarrow$ **P(i) = ?**

A) $1 + 3i$

B) $1 + 5i$

C) $2 + 3i$

D) $4 - i$

E) $5 + 3i$

3. $(1 - i) \cdot z + i\bar{z} = 5 + i$ $\Rightarrow$ $|z| = ?$

A) $2\sqrt{5}$

B) $\sqrt{10}$

C) $3\sqrt{2}$

D) $5\sqrt{3}$

E) $2\sqrt{15}$

4. $z = (1 - i)^{22} + (1 + i)^{22}$ $\Rightarrow$ **z = ?**

A) -2^{11}

B) $-2^{11}i$

C) 0

D) 2^{11}

E) $2^{11}i$

5. $z = x + yi$,

$3z + 5 = \bar{z} + 13 - 4i$ $\Rightarrow$ **x + y = ?**

A) -5

B) -3

C) -2

D) 1

E) 3

6. $z = x + yi$,

$(z - 1) \cdot (3 + i) = 3 + 11i$ $\Rightarrow$ **x + y = ?**

A) -8

B) -3

C) 2

D) 6

E) 7

7. $z = \dfrac{(2-i)^2 \cdot (4+2i)}{3+4i} \Rightarrow$ **$|z| = ?$**

A) $2\sqrt{5}$

B) $3\sqrt{5}$

C) $4\sqrt{5}$

D) $5\sqrt{5}$

E) $6\sqrt{5}$

8. $z = x + yi,$

$z \cdot (1-i) = \overline{z} + 1 - i \Rightarrow$ **$x + y = ?$**

A) 2

B) 3

C) 4

D) 5

E) 6

9. $z = \dfrac{3+2i}{4-i} \Rightarrow$ **$\mathrm{Im}(z) = ?$**

A) $\dfrac{7}{17}$

B) $\dfrac{11}{17}$

C) $\dfrac{12}{17}$

D) $\dfrac{18}{17}$

E) $\dfrac{24}{17}$

10. $\left.\begin{array}{l} z_1 = 2 - i \\ z_2 = 4 + 9i \end{array}\right\} \Rightarrow$ **$|z_1 + z_2| = ?$**

A) 4

B) 5

C) 7

D) 8

E) 10

11. $z = -6 + 6i$

$z = r\mathrm{cis}\theta \quad \Rightarrow$ **$\theta = ?$**

A) $\dfrac{3\pi}{4}$

B) $\dfrac{5\pi}{4}$

C) $\dfrac{3\pi}{2}$

D) $\dfrac{7\pi}{4}$

E) $\dfrac{11\pi}{6}$

12. $z = x + yi$

$2z + 3 - 7i = \overline{z} + 8 - 4i \Rightarrow$ **$x + y = ?$**

A) -1

B) 2

C) 3

D) 5

E) 6

YANITLAR / ANSWERS

1. C 2. B 3. B 4. C 5. E 6. D 7. A 8. C
9. B 10. E 11. A 12. E

1. $\dfrac{i^{55} - 4\cdot i^{37} + 3\cdot i^{72}}{2+i^{131}} = ?$

A) $\frac{2}{5} - \frac{11}{5}i$

B) $\frac{3}{5} + \frac{13}{5}i$

C) $\frac{11}{5} - \frac{7}{5}i$

D) $\frac{1}{7} - \frac{11}{7}i$

E) $\frac{3}{10} - \frac{17}{10}i$

2. $z = x + yi$,

$z = \dfrac{2+i}{1-i} + \dfrac{4-i}{2+i} \Rightarrow$ **Re(z) = ?**

A) $\frac{17}{10}$

B) $\frac{19}{10}$

C) $\frac{21}{10}$

D) $\frac{23}{10}$

E) $\frac{27}{10}$

3. $z = \dfrac{1}{1+i} - \dfrac{1}{1-i} \Rightarrow$ **$|z|$ = ?**

A) 1

B) 0

C) $\sqrt{2}$

D) $\sqrt{3}$

E) 2

4. $-|5i\cdot z| + |4\bar{z}| + |3i\cdot \bar{z}| = 8 \Rightarrow$ **$|z|$ = ?**

A) 1

B) 2

C) 3

D) 4

E) 5

5. $z_1 = 5 - 3i$

$z_2 = 4 + 2i \Rightarrow$ **$|z_1 - z_2|$ = ?**

A) $\sqrt{17}$

B) $\sqrt{26}$

C) $\sqrt{37}$

D) $\sqrt{47}$

E) $\sqrt{65}$

6. $x \in R^+$,

$z = \dfrac{(2-i)\cdot(x+4i)}{3-4i}$,

$|z| = 2 \Rightarrow$ **x = ?**

A) $\frac{7}{2}$

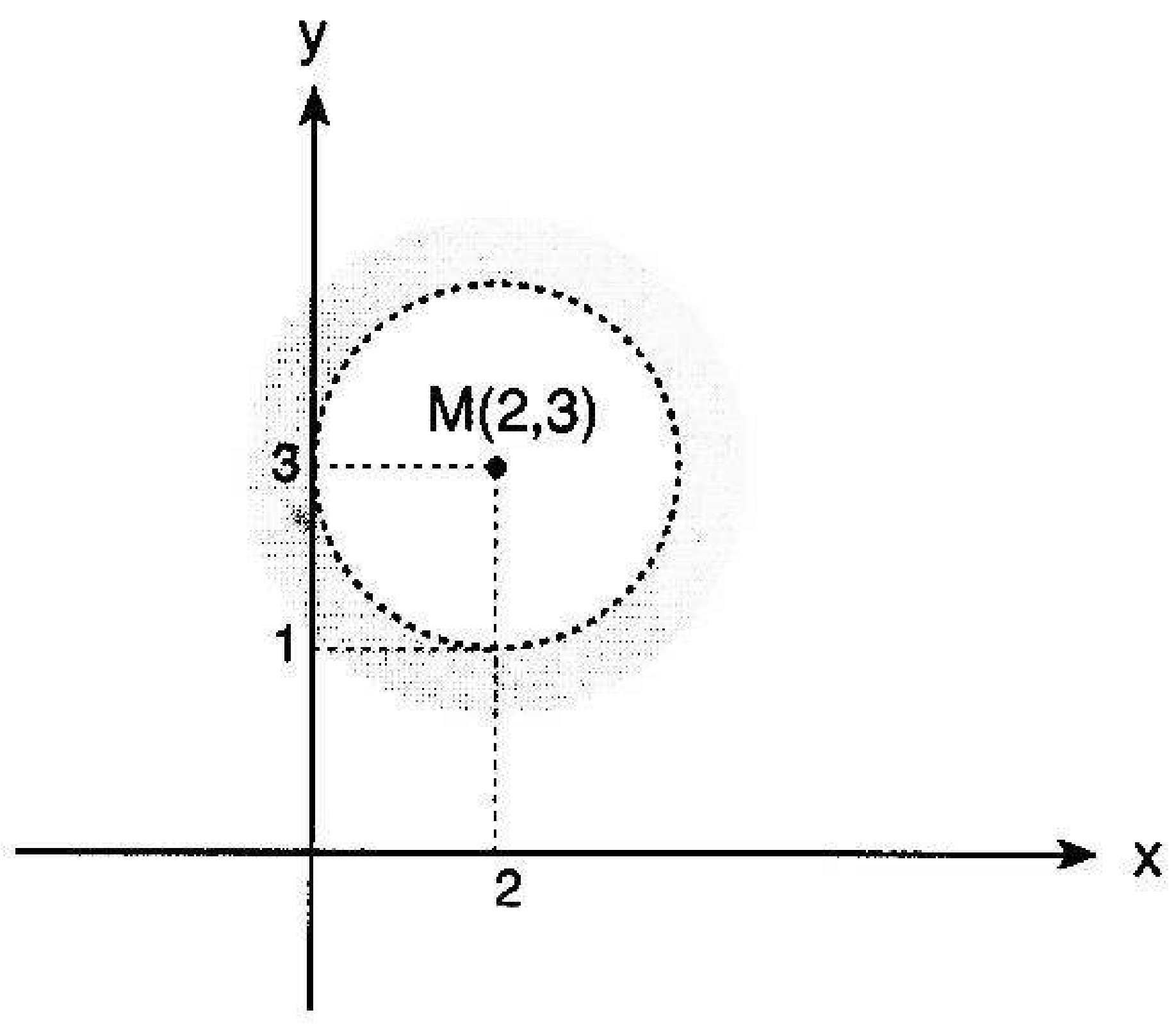
B) 3

C) $\frac{5}{2}$

D) 2

E) $\frac{3}{2}$

7.



Taralı bölge / The shaded region = ?

A) $|z-(2+3i)| \le 2$

B) $|z-2-3i| \ge 2$

C) $|z+2-3i| > 2$

D) $|z-2-3i| > 2$

E) $|z-2-3i| < 2$

---

8. $|z| + z = 2 + \frac{i}{2} \Rightarrow$ **Re(z) = ?**

A) $-\frac{15}{16}$

B) 1

C) $\frac{15}{16}$

D) 0

E) –1

---

9. $z = x + yi$,

$z \cdot (3-i) + 2 + 5i = -1 + 16i \Rightarrow$ **x + y = ?**

A) 1

B) 2

C) 3

D) 4

E) 5

---

10. $w^2 = -1 - \sqrt{3}i \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\sqrt{2}\,\text{cis}\frac{2\pi}{3},\ \sqrt{2}\,\text{cis}\frac{5\pi}{3}\right\}$

B) $\left\{\sqrt{2}\,\text{cis}\frac{5\pi}{6},\ \sqrt{2}\,\text{cis}\frac{11\pi}{6}\right\}$

C) $\left\{\sqrt{2}\,\text{cis}\frac{\pi}{3},\ \sqrt{2}\,\text{cis}\frac{7\pi}{3}\right\}$

D) $\left\{\sqrt{2}\,\text{cis}\frac{\pi}{6},\ \sqrt{2}\,\text{cis}\frac{4\pi}{3}\right\}$

E) $\left\{\sqrt{2}\,\text{cis}\frac{\pi}{4},\ \sqrt{2}\,\text{cis}\frac{3\pi}{4}\right\}$

---

11. $\text{Im}\left(\frac{z_2}{z_1}\right) = ?$

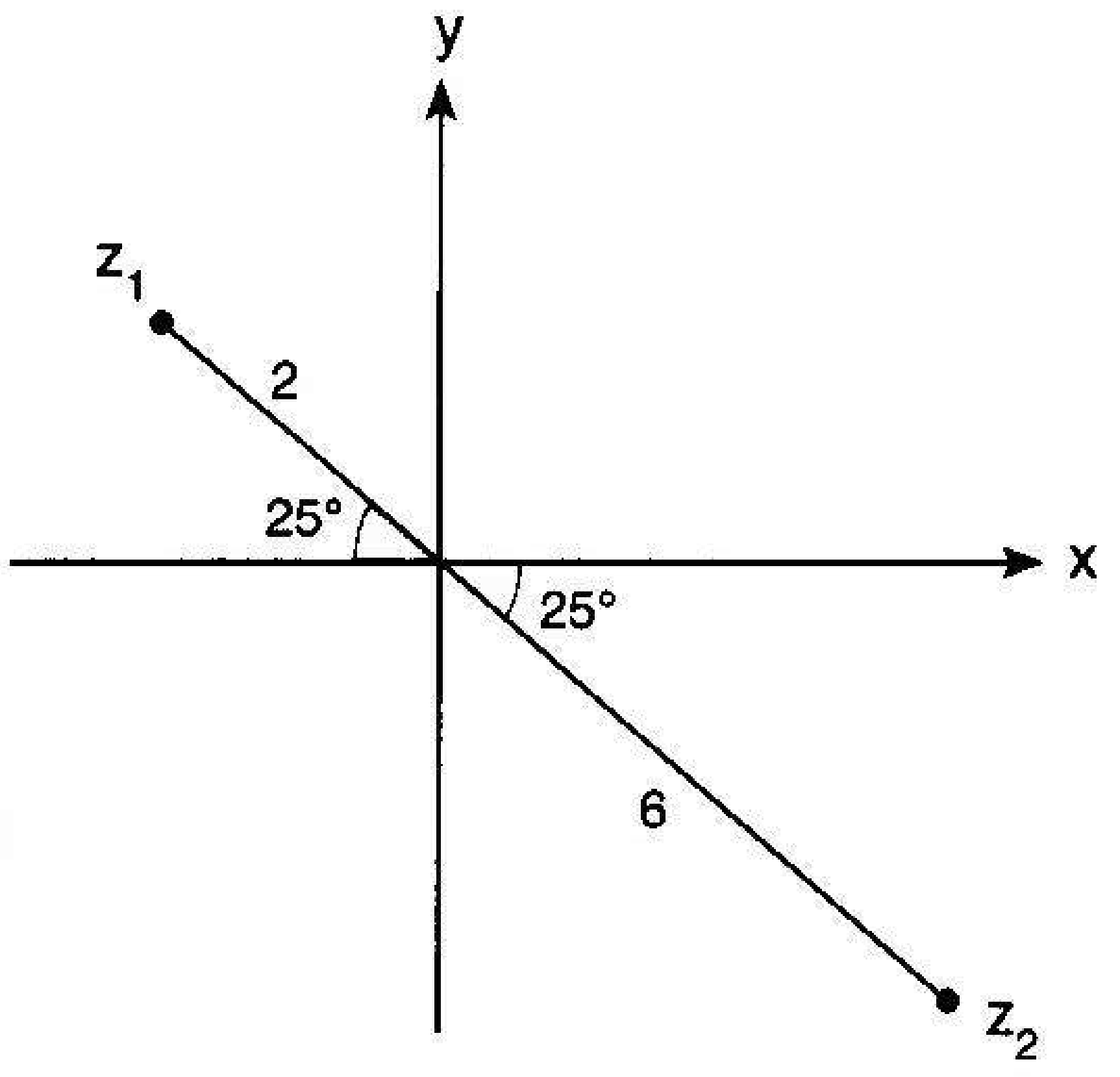
A) 1

B) 0

C) –1

D) –2

E) –3



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12. $z_1 = 2 - 2\sqrt{3}i$

$z_2 = -1 + i \quad \Rightarrow \quad \text{Arg}\left(\frac{z_1^3}{z_2}\right) = ?$

A) $\frac{\pi}{15}$

B) $\frac{\pi}{12}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{3}$

E) $\frac{\pi}{2}$

YANITLAR / ANSWERS

1. C	2. B	3. A	4. D	5. B	6. D	7. D	8. C
9. A	10. A	11. B	12. C				

LOGARİTMA
LOGARITHM

BÖLÜM
CHAPTER **4**

LOGARİTMİK FONKSİYONLAR / LOGARITHMIC FUNCTIONS

$a > 0$, $a \neq 1$ $f(x) = a^x \Rightarrow f(x)$ üstel fonksiyondur. / *$f(x)$ is an exponential function.*

Tanım: $a > 0$, $a \neq 1$ ve $x > 0$ olmak üzere, $f(x) = a^x$ fonksiyonunun tersi x'in a tabanında logaritması olarak adlandırılır. $y = \log_a x$ ile gösterilir.

Definition: *The inverse of $f(x) = a^x$ is called the logarithm of x with the base a, where $a > 0$, $a \neq 1$ and $x > 0$. It is denoted by $y = \log_a x$.*

ÖRNEK / EXAMPLE 1

$\log_2 x = 4 \Rightarrow$ **$x = ?$**

A) 2　　B) 4　　C) 8　　D) 16　　E) 32

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\log_2 x = 4$　　[Verilen eşitlik / *Given equality*]

$x = 2^4$　　[Logaritma tanımı / *Definition of logarithm*]

$x = 16$　　[Sonuç / *Conclusion*]

PEKİŞTİRME / REINFORCE

$\log_{0,1} x = 2 \Rightarrow x = ?$

YANIT / ANSWER : 0,01

LOGARİTMANIN ÖZELLİKLERİ / PROPERTIES of LOGARITHM

Özellik / *Property* 1 : $\log_a 1 = 0$

ÖRNEK / EXAMPLE 2

$\log_3(\log_2(x + 1)) = 0 \Rightarrow$ **$x = ?$**

A) 1　　B) 2　　C) 3　　D) 4　　E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$\log_3 (\log_2 (x + 1)) = 0$　　[Verilen eşitlik / *Given equality*]

$\log_2 (x + 1) = 3^0 = 1$　　[1. özellik / *Property 1*]

$x + 1 = 2$　　[1. özellik / *Property 1*]

$x = 1$　　[Sonuç / *Conclusion*]

Özellik / *Property* 2: $\log_a a = 1$

ÖRNEK / EXAMPLE 3

$\log_3 (x - 5) = 1 \Rightarrow$ **x = ?**

A) 6 B) 7 C) 8 D) 9 E) 10

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\log_3 (x - 5) = 1$ [Verilen eşitlik / *Given equality*]

$x - 5 = 3$ [2. özellik / *Property 2*]

$x = 8$ [Sonuç / *Conclusion*]

Özellik / *Property* 3: $\log_a x^n = n \cdot \log_a x$

PEKİŞTİRME / REINFORCE

$\log_3 27 = x \Rightarrow x = ?$

YANIT / ANSWER : 3

ÖRNEK / EXAMPLE 4

$\log_4 16 = x \Rightarrow$ **x = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$
\begin{aligned}
x = \log_4 16 &= \log_4 4^2 \\
&= 2\log_4 4 \\
&= 2
\end{aligned}
$$

Özellik / *Property* 4: $\log_{a^n} x^m = \frac{m}{n} \cdot \log_a x$

ÖRNEK / EXAMPLE 5

$2^{\log_4 8+1} = ?$

A) 1 B) $\sqrt{2}$ C) $2\sqrt{2}$ D) $3\sqrt{2}$ E) $4\sqrt{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$2^{\log_4 8+1} = 2^{\log_4 8} \cdot 2^1$$

$$= 2^{\log_{2^2} 2^3} \cdot 2$$

$$= 2^{\frac{3}{2}\log_2 2} \cdot 2$$

$$= 2\sqrt{2} \cdot 2$$

$$= 4\sqrt{2}$$

PEKİŞTİRME / REINFORCE

$2^{\log_3 (6+3)} = ?$

YANIT / ANSWER : 4

Özellik / *Property* 5: $a^{\log_a x} = x$, $a^{\log_x b} = b^{\log_x a}$

ÖRNEK / EXAMPLE 6

$4^{\log_4 x} + 7^{\log_7 3} = \log_5 625 \Rightarrow$ **x = ?**

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$4^{\log_4 x} + 7^{\log_7 3} = \log_5 625$$

$$x + 3 = 4$$

$$x = 1$$

PEKİŞTİRME / REINFORCE

$3^{\log_x 2} = 4^{\log_5 3} \Rightarrow x = ?$

YANIT / ANSWER : $\sqrt{5}$

Özellik / *Property* **6**: $\log_a (x \cdot y) = \log_a x + \log_a y$

ÖRNEK / EXAMPLE 7

$\log_4 15! = x \Rightarrow \log_4 16! = ?$

A) $x + 1$ B) $x + 2$ C) $x + 3$

D) $x + 4$ E) $x + 5$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned}\log_4 16! &= \log_4 (16 \cdot 15!)\\ &= \log_4 16 + \log_4 15!\\ &= \log_4 4^2 + x\\ &= 2 + x\end{aligned}$$

Özellik / *Property* **7**: $\log_a \frac{x}{y} = \log_a x - \log_a y$

ÖRNEK / EXAMPLE 8

$$\left.\begin{aligned}\log_2 (x \cdot y^2) &= 4\\ \log_2 \left(\frac{x}{y}\right) &= -5\end{aligned}\right\} \Rightarrow x = ?$$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{16}$ E) $\frac{1}{32}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\log_2 (x \cdot y^2) = 4 \Rightarrow \log_2 x + 2\log_2 y = 4$$

$$\log_2 \left(\frac{x}{y}\right) = -5 \Rightarrow \underline{2 \,/\, \log_2 x - \log_2 y = -5}$$

$$3\log_2 x = -6$$

$$\log_2 x = -2$$

$$x = 2^{-2} = \frac{1}{4}$$

Özellik / *Property* 8: Taban Değiştirme / *Changing a Base*

$$\log_a x = \frac{\log_b x}{\log_b a}$$

ÖRNEK / EXAMPLE 9

$\log_2 3 = a \Rightarrow$ $\log_6 24 = ?$

A) $\frac{a+2}{a+1}$ B) $\frac{a+4}{a+2}$ C) $\frac{a+3}{a+1}$

D) $\frac{a+4}{a+3}$ E) $\frac{a+3}{a+5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\log_6 24 = \frac{\log_2 24}{\log_2 6}$$

$$= \frac{\log_2 3 + \log_2 8}{\log_2 2 + \log_2 3}$$

$$= \frac{a+3}{1+a}$$

Özellik / *Property* 9: $\log_a x = \frac{1}{\log_x a}$

ÖRNEK / EXAMPLE 10

$$\frac{1}{\log_3 84} + \frac{1}{\log_4 84} + \frac{1}{\log_7 84} = ?$$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{1}{\log_3 84} + \frac{1}{\log_4 84} + \frac{1}{\log_7 84} = \log_{84} 3 + \log_{84} 4 + \log_{84} 7$$

$$= \log_{84}(3 \cdot 4 \cdot 7)$$

$$= \log_{84} 84 = 1$$

PEKİŞTİRME / REINFORCE

$\log_3 \frac{2}{3} + \log_3 \frac{3}{4} + \ldots + \log_3 \frac{17}{18} = ?$

YANIT / ANSWER : −2

Özellik / *Property* 10: $\log_{10} x = \log x$

ÖRNEK / EXAMPLE 11

$\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \ldots + \log \frac{101}{100} = ?$

A) −3 B) −2 C) −1 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned} \log \frac{1}{2} + \log \frac{2}{3} + \ldots + \log \frac{101}{100} &= \log\left(\frac{1}{2} \cdot \frac{2}{3} \cdot \ldots \cdot \frac{101}{100}\right) \\ &= \log \frac{1}{100} \\ &= \log 10^{-2} \\ &= -2 \log 10 = -2 \end{aligned}$$

Özellik / *Property* 11: Doğal Logaritma / *Natural Logarithm*

$\log_e x = \ln x, \quad e = 2,71\ldots$

ÖRNEK / EXAMPLE 12

$\ln x^2 = 2t \Rightarrow$ **$\log x^3 = ?$**

A) $3t \cdot \log 3e$ B) $t \cdot \log e$ C) $3t \cdot \log e$

D) $t \cdot \ln 3$ E) $3t \cdot \log 3$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\ln x^2 = 2t$

$2 \ln x = 2t$

$\ln x = t$

$$x = e^t \Rightarrow \log x^3 = \log e^{3t} = 3t \cdot \log e$$

Özellik / *Property* 12: $\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$

ÖRNEK / EXAMPLE 13

$\log_4 9 \cdot \log_{27} 125 \cdot \log_5 2 = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\begin{aligned}\log_4 9 \cdot \log_{27} 125 \cdot \log_5 2 &= \log_{2^2} 3^2 \cdot \log_{3^3} 5^3 \cdot \log_5 2\\ &= \frac{2}{2} \cdot \log_2 3 \cdot \frac{3}{3} \log_3 5 \cdot \log_5 2\\ &= \log_2 3 \cdot \log_3 5 \cdot \log_5 2\\ &= \log_2 2 = 1\end{aligned}$$

PEKİŞTİRME / REINFORCE

$\log_3 4 \cdot \log_2 9 = ?$

YANIT / ANSWER : 4

ÖRNEK / EXAMPLE 14

$\ln 2 \cdot \log_4 9 \cdot \log_3 e = ?$

A) e B) e^2 C) 1 D) –1 E) 0

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{aligned}\ln 2 \cdot \log_4 9 \cdot \log_3 e &= \log_e 2 \cdot \log_2 3 \cdot \log_3 e\\ &= \log_e e\\ &= 1\end{aligned}$$

LOGARİTMA FONKSİYONUNUN TERSİ / INVERSE of a LOGARITHM FUNCTION

$$f(x) = \log_a x \Rightarrow f^{-1}(x) = a^x$$

PEKİŞTİRME / REINFORCE

$f(x) = \ln 2x \Rightarrow f^{-1}(x) = ?$

YANIT / ANSWER : $e^x / 2$

ÖRNEK / EXAMPLE 15

$f(x) = \log_3(4x - 5) \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\frac{3^x+5}{4}$ B) $\frac{3^x+4}{5}$ C) $\frac{4^x+3}{5}$

D) $\frac{4^x+5}{3}$ E) $\frac{5^x+3}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$f(x) = \log_3(4x-5)$$

$$3^y = 4x - 5$$

$$3^y + 5 = 4x$$

$$x = \frac{3^y + 5}{4}$$

$$f^{-1}(x) = \frac{3^x + 5}{4}$$

ÖRNEK / EXAMPLE 16

$f(x) = \log_3(ax + 1)$,

$f(2) = 2 \Rightarrow$ **$a = ?$**

A) 2 B) 3 C) 4 D) 6 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$f(2) = \log_3(2a + 1) = 2$ [Verilen eşitlik / *Given equality*]

$2a + 1 = 9$ [1. özellik / *Property 1*]

$2a = 8$ [Sadeleştir. / *Simplify.*]

$a = 4$ [Sonuç / *Conclusion*]

LOGARİTMİK DENKLEMLER / LOGARITHMIC EQUATIONS

Logaritma terimi içeren ve logaritma özellikleri kullanılarak çözülen denklemlere *logaritmik denklemler* denir.

Logaritmik denklemi çözmek için, denklem üstel biçimde yazılır ve değişken için çözüm yapılır.

The equations involving a logarithmic term are called logarithmic equations that are solved by using the properties of logarithm.

To solve a logarithmic equation, rewrite the equation in exponential form and solve for the variable.

NOT / NOTE

$\log_a f(x)$ fonksiyonunda

In the function $\log_a f(x)$

$a > 0$

$a \neq 1$

$f(x) > 0$

ÖRNEK / EXAMPLE 17

$\log_2(3x-2) = 4 + \log_2(x-5) \Rightarrow \mathbf{x = ?}$

A) 4 B) 5 C) 6 D) 7 E) 8

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\log_2(3x-2) = 4 + \log_2(x-5)$ [4 yerine $\log_2 16$ yaz. / *Substitute $\log_2 16$ for 4.*]

$\log_2(3x-2) = \log_2 16(x-5)$ [Çarpma özelliği / *Multiplication property*]

$3x - 2 = 16x - 80$ [Sadeleştir. / *Simplify.*]

$78 = 13x$ [Benzer terimleri topla. / *Combine like terms.*]

$x = 6$ [Sonuç / *Conclusion*]

ÖRNEK / EXAMPLE 18

$(\log_3 x)^2 - 4\log_3 x + 3 = 0 \Rightarrow \mathbf{\sum x = ?}$

A) 26 B) 28 C) 30 D) 32 E) 34

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$t = \log_3 x \Rightarrow t^2 - 4t + 3 = 0$

$(t-3)\cdot(t-1) = 0$

$t = 3 \quad , \quad t = 1$

$\log_3 x = 3 \quad , \quad \log_3 x = 1$

$x = 27 \qquad x = 3$

$\Rightarrow \sum x = 27 + 3 = 30$

LOGARİTMİK EŞİTSİZLİKLER / LOGARITHMIC INEQUALITIES

$a > 1, \quad \log_a f(x) < \log_a g(x) \Rightarrow 0 < f(x) < g(x)$

$0 < a < 1, \quad \log_a f(x) < \log_a g(x) \Rightarrow f(x) > g(x) > 0$

ÖRNEK / EXAMPLE 19

$x \in Z$, $\log_4(x-1) > 2 + \log_4(2x-1) \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$ B) {3} C) {4} D) {5} E) {6}

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$\log_4(x-1) > 2 + \log_4(2x-1)$

$\log_4(x-1) > \log_4(32x-16)$

$x - 1 > 32x - 16$

$15 > 31x \Rightarrow \frac{15}{31} > x$

$x - 1 > 0 \qquad 2x - 1 > 0$

$x > 1 \qquad x > \frac{1}{2}$

$\Rightarrow x > 1$ ve(and) $x < \frac{15}{31}$

$\Rightarrow$ ÇK(SS) = $\varnothing$

		15/31		1/2		1	
x − 1	−		−		−	○	+
2x − 1	−		−	○	+		+
x = 15/31	−	○	+		+		+

$\Rightarrow$ ÇK(SS) = $\varnothing$

ÖRNEK / EXAMPLE 20

$\log_{\frac{1}{2}}(2x-3) < \log_{\frac{1}{2}}(3x+1) + 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left(-\frac{3}{4}, \infty\right)$ B) $(3, \infty)$ C) $(7, \infty)$ D) $(8, \infty)$ E) $(12, \infty)$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\log_{\frac{1}{2}}(2x-3) < \log_{\frac{1}{2}}(3x+1) + \log_{\frac{1}{2}}\frac{1}{2}$

$\log_{\frac{1}{2}}(2x-3) < \log_{\frac{1}{2}}\left(\frac{3x+1}{2}\right)$

$2x - 3 > \frac{3x+1}{2}$

$2x - 3 - \frac{3x+1}{2} > 0$

$x - 7 > 0 \Rightarrow x > 7$

$2x - 3 > 0 \quad , \quad 3x + 1 > 0$

$x > \frac{3}{2} \qquad x > -\frac{1}{3} \quad \Rightarrow x > 7$

ÇK(SS) = $(7, \infty)$

		−1/3		3/2		7	
x − 7	−		−		−	○	+
2x − 3	−		−	○	+		+
3x + 1	−	○	+		+		+

$\Rightarrow$ ÇK(SS) = $(7, \infty)$

ÖRNEK / EXAMPLE 21

YILDIZ TEKNİK ÜNİ. YÖS 2013

$\frac{1}{1+\log_{1/5} 15} = ?$

A) $\log_3 5$ B) $\log_5 3$ C) $-\log_3 5$

D) $-\log 5^{1/5}$ E) $\log_{1/5} 3$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\frac{1}{1+\log_{1/5} 15} = \frac{1}{1-(\log_5 5+\log_5 3)}$$

$$= \frac{1}{-\log_5 3}$$

$$= -\log_3 5$$

ÖRNEK / EXAMPLE 22

İSTANBUL ÜNİV. YÖS 2013

$\log_x 4 = 3$

$\log_2 x = y$

$\Rightarrow 3x^3y = ?$

A) $\frac{81}{2}$ B) 24 C) 12 D) 8 E) 6

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\log_x 4 = 3 \Rightarrow 4 = x^3 \Rightarrow x = 2^{\frac{2}{3}}$$

$$\log_2 x = y \Rightarrow 2^y = x \Rightarrow y = \frac{2}{3}$$

$$3x^3y = 3\cdot(2^{\frac{2}{3}})^3\cdot\frac{2}{3} = 3\cdot4\cdot\frac{2}{3} = 8$$

ÖRNEK / EXAMPLE 23

MARMARA ÜNİV. YÖS 2016

$9^{(\log_3 2)+1} = ?$

A) 26 B) 27 C) 36 D) 40 E) 48

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$(3^2)^{(\log_3 2)+1} = 3^{2(\log_3 2)+2} = 3^{\log_3 2^2} \cdot 3^2$$

$$= 4 \cdot 9 = 36$$

ÖRNEK / EXAMPLE 24

ANKARA ÜNİV. YÖS 2016

$\log_5 [\log_3(x-1)] = 0 \Rightarrow x = ?$

A) 5 B) 4 C) 3 D) 2 E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\log_5\left[\log_3(x-1)\right] = 0$$

$$5^0 = \log_3(x-1)$$

$$1 = \log_3(x-1)$$

$$3^1 = x-1 \Rightarrow x = 4$$

1. $\log_{0,01}100000 = x \Rightarrow$ **$x = ?$**

A) –1

B) $-\frac{3}{2}$

C) –2

D) $-\frac{5}{2}$

E) –3

2. $3\log_2 x = 6 \Rightarrow$ **$x = ?$**

A) 2

B) 4

C) 6

D) 8

E) 12

3. $f(x) = 3 + \log_3(x - 2) \Rightarrow$ **$f(11) = ?$**

A) 6

B) 5

C) 4

D) 3

E) 2

4. $f(x) = \log_3(2x - 3)$,
$g(x) = 5^x \Rightarrow$ **$fog^{-1}(25) = ?$**

A) –1

B) 0

C) 1

D) 2

E) 3

5. $5 = 3^a$ ve(and) $2 = 3^b \Rightarrow$ **$\log_3 500 = ?$**

A) a + b

B) 2a + 2b

C) 2a + 3b

D) 3a + 2b

E) 3a + 3b

6. $$\left.\begin{array}{l} x = \log_3 11 \\ y = \log_5 7 \\ z = \log_2 18 \end{array}\right\} \Rightarrow \mathbf{? < ? < ?}$$

A) x < y < z

B) z < y < x

C) y < z < x

D) y < x < z

E) x < z < y

7. $\log_2 3 = a$ ve (and) $\log_2 5 = b \Rightarrow \mathbf{\log_6 15 = ?}$

A) $\dfrac{2a+b}{1+2a}$

B) $\dfrac{a+b}{1+a}$

C) $\dfrac{a+2b}{3+a}$

D) $\dfrac{a+b}{2+a}$

E) $\dfrac{3a+b}{3+a}$

---

8. $3^{\log_x 5} + 5^{\log_x 3} = 18 \Rightarrow \mathbf{x = ?}$

A) $\sqrt{3}$

B) $\sqrt{5}$

C) $\sqrt{6}$

D) $\sqrt{7}$

E) $\sqrt{10}$

---

9. $\log 5 = a \Rightarrow \mathbf{\log 4 = ?}$

A) $1 - a$

B) $2 - 2a$

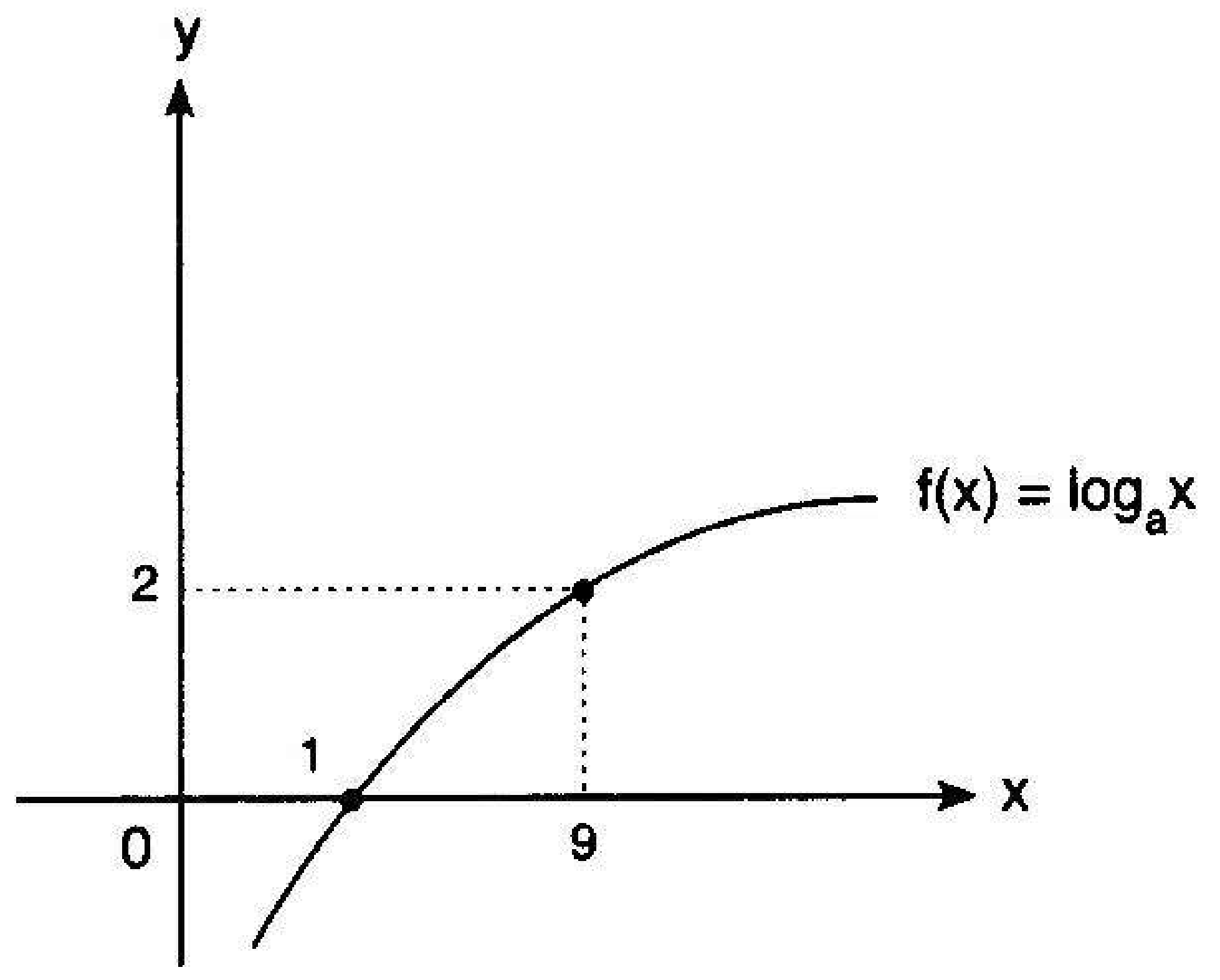
C) $3 - 3a$

D) $2 + a$

E) $1 + 2a$

---

10. $f\left(\dfrac{1}{3}\right) = ?$



A) -1

B) $-\dfrac{1}{2}$

C) 0

D) $\dfrac{1}{2}$

E) 3

---

11. $x \in \mathbb{Z}$,

$\log_3(2x - 3) < 2 \Rightarrow \mathbf{\sum x = ?}$

A) 12

B) 14

C) 15

D) 21

E) 22

---

12. $x^{\ln x} - e^{12} \cdot x = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{e, e^3\}$

B) $\left\{\dfrac{1}{e^3}, e^4\right\}$

C) $\{1, e^4\}$

D) $\left\{1, \dfrac{1}{e^3}, e^4\right\}$

E) $\{e^4\}$

YANITLAR / ANSWERS

1. D	2. B	3. B	4. B	5. D	6. D	7. B	8. B
9. B	10. A	11. B	12. B				

1. $\log_{0,01} 100000 = x$

$$\log_{10^{-2}} 10^5 = x$$

$$-\frac{5}{2} = x$$

Yanıt / The Answer is D

2. $3 \cdot \log_2 x = 6$

$$\log_2 x = 2$$

$$x = 4$$

Yanıt / The Answer is B

3. $f(x) = 3 + \log_3 (x - 2)$

$$\Rightarrow f(11) = 3 + \log_3 9$$

$$= 3 + 2$$

$$= 5$$

Yanıt / The Answer is B

4. $g(x) = 5^x \Rightarrow x = \log_5 y$

$$g^{-1}(x) = \log_5 y$$

$$fog^{-1}(25) = f(\log_5 25)$$

$$= f(2)$$

$$= \log_3 (2 \cdot 2 - 3) = 0$$

Yanıt / The Answer is B

5. $3^a = 5 \Rightarrow a = \log_3 5$

$$3^b = 2 \Rightarrow b = \log_3 2$$

$$\log_3 500 = \log_3 (5^3 \cdot 2^2)$$

$$= 3 \log_3 5 + 2\log_3 2$$

$$= 3a + 2b$$

Yanıt / The Answer is D

6.

$$\left.\begin{array}{l} x = \log_3 11 \Rightarrow 3^x = 11 \Rightarrow 2 < x < 3 \\ y = \log_5 7 \Rightarrow 5^y = 7 \Rightarrow 1 < y < 2 \\ z = \log_2 18 \Rightarrow 2^z = 18 \Rightarrow 4 < z < 5 \end{array}\right\} \Rightarrow y < x < z$$

Yanıt / The Answer is D

7. $$\log_6 15 = \frac{\log_2 15}{\log_2 6}$$

$$= \frac{\log_2 3 + \log_2 5}{\log_2 3 + \log_2 2}$$

$$= \frac{a+b}{a+1}$$

Yanıt / The Answer is B

---

8. $3^{\log_x 5} + 5^{\log_x 3} = 18$

$3^{\log_x 5} + 3^{\log_x 5} = 18$

$3^{\log_x 5} = 9$

$\log_x 5 = 2$

$x^2 = 5$

$x = \sqrt{5}$

Yanıt / The Answer is B

---

9. $\log 4 = 2\log 2$

$= 2\,[\log 10 - \log 5]$

$= 2\,[1 - a]$

$= 2 - 2a$

Yanıt / The Answer is B

---

10. $x = 9 \Rightarrow y = 2$

$f(x) = \log_a x \Rightarrow f(9) = \log_a 9 = 2$

$a^2 = 9$

$a = 3$

$$f(x) = \log_3 x \Rightarrow f\left(\frac{1}{3}\right) = \log_3 \frac{1}{3}$$

$= \log_3 3^{-1}$

$= -1$

Yanıt / The Answer is A

---

11. $\log_3 (2x-3) < 2$

$2x - 3 < 9$

$2x < 12$

$x < 6$

$$2x - 3 > 0 \Rightarrow x > \frac{3}{2}$$

$$\frac{3}{2} < x < 6 \Rightarrow \sum x = 2 + 3 + 4 + 5 = 14$$

Yanıt / The Answer is B

---

12. $x^{\ln x} - e^{12}x = 0$

$x^{\ln x} = e^{12}x$

$\ln x \cdot \ln x = \ln x + \ln e^{12}$

$(\ln x)^2 = \ln x + 12$

$(\ln x)^2 - \ln x - 12 = 0$

$\ln x = t \Rightarrow t^2 - t - 12 = 0$

$(t-4)\cdot(t+3) = 0$

$t = 4, \quad t = -3$

$$\left.\begin{array}{l} \ln x = 4 \Rightarrow x = e^4 \\ \ln x = -3 \Rightarrow x = e^{-3} \end{array}\right\} \Rightarrow \text{ÇK(SS)} = \left\{e^4, \frac{1}{e^3}\right\}$$

Yanıt / The Answer is B

1. $\log_2 \frac{1}{16} + \log_3 27 = ?$

A) –5

B) –4

C) –3

D) –1

E) 0

2. $\log_5 x \cdot \log_{25} x \cdot \log_{125} x = \frac{1}{6} \Rightarrow x = ?$

A) 1

B) 2

C) 3

D) 4

E) 5

3. $9^{\log_3 x} = 8 \Rightarrow \log_2 x = ?$

A) $\frac{1}{2}$

B) 1

C) $\frac{3}{2}$

D) 2

E) $\frac{5}{2}$

4. $\log_4 9 \cdot \log_{\sqrt{3}} 125 \cdot \log_{25} 8 = ?$

A) 5

B) 6

C) 7

D) 8

E) 9

5. $\log_3 8! = a \Rightarrow \log_{27} 9! = ?$

A) $\frac{a+1}{2}$

B) $\frac{a+3}{2}$

C) $\frac{a+1}{3}$

D) $\frac{a+2}{3}$

E) $\frac{a+5}{4}$

6. $\left.\begin{array}{l} \log_2\left(\frac{x}{y^2}\right)=0 \\ \log_2(x \cdot y)=3 \end{array}\right\} \Rightarrow x=?$

A) 2

B) 4

C) 8

D) 16

E) 32

7. $\log_3 4 = a \Rightarrow \mathbf{\log_4 12 = ?}$

A) $\dfrac{2a+2}{a}$

B) $\dfrac{3a+1}{a}$

C) $\dfrac{a+1}{a}$

D) $\dfrac{a+3}{a+1}$

E) $\dfrac{a-2}{a+1}$

8. $2^{\log_x 3} + 3^{\log_x 2} = 18 \Rightarrow \mathbf{x = ?}$

A) 4

B) $\sqrt{5}$

C) $\sqrt{3}$

D) $\sqrt{2}$

E) 2

9. $\log_5 2 + \log_5 x - \log_5 4 = 1 \Rightarrow \mathbf{x = ?}$

A) $\dfrac{1}{5}$

B) $\dfrac{5}{4}$

C) $\dfrac{5}{2}$

D) 5

E) 10

10. $f(x) = \log_3(4x - 1) \Rightarrow \mathbf{f^{-1}(3) = ?}$

A) 5

B) 6

C) 7

D) 8

E) 9

11. $x \in Z$,

$\log_3(x + 4) < \log_9 16 \Rightarrow \mathbf{\sum x = ?}$

A) –2

B) –3

C) –4

D) –5

E) –6

12. $\log_{\frac{1}{3}}(x+5) < \log_3 \dfrac{1}{7} \Rightarrow \mathbf{ÇK(SS) = ?}$

A) $(-\infty, 2)$

B) $(-\infty, 3)$

C) $(2, \infty)$

D) $(3, \infty)$

E) $(4, \infty)$

YANITLAR / ANSWERS							
1. D	2. E	3. C	4. E	5. D	6. B	7. C	8. D
9. E	10. C	11. E	12. C				

1. $\log_2 \frac{1}{16} + \log_3 27 = \log_2 2^{-4} + \log_3 3^3$

$= -4 + 3$

$= -1$

Yanıt / The Answer is D

2. $\log_5 x \cdot \frac{1}{2}\log_5 x \cdot \frac{1}{3}\log_5 x = \frac{1}{6}$

$(\log_5 x)^3 = 1$

$\log_5 x = 1$

$x = 5$

Yanıt / The Answer is E

3. $9^{\log_3 x} = 8 \Rightarrow \log_3 x = \log_9 8$

$= \log_{3^2} 2^3 \Rightarrow x = 2^{\frac{3}{2}}$

$\log_2 x = \log_2 2^{\frac{3}{2}} = \frac{3}{2}$

Yanıt / The Answer is C

4. $\log_4 9 \cdot \log_{\sqrt{3}} 125 \cdot \log_{25} 8 = \log_{2^2} 3^2 \cdot \log_{3^{\frac{1}{2}}} 5^3 \cdot \log_{5^2} 2^3$

$= \log_2 3 \cdot 6 \cdot \log_3 5 \cdot \frac{3}{2} \cdot \log_5 2$

$= 6 \cdot \frac{3}{2} \cdot \log_2 2$

$= 9$

Yanıt / The Answer is E

5. $\log_{27} 9! = \log_{27} 9 + \log_{27} 8!$

$= \frac{2}{3} + \frac{1}{3}\log_3 8!$

$= \frac{2}{3} + \frac{1}{3}a$

$= \frac{2+a}{3}$

Yanıt / The Answer is D

6. $\log_2 \left(\frac{x}{y^2}\right) = 0 \Rightarrow \log_2 x - 2\log_2 y = 0$

$\log_2 (x \cdot y) = 3 \Rightarrow \underline{2 / \log_2 x + \log_2 y = 3}$

$3\log_2 x = 6$

$\log_2 x = 2$

$x = 4$

Yanıt / The Answer is B

7. $\log_4 12 = \dfrac{\log_3 12}{\log_3 4}$

$= \dfrac{\log_3 3 + \log_3 4}{\log_3 4}$

$= \dfrac{1+a}{a}$

Yanıt / The Answer is C

8. $2^{\log_x 3} + 3^{\log_x 2} = 18$

$3^{\log_x 2} + 3^{\log_x 2} = 18$

$3^{\log_x 2} = 9$

$\log_x 2 = 2$

$x^2 = 2$

$x = \sqrt{2}$

Yanıt / The Answer is D

9. $\log_5 2 + \log_5 x - \log_5 4 = 1$

$\log_5 \dfrac{2x}{4} = 1$

$\dfrac{2x}{4} = 5$

$x = 10$

Yanıt / The Answer is E

10. $f(x) = \log_3 (4x - 1) \Rightarrow f^{-1} (\log_3 (4x - 1)) = x$

$\log_3 (4x - 1) = 3$

$4x - 1 = 27$

$4x = 28$

$x = 7$

Yanıt / The Answer is C

11. $\log_3 (x+4) < \log_{3^2} 4^2$

$\log_3 (x+4) < \log_3 4$

$x+4 < 4$

$x < 0$

$x+4 > 0 \Rightarrow x > -4$

$\sum x = -3-2-1 = -6$

Yanıt / The Answer is E

12. $\log_{\frac{1}{3}} (x+5) < \log_3 \dfrac{1}{7}$

$\log_{\frac{1}{3}} (x+5) < \log_{\frac{1}{3}} 7$

$x+5 > 7$

$x > 2$

$\Rightarrow$ ÇK(SS) = $(2, \infty)$

Yanıt / The Answer is C

1. $\log_5 x = -3 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: $\frac{1}{125}$

2. $3\log_4(x + 2) = 9 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **62**

3. $\log_2(x - 4) = 5 \Rightarrow \mathbf{\log_6 x = ?}$

Yanıt / The Answer is: **2**

4. $\log_3(x + 1) = 2 \Rightarrow \mathbf{\log_2 x = ?}$

Yanıt / The Answer is: **3**

5. $\left.\begin{array}{r} \log(x \cdot y) = 3 \\ \log\dfrac{x}{y} = 1 \end{array}\right\} \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **100**

6. $3^{\log_9 125} = 5^{x+1} \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: $\frac{1}{2}$

7. $\log 2 = a \Rightarrow \mathbf{\log 25 = ?}$

Yanıt / The Answer is: **2 – 2a**

8. $f(x) = \log_3(2x - 1) \Rightarrow \mathbf{f^{-1}(3) = ?}$

Yanıt / The Answer is: **14**

9. $\log_5(\log_4(\log_3(\log_2(x)))) = 0 \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: $\mathbf{2^{81}}$

10. $3^{\log_2 x} = 7^{\log_2 3} \Rightarrow \mathbf{x = ?}$

Yanıt / The Answer is: **7**

11. $\mathbf{\log_3(\log_2 27 \cdot \log_3 8) = ?}$

Yanıt / The Answer is: **2**

12. $x \in Z$,

$\log_5 x + \log_5(x + 5) \le \log_5 3 + \log_5 2 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is: **{1}**

1. $\log_{\frac{1}{3}} x = -\log_3 4 \Rightarrow$ **x = ?**

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 4

2. $\log_3(\log_5(\log_2(5x + 2))) = 0 \Rightarrow$ **x = ?**

A) 6 B) 7 C) 8 D) 9 E) 10

3. $\log_9 x = -\frac{1}{2} \Rightarrow$ **x = ?**

A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) 1 D) 3 E) 9

4. $\log_2(2 + 5\log_3 x) = 5 \Rightarrow$ **$\log_9 x$ = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

5. $\log_{25}81 \cdot \log_8 49 \cdot \log_9 125 \cdot \log_7 4 = ?$

A) 3 B) $\frac{7}{2}$ C) 4 D) $\frac{9}{2}$ E) 5

6. $\log_2 5 = a \Rightarrow$ **log20 = ?**

A) $\frac{1+a}{a}$ B) $\frac{2+a}{1+a}$ C) $\frac{3+a}{2+a}$ D) $\frac{1+2a}{2+a}$ E) $\frac{2+a}{2a}$

7. $f(x) = \log_2(4x + m)$,

$f^{-1}(3) = -1 \Rightarrow$ **m = ?**

A) 4 B) 8 C) 12

D) 16 E) 18

8. log2 = m ve(and) log3 = n $\Rightarrow$ $\mathbf{\log_{24}12 = ?}$

A) $\frac{m+n}{2m+n}$ B) $\frac{2m+n}{m+2n}$ C) $\frac{2m+n}{3m+n}$

D) $\frac{3m+n}{m+n}$ E) $\frac{m+3n}{m+n}$

9. $\mathbf{\log_{\sqrt{3}}4 \cdot \log_8 25 \cdot \log_5 27 = ?}$

A) 2 B) 4 C) 6

D) 8 E) 9

10. $x \in Z$,

$(\log_3(x-3))^2 - 4 \cdot \log_3(x-3) - 5 = 0$

$\Rightarrow$ **x = ?**

A) 30 B) 84 C) 246

D) 732 E) 750

11. $(\log_2 x)^2 - 5\log_2 x + 6 = 0 \Rightarrow \sum x = ?$

A) 8 B) 9 C) 10

D) 12 E) 15

12. $x \in Z$,

$\log_3(\log_{\frac{1}{2}}(x+1)) < 0 \Rightarrow$ **min(x) = ?**

A) 4 B) 3 C) 2

D) 1 E) 0

YANITLAR / ANSWERS

1. E	**2. A**	**3. B**	**4. A**	**5. C**	**6. B**	**7. C**	**8. C**
9. D	**10. C**	**11. D**	**12. E**				

1. $\log_2 x - 2 = \log_2 y \Rightarrow \mathbf{x = ?}$

A) y
B) 2y
C) 3y
D) 4y
E) 5y

2. $\left.\begin{array}{r} \ln(ab) = 5 \\ \ln\left(\frac{a}{b}\right) = 1 \end{array}\right\} \Rightarrow \mathbf{a = ?}$

A) 1
B) e
C) e^2
D) e^3
E) e^4

3. $\dfrac{\log_3 x}{\log_x 3} = 4 \Rightarrow \prod x = ?$

A) 1
B) 2
C) 3
D) 8
E) 9

4. $2^{4+\ln x} + 2^{\ln x} = 272 \Rightarrow \mathbf{x = ?}$

A) e^5
B) e^4
C) e^3
D) e^2
E) e

5. $\log_3(4x - 2) = 1 + \log_3(x + 7)$

$\Rightarrow \mathbf{x = ?}$

A) 23
B) 25
C) 27
D) 29
E) 32

6. $\log_3 5 = a \Rightarrow \mathbf{\log_{15} 25 = ?}$

A) $\dfrac{a}{a+2}$
B) $\dfrac{2a}{a+2}$
C) $\dfrac{a+1}{a}$
D) $\dfrac{2a}{a+1}$
E) $\dfrac{3a}{a+2}$

7. $\log_5 3 = a,\ \log_5 7 = b \Rightarrow \mathbf{\log_{35} 9 = ?}$

A) $\dfrac{a+1}{3+b}$ B) $\dfrac{a}{2+b}$ C) $\dfrac{2a}{1+b}$

D) $\dfrac{a+1}{b}$ E) $\dfrac{3a}{2+b}$

8. $f(x) = 3^{4x-1} \Rightarrow \mathbf{f^{-1}(\sqrt{3}) = ?}$

A) $\dfrac{1}{8}$ B) $\dfrac{1}{4}$ C) $\dfrac{3}{8}$

D) $\dfrac{1}{2}$ E) $\dfrac{5}{8}$

9. $f(x) = 2\log_3 x - 5 \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $\sqrt{3^{x+2}}$ B) $\sqrt{3^{x+3}}$ C) $\sqrt{3^{x+4}}$

D) $\sqrt{3^{x+5}}$ E) $\sqrt{3^{x+6}}$

10.
$$\left.\begin{aligned} x &= \log_4 \frac{1}{15} \\ y &= \log_3 \frac{1}{2} \\ z &= \log_5 \frac{1}{30} \end{aligned}\right\} \Rightarrow \mathbf{? < ? < ?}$$

A) $x < y < z$ B) $z < x < y$ C) $z < y < x$

D) $y < x < z$ E) $x < z < y$

11. $\log_3 x = \dfrac{1}{8} \Rightarrow \mathbf{\log_9 3x^4 = ?}$

A) $\dfrac{1}{4}$ B) $\dfrac{1}{2}$ C) $\dfrac{3}{4}$

D) 1 E) $\dfrac{5}{4}$

12. $\mathbf{f\left(\dfrac{9}{16}\right) = ?}$

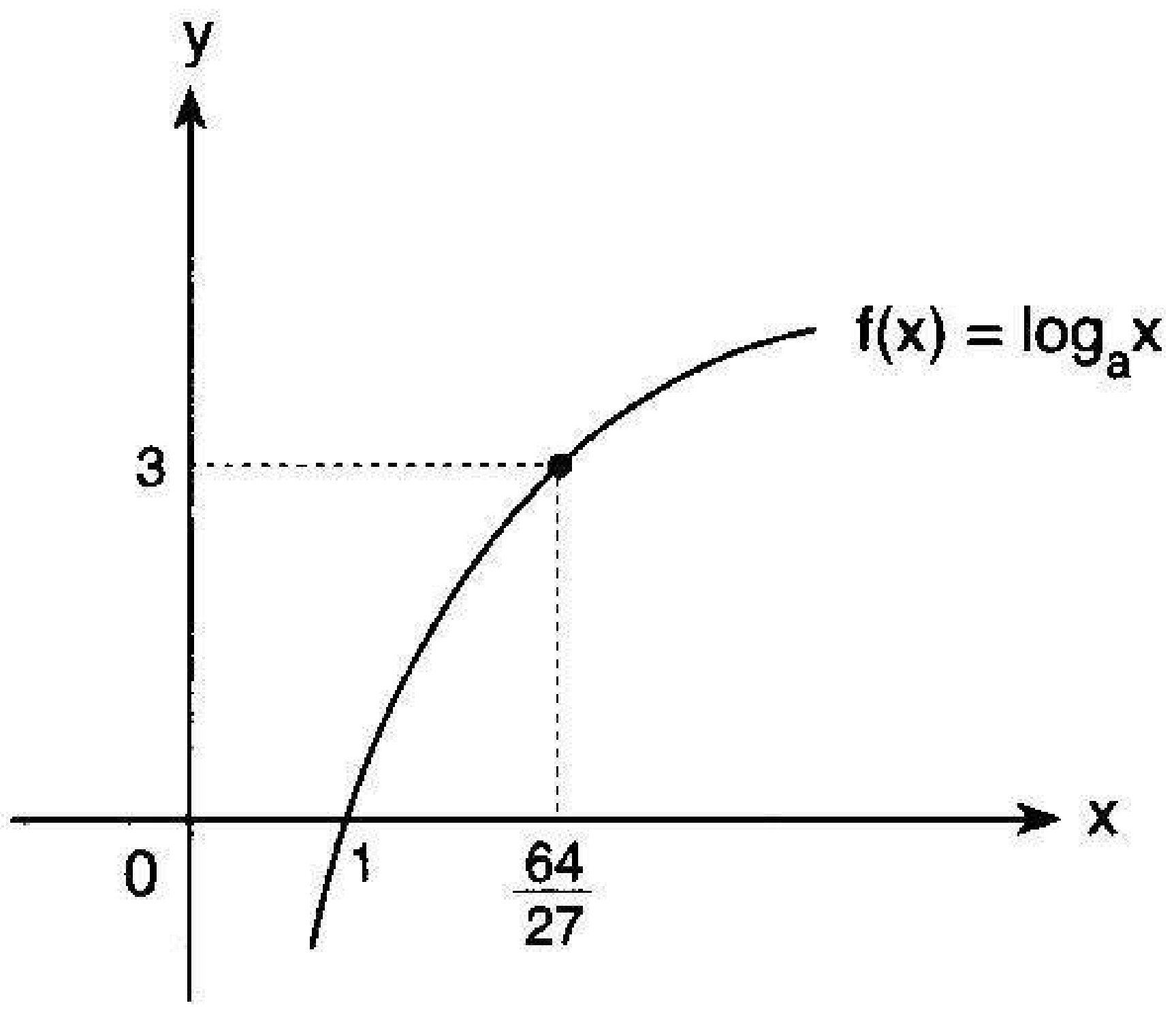
A) 2

B) 1

C) 0

D) –1

E) –2



YANITLAR / ANSWERS							
1. D	2. D	3. A	4. B	5. A	6. D	7. C	8. C
9. D	10. B	11. C	12. E				

1. $\log(2x + 16) = 1 + \log 2 + \log(x - 1) \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $\ln 6 \cdot \log_6 8 \cdot \log_2 e = ?$

A) 3 B) 5 C) 6 D) 7 E) 8

3. $\left.\begin{matrix}\log_3 2 = a \\ \log_2 5 = b\end{matrix}\right\} \Rightarrow \log_5 6 = ?$

A) $\frac{1+ab}{a}$ B) $\frac{1+2a}{b}$ C) $\frac{1+a}{ab}$ D) $\frac{2+a}{ab}$ E) $\frac{1+b}{2a}$

4. $\log_3\sqrt{7} \cdot \log_{49} 8 \cdot \log_4 27 = ?$

A) $\frac{3}{4}$ B) $\frac{7}{8}$ C) 1 D) $\frac{9}{8}$ E) $\frac{5}{4}$

5. $\left.\begin{matrix}\log_3 5 = a \\ \log_2 3 = b\end{matrix}\right\} \Rightarrow \log_6 5 = ?$

A) $\frac{ab}{b+1}$ B) $\frac{2a}{b+3}$ C) $\frac{3b}{b+2}$ D) $\frac{ab}{b+2}$ E) $\frac{ab}{a+4}$

6. $\left.\begin{matrix}\log_3 5 = a \\ \log_3 2 = b\end{matrix}\right\} \Rightarrow \log 25 = ?$

A) $\frac{a}{a+b}$ B) $\frac{2a}{a+b}$ C) $\frac{2a}{a+2b}$ D) $\frac{3a}{a+b}$ E) $\frac{a}{3a+b}$

7. $\log_3 2 = a$ ve (and) $\log_3 5 = b \Rightarrow$ **$\log 50 = ?$**

A) $\dfrac{a+2b}{a+b}$ B) $\dfrac{2a+b}{a+3b}$ C) $\dfrac{a+b}{a+2b}$

D) $\dfrac{3a+b}{a+b}$ E) $\dfrac{a+3b}{a+b}$

8. $f(x) = \sqrt{5^{x+1}} \Rightarrow$ **$f^{-1}(x) = ?$**

A) $2\log_5 x + 2$

B) $2\log_5 x - 1$

C) $5\log_2 x - 1$

D) $5\log_2 x + 3$

E) $\log_2 x + 5$

9. $3\sqrt{\ln x} - 2\ln\sqrt{x} = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{1, e^3\}$ B) $\{e^3, e^9\}$ C) $\{e, e^3\}$

D) $\{e, e^9\}$ E) $\{1, e^9\}$

10. $\log_a 3 = 2$,

$\log_3(a \cdot 3^b) = 7 \Rightarrow$ **$b = ?$**

A) $\dfrac{11}{2}$ B) 6 C) $\dfrac{13}{2}$

D) 7 E) $\dfrac{15}{2}$

11. $\left.\begin{array}{l}\log_4 7 = a\\ \log_4 5 = b\\ \log_4 3 = c\end{array}\right\} \Rightarrow$ **$\log_4 525 = ?$**

A) a + 2b + c B) 2a + b + c C) a + 3b + 2c

D) a + 2b + 2c E) a + b + c

12. $f(x) = \log_2(3x + 1)$,

$g(x) = 2x - 5 \Rightarrow$ **$(fog)^{-1}(4) = ?$**

A) 5 B) 3 C) 1

D) –3 E) –5

YANITLAR / ANSWERS

1. B **2. A** **3. C** **4. D** **5. A** **6. B** **7. A** **8. B**

9. E **10. C** **11. A** **12. A**

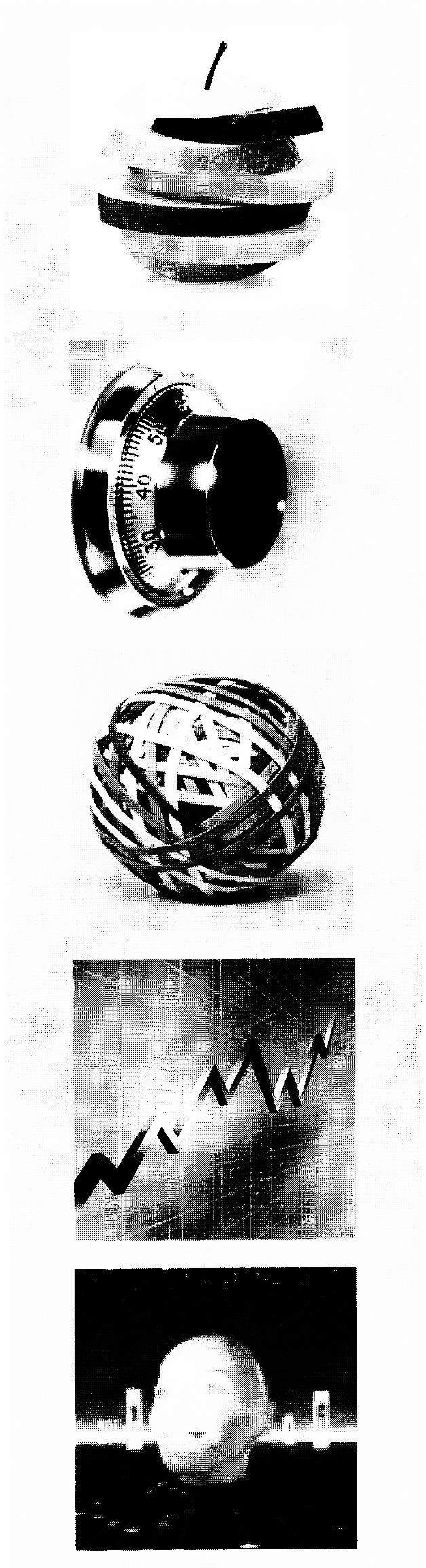
ÜNİTE 8
UNIT

Bu ünitede; tümevarım, diziler ve seriler; permütasyon, kombinasyon ve binom açılımı; matris ve determinant ayrıntılı açıklanmıştır.

Bu ünite; 96 çözümlü örnek, 36 alıştırma ve 96 test sorusundan oluşmaktadır.

In this unit, induction, sequences and series; permutation, combination and binomial expansion; matrix and determinant are explained in detailed.

This unit includes; 96 examples, 36 exercises, and 96 test questions.



- **TÜMEVARIM, DİZİLER ve SERİLER**
  INDUCTION, SEQUENCES, and SERIES .... 557 – 584

- **BİNOM AÇILIMI**
  BINOMIAL EXPANSION .... 585 – 596

- **MATRİS ve DETERMİNANT**
  MATRIX and DETERMINANT .... 597 – 626

BU SAYFADA KONU MATERYALİ YOKTUR.

NO SUBJECT MATERIAL ON THIS PAGE.

TÜMEVARIM, DİZİLER ve SERİLER
INDUCTION, SEQUENCES, and SERIES

BÖLÜM
CHAPTER
1

TÜMEVARIM / INDUCTION

Tanım: Tümevarım matematikte kullanılan ispat yöntemlerinden biridir.

$\forall n \in N$ olmak üzere P(n) açık önermesinin doğruluğunu tümevarım yöntemiyle kanıtlamak için,

1. $n = 1$ için P(1) önermesinin doğruluğu gösterilir.
2. $k \in N^+$ olmak üzere, P(k) önermesi doğru varsayılır.
3. P(k + 1) önermesinin doğruluğu gösterilir.

Bu durumda, $\forall n \in N^+$ için P(n) doğrudur denir.

***Definition:** Mathematical induction is a method of proving statements.*

Let P(n) be an open sentence involving $\forall \in N$

1. *Prove that P(1) is true for $n = 1$.*
2. *Assume that P(k) is true for $n = k$.*
3. *Show that P(k + 1) is true for $n = k + 1$*

In this situation, for $\forall n \in N^+$, P(n) is true.

TOPLAM ve ÇARPIM SEMBOLLERİ
SUMMATION and MULTIPLICATION SYMBOLS

A. TOPLAM SEMBOLÜ / SUMMATION SYMBOL (Σ)

$\forall n \in N^+$ ve $a_k \in R$ olmak üzere,

$a_1 + a_2 + a_3 + \ldots + a_n$ toplamını daha kısa olarak $\sum_{k=1}^{n} a_k$ biçiminde gösterilir.

For all $\forall n \in N^+$, $a_k \in R$

the sum $a_1 + a_2 + a_3 + \ldots + a_n$ can be represented by the symbol $\sum_{k=1}^{n} a_k$.

$\sum$ gösterimi sigma diye okunur. k ya da *toplama indisi* denir.

Representation $\sum$ read "sigma." k is called the index of summation.

NOT / NOTE

$\sum_{k=1}^{5} a_k$ ifadesinde k harfi yerine başka harfler kullanılabilir ve alt sınır 1 olmayabilir.

The letter k can be replaced by any other symbols and upper limit cannot be 1.

$$\sum_{i=5}^{20} i, \quad \sum_{i=-3}^{30} i, \quad \sum_{p=-7}^{13} 3^p$$

PEKİŞTİRME / REINFORCE

$\sum_{k=1}^{3} k^2 = ?$

YANIT / ANSWER : 14

ÖRNEK / EXAMPLE 1

$\sum_{k=1}^{7} k = ?$

A) 18 B) 21 C) 28 D) 32 E) 48

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\sum_{k=1}^{7} k = 1+2+3+4+5+6+7 = 28$

ÖRNEK / EXAMPLE 2

$\sum_{k=1}^{4} k^2 = ?$

A) 30 B) 35 C) 40 D) 55 E) 68

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\begin{aligned}\sum_{k=1}^{4} k^2 &= 1^2+2^2+3^2+4^2 \\ &= 1+4+9+16 \\ &= 30\end{aligned}$$

ÖRNEK / EXAMPLE 3

$$\sum_{k=1}^{7} \log_2\left(1+\frac{1}{k}\right)=?$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer C

$$\sum_{k=1}^{7} \log_2\left(1+\frac{1}{k}\right)=\sum_{k=1}^{7} \log_2\left(\frac{k+1}{k}\right)$$

$$=\log_2\frac{2}{1}+\log_2\frac{3}{2}+\log_2\frac{4}{3}+\ldots+\log_2\frac{8}{7}$$

$$=\log_2\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\ldots\cdot\frac{8}{7}$$

$$=\log_2 8=3$$

ÖRNEK / EXAMPLE 4

$$\sum_{k=-3}^{7} k=?$$

A) 7 B) 9 C) 12 D) 15 E) 22

ÇÖZÜM / SOLUTION

Yanıt / Answer E

$$\sum_{k=-3}^{7} k=-3-2-1+0+1+2+3+4+5+6+7=22$$

Toplam Sembolünün Özellikleri / Properties of Summation Notation

1. $\sum_{k=1}^{n}(a_k \mp b_k)=\sum_{k=1}^{n}a_k \mp \sum_{k=1}^{n}b_k$

2. $\sum_{k=1}^{n}(c\cdot a_k)=c\cdot\sum_{k=1}^{n}a_k$

3. $\sum_{k=1}^{n}c=n\cdot c$

4. *For* $1<m<n$ için, $\sum_{k=1}^{n}a_k=\sum_{k=1}^{m}a_k+\sum_{k=m+1}^{n}a_k$

5. $\sum_{k=r}^{n}a_k=\sum_{k=r\mp p}^{n\mp p}a_{k\pm p}$

ÖRNEK / EXAMPLE 5

$$\sum_{k=1}^{8}3=?$$

A) 15 B) 18 C) 20 D) 24 E) 30

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\sum_{k=1}^{8}3=8\cdot3=24$$

ÖRNEK / EXAMPLE 6

$$\sum_{m=0}^{5}12=?$$

A) 30 B) 35 C) 40 D) 60 E) 72

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\sum_{m=0}^{5}12=\sum_{m=1}^{5+1}12=6\cdot12=72$$

Bazı Önemli Toplam Formülleri / Some Important Summation Formulas

1. $\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

2. $\sum_{k=1}^{n} (2k-1) = 1+3+5+\ldots+(2n-1) = n^2$

3. $\sum_{k=1}^{n} 2k = 2+4+6+8+\ldots+2n = n(n+1)$

4. $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

5. $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+4^3+\ldots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

6. $\sum_{k=0}^{n-1} r^k = 1+r+r^2+\cdots+r^{n-1} = \frac{1-r^n}{1-r} \qquad (r \neq 1)$

7. $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)} = \frac{n}{n+1}$

8. $\sum_{k=1}^{n} k\cdot k! = (n+1)!-1$

9. $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$

10. $\sum_{k=1}^{n} k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

ÖRNEK / EXAMPLE 7

$\sum_{k=1}^{20} k = ?$

A) 120 B) 180 C) 210 D) 340 E) 420

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\sum_{k=1}^{20} k = \frac{20\cdot 21}{2} = 210$$

PEKİŞTİRME / REINFORCE

$\sum\limits_{k=3}^{13} k = ?$

YANIT / ANSWER : 88

ÖRNEK / EXAMPLE 8

$\sum\limits_{n=-3}^{18} n = ?$

A) 165 B) 170 C) 185 D) 196 E) 204

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\sum_{n=-3}^{18} n = \sum_{n=-3+4}^{18+4} (n-4)$$

$$\sum_{n=-3+4}^{18+4} (n-4) = \sum_{n=1}^{22} n - \sum_{n=1}^{22} 4$$

$$= \frac{22 \cdot 23}{2} - 22 \cdot 4$$

$$= 253 - 88 = 165$$

ÖRNEK / EXAMPLE 9

$\sum\limits_{k=1}^{16} (3k-4) = ?$

A) 472 B) 466 C) 408 D) 344 E) 342

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\sum_{k=1}^{16} (3k-4) = \sum_{k=1}^{16} 3k - \sum_{k=1}^{16} 4$$

$$= 3 \cdot \frac{16 \cdot 17}{2} - 4 \cdot 16$$

$$= 344$$

ÖRNEK / EXAMPLE 10

$f(x)=x+3 \Rightarrow \sum_{n=1}^{10}[f(n)]^2=?$

A) 705 B) 745 C) 795 D) 800 E) 805

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$f(n)=n+3 \Rightarrow (f(n))^2=n^2+6n+9$$

$$\sum_{n=1}^{10}(f(n))^2=\sum_{n=1}^{10}(n^2+6n+9)$$

$$=\sum_{n=1}^{10}n^2+\sum_{n=1}^{10}6n+\sum_{n=1}^{10}9$$

$$=\frac{10\cdot 11\cdot 21}{6}+6\cdot\frac{10\cdot 11}{2}+9\cdot 10$$

$$=385+330+90=805$$

ÖRNEK / EXAMPLE 11

$\sum_{k=1}^{20}\frac{1}{k}-\sum_{k=1}^{20}\frac{1}{k+1}=?$

A) $\frac{1}{18}$ B) $\frac{1}{20}$ C) $\frac{19}{20}$ D) $\frac{20}{21}$ E) $\frac{9}{10}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\sum_{k=1}^{20}\frac{1}{k}-\sum_{k=1}^{20}\frac{1}{k+1}=\sum_{k=1}^{20}\frac{1}{k}-\frac{1}{k+1}$$

$$=\sum_{k=1}^{20}\frac{1}{k\cdot(k+1)}=\frac{20}{21}$$

Düşünün / Think about it

$\sum_{n=1}^{1913} n \equiv x \pmod{1916} \Rightarrow x=?$

Yanıt / Answer

961

B. ÇARPIM SEMBOLÜ / MULTIPLICATION SYMBOL (Π)

Tanım: $\forall k \in N^+$ ve $a_k \in R$ olmak üzere, $a_1 \cdot a_2 \cdot a_3 \cdot a_4 \ldots\ldots \cdot a_n$ çarpımı kısa olarak $\prod\limits_{k=1}^{n} a_k$ yazılır. $\prod$ gösterimi pi diye okunur. k ye ise, çarpma indisi denir.

Definition: *For all* $\forall k \in N^+$, $a_k \in R$
The product $a_1 \cdot a_2 \cdot a_3 \cdot a_4 \ldots\ldots \cdot a_n$ *can be expressed as the notation* $\prod\limits_{k=1}^{n} a_k$. *The notation* $\prod$ *read "pi". k is called the index of multiplication.*

Çarpım Sembolünün Özellikleri / Properties of Multiplication Symbol

1. $\prod\limits_{k=1}^{n} c = c^n \quad (c \in R)$

2. $\prod\limits_{k=1}^{n} c \cdot a_k = c^n \cdot \prod\limits_{k=1}^{n} a_k \quad (c \in R)$

3. $\prod\limits_{k=1}^{n} (a_k \cdot b_k) = \prod\limits_{k=1}^{n} a_k \cdot \prod\limits_{k=1}^{n} b_k$

4. $1 < m < n \Rightarrow \prod\limits_{k=1}^{n} a_k = \prod\limits_{k=1}^{m} a_k \cdot \prod\limits_{k=m+1}^{n} a_k$

5. $\prod\limits_{k=1}^{n} \dfrac{a_k}{b_k} = \left(\dfrac{\prod\limits_{k=1}^{n} a_k}{\prod\limits_{k=1}^{n} b_k} \right) \quad (\forall b \neq 0)$

6. $\prod\limits_{k=p}^{n} a_k = \prod\limits_{k=p\mp r}^{n\mp r} (a_{k\pm r})$

Bazı Önemli Çarpım Formülleri / Some Important Multiplication Formulas

1. $\prod_{k=1}^{n} k = n!$

2. $\prod_{k=p}^{n} \log_k (k+1) = \log_p (n+1)$

3. $\prod_{k=1}^{n} r^k = r^{\frac{n\cdot(n+1)}{2}}$

4. $\prod_{i=2}^{n} \left(\frac{i^2-1}{i^2}\right) = \frac{n+1}{2n}$

ÖRNEK / EXAMPLE 12

$\prod_{k=1}^{4} 3k = ?$

A) 1280 B) 1440 C) 1620 D) 1944 E) 2100

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\prod_{k=1}^{4} 3k = 3\cdot 1\cdot 3\cdot 2\cdot 3\cdot 3\cdot 3\cdot 4 = 1944$

ÖRNEK / EXAMPLE 13

$$\prod_{k=-2}^{5}(k+3)=?$$

A) 8! B) 10! C) 11! D) 12! E) 13!

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\prod_{k=-2}^{5}(k+3)=\prod_{k=-2+3}^{5+3}(k+3-3)=\prod_{k=1}^{8}k=8!$$

ÖRNEK / EXAMPLE 14

$$\prod_{k=1}^{n}3^k=3^{105}\Rightarrow \mathbf{n=?}$$

A) 10 B) 14 C) 19 D) 20 E) 21

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\prod_{k=1}^{n}3^k=3^{105}\Rightarrow 3^{\frac{n(n+1)}{2}}=3^{105}$$

$n^2+n-210=0$

$(n-14)\cdot(n+15)=0$

$n=14\ ,\ n=-22$

$n\in N^+\Rightarrow n=14$

PEKİŞTİRME / REINFORCE

$$\prod_{p=1}^{4}p=?$$

YANIT / ANSWER : 24

DİZİLER ve SERİLER / SEQUENCES and SERIES

Tanım: $f: N^+ \rightarrow R$, $f(n) = a_n$ biçiminde tanımlı her fonksiyona *dizi* denir.

$n = 1, 2, 3, 4 \ldots$ sayılarına karşılık gelen $f(1) = a_1$, $f(2) = a_2$, $f(3) = a_3$, … $f(n) = a_n$ reel sayılarına *dizinin terimleri* denir. $f(n) = a_n$ reel sayısına da dizinin *genel terimi* denir.

$f(1) = a_1$, $f(2) = a_2$, …… $f(n) = a_n$ ise, f dizisi genel olarak $(a_1, a_2, a_3, \ldots a_n, \ldots)$ veya kısaca (a_n) ile gösterilir.

Definition: *A function which is defined in the form;*

$f: N^+ \rightarrow N$, $f(n) = a_n$ is called a sequence. The real numbers $f(1) = a_1 \ldots f(n) = a_n$ which correspond to the numbers. $n = 1, 2, 3, 4 \ldots$ are called the terms of the sequence.

$f(n) = a_n$ is called the general term of the sequence.

Let $f(1) = a_1$, $f(2) = a_2$, …… $f(n) = a_n$.

The sequence is denoted by $(a_1, a_2, a_3, \ldots a_n, \ldots)$ in general or it is denoted by (a_n), briefly.

Tanım: (a_n) bir reel terimli dizi olmak üzere, genel terimi, $(S_n) = a_1 + a_2 + a_3 + \ldots + a_n$ olan (S_n) dizisi gözönüne alınırsa $((a_n), (S_n))$ sıralı ikilisine bir *seri* denir. (a_n) terimine serinin genel terimi, (S_n) dizisine de serinin *kısmi toplamlar dizisi* adı verilir.

Definition: *Provided that (a_n) is a sequence, the terms of which is real number, let (S_n) be the sequence with the general term. $(S_n) = a_1 + a_2 + a_3 + \ldots + a_n$.*

Then the ordered pair $((a_n), (S_n))$ is called a series. (a_n) is called the general term of the series and (S_n) is called the partial sum sequence of the series.

ÖRNEK / EXAMPLE 15

$$(a_n) = \left(\frac{n+6}{n^2-1}\right) \Rightarrow a_4 = ?$$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$a_4 = \frac{4+6}{16-1} = \frac{10}{15} = \frac{2}{3}$$

ÖRNEK / EXAMPLE 16

$a_{n+1} = a_n + r,$

$a_1 = 4,$

$r = 4 \Rightarrow \mathbf{a_{18} = ?}$

A) 56 B) 60 C) 66 D) 72 E) 76

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$a_{n+1} = a_n + r \Rightarrow a_{18} = a_{17} + 4$$
$$a_{17} = a_{16} + 4$$
$$\vdots$$
$$a_2 = a_1 + 4$$
$$\Rightarrow a_{18} = a_1 + 17 \cdot 4$$
$$a_{18} = 4 + 68 = 72$$

ÖRNEK / EXAMPLE 17

$a_n = 2^n \cdot n!,$

$a_{n+1} = k \cdot a_n$

$\Rightarrow \mathbf{k = ?}$

A) n + 1 B) n + 2 C) 2n

D) 2n + 1 E) 2n + 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$a_{n+1} = 2^{n+1} \cdot (n+1)! = 2 \cdot 2^n \cdot (n+1) \cdot n!$$
$$= 2 \cdot (n+1) \cdot 2^n \cdot n!$$
$$= (2n+2) \cdot a_n$$
$$\Rightarrow k = 2n + 2$$

ÖRNEK / EXAMPLE 18

YILDIZ TEKNİK ÜNİ. YÖS 2011

$n > 1,\ a_1 = 1$

$a_n = na_{n-1}$

$a_4 = ?$

A) 4 B) 10 C) 12 D) 24 E) 120

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$a_2 = 2 \cdot a_1 = 2$

$a_3 = 3 \cdot a_2 = 6$

$a_4 = 4 \cdot a_3 = 24$

ÖRNEK / EXAMPLE 19

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$\sum_{n=-4}^{25} (2n+3) = ?$$

A) 574 B) 600 C) 645 D) 682 E) 720

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\sum_{n=-4}^{25} (2n+3) = \sum_{n=1}^{30} (2(n-5)+3)$$

$$= \sum_{n=1}^{30} (2n-7)$$

$$= 30 \cdot 31 - 7 \cdot 30$$

$$= 930 - 210$$

$$= 720$$

ÖRNEK / EXAMPLE 20

YILDIZ TEKNİK ÜNİ. YÖS 2013

$n > 1,\ a_1 = 1$

$a_n = na_{n-1}$

$\Rightarrow\ a_{20} = ?$

A) 20! B) 20^n C) 34110

D) 19×20 E) 20^{20}

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$n > 1,\ a_1 = 1$

$$a_n = na_{n-1} \Rightarrow \frac{a_n}{a_{n-1}} = n$$

$$\frac{a_2}{a_1}\cdot\frac{a_3}{a_2}\cdot\ldots\cdot\frac{a_{20}}{a_{19}} = 2\cdot3\cdot\ldots\cdot20$$

$$\frac{a_{20}}{a_1} = 20!$$

$$a_{20} = 20!$$

ÖRNEK / EXAMPLE 21

MARMARA ÜNİ. YÖS 2016

$$\sum_{m=1}^{10}\prod_{n=2}^{3} n^2 = ?$$

A) 36 B) 72 C) 90 D) 180 E) 360

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\sum_{m=1}^{10}\prod_{n=2}^{3} n^2 = \prod_{n=2}^{3} n^2 = 2^2\cdot3^2 = 4\cdot9 = 36$$

$$\sum_{m=1}^{10} 36 = 36\cdot10 = 360$$

1. $\sum_{k=1}^{15} k = ?$

A) 90
B) 105
C) 120
D) 135
E) 150

2. $\sum_{k=1}^{12} k^2 = ?$

A) 640
B) 650
C) 660
D) 670
E) 680

3. $\sum_{k=1}^{35} \left(\frac{1}{k} - \frac{1}{k+1}\right) = ?$

A) $\frac{33}{34}$
B) $\frac{34}{35}$
C) $\frac{35}{36}$
D) $\frac{36}{37}$
E) $\frac{37}{38}$

4. $\sum_{k=1}^{99} \log \frac{k}{k+1} = ?$

A) –2
B) –3
C) –4
D) –5
E) –6

5. $i = \sqrt{-1}$

$\Rightarrow \sum_{k=1}^{40} i^k = ?$

A) 1 – i
B) –i
C) –1
D) 0
E) i

6. $\sum_{m=1}^{3} \sum_{k=0}^{2} \sum_{i=2}^{4} (k+i+m) = ?$

A) 124
B) 136
C) 144
D) 158
E) 162

7. $\prod_{k=1}^{25} k = ?$

A) 22!

B) 23!

C) 24!

D) 25!

E) 26!

8. $\prod_{k=5}^{29} \left(\frac{k+2}{k+3}\right) = ?$

A) $\frac{3}{28}$

B) $\frac{4}{29}$

C) $\frac{1}{6}$

D) $\frac{6}{31}$

E) $\frac{7}{32}$

9. $\prod_{k=1}^{32} 2^k = 2^x \Rightarrow x = ?$

A) 515

B) 518

C) 520

D) 525

E) 528

10. $f(x) = x^2 - 5x + c$, ÇK(SS) = $\{x_1, x_2\}$,

$\prod_{k=1}^{2} x_k = 8 \Rightarrow c = ?$

A) –8

B) –4

C) 2

D) 4

E) 8

11. $\sum_{k=1}^{10} \prod_{m=1}^{2} (k+m) = ?$

A) 550

B) 560

C) 570

D) 580

E) 590

12. $\prod_{k=2}^{60} \left(1 + \frac{1}{k-1}\right) = ?$

A) 59

B) 60

C) 61

D) 62

E) 63

YANITLAR / ANSWERS

1. C	2. B	3. C	4. A	5. D	6. E	7. D	8. E
9. E	10. E	11. C	12. B				

1. $$\sum_{k=1}^{15} k = \frac{15\cdot 16}{2}$$

$$= 120$$

Yanıt / The Answer is C

2. $$\sum_{k=1}^{12} k^2 = \frac{12\cdot 13\cdot 25}{6}$$

$$= 650$$

Yanıt / The Answer is B

3. $$\sum_{k=1}^{35}\left(\frac{1}{k}-\frac{1}{k+1}\right)=\sum_{k=1}^{35}\frac{(k+1)-k}{k\cdot(k+1)}$$

$$=\sum_{k=1}^{35}\frac{1}{k\cdot(k+1)}$$

$$=\frac{35}{36}$$

Yanıt / The Answer is C

4. $$\sum_{k=1}^{99}\log\frac{k}{k+1}=\log\frac{1}{2}+\log\frac{2}{3}+\ldots+\log\frac{99}{100}$$

$$=\log\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\ldots\cdot\frac{99}{100}\right)$$

$$=\log\frac{1}{100}=-2$$

Yanıt / The Answer is A

5. $$\sum_{k=1}^{40} i^k = i^1+i^2+\ldots+i^{40}$$

$$\left.\begin{array}{l} i^1=i\\ i^2=-1\\ i^3=-i\\ i^4=1 \end{array}\right\}\Rightarrow i+(-1)+(-i)+1+\ldots+1=0$$

Yanıt / The Answer is D

6. $$\sum_{m=1}^{3}\sum_{k=0}^{2}\sum_{i=2}^{4}(k+i+m)=\sum_{m=1}^{3}\sum_{k=0}^{2}(9+3k+3m)$$

$$=\sum_{m=1}^{3}(27+9m+9)$$

$$=\sum_{m=1}^{3}(36+9m)$$

$$=3\cdot 36+9\cdot 6$$

$$=108+54=162$$

Yanıt / The Answer is E

7. $\prod_{k=1}^{25} k = 25!$

Yanıt / The Answer is D

8. $\prod_{k=5}^{29} \frac{k+2}{k+3} = \frac{7}{8}\cdot\frac{8}{9}\cdot\frac{9}{10}\cdot\ldots\cdot\frac{31}{32}$

$= \frac{7}{32}$

Yanıt / The Answer is E

9. $\prod_{k=1}^{32} 2^k = 2^{\frac{32\cdot 33}{2}} = 2^x$

$2^{528} = 2^x$

$x = 528$

Yanıt / The Answer is E

10. $\prod_{k=1}^{2} x_k = x_1 \cdot x_2 = 8$

$f(x) = x^2 - 5x + c \Rightarrow x_1 \cdot x_2 = \frac{c}{1}$

$c = 8$

Yanıt / The Answer is E

11. $\sum_{k=1}^{10} \prod_{m=1}^{2} (k+m) = \sum_{k=1}^{10} (k+1)\cdot(k+2)$

$\sum_{k=1}^{10} (k^2+3k+2) = \sum_{k=1}^{10} k^2 + \sum_{k=1}^{10} 3k + \sum_{k=1}^{10} 2$

$= \frac{10\cdot 11\cdot 21}{6} + 3\cdot\frac{10\cdot 11}{2} + 2\cdot 10$

$= 385 + 165 + 20$

$= 570$

Yanıt / The Answer is C

12. $\prod_{k=2}^{60} \left(1+\frac{1}{k-1}\right) = \prod_{k=2}^{60} \left(\frac{k}{k-1}\right)$

$= \frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\ldots\cdot\frac{60}{59} = 60$

Yanıt / The Answer is B

1. $\sum_{k=45^\circ}^{315^\circ} \sin k = ?$

Yanıt / The Answer is: **0**

2. $\sum_{k=5}^{103} \log\left(1-\frac{1}{k-3}\right) = ?$

Yanıt / The Answer is: **–2**

3. $\ln\left(\prod_{k=10}^{30} e^{k}\right) = ?$

Yanıt / The Answer is: **420**

4. $\sum_{m=0}^{7}\prod_{k=1}^{9}\left(1-\frac{1}{k+3}\right) = ?$

Yanıt / The Answer is: **2**

5. $\sum_{k=4}^{51}\prod_{n=1}^{15}\left(\frac{n}{n+1}\right) = ?$

Yanıt / The Answer is: **3**

6. $\prod_{k=1}^{10}\left(\frac{3}{7}\right)^{k} = \left(\frac{7}{3}\right)^{x} \Rightarrow x = ?$

Yanıt / The Answer is: **–55**

7. $\sum_{k=1}^{20}(3k-2)=?$

Yanıt / The Answer is: **590**

8. $\prod_{k=-2}^{11}(15-3k)=?$

Yanıt / The Answer is: **0**

9. $\sum_{k=6}^{n}\log_5\left(1-\frac{1}{k}\right)=-1 \Rightarrow n=?$

Yanıt / The Answer is: **25**

10. $\prod_{n=91}^{179}\tan x=?$

Yanıt / The Answer is: **–1**

11. $a_n=(3n-1) \Rightarrow \frac{a_5}{a_2}=?$

Yanıt / The Answer is: $\frac{14}{5}$

12. $a_n=3^n \Rightarrow \frac{a_{n+1}}{a_n}=?$

Yanıt / The Answer is: **3**

1. $\sum_{k=1}^{10} k^2 = ?$

A) 332

B) 348

C) 356

D) 385

E) 406

2. $\sum_{k=1}^{15} (3k+2) = ?$

A) 380

B) 390

C) 400

D) 410

E) 420

3. $\sum_{k=1}^{25} (2k-4) = ?$

A) 520

B) 530

C) 540

D) 550

E) 560

4. $\sum_{k=1}^{n} (k-2) = 20 \Rightarrow n = ?$

A) 5

B) 6

C) 7

D) 8

E) 9

5. $\sum_{k=1}^{n} (2k+1) = 120 \Rightarrow n = ?$

A) 6

B) 7

C) 8

D) 9

E) 10

6. $\sum_{k=-2}^{16} (k+3) = ?$

A) 170

B) 180

C) 190

D) 200

E) 210

7. $\sum_{k=-5}^{6} k^3 = ?$

A) 184
B) 196
C) 198
D) 208
E) 216

8. $f(x) = 2x - 5 \Rightarrow$

$\sum_{k=1}^{3} f(k) = ?$

A) –4
B) –3
C) –2
D) –1
E) 0

9. $\sum_{k=-7}^{43} a = 153 \Rightarrow a = ?$

A) 3
B) 4
C) 5
D) 6
E) 7

10. $\sum_{k=n}^{n+3} (k+2) = 26 \Rightarrow n = ?$

A) 3
B) 4
C) 5
D) 6
E) 7

11. $\sum_{m=1}^{3} \sum_{k=-1}^{1} (2k+m) = ?$

A) 10
B) 12
C) 15
D) 16
E) 18

12. $f(x) = x^2 + 3x - 1 \Rightarrow$

$\sum_{k=1}^{2} f(k) = ?$

A) 9
B) 10
C) 12
D) 14
E) 15

YANITLAR / ANSWERS

1. D 2. B 3. D 4. D 5. E 6. C 7. E 8. B
9. A 10. A 11. E 12. C

1. $\prod_{k=1}^{29} \frac{k}{k+1} = ?$

A) $\frac{1}{27}$

B) $\frac{1}{28}$

C) $\frac{1}{29}$

D) $\frac{1}{30}$

E) $\frac{1}{31}$

2. $\prod_{k=3}^{25} 3^k = 3^x \Rightarrow x = ?$

A) 318

B) 320

C) 322

D) 332

E) 338

3. $\prod_{k=1}^{12} (2 \cdot k) = ?$

A) $2^{13} \cdot 12!$

B) $2^{12} \cdot 12!$

C) $2^{11} \cdot 11!$

D) $2^{12} \cdot 13!$

E) $2^{12} \cdot 78$

4. $\prod_{k=1}^{10} [k \cdot (k+1)] = ?$

A) $9! \cdot 10!$

B) $10! \cdot 11!$

C) $11! \cdot 12!$

D) $12! \cdot 13!$

E) $13! \cdot 14!$

5. $\prod_{k=8}^{25} k = ?$

A) $\frac{25!}{7!}$

B) $\frac{25!}{8!}$

C) $\frac{24!}{7!}$

D) $\frac{24!}{8!}$

E) $\frac{26!}{8!}$

6. $\prod_{k=-3}^{15} (k+5) = ?$

A) 15!

B) 16!

C) 18!

D) 19!

E) 20!

7. $\prod_{k=2}^{9}\left(\sin\frac{\pi k}{2}\right)=?$

A) –2

B) –1

C) 0

D) 1

E) 2

8. $\prod_{k=5}^{20} e^{\ln(k-4)}=?$

A) 13!

B) 14!

C) 15!

D) 16!

E) 17!

9. $\prod_{k=4}^{63}\left(\log_k(k+1)\right)=?$

A) 1

B) 2

C) 3

D) 4

E) 5

10. $\prod_{k=2}^{3}\prod_{i=1}^{2}(i^2-1)\cdot(k+1)=?$

A) –156

B) 0

C) 98

D) 158

E) 186

11. $\sum_{k=1}^{20}\prod_{m=2}^{3}3=?$

A) 153

B) 162

C) 171

D) 180

E) 189

12. $\prod_{k=1}^{5}\sum_{m=1}^{3}\frac{m}{k}=?$

A) $\frac{324}{5}$

B) $\frac{328}{5}$

C) $\frac{332}{5}$

D) $\frac{338}{5}$

E) $\frac{342}{5}$

YANITLAR / ANSWERS

1. D	2. C	3. B	4. B	5. A	6. E	7. C	8. D
9. C	10. B	11. D	12. A				

1. $f(x) = 3x + 5,$

$x_1 = -2,\ \ x_2 = 1,\ \ x_3 = 4 \Rightarrow \sum_{k=1}^{3}\left[\mathbf{2x_k + f(x_k)}\right] = \mathbf{?}$

A) 25 B) 30 C) 32 D) 35 E) 36

2. $\sum_{k=1}^{23}(2k) = A,\ \sum_{k=8}^{23}(2k) = B \Rightarrow \mathbf{A-B=?}$

A) 52 B) 54 C) 56 D) 58 E) 60

3. $\sum_{m=1}^{4}\prod_{k=2}^{3}\mathbf{2^k = ?}$

A) 108 B) 116 C) 124 D) 128 E) 130

4. $\sum_{k=1}^{n}(2k-5) = 77 \Rightarrow \mathbf{n=?}$

A) 9 B) 10 C) 11 D) 12 E) 13

5. $f(n) = \sum_{k=1}^{n} 3k \Rightarrow \mathbf{f(12)=?}$

A) 234 B) 238 C) 244 D) 246 E) 250

6. $\sum_{k=1}^{3}\sum_{i=1}^{3}\mathbf{(3k-i+2)=?}$

A) 46 B) 48 C) 50 D) 52 E) 54

7. $\prod\limits_{k=1}^{21}(a-3)=8^{7} \Rightarrow$ **a = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

8. $\sum\limits_{k=1}^{4}\sum\limits_{m=1}^{k} m=?$

A) 18 B) 20 C) 21 D) 24 E) 25

9. $\prod\limits_{k=5}^{124}\log_k(k+1)=?$

A) 1 B) 2 C) 3 D) 4 E) 5

10. $\prod\limits_{i=1}^{2}\sum\limits_{k=2}^{4}(i+k-1)=?$

A) 100 B) 108 C) 120 D) 132 E) 150

11. $\prod\limits_{k=1}^{15}\left(\frac{2}{5}\right)^{k}=\left(\frac{5}{2}\right)^{x} \Rightarrow$ **x = ?**

A) –40 B) –60 C) –80 D) –100 E) –120

12. $2x^2+6x-8=0$, ÇK(SS) = $\{x_1, x_2\} \Rightarrow$

$\sum\limits_{k=1}^{2}(x_k)^2=?$

A) 12 B) 16 C) 17 D) 18 E) 20

YANITLAR / ANSWERS

1. B	2. C	3. D	4. C	5. A	6. E	7. D	8. B
9. C	10. B	11. E	12. C				

1. $(a_n) = \begin{cases} 3n-1 & n=2k \\ 5n+1 & n=2k+1 \end{cases}$

$\Rightarrow \mathbf{a_{11} + a_{12} = ?}$

A) 88
B) 91
C) 93
D) 95
E) 97

---

2. $(a_n) = \dfrac{3n-1}{2n+1} \Rightarrow \mathbf{a_{15} = ?}$

A) $\dfrac{40}{31}$
B) $\dfrac{41}{31}$
C) $\dfrac{43}{31}$
D) $\dfrac{44}{31}$
E) $\dfrac{45}{31}$

---

3. $(a_n) = \dfrac{3n-2}{2n+1} \Rightarrow \mathbf{(a_{n+2}) = ?}$

A) $\dfrac{3n}{2n+3}$
B) $\dfrac{3n+1}{2n+1}$
C) $\dfrac{3n+2}{2n+2}$
D) $\dfrac{3n+3}{2n+3}$
E) $\dfrac{3n+4}{2n+5}$

---

4. $(a_n) = \dfrac{3^n \cdot n!}{n+1} \Rightarrow \mathbf{\dfrac{a_{n-2}}{a_{n-1}} = ?}$

A) $\dfrac{n!}{3(n-1)^2}$
B) $\dfrac{n}{3(n-1)^2}$
C) $\dfrac{n}{(n-1)^2}$
D) $\dfrac{3n}{(n-1)^2}$
E) $\dfrac{3}{(n-1)^2}$

---

5. $a_1 = 4,$

$a_{n+1} = a_n + n \Rightarrow \mathbf{a_{12} = ?}$

A) 65
B) 66
C) 68
D) 70
E) 72

---

6. $a_{2n-1} = \dfrac{4n+1}{2n-1} \Rightarrow \mathbf{a_n = ?}$

A) $\left(\dfrac{2n+2}{n}\right)$
B) $\left(\dfrac{2n+3}{n}\right)$
C) $\left(\dfrac{4n+5}{2n}\right)$
D) $\left(\dfrac{8n-3}{4n+3}\right)$
E) $\left(\dfrac{8n-7}{4n+3}\right)$

7. $a_n = a_{n-1} + r,$

$a_3 = 4,\ a_{12} = 13 \Rightarrow \mathbf{a_{23} = ?}$

A) 21

B) 22

C) 23

D) 24

E) 25

---

8. $\left.\begin{array}{l} a_n = \left(a_{n-1}\right)\cdot r \\ a_1 = 6,\ r = \dfrac{1}{3} \end{array}\right\} \Rightarrow \mathbf{a_{20} = ?}$

A) $\dfrac{2}{3^{20}}$

B) $\dfrac{2}{3^{19}}$

C) $\dfrac{1}{3^{18}}$

D) $\dfrac{2}{3^{18}}$

E) $\dfrac{1}{3^{17}}$

---

9. $\left(a_n\right) = \left(a_{n-1}\right) + r$

$a_1 = 6,\ r = \dfrac{4}{3} \Rightarrow \mathbf{a_{13} = ?}$

A) 22

B) 24

C) 26

D) 28

E) 30

---

10. $\left(a_n\right) = \dfrac{1}{(n+1)(n+2)} \Rightarrow \sum\limits_{n=1}^{18} \left(a_n\right) = \mathbf{?}$

A) $\dfrac{9}{20}$ B) $\dfrac{11}{20}$ C) $\dfrac{3}{5}$

D) $\dfrac{7}{10}$ E) $\dfrac{2}{5}$

---

11. $\sum\limits_{n=2}^{\infty} \dfrac{1}{3^n} = \mathbf{?}$

A) $\dfrac{1}{6}$

B) $\dfrac{1}{3}$

C) $\dfrac{1}{2}$

D) 0

E) 1

---

12. $\sum\limits_{n=0}^{\infty} \left(\dfrac{2}{3}\right)^n = \mathbf{?}$

A) $\dfrac{1}{6}$

B) 1

C) $\dfrac{3}{2}$

D) 2

E) 3

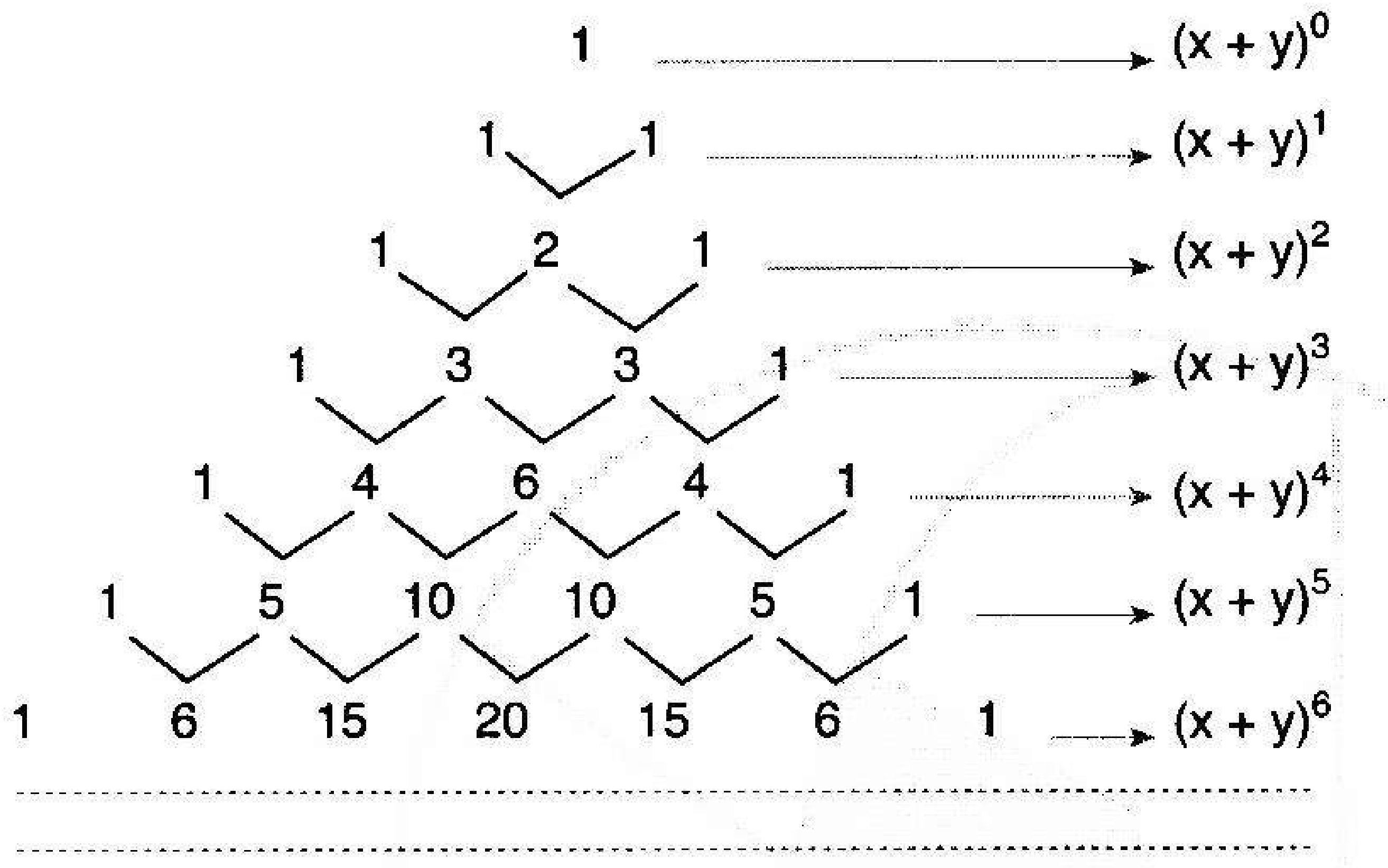
YANITLAR / ANSWERS

1. B	**2. D**	**3. E**	**4. B**	**5. D**	**6. B**	**7. D**	**8. D**
9. A	**10. A**	**11. A**	**12. E**				

BİNOM AÇILIMI
BINOMIAL EXPANSION

BÖLÜM
CHAPTER 2

BİNOM AÇILIMI ve PASCAL ÜÇGENİ
BINOMIAL EXPANSION and PASCAL'S TRIANGLE



$$(x+y)^n=\binom{n}{0}x^n+\binom{n}{1}x^{n-1}y+\binom{n}{2}x^{n-2}y^2+\ldots+\binom{n}{n}y^n$$

Binom Açılımının Özellikleri / Properties of Binomial Expansion

1. $(x + y)^n$ nin açılımında, $n + 1$ tane terim vardır.
   There are $(n + 1)$ terms in the expansion $(x + y)^n$.
2. $(x + y)^n$ açılımında, her terimdeki x ve y nin üslerinin toplamı n'ye eşittir.
   In each terms the sum of the power of x and y is equal to n in the expansion $(x + y)^n$.
3. Baştan $(r + 1)$. terim, $\binom{n}{r}x^{n-r}y^r$ dir.
   The $(r + 1)$. term from beginning is $\binom{n}{r}x^{n-r}y^r$.

ÖRNEK / EXAMPLE 1

$(x - 3y)^5 = Ax^5 + Bx^4y^1 + Cx^3y^2 + Dx^2y^3 + Exy^4 + Fy^5$

$\Rightarrow$ **C = ?**

A) 10 B) 20 C) 50 D) 90 E) 120

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$Cx^3y^2=\binom{5}{2}\cdot x^3\cdot(-3y)^2$$

$$=\frac{5!}{(5-2)!\cdot 2!}\cdot x^3\cdot(9y^2)=90x^3\cdot y^2 \Rightarrow C=90$$

NOT / NOTE

Pascal üçgeninin her bir satırındaki ilk ve son sayı 1'dir. Her satırdaki diğer sayılar, üzerinde yer alan iki sayının toplamından elde edilir.

The first and last number in each row of Pascal's triangle is 1. Every other number in each row is formed by adding the two numbers immediately above the number.

NOT / NOTE

Her bir açılımda, x ve y simetriktir. x'in kuvveti 1 azalırken, y'nin kuvveti 1 artar.

In each expansion, x and y have symmetrical roles. The powers of x decrease by 1 in successive terms, whereas the powers of y increase by 1.

PEKİŞTİRME / REINFORCE

$(x + y)^4 = \ldots\ldots + Ax^2y^2 \Rightarrow A = ?$

YANIT / ANSWER : 6

ÖRNEK / EXAMPLE 2

$$\left(x-\frac{1}{x}\right)^6 = Ax^6+Bx^4+Cx^2+D+Ex^{-2}+Fx^{-4}+Hx^{-6} \Rightarrow \mathbf{D=?}$$

A) 40　　B) 20　　C) 10　　D) –10　　E) –20

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$Dx^0 = \binom{6}{r}\cdot x^{6-r}\cdot\left(-\frac{1}{x}\right)^r$$

$$6-r-r=0$$

$$2r=6$$

$$r=3 \Rightarrow D=\binom{6}{3}\cdot(-1)^3=\frac{6\cdot5\cdot4}{3\cdot2}\cdot(-1)=-20$$

▼ $p,\ q,\ r \in N$,

$p+q+r=n \Rightarrow$

$$(ax+by+cz)^n=(ax)^p\cdot(by)^q\cdot(cz)^r\cdot\frac{n!}{p!\cdot q!\cdot r!}$$

ÖRNEK / EXAMPLE 3

$(2x + 3y + z)^5 = \ldots\ldots\ldots + A\cdot x^2y^2z + \ldots\ldots\ldots$

$\Rightarrow$ **A = ?**

A) 1010　　B) 1020　　C) 1040　　D) 1060　　E) 1080

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$A\cdot x^2y^2z=(2x)^2\cdot(3y)^2\cdot z\cdot\frac{5!}{2!\cdot2!\cdot1!}$$

$$=4x^2\cdot9y^2\cdot z\cdot30$$

$$=1080\cdot x^2y^2z \Rightarrow A=1080$$

ÖRNEK / EXAMPLE 4

$(x+y+z)^7 = \ldots\ldots + Ax^3y^2z^2 \Rightarrow$ **A = ?**

A) 100 B) 110 C) 120 D) 180 E) 210

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$y+z=t \Rightarrow (x+t)^7 = \ldots\ldots + Kx^3t^4$

$Kx^3t^4 = \binom{7}{4}x^3t^4 \Rightarrow K = \frac{7\cdot 6\cdot 5\cdot 4}{4\cdot 3\cdot 2\cdot 1} = 35$

$t^4 = (y+z)^4 = \binom{4}{2}y^2z^2 \Rightarrow A = 35\cdot\binom{4}{2} = 35\cdot 6 = 210$

ÖRNEK / EXAMPLE 5

$\left(x - \frac{2}{x}\right)^5 = Ax^5 + Bx^3 + Cx + Dx^{-1} + Ex^{-3} + Fx^{-5}$

$\Rightarrow$ **C = ?**

A) –36 B) –20 C) 24 D) 40 E) 64

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$C\cdot x = \binom{5}{3}\cdot x^3\cdot\left(\frac{-2}{x}\right)^2$

$C\cdot x = \frac{5!}{(5-3)!\cdot 3!}\cdot x^3\cdot\frac{4}{x^2}$

$C\cdot x = 40x \Rightarrow C = 40$

ÖRNEK / EXAMPLE 6

$(2x + 1)^4 = Ax^4 + Bx^3 + Cx^2 + Dx + 1 \Rightarrow$ **A + B + C + D = ?**

A) 20 B) 30 C) 50 D) 60 E) 80

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$(2x + 1)^4 = (2x)^4 + 4\cdot(2x)^3 + 6\cdot(2x)^2 + 4\cdot(2x) + 1$$

$$= 16x^4 + 32x^3 + 24x^2 + 8x + 1$$

$\Rightarrow$ A = 16, B = 32, C = 24, D = 8

$\Rightarrow$ A + B + C + D = 16 + 32 + 24 + 8 = 80

ÖRNEK / EXAMPLE 7

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$(x + 2y)^n = \ldots\ldots + Ax^6y^3 + \ldots\ldots$

$\Rightarrow$ **n + A = ?**

A) 681 B) 682 C) 683 D) 684 E) 685

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

n = 6 + 3 = 9

$$A = \binom{9}{6}\cdot 2^3 = 672$$

n + A = 681

1. $(x+y)^4 = Ax^4 + Bx^3y + Cxy^2 + Dxy^3 + Ey^4$

$\Rightarrow$ **A + B + C = ?**

A) 9 B) 10 C) 11 D) 12 E) 13

2. $(x+y)^{10} = Ax^{10} + Bx^9y^8 + \ldots + Kx^3y^7 + \ldots + My^{10}$

$\Rightarrow$ **K = ?**

A) 120 B) 140 C) 160 D) 240 E) 300

3. $\left(x - \frac{2}{x}\right)^{10} = Ax^{10} + Bx^8 + \ldots + Kx^{-2} + \ldots + Mx^{-10}$

$\Rightarrow$ **K = ?**

A) $210 \cdot 2^5$ B) $210 \cdot 2^6$ C) $105 \cdot 2^6$ D) $100 \cdot 2^6$ E) $120 \cdot 2^7$

4. $\left(x - \frac{1}{x}\right)^{12} = Ax^{12} + Bx^{10} + \ldots + Kx^6 + \ldots + Mx^{-12}$

$\Rightarrow$ **K = ?**

A) 120 B) 200 C) 220 D) –120 E) –220

5. $\binom{n}{0}4^0 + \binom{n}{1}4^1 + \ldots + \binom{n}{n}4^n = 625 \Rightarrow$ **n = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

6. $(x^3 + y^2)^{10} = Ax^{30} + Bx^{27}y^2 + \ldots + Mx^{15}y^{10} + \ldots + Ky^{20}$

$\Rightarrow$ **M = ?**

A) 200 B) 240 C) 252 D) 256 E) 320

7. $\left(x-\frac{3}{x}\right)^8 = \ldots + a\cdot x^4 \ldots \Rightarrow$ **$a = ?$**

A) 252
B) 256
C) 260
D) 264
E) 268

8. $\left.\begin{array}{l} x^3+y^3=19 \\ x^2y+xy^2=15 \end{array}\right\} \Rightarrow$ **$x+y=?$**

A) 2
B) 3
C) 4
D) 5
E) 6

9. $\left.\begin{array}{l} x^2-xy=98 \\ xy-y^2=17 \end{array}\right\} \Rightarrow$ **$|x-y|=?$**

A) 5
B) 6
C) 7
D) 8
E) 9

10. $(\sqrt{x}+3)^6 = x^3 + \ldots + A\cdot x^2 + \ldots \Rightarrow$ **$A = ?$**

A) 110
B) 115
C) 125
D) 135
E) 140

11. $(x^2-2y+3z)^6 = \ldots + A\cdot x^6y^2\cdot z + \ldots \Rightarrow$ **$A = ?$**

A) 500
B) 600
C) 675
D) 720
E) 800

12. $(2a-2b+c)^8 = \ldots + x\cdot a^2\cdot b\cdot c^5 + \ldots \Rightarrow$ **$x = ?$**

A) −1344
B) −1200
C) −1000
D) 1120
E) 1342

YANITLAR / ANSWERS

1. C	**2. A**	**3. B**	**4. E**	**5. A**	**6. C**	**7. A**	**8. C**
9. E	**10. D**	**11. D**	**12. A**				

1.

$$(x+y)^4=\underbrace{\binom{4}{0}}_{A}x^4+\underbrace{\binom{4}{1}}_{B}x^3y+\underbrace{\binom{4}{2}}_{C}x^2y^2+\binom{4}{3}xy^3+\binom{4}{4}y^4$$

$$\left.\begin{aligned}A&=\binom{4}{0}=\frac{4!}{(4-0)!\cdot 0!}=1\\ B&=\binom{4}{1}=\frac{4!}{(4-1)!\cdot 1!}=4\\ C&=\binom{4}{2}=\frac{4!}{(4-2)!\cdot 2!}=6\end{aligned}\right\}\Rightarrow A+B+C=1+4+6=11$$

Yanıt / The Answer is C

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2. $r=7 \Rightarrow K=\binom{10}{7}=\dfrac{10!}{(10-7)!\cdot 7!}=120$

Yanıt / The Answer is A

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3. $(r+1)$. terim/term $=\dbinom{10}{r}\cdot x^{10-r}\cdot\left(\dfrac{-2}{x}\right)^r$

$$\Rightarrow x^{-2}=x^{10-r}\cdot\left(x^{-1}\right)^r$$

$$-2=10-r-r$$

$$r=6$$

$$K\cdot x^{-2}=\binom{10}{6}\cdot x^{10-6}\cdot\left(\frac{2}{x}\right)^6$$

$$=\frac{10!}{(10-6)!\cdot 6!}\cdot x^4\cdot\frac{2^6}{x^{-6}}$$

$$=210\cdot 2^6\cdot x^{-2}\Rightarrow K=210\cdot 2^6$$

Yanıt / The Answer is B

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4. $(r+1)$. terim / term $=\dbinom{12}{r}\cdot x^{12-r}\cdot\left(\dfrac{1}{x}\right)^r$

$$\Rightarrow K\cdot x^6=\binom{12}{r}\cdot x^{12-r}\cdot(-x)^{-r}$$

$$x^6=x^{12-r}\cdot x^{-r}\Rightarrow 6=12-2r\Rightarrow r=3$$

$$K\cdot x^6=\binom{12}{3}\cdot x^{12-3}\cdot\left(-\frac{1}{x}\right)^3$$

$$K\cdot x^6=\frac{12!}{(12-3)!\cdot 3!}\cdot x^9\cdot\left(-x^{-3}\right)$$

$$K\cdot x^6=-220\cdot x^6\Rightarrow K=-220$$

Yanıt / The Answer is E

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5. $\dbinom{n}{0}4^0+\dbinom{n}{1}4^1+\ldots+\dbinom{n}{n}4^n=(1+4)^n$

$$=5^n$$

$$5^n=625$$

$$n=4$$

Yanıt / The Answer is A

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6. $M\cdot x^{15}\cdot y^{10}=\dbinom{10}{r}\cdot\left(x^3\right)^{10-r}\cdot\left(y^2\right)^r$

$$y^{10}=\left(y^2\right)^r$$

$$10=2r$$

$$r=5$$

$$\Rightarrow M=\binom{10}{5}=\frac{10!}{5!\cdot(10-5)!}=252$$

Yanıt / The Answer is C

7. $\binom{8}{r} x^{8-r} \cdot \left(\frac{-3}{x}\right)^{r} \Rightarrow 8-r-r=4$

$2r=4$

$r=2$

$\Rightarrow a=\binom{8}{2}\cdot(-3)^2=\frac{8\cdot 7}{2}\cdot 9$

$=28\cdot 9=252$

Yanıt / The Answer is A

8.

$$\begin{array}{rl} & x^3+y^3=19 \\ 3\,/ & x^2y+xy^2=15 \\ \hline & x^3+3x^2y+3xy^2+y^3=64 \end{array}$$

$(x+y)^3=64$

$x+y=4$

Yanıt / The Answer is C

9.

$$\begin{array}{rl} & x^2-xy=98 \\ -\,/ & xy-y^2=17 \\ \hline & x^2-2xy+y^2=81 \end{array}$$

$(x-y)^2=81$

$|x-y|=9$

Yanıt / The Answer is E

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10. $\binom{6}{r}\cdot\left(\sqrt{x}\right)^{6-r}\cdot 3^{r}=A\cdot x^2$

$\frac{6-r}{2}=2 \Rightarrow 6-r=4$

$r=2$

$A=\binom{6}{2}\cdot 3^2=15\cdot 9=135$

Yanıt / The Answer is D

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11. $\binom{6}{r}\cdot\binom{r}{k}\cdot(x^2)^{6-r}\cdot(-2y)^{r-k}\cdot(3z)^{k}=Ax^6y^2z$

$k=1\,,\ r=3 \Rightarrow$

$A=\binom{6}{3}\cdot\binom{3}{1}\cdot(-2)^2\cdot 3=720$

Yanıt / The Answer is D

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12. $(2a-2b+c)^8=\ldots+x\cdot a^2\cdot b\cdot c^5+\ldots$

$\binom{8}{r}\cdot\binom{r}{k}\cdot(2a)^{8-r}\cdot(-2b)^{r-k}\cdot(c)^{k}=x\cdot a^2\cdot b\cdot c^5$

$k=5\,,\ r=6$

$\Rightarrow\ x=\binom{8}{6}\cdot\binom{6}{5}\cdot(2)^2\cdot(-2)^1=-1344$

Yanıt / The Answer is A

1. $(x-3)^5 = \ldots + a \cdot x^3 + \ldots \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **90**

2. $(x-3)^4 = \ldots + a \cdot x^3 \ldots \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **–12**

3. $(x-2)^7 = \ldots + a \cdot x^4 \ldots \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **–280**

4. $(x-3)^7 = \ldots + a \cdot x^5 \ldots \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **189**

5. $\left(x - \frac{3}{x}\right)^6 = \ldots + a \cdot x^2 \ldots \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **135**

6. $\left(x + \frac{2}{x}\right)^7 = \ldots + a \cdot x^5 \ldots \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **14**

7. $(4-x)^5 = 4^5 \ldots + A \cdot x^3 + \ldots - x^5 \Rightarrow \mathbf{A = ?}$

Yanıt / The Answer is: **–160**

8. $x^4 + y^4 = 41$

$2x^3y + 3x^2y^2 + 2xy^3 = 20$

$\Rightarrow \mathbf{x + y = ?}$

Yanıt / The Answer is: **–12**

9. $(2a-3b)^6 = 64a^6 \ldots + x \cdot a^3 \cdot b^3 + \ldots$

Yanıt / The Answer is: **–4320**

10. $(2x - y^2)^8 = \ldots + k \cdot x^3 y^{10} \ldots \Rightarrow \mathbf{k = ?}$

Yanıt / The Answer is: **–448**

11. $\left.\begin{array}{l} x = a - \sqrt{2} \\ y = a + \sqrt{2} \end{array}\right\} \Rightarrow \mathbf{x^3 + 3x^2y + 3xy^2 + y^3 = ?}$

Yanıt / The Answer is: **$8a^3$**

12. $(a - 2b + c)^{10} = \ldots + m \cdot a \cdot b^2 \cdot c^7 + \ldots \Rightarrow \mathbf{m = ?}$

Yanıt / The Answer is: **1440**

1. $(x-y)^n = \ldots + a\cdot x^3\cdot y^5 \ldots \Rightarrow$ **n = ?**

A) 3
B) 4
C) 5
D) 7
E) 8

2. $(x+y)^n = \ldots + a\cdot x^3\cdot y^7 \ldots \Rightarrow$ **n = ?**

A) 7
B) 8
C) 9
D) 10
E) 11

3. $(x+2)^4 = \ldots + a\cdot x^2 \ldots \Rightarrow$ **a = ?**

A) 12
B) 16
C) 18
D) 20
E) 24

4. $(x-3)^6 = \ldots + a\cdot x^5 + \ldots \Rightarrow$ **a = ?**

A) –20
B) –18
C) –16
D) –15
E) –14

5. $(x-2y)^5 = \ldots + a\cdot x^4\cdot y^n \ldots \Rightarrow$ **a + n = ?**

A) –6
B) –7
C) –8
D) –9
E) –10

6. $(x+y)^7 = \ldots + a\cdot x^4\cdot y^n \ldots \Rightarrow$ **a + n = ?**

A) 34
B) 35
C) 36
D) 37
E) 38

7. $\left(x+\frac{1}{x}\right)^5 = \ldots + a \cdot x \ldots \Rightarrow$ **a = ?**

A) 5
B) 10
C) 15
D) 20
E) 25

8. $\left(x-\frac{2}{x}\right)^6 = \ldots + a \cdot x^2 \ldots \Rightarrow$ **a = ?**

A) 60
B) 64
C) 68
D) 70
E) 72

9. $(x-5)^4 = \ldots + a \cdot x^3 \ldots \Rightarrow$ **a = ?**

A) –15
B) –20
C) –25
D) –30
E) –35

10. $\left.\begin{array}{l} 3ab^2 + a^3 = 35 \\ 3a^2b + b^3 = 27 \end{array}\right\} \Rightarrow$ **a – b = ?**

A) 2
B) 3
C) 4
D) 5
E) 6

11. $\left.\begin{array}{l} a^3 + b^3 = 62 \\ ab(a+b) = 21 \end{array}\right\} \Rightarrow$ **a + b = ?**

A) 2
B) 3
C) 4
D) 5
E) 6

12. $\left.\begin{array}{l} a^3 - b^3 = 12 \\ ab(b-a) = 5 \end{array}\right\} \Rightarrow$ **a – b = ?**

A) 3
B) 4
C) 5
D) 6
E) 7

YANITLAR / ANSWERS

1. E	**2. D**	**3. E**	**4. B**	**5. D**	**6. E**	**7. B**	**8. A**
9. B	**10. A**	**11. D**	**12. A**				

MATRİS ve DETERMİNANT
MATRIX and DETERMINANT

BÖLÜM
CHAPTER **3**

MATRİS / MATRIX

$$\xrightarrow{\quad}\ \text{m satır / rows}$$

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n}\\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n}\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn}\end{bmatrix}_{mxn}\Big\downarrow \ \text{n sütun / columns}$$

Yukarıdaki n tane sütun, m tane satırdan oluşan dikdörtgen tabloya *matris* denir.

The rectangle array composed of n columns (vertical lines) and m rows (horizantal lines) is called a matrix.

$a_{ij} \in R \ (1 \le i \le m,\ 1 \le j \le n)$

m : Satır sayısı / *Number of rows*

n : Sütun sayısı / *Number of columns*

Örneğin / *For example;*

$$A=\begin{bmatrix} 1 & 2 & 3 & 4\\ 0 & -2 & 1 & -3\\ 4 & 2 & 0 & 1\end{bmatrix}$$

A bir 3 x 4 matristir. / *A is a 3 x 4 matrix.*

MATRİS ÇEŞİTLERİ / KINDS of MATRIX

KARE MATRİS / SQUARE MATRIX

Tanım: Satır ve sütun sayısı aynı olan matrise *kare matris* denir.

Definition: *A matrix having equal number of rows and columns is called a square matrix.*

Kare Matrislere Örnekler / *Examples for square matrices:*

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{bmatrix}\longrightarrow \text{köşegen / } \textit{diagonal}$$

$$A=\begin{bmatrix} 2 & 3 & -4 & 1\\ 0 & 5 & -1 & 5\\ 0 & 0 & -3 & 1\\ 0 & 0 & 0 & 4\end{bmatrix}\longrightarrow \begin{matrix}\text{köşegen}\\ \textit{diagonal}\end{matrix} \qquad B=\begin{bmatrix} -1 & 0 & 0\\ 3 & 4 & 0\\ -1 & -7 & 2\end{bmatrix}\longrightarrow \begin{matrix}\text{köşegen}\\ \textit{diagonal}\end{matrix} \qquad C=\begin{bmatrix} 2 & 0 & 0 & 0\\ 0 & 3 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 6\end{bmatrix}\longrightarrow \begin{matrix}\text{köşegen}\\ \textit{diagonal}\end{matrix}$$

NOT / NOTE

Tek satıra sahip olan matrise satır matrisi, tek sütuna sahip matrise ise sütün matrisi denir

A matrix that has only one row is called a row matris, and a matris that has only one column is called a column matrix.

BİRİM MATRİS / IDENTITY MATRIX

Tanım: Köşegen (esas köşegen) üzerindeki elemanları 1 ve diğer elemanları 0 olan kare matrise *birim matris* denir. I veya I_n ile gösterilir.

Definition: *The n x n matrix that consists of 1's on its main diagonal and 0's elsewhere is called the identity matrix of order n and is denoted by I or I_n.*

Örneğin / *For example;*

$$I_2=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

I_2 ve I_3 birim matrislerdir. / *I_2 and I_3 are unit matrices.*

NOT / NOTE

$1 \le i \le m$ ve $1 \le j \le n$ olmak üzere, m x n boyutlu matrisler için $a_{ij} = b_{ij}$ ise, $A = [a_{ij}]$ ve $B = [b_{ij}]$ matrisleri eşittir. Başka bir deyişle, 2 matrisin uygun hücreleri eşitse bu matrisler eşittir.

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if they have the same order (mxn) and $a_{ij} = b_{ij}$ for $1 \le i \le m$ and $1 \le j \le n$. In other words, two matrices are equal if their corresponding entries are equal.

MATRİSLERİN EŞİTLİĞİ / EQUALITY of MATRICES

$$\begin{bmatrix}x&y&z\\m&n&t\\a&b&c\end{bmatrix}=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}\Rightarrow\begin{cases}x=a_{11},\ y=a_{12},\ z=a_{13}\\m=a_{21},\ n=a_{22},\ t=a_{23}\\a=a_{31},\ b=a_{32},\ c=a_{33}\end{cases}$$

PEKİŞTİRME / REINFORCE

$X = [\,a-1 \quad b+3\,]$

$Y = [\,5 \quad 2b-5\,]$

$X = Y \Rightarrow a\cdot b = ?$

YANIT / ANSWER : 48

ÖRNEK / EXAMPLE 1

$$A=\begin{bmatrix}x+1&2\\-3&2y\end{bmatrix}\qquad B=\begin{bmatrix}z-2&z\\-3&4x\end{bmatrix}$$

$A = B \Rightarrow$ **$x + y + z = ?$**

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$A = B \Rightarrow x + 1 = z - 2 \ , \quad z = 2 \ , \quad 2y = 4x$

$x + 1 = 2 - 2$

$x = -1$

$y = 2x = -2$

$x + y + z = -1 - 2 + 2 = -1$

MATRİSLERİN TOPLAMI ve FARKI / SUM and DIFFERENCE of MATRICES

$A=[a_{ij}]_{m\times n},\ B=[b_{ij}]_{m\times n}\ \Rightarrow\ A+B=[a_{ij}+b_{ij}]_{m\times n}\ ,\quad A-B=[a_{ij}-b_{ij}]_{m\times n}$

ÖRNEK / EXAMPLE 2

$$A=\begin{bmatrix}\sqrt{50} & 3\\ x & 1\end{bmatrix},\ B=\begin{bmatrix}-\sqrt{18} & 5\\ 2 & 3\end{bmatrix},\ C=\begin{bmatrix}2^{\frac{x}{2}} & 8\\ y & \log_2 z\end{bmatrix}$$

$A+B=C\ \Rightarrow\ \mathbf{x+y+z=?}$

A) 24 B) 25 C) 26 D) 27 E) 28

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$A+B=\begin{bmatrix}\sqrt{50}-\sqrt{18} & 3+5\\ x+2 & 1+3\end{bmatrix}=\begin{bmatrix}2\sqrt{2} & 8\\ x+2 & 4\end{bmatrix}=\begin{bmatrix}2^{\frac{x}{2}} & 8\\ y & \log_2 z\end{bmatrix}$$

$\Rightarrow 2^{\frac{x}{2}}=2\sqrt{2}=2^{\frac{3}{2}}\ \Rightarrow x=3$

$y=x+2=5$

$\log_2 z=4\Rightarrow z=16$

$x+y+z=3+5+16=24$

PEKİŞTİRME / REINFORCE

$$W=\begin{bmatrix}i^3+2i\\ 5\end{bmatrix},\quad T=\begin{bmatrix}x+3\\ y-2\end{bmatrix}$$

$W=T\ \Rightarrow\ x+y=?$

YANIT / ANSWER : $i+4$

BİR MATRİSİN bir SKALERLE ÇARPIMI PRODUCT of SCALAR and MATRIX

$\alpha\in R$

$$\alpha\cdot\begin{bmatrix}a & b & c\\ x & y & z\\ m & n & t\end{bmatrix}=\begin{bmatrix}\alpha\cdot a & \alpha\cdot b & \alpha\cdot c\\ \alpha\cdot x & \alpha\cdot y & \alpha\cdot z\\ \alpha\cdot m & \alpha\cdot n & \alpha\cdot t\end{bmatrix}$$

ÖRNEK / EXAMPLE 3

$$A=\begin{bmatrix}-3 & 0\\ 4 & 2\\ 3 & 1\end{bmatrix},\quad B=\begin{bmatrix}4 & 5\\ -1 & 2\\ 3 & 1\end{bmatrix},\ 2A+x=B \Rightarrow x=?$$

A) $\begin{bmatrix}0 & -2\\ 3 & 4\\ 6 & 2\end{bmatrix}$

B) $\begin{bmatrix}-2 & 5\\ 7 & 6\\ 9 & 3\end{bmatrix}$

C) $\begin{bmatrix}9 & 3\\ -7 & 8\\ 6 & 3\end{bmatrix}$

D) $\begin{bmatrix}10 & 5\\ -9 & -2\\ -3 & -1\end{bmatrix}$

E) $\begin{bmatrix}-1 & 3\\ 4 & 8\\ -2 & 10\end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$2A+x=B \Rightarrow x=B-2A=\begin{bmatrix}4 & 5\\ -1 & 2\\ 3 & 1\end{bmatrix}-2\begin{bmatrix}-3 & 0\\ 4 & 2\\ 3 & 1\end{bmatrix}=\begin{bmatrix}10 & 5\\ -9 & -2\\ -3 & -1\end{bmatrix}$$

MATRİSLERİN ÇARPIMI / PRODUCT of MATRICES

$A=[a_{ij}]_{m\times n}$ ve $B=[b_{ij}]_{n\times p}$

A ve B matrislerinin çarpımı, $A\cdot B=AB$ biçiminde gösterilir.

Multiplication of matrices A and B is denoted by $A\cdot B=AB$.

Örneğin / *For example:*

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix},\quad B=\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}$$

$$\Rightarrow A\cdot B=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\cdot\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22}\\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22}\end{bmatrix}$$

NOTLAR / NOTES

- ✓ İki matrisin çarpılabilmesi için; birinci matrisin sütun sayısı, ikinci matrisin satır sayısına eşit olmalıdır.

  For the product of two matrices, the number of columns of the first matrix must equal the number of rows of the second matrix.
- ✓ $AB \neq BA$
- ✓ $(AB)\cdot C = A\cdot(BC)$
- ✓ $(B+C)\cdot A = BA + CA$
- ✓ $k\cdot(AB) = (kA)\cdot B = A\cdot(kB)\ (k\in R)$
- ✓ $IA = AI = A$

ÖRNEK / EXAMPLE 4

$$\begin{bmatrix} -1 & 0 & 3 \\ 4 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 2 \end{bmatrix} = ?$$

A) $\begin{bmatrix} 4 & 7 \\ 12 & 8 \end{bmatrix}$ B) $\begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$ C) $\begin{bmatrix} -4 & 3 \\ 7 & 14 \end{bmatrix}$ D) $\begin{bmatrix} -2 & 5 \\ 3 & 9 \end{bmatrix}$ E) $\begin{bmatrix} -6 & 4 \\ 8 & 10 \end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{bmatrix} -1 & 0 & 3 \\ 4 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} (-1)\cdot 1 + 0\cdot 2 + (3)\cdot(-1) & (-1)\cdot 3 + 0\cdot 0 + 3\cdot 2 \\ 4\cdot 1 + 2\cdot 2 + 1\cdot(-1) & 4\cdot 3 + 2\cdot 0 + 1\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 3 \\ 7 & 14 \end{bmatrix}$$

ÖRNEK / EXAMPLE 5

$$\begin{bmatrix} -1 & 4 \\ 0 & 2 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} = ?$$

A) $\begin{bmatrix} 9 & 1 \\ 4 & 2 \\ -2 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 8 & 7 \\ 3 & -2 \\ 4 & 5 \end{bmatrix}$ C) $\begin{bmatrix} 1 & -2 \\ 4 & -4 \\ 3 & 6 \end{bmatrix}$ D) $\begin{bmatrix} 2 & -3 \\ 5 & -2 \\ 7 & 4 \end{bmatrix}$ E) $\begin{bmatrix} 4 & -11 \\ 4 & -6 \\ 8 & 3 \end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{bmatrix} -1 & 4 \\ 0 & 2 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} (-1)\cdot 4 + 4\cdot 2 & (-1)\cdot(-1) + 4\cdot(-3) \\ 0\cdot 4 + 2\cdot 2 & 0\cdot(-1) + 2\cdot(-3) \\ 3\cdot 4 + (-2)\cdot 2 & 3\cdot(-1) + (-2)\cdot(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -11 \\ 4 & -6 \\ 8 & 3 \end{bmatrix}$$

PEKİŞTİRME / REINFORCE

$$\begin{bmatrix} -1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} = ?$$

YANIT / ANSWER : 15

NOT / NOTE

✓ $I_n^k = I_n$

MATRİSİN KUVVETİ / POWER of a MATRIX

$$A^n = \underbrace{A \cdot A \cdot A \cdot \ldots \cdot A}_{\text{n tane / times}}$$

ÖRNEK / EXAMPLE 6

$$A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} \Rightarrow A^{32} = ?$$

A) I_2 B) $2^{16} \cdot I_2$ C) $3^{16} \cdot I_2$ D) $4^{16} \cdot I_2$ E) $5^{16} \cdot I_2$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$A^{32} = (A^2)^{16} = (I_2)^{16} = I_2$$

PEKİŞTİRME / REINFORCE

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \Rightarrow A^{30} = ?$$

YANIT / ANSWER : I_2

ÖRNEK / EXAMPLE 7

$$A = \begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix} \Rightarrow A^{100} = ?$$

A) $10^{50} \cdot I_2$ B) $12^{50} \cdot I_2$ C) $12^{100} \cdot I_2$ D) $13^{50} \cdot I_2$ E) $14^{50} \cdot I_2$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$A^2 = A \cdot A = \begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = 14I_2$$

$$A^{100} = (A^2)^{50} = (14I_2)^{50} = 14^{50}\, I_2$$

ÖRNEK / EXAMPLE 8

$$A=\begin{bmatrix}-2 & 1\\ 4 & 3\end{bmatrix},\ f(x)=x^2-3x+1 \Rightarrow \mathbf{f(A)=?}$$

A) $\begin{bmatrix}15 & -2\\ -8 & 5\end{bmatrix}$ B) $\begin{bmatrix}13 & -1\\ -7 & 6\end{bmatrix}$ C) $\begin{bmatrix}10 & -8\\ -9 & 12\end{bmatrix}$

D) $\begin{bmatrix}16 & -3\\ -6 & 8\end{bmatrix}$ E) $\begin{bmatrix}14 & -1\\ -5 & 9\end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$f(A)=A^2-3A+1=\begin{bmatrix}-2 & 1\\ 4 & 3\end{bmatrix}\cdot\begin{bmatrix}-2 & 1\\ 4 & 3\end{bmatrix}-3\begin{bmatrix}-2 & 1\\ 4 & 3\end{bmatrix}+1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 1\\ 4 & 13\end{bmatrix}+\begin{bmatrix}6 & -3\\ -12 & -9\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}15 & -2\\ -8 & 5\end{bmatrix}$$

ÖRNEK / EXAMPLE 9

$$A=\begin{bmatrix}-1 & 4\\ 5 & 2\end{bmatrix},\ f(x)=x^2+2x+5 \Rightarrow \mathbf{f(A)=?}$$

A) $\begin{bmatrix}16 & 24\\ 12 & 32\end{bmatrix}$ B) $\begin{bmatrix}17 & 21\\ 30 & 26\end{bmatrix}$ C) $\begin{bmatrix}19 & 12\\ 15 & 28\end{bmatrix}$

D) $\begin{bmatrix}24 & 12\\ 15 & 33\end{bmatrix}$ E) $\begin{bmatrix}24 & 17\\ 20 & 33\end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$f(A)=A^2+2A+5I=\begin{bmatrix}-1 & 4\\ 5 & 2\end{bmatrix}\cdot\begin{bmatrix}-1 & 4\\ 5 & 2\end{bmatrix}+2\cdot\begin{bmatrix}-1 & 4\\ 5 & 2\end{bmatrix}+5\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}21 & 4\\ 5 & 24\end{bmatrix}+\begin{bmatrix}-2 & 8\\ 10 & 4\end{bmatrix}+\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

$$=\begin{bmatrix}24 & 12\\ 15 & 33\end{bmatrix}$$

NOT / NOTE

- ✓ $(A+B)^T = A^T + B^T$
- ✓ $(A \cdot B)^T = B^T \cdot A^T$
- ✓ $(A^T)^T = A$
- ✓ $(k \cdot A)^T = k \cdot A^T$

MATRİSİN TRANSPOZU / TRANSPOZE of a MATRIX

Bir A matrisinin transpozu A^t olarak gösterilir ve satırların yerine sütunların, sütunların yerine satırların yazılması ile elde edilir.

The transpoze of a matrix A is denoted by A^t. To find the transpose of a matrix, rows of matrix are written instead of columns and columns of matrix are written instead of rows.

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ -2 & 3 & -1 & 2 \\ 6 & -4 & -3 & 5 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 2 & -2 & 6 \\ 1 & 3 & -4 \\ 3 & -1 & -3 \\ 4 & 2 & 5 \end{bmatrix}$$

PEKİŞTİRME / REINFORCE

$A = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix} \Rightarrow A \cdot A^T = ?$

ÖRNEK / EXAMPLE 10

$$A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} \Rightarrow \mathbf{B^T \cdot A = ?}$$

A) $\begin{bmatrix} 7 & 10 \\ 1 & 17 \end{bmatrix}$ B) $\begin{bmatrix} 12 & 3 \\ 10 & -3 \end{bmatrix}$ C) $\begin{bmatrix} 4 & 9 \\ -15 & 11 \end{bmatrix}$

D) $\begin{bmatrix} 13 & 4 \\ -11 & 16 \end{bmatrix}$ E) $\begin{bmatrix} 8 & -7 \\ 14 & 9 \end{bmatrix}$

YANIT / ANSWER : 14

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$B = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B^T \cdot A = \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 3 \\ 10 & -3 \end{bmatrix}$$

MATRİSİN TERSİ / THE INVERSE of a MATRIX

Bir matrisin tersi A^{-1} ile gösterilir. / *Inverse of a matrix is denoted by A^{-1}.*

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

NOTLAR / NOTES

- ✓ $A \cdot A^{-1} = I_n$
- ✓ $(A^T)^{-1} = (A^{-1})^T$
- ✓ $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$
- ✓ $(A^{-1})^{-1} = A$

ÖRNEK / EXAMPLE 11

$$A=\begin{bmatrix} 3 & -2 \\ -8 & 5 \end{bmatrix} \Rightarrow A^{-1}=?$$

A) $\begin{bmatrix} -5 & 2 \\ 8 & -3 \end{bmatrix}$ B) $\begin{bmatrix} 5 & -2 \\ -8 & 3 \end{bmatrix}$ C) $\begin{bmatrix} -5 & -2 \\ -8 & -3 \end{bmatrix}$

D) $\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}$ E) $\begin{bmatrix} 8 & -3 \\ -2 & 5 \end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$A^{-1}=\frac{1}{5 \cdot 3-(-8)(-2)}\begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}=\begin{bmatrix} -5 & -2 \\ -8 & -3 \end{bmatrix}$$

BİR MATRİSİN DETERMİNANTI / THE DETERMINANT of a MATRIX

Tanım: Bir kare matrisinin ilişkilendirildiği reel sayıya o matrisin *determinantı* denir ve detA veya IAI ile gösterilir.

***Definition:** Every square matrix can be associated with a real number called its determinant and determinant of a matrix is denoted by **detA** or **IAI**.*

a. 2 x 2'lik Matrisin Determinantı / The Determinant of a 2 x 2 Matrix

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |A|=\det A=\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$$

PEKİŞTİRME / REINFORCE

$$x=\begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix} \Rightarrow |x|=?$$

YANIT / ANSWER : –5

ÖRNEK / EXAMPLE 12

$$A=\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow |A|=?$$

A) –16 B) –14 C) –12 D) –10 E) –8

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$A=\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow |A|=(-1)\cdot 4-3\cdot 2=-10$$

b. 3 x 3'lük Matrisin Determinantı / The Determinant of a 3 x 3 Matrix (Sarrus Kuralı / Rule of Sarrus)

$$A=\begin{bmatrix} a & b & c \\ x & y & z \\ m & n & p \end{bmatrix} \Rightarrow \det A=|A|=\left|\begin{array}{ccc} a & b & c \\ x & y & z \\ m & n & p \\ a & b & c \\ x & y & z \end{array}\right|\begin{array}{c} - \\ \\ \\ \\ + \end{array}=(a\cdot y\cdot p+x\cdot n\cdot c+m\cdot b\cdot z)-(m\cdot y\cdot c+a\cdot n\cdot z+x\cdot b\cdot p)$$

ÖRNEK / EXAMPLE 13

$$A=\begin{bmatrix} -3 & 1 & 5 \\ 2 & 4 & 3 \\ 0 & 1 & -1 \end{bmatrix} \Rightarrow |A|=?$$

A) 33 B) 34 C) 35 D) 36 E) 37

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\begin{array}{r|rrr|r} & -3 & 1 & 5 & \\ & 2 & 4 & 3 & \\ 0 & 0 & 1 & -1 & 12 \\ -9 & -3 & 1 & 5 & 10 \\ \underline{-2} & 2 & 4 & 3 & \underline{0} \\ -11 & & & & 22 \end{array} \Rightarrow |A|=22+11=33$$

c. n x n lik Matrisin Determinantı / The Determinant of a n x n Matrix

Minör: A kare matrisinin i. satır ve j. sütununun silinmesiyle elde edilen matrisin determinantına bu matrisin *küçüğü (minörü)* denir. M_{ij} ile gösterilir.

Minor: *If A is a square matrix, the minor M_{ij} of the entry aij is the determinant of the matrix obtained by deleting the i^{th} row and j^{th} column of A.*

Etkinlikler / Activities

a. $A=\begin{bmatrix}1 & 3\\ 4 & 1\end{bmatrix} \Rightarrow M_{11}=[1]$

b. $A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 3\\ 2 & 1 & 1\end{bmatrix} \Rightarrow M_{21}=\begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}$

Kofaktör: Bir matrisin i. satır, j. sütununun kofaktörü $A_{ij}=(-1)^{i+j}M_{ij}$ ile gösterilir.

Cofactor: *The cofaktor A_{ij} of the entry a_{ij} is given by $A_{ij}=(-1)^{i+j}M_{ij}$.*

Etkinlik / Activitiy

$$A=\begin{bmatrix}-1 & 1 & 3\\ 0 & 2 & 1\\ 3 & 1 & 2\end{bmatrix} \Rightarrow A_{11}=(-1)^2\cdot M_{11}=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

▼ **$|A|=(-1)^{i+j}M_{ij}\cdot a_{ij}$**

ÖRNEK / EXAMPLE 14

$$A=\begin{bmatrix}-1 & 0 & 3 & 1\\ 0 & 2 & -1 & 2\\ 0 & 0 & 0 & 3\\ -1 & 3 & 1 & 2\end{bmatrix} \Rightarrow |A|=?$$

A) –3 B) –2 C) 0 D) 2 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$|A|=0\cdot(-1)^{3+1}\begin{vmatrix}0 & 3 & 1\\ 2 & -1 & 2\\ 3 & 1 & 2\end{vmatrix}+0\cdot(-1)^{3+2}\begin{vmatrix}-1 & 3 & 1\\ 0 & -1 & 2\\ -1 & 1 & 2\end{vmatrix}+0\cdot(-1)^{3+3}\begin{vmatrix}-1 & 0 & 1\\ 0 & 2 & 2\\ -1 & 3 & 2\end{vmatrix}+3\cdot\begin{vmatrix}-1 & 0 & 3\\ 0 & 2 & -1\\ -1 & 3 & 1\end{vmatrix}\cdot(-1)^{3+4}$$

$$=(-3)\cdot\begin{array}{r|rrr|r} & -1 & 0 & 3 & \\ & 0 & 2 & -1 & \\ -6 & -1 & 3 & 1 & -2\\ 3 & -1 & 0 & 3 & 0\\ \underline{0} & 0 & 2 & -1 & \underline{0}\\ -3 & & & & -2\end{array} \Rightarrow |A|=(-3)\cdot[-2-(-3)]=-3$$

Determinantın Özellikleri / Properties of Determinant

1. $|A| = |A^T|$

2. $|A \cdot B| = |A| \cdot |B|$

3. $|A^n| = |A|^n$

4. $|A^{-1}| = \frac{1}{|A|}$

5. Bir kare matrisin herhangi satır veya sütunu diğeri ile yer değiştirirse, determinantın değeri işaret değiştirir.
   If any two rows or columns of a square matrix is interchanged with eachother, the sign of its determinant changes.

6. A kare matrisinin herhangi satır veya sütununun tamamı 0 ise, $|A| = 0$ dır.
   If the whole entries in any row or column of a square matrix A is 0, then $|A| = 0$.

7. A kare matrisinin herhangi iki satır veya sütunu aynı ise, determinantı 0 dır.
   If any two rows or columns of a square matrix A are the same, determinant will be 0.

8. Bir determinantın herhangi iki satır veya sütununun tamamı orantılı ise, determinantı sıfırdır.
   If the entries of any two rows or columns of a determinant are proportional, the determinant will be zero.

9. A kare matrisinin herhangi satır veya sütününun tamamı k reel sayısı ile çarpılırsa, matrisin determinantı $k \cdot |A|$ olur.
   When the entries of any row or column of a square matrix A are multiplied by a real number k, the new determinant becomes $k \cdot |A|$.

10. 2 x 2'lik A matrisi ve $k \in R$ için, $|kA| = k^2|A|$ dir.
    For a 2 x 2 matrix A and $k \in R$, $|kA| = k^2|A|$.

11. Bir matrisin herhangi satır veya sütunu k reel sayısı ile çarpılır ve başka sütun veya satıra eklenirse determinant değişmez.
    The value of a determinant does not change if its any row or column is multiplied by a real number k and then added to another row or column, respectively.

12. Bir matrisin köşegeninin altındaki veya üstündeki tüm elemanlar sıfır ise, determinant değeri, köşegendeki elemanların çarpımına veya negatif işaretlisine eşittir.
    If the entries above or below a diagonal of a determinant are all 0, then the determinant is equal to either the product of entries on this diagonal or its minus sign.

ÖRNEK / EXAMPLE 15

YILDIZ TEKNİK ÜNİV. YÖS 2013

$A^{-1}=\begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix} \Rightarrow A=?$

A) $\begin{bmatrix}-3/2 & 3/4\\ 1/2 & 1/2\end{bmatrix}$ B) $\begin{bmatrix}1/3 & 1/4\\ 2/3 & 1\end{bmatrix}$ C) $\begin{bmatrix}-1/4 & 1/4\\ 0 & -1\end{bmatrix}$ D) $\begin{bmatrix}1/2 & -3/4\\ -1/2 & 3/4\end{bmatrix}$ E) $\begin{bmatrix}1/2 & 1/4\\ 1/2 & 3/4\end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, $A\cdot A^{-1}=I$

$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\cdot\begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \Rightarrow$

$$\begin{array}{rl|rl} & 3a-2b=1 & & 3c-2d=0\\ + & -a+2b=0 & + & -c+2d=1\\ \hline & 2a=1 & & 2c=1\\ & a=\frac{1}{2},\ b\frac{1}{4} & & c=\frac{1}{2},\ d=\frac{3}{4} \end{array} \quad \Rightarrow A=\begin{bmatrix}1/2 & 1/4\\ 1/2 & 3/4\end{bmatrix}$$

ÖRNEK / EXAMPLE 16

ANKARA ÜNİ. YÖS 2013

$A=\begin{bmatrix}1 & 0 & -1\\ 2 & -2 & 0\end{bmatrix} \Rightarrow A^T\cdot A=?$

A) $\begin{bmatrix}5 & -4 & -1\\ -4 & 4 & 0\\ -1 & 0 & 1\end{bmatrix}$ B) $\begin{bmatrix}5 & -4 & -1\\ 4 & -4 & 0\\ -1 & 0 & 1\end{bmatrix}$ C) $\begin{bmatrix}5 & 4 & -1\\ -4 & 4 & 0\\ -1 & 0 & -1\end{bmatrix}$

D) $\begin{bmatrix}5 & -4 & 1\\ 4 & -4 & 0\\ -1 & 0 & -1\end{bmatrix}$ E) $\begin{bmatrix}5 & 4 & 1\\ 4 & 4 & 0\\ 1 & 0 & 1\end{bmatrix}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$A=\begin{bmatrix}1 & 0 & -1\\ 2 & -2 & 0\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}1 & 2\\ 0 & -2\\ -1 & 0\end{bmatrix}$

$A^T\cdot A=\begin{bmatrix}1 & 2\\ 0 & -2\\ -1 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & -1\\ 2 & -2 & 0\end{bmatrix}=\begin{bmatrix}5 & -4 & -1\\ -4 & 4 & 0\\ -1 & 0 & 1\end{bmatrix}$

ÖRNEK / EXAMPLE 17

İSTANBUL ÜNİ. YÖS 2013

$$\begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix}^2 = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix} \Rightarrow (x,y) = ?$$

A) (1, 2) B) (1, 0) C) (2, 0) D) (2, 1) E) (2, 3)

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} \cdot \begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x^2+2y & 2x-2 \\ xy-y & 2y+1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

$2x-2=2$ $2y+1=3$

$2x=4$ $2y=2$

$x=2$ $y=1$

$\Rightarrow (x,y)=(2,1)$

ÖRNEK / EXAMPLE 18

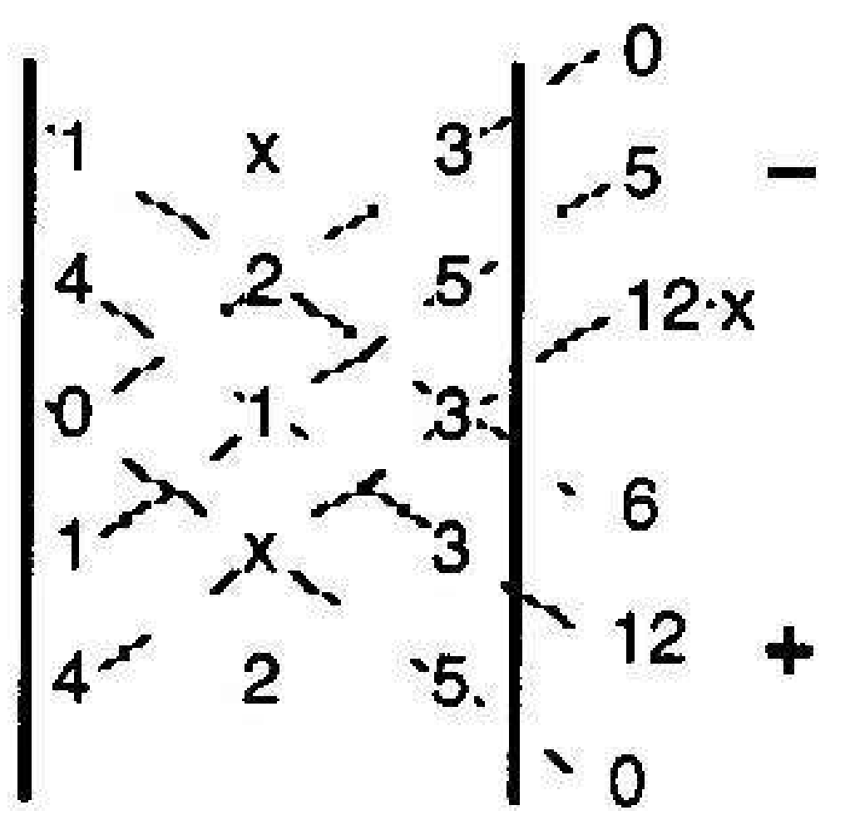
ÇANAKKALE 18 MART ÜNİV. YÖS 2015

$$\begin{vmatrix} 1 & x & 3 \\ 4 & 2 & 5 \\ 0 & 1 & 3 \end{vmatrix} = 25 \Rightarrow x = ?$$

A) 3 B) 2 C) 1 D) 0 E) −1

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**



$6 + 12 - (12x + 5) = 25$

$13 - 12x = 25$

$x = -1$

1. $\begin{bmatrix} 5 & y \\ 2x & 4 \end{bmatrix} = \begin{bmatrix} 5 & z-1 \\ 4y & z \end{bmatrix} \Rightarrow \mathbf{x+y+z=?}$

A) 11 B) 12 C) 13 D) 14 E) 15

2. $\begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{3} & \mathbf{1} & \mathbf{2} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{3} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} \\ \mathbf{4} & \mathbf{1} \end{bmatrix} = \mathbf{?}$

A) $\begin{bmatrix} 7 & -2 \\ 16 & 4 \end{bmatrix}$ B) $\begin{bmatrix} 6 & -3 \\ 14 & 3 \end{bmatrix}$ C) $\begin{bmatrix} -6 & 5 \\ 4 & 16 \end{bmatrix}$ D) $\begin{bmatrix} -10 & 9 \\ 12 & 5 \end{bmatrix}$ E) $\begin{bmatrix} -3 & 4 \\ 2 & 12 \end{bmatrix}$

3. $A = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}, \ B = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \Rightarrow \mathbf{A^T \cdot B = ?}$

A) $\begin{bmatrix} 5 & 4 \\ -2 & 3 \\ 6 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -10 & 3 \\ 5 & 1 \\ 4 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 15 \\ 3 & 4 \\ 6 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 12 & 1 \\ 3 & 7 \\ -1 & 6 \end{bmatrix}$ E) $\begin{bmatrix} 14 & 0 \\ 5 & 4 \\ 0 & 7 \end{bmatrix}$

4. $A = \begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix}, \ f(x) = x^2 - 3x + 5 \Rightarrow \mathbf{f(A) = ?}$

A) $\begin{bmatrix} -15 & 16 \\ -30 & 28 \end{bmatrix}$ B) $\begin{bmatrix} -12 & 8 \\ -20 & 16 \end{bmatrix}$ C) $\begin{bmatrix} 15 & -16 \\ -18 & 25 \end{bmatrix}$ D) $\begin{bmatrix} 43 & -16 \\ -20 & 23 \end{bmatrix}$ E) $\begin{bmatrix} 23 & -14 \\ 19 & -15 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} \Rightarrow \mathbf{A^{50} = ?}$

A) I_2 B) $2^{25} \cdot I_2$ C) $2^{50} \cdot I_2$ D) $2^{75} \cdot I_2$ E) $2^{100} \cdot I_2$

6. $A = \begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}, \ A \cdot x = I_2 \Rightarrow \mathbf{x = ?}$

A) $\begin{bmatrix} -3 & 5 \\ 4 & -7 \end{bmatrix}$ B) $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$ C) $\begin{bmatrix} -4 & 5 \\ 3 & -7 \end{bmatrix}$ D) $\begin{bmatrix} 5 & -4 \\ -7 & 3 \end{bmatrix}$ E) $\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

7. $A=\begin{bmatrix}3 & 2\\ -1 & 4\end{bmatrix}$, $B=\begin{bmatrix}12 & 1\\ 10 & -5\end{bmatrix}$, $A\cdot x=B \Rightarrow$ **x = ?**

A) $\begin{bmatrix}-2 & 1\\ -3 & 1\end{bmatrix}$ B) $\begin{bmatrix}2 & 1\\ 3 & -1\end{bmatrix}$ C) $\begin{bmatrix}2 & -1\\ -3 & 1\end{bmatrix}$

D) $\begin{bmatrix}1 & -1\\ 3 & 2\end{bmatrix}$ E) $\begin{bmatrix}3 & -2\\ 1 & -1\end{bmatrix}$

8. $A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}-1 & 4\\ 3 & 2\end{bmatrix} \Rightarrow$ **$B\cdot A^{-1}=?$**

A) $\begin{bmatrix}-5 & 14\\ 1 & 0\end{bmatrix}$ B) $\begin{bmatrix}-6 & 12\\ 2 & 3\end{bmatrix}$ C) $\begin{bmatrix}5 & -7\\ 13 & 0\end{bmatrix}$

D) $\begin{bmatrix}4 & -8\\ -2 & 0\end{bmatrix}$ E) $\begin{bmatrix}-3 & 15\\ -3 & 0\end{bmatrix}$

9. $A=\begin{bmatrix}-2 & 3 & 1\\ 0 & 4 & -1\\ 2 & 1 & 4\end{bmatrix} \Rightarrow |A|=?$

A) –44 B) –46 C) –48

D) –50 E) –52

10. $\begin{vmatrix}x & 0 & 0\\ -1 & 3 & x\\ 2 & 4 & 0\end{vmatrix}=-100,\ x\in R^{+} \Rightarrow$ **x = ?**

A) 5 B) 7 C) 9

D) 11 E) 13

11. $A=\begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix} \Rightarrow$ **$A^5=?$**

A) $\begin{bmatrix}-20 & 0\\ 0 & -20\end{bmatrix}$ B) $\begin{bmatrix}4^5 & 0\\ 0 & 4^5\end{bmatrix}$ C) $\begin{bmatrix}1 & -20\\ 0 & 1\end{bmatrix}$

D) $\begin{bmatrix}-20 & 0\\ 1 & -20\end{bmatrix}$ E) $\begin{bmatrix}-4^5 & 0\\ 0 & -4^5\end{bmatrix}$

12. $A=\begin{bmatrix}-2 & 1 & 3\\ 0 & -1 & 4\\ 1 & 2 & -3\end{bmatrix} \Rightarrow |A|=?$

A) 13 B) 14 C) 16

D) 17 E) 19

YANITLAR / ANSWERS

1. C	**2. A**	**3. E**	**4. D**	**5. A**	**6. B**	**7. B**	**8. A**
9. C	**10. A**	**11. C**	**12. D**				

1. $$\left.\begin{aligned} y &= z-1 \\ 2x &= 4y \\ z &= 4 \end{aligned}\right\} \Rightarrow \begin{aligned} y &= 3, \\ x &= 2y = 6 \\ z &= 4 \end{aligned}$$

$$\Rightarrow x+y+z = 6+3+4 = 13$$

Yanıt / The Answer is C

2. $$\begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ 16 & 4 \end{bmatrix}$$

Yanıt / The Answer is A

3. $$A^T \cdot B = \begin{bmatrix} -2 & 4 \\ 1 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 5 & 4 \\ 0 & 7 \end{bmatrix}$$

Yanıt / The Answer is E

4. $$f(A) = A^2 - 3A + 5$$

$$= \begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix} - 3\begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix} + 5I$$

$$= \begin{bmatrix} 29 & -4 \\ -5 & 24 \end{bmatrix} + \begin{bmatrix} 14 & -12 \\ -15 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 43 & -16 \\ -20 & 23 \end{bmatrix}$$

Yanıt / The Answer is D

5. $$A^2 = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$A^{50} = (I_2)^{25} = I_2$$

Yanıt / The Answer is A

6. $$A \cdot x = I_2 \Rightarrow$$

$$x = A^{-1} \cdot I_2 = A^{-1} = \frac{1}{3 \cdot 7 - 5 \cdot 4} \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

Yanıt / The Answer is B

7. $A \cdot x = B \Rightarrow x = A^{-1}B$

$$A^{-1} = \frac{1}{12+2}\begin{bmatrix}4 & -2\\ 1 & 3\end{bmatrix} = \frac{1}{14}\begin{bmatrix}4 & -2\\ 1 & 3\end{bmatrix}$$

$$A^{-1}B = \frac{1}{14}\begin{bmatrix}4 & -2\\ 1 & 3\end{bmatrix} \cdot \begin{bmatrix}12 & 1\\ 10 & -5\end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix}28 & 14\\ 42 & -14\end{bmatrix} = \begin{bmatrix}2 & 1\\ 3 & -1\end{bmatrix}$$

Yanıt / The Answer is B

8. $$B \cdot A^{-1} = \begin{bmatrix}-1 & 4\\ 3 & 2\end{bmatrix} \cdot \begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix} = \begin{bmatrix}-5 & 14\\ 1 & 0\end{bmatrix}$$

Yanıt / The Answer is A

9.

$$\begin{array}{r|rrr|r} & -2 & 3 & 1 & \\ & 0 & 4 & -1 & \\ 8 & 2 & 1 & 4 & -32 \\ 2 & -2 & 3 & 1 & 0 \\ 0 & 0 & 4 & -1 & -6 \\ \hline 10 & & & & -38 \end{array}$$

$\Rightarrow |A| = -48$

Yanıt / The Answer is C

10. $$(-1)^{1+1}x\begin{vmatrix}3 & x\\ 4 & 0\end{vmatrix} = -100$$

$$-x \cdot 4x = -100$$

$$x^2 = 25$$

$$x = 5$$

Yanıt / The Answer is A

11. $$A^2 = \begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix} \cdot \begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -8\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow A^5 = \begin{bmatrix}1 & -20\\ 0 & 1\end{bmatrix}$$

Yanıt / The Answer is C

12.

$$\begin{array}{r|rrr|r} & -2 & 1 & 3 & \\ & 0 & -1 & 4 & \\ -3 & 1 & 2 & -3 & -6 \\ -16 & 1 & 1 & 3 & 0 \\ 0 & 0 & -1 & 4 & 4 \\ \hline -19 & & & & -2 \end{array}$$

$\Rightarrow |A| = -2 + 19 = 17$

Yanıt / The Answer is D

1. $\begin{bmatrix} -2 & x & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 1 & y \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 3 \end{bmatrix} \Rightarrow \mathbf{x+y=?}$

A) 4 B) 6 C) 8

D) 10 E) 12

---

2. $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 4 \end{bmatrix}, \; B = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow \mathbf{A^T \cdot B = ?}$

A) $\begin{bmatrix} 1 & -3 \\ 4 & 12 \\ 7 & 15 \end{bmatrix}$ B) $\begin{bmatrix} -2 & 5 \\ 7 & 12 \\ 8 & 10 \end{bmatrix}$ C) $\begin{bmatrix} 4 & 18 \\ 3 & 11 \\ 9 & 13 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 20 \\ 2 & 9 \\ 11 & 15 \end{bmatrix}$ E) $\begin{bmatrix} 4 & 15 \\ 1 & 16 \\ 8 & 7 \end{bmatrix}$

---

3. $\begin{bmatrix} -2 & 3 & 4 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix} = ?$

A) $\begin{bmatrix} 18 & 8 \\ -7 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 16 & -5 \\ -5 & -3 \end{bmatrix}$ C) $\begin{bmatrix} 12 & 4 \\ -3 & -5 \end{bmatrix}$

D) $\begin{bmatrix} 20 & 18 \\ -7 & -9 \end{bmatrix}$ E) $\begin{bmatrix} 16 & 8 \\ -7 & 5 \end{bmatrix}$

---

4. $A = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix} \Rightarrow \mathbf{2 \cdot A + A^T = ?}$

A) $\begin{bmatrix} -11 & 4 \\ 10 & 3 \end{bmatrix}$ B) $\begin{bmatrix} -12 & 9 \\ 8 & 5 \end{bmatrix}$ C) $\begin{bmatrix} -6 & 8 \\ 7 & 6 \end{bmatrix}$

D) $\begin{bmatrix} -9 & 10 \\ 8 & 3 \end{bmatrix}$ E) $\begin{bmatrix} -10 & 5 \\ 7 & 4 \end{bmatrix}$

---

5. $A = \begin{bmatrix} 3 & 13 \\ 1 & 4 \end{bmatrix} \Rightarrow \mathbf{A^{-1} = ?}$

A) $\begin{bmatrix} -3 & 1 \\ 13 & -4 \end{bmatrix}$ B) $\begin{bmatrix} -13 & 4 \\ 1 & -3 \end{bmatrix}$ C) $\begin{bmatrix} 3 & -4 \\ 13 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 4 & 13 \\ 1 & 3 \end{bmatrix}$ E) $\begin{bmatrix} -4 & 13 \\ 1 & -3 \end{bmatrix}$

---

6. $A = \begin{bmatrix} -3 & 4 \\ 1 & -1 \end{bmatrix} \Rightarrow \mathbf{A^2 + A = ?}$

A) $\begin{bmatrix} 11 & -6 \\ -4 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 8 & -7 \\ -1 & 5 \end{bmatrix}$ C) $\begin{bmatrix} 10 & -12 \\ -3 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 9 & -5 \\ -6 & 7 \end{bmatrix}$ E) $\begin{bmatrix} 10 & -9 \\ -2 & 6 \end{bmatrix}$

7. $\begin{vmatrix} x-1 & x \\ -2 & 3 \end{vmatrix} = 32 \Rightarrow \mathbf{x=?}$

A) 5 B) 6 C) 7 D) 8 E) 9

8. $\begin{vmatrix} x & -2 & 1 \\ 3 & -1 & x \\ 4 & 0 & 2 \end{vmatrix} = -24 \Rightarrow \mathbf{x=?}$

A) 4 B) 5 C) 6 D) 7 E) 8

9. $A = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \Rightarrow \mathbf{A^{15}} = \mathbf{?}$

A) $\begin{bmatrix} 1 & -48 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & -54 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & -60 \\ 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -80 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & -100 \\ 0 & 1 \end{bmatrix}$

10. $\begin{vmatrix} \mathbf{2007} & \mathbf{2006} \\ \mathbf{2008} & \mathbf{2005} \end{vmatrix} = \mathbf{?}$

A) –4013 B) 1 C) 2008 D) 4013 E) 40113

11. $A = \begin{bmatrix} 3 & 8 \\ -2 & 5 \end{bmatrix} \Rightarrow \left|\mathbf{A^T}\right| = \mathbf{?}$

A) 28 B) 29 C) 30 D) 31 E) 32

12. $A = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix},\ B = \begin{bmatrix} 3 & x \\ 2 & 10 \end{bmatrix}$

$|A| = |B| \Rightarrow \mathbf{x = ?}$

A) –2 B) 0 C) 2 D) 4 E) 6

YANITLAR / ANSWERS

1. B 2. C 3. A 4. D 5. E 6. C 7. C 8. A
9. C 10. A 11. D 12. C

1. $-2\cdot 0 + x\cdot 1 + 4\cdot 2 = 9$

$x = 1$

$(-2)\cdot 3 + x\cdot y + 4\cdot 1 = 3$

$-2 + y = 3 \Rightarrow y = 5$

$\Rightarrow x + y = 6$

Yanıt / The Answer is B

---

2. $$A^T \cdot B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 18 \\ 3 & 11 \\ 9 & 13 \end{bmatrix}$$

Yanıt / The Answer is C

---

3. $$\begin{bmatrix} -2 & 3 & 4 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ -7 & -1 \end{bmatrix}$$

Yanıt / The Answer is A

---

4. $$2A + A^T = \begin{bmatrix} -6 & 8 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 10 \\ 8 & 3 \end{bmatrix}$$

Yanıt / The Answer is D

---

5. $|A| = 12 - 13 = -1$

$$A^{-1} = -\begin{bmatrix} 4 & -13 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -4 & 13 \\ 1 & -3 \end{bmatrix}$$

Yanıt / The Answer is E

---

6. $$A^2 + A = \begin{bmatrix} -3 & 4 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -12 \\ -3 & 4 \end{bmatrix}$$

Yanıt / The Answer is C

7. $(x-1)\cdot 3+2x=32$

$$5x=35$$

$$x=7$$

Yanıt / The Answer is C

8.

$$\begin{array}{r|ccc|r} & x & -2 & 1 & \\ & 3 & -1 & x & \\ -4 & 4 & 0 & 2 & -2x \\ 0 & x & -2 & 1 & 0 \\ \underline{-12} & 3 & -1 & x & \underline{-8x} \\ \multicolumn{1}{r}{-16} & & & & \multicolumn{1}{r}{-10x} \end{array}$$

$$-10x+16=-24$$

$$x=4$$

Yanıt / The Answer is A

9. $A^2=\begin{bmatrix}1 & -4\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & -4\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & -8\\0 & 1\end{bmatrix}$

$$\Rightarrow A^{15}=\begin{bmatrix}1 & -60\\0 & 1\end{bmatrix}$$

Yanıt / The Answer is C

10. $\begin{vmatrix}2007 & 2006\\2008 & 2005\end{vmatrix}=2007\cdot 2005-2008\cdot 2006$

$$2007=t \Rightarrow t(t-2)-(t+1)(t-1)$$

$$=t^2-2t-t^2+1$$

$$=1-2\cdot 2007$$

$$=-4013$$

Yanıt / The Answer is A

11. $|A^T|=|A|=3\cdot 5-(-2)\cdot 8=31$

Yanıt / The Answer is D

12. $|A|=18+8=26$

$|B|=30-2x$

$30-2x=26$

$x=2$

Yanıt / The Answer is C

1. $A=\begin{bmatrix}3 & x\\ z & y\end{bmatrix}$ $B=\begin{bmatrix}3 & -2\\ x+1 & 3z\end{bmatrix}$

$A=B \Rightarrow \mathbf{x+y+z=?}$

Yanıt / The Answer is: **–6**

2. $\begin{bmatrix}-2 & 3\\ 4 & 5\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}14\\ 16\end{bmatrix} \Rightarrow \mathbf{x+y=?}$

Yanıt / The Answer is: **3**

3. $A=\begin{bmatrix}-3 & 2\\ 5 & -1\end{bmatrix}$ $B=\begin{bmatrix}-2 & 5\\ 4 & 0\end{bmatrix}$

$3A-2B=C \Rightarrow \mathbf{C=?}$

Yanıt / The Answer is: $\begin{bmatrix}\mathbf{-5} & \mathbf{-4}\\ \mathbf{7} & \mathbf{-3}\end{bmatrix}$

4. $A=\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix} \Rightarrow \mathbf{A^{-1}=?}$

Yanıt / The Answer is: $\begin{bmatrix}\mathbf{2} & \mathbf{1}\\ \mathbf{5} & \mathbf{3}\end{bmatrix}$

5. $A=\begin{bmatrix}-2 & 3 & 1\\ 2 & 5 & -3\end{bmatrix}$, $B=\begin{bmatrix}-1 & 4 & -1\\ 2 & -2 & 1\end{bmatrix} \Rightarrow \mathbf{A\cdot B^{T}=?}$

Yanıt / The Answer is: $\begin{bmatrix}\mathbf{13} & \mathbf{-9}\\ \mathbf{21} & \mathbf{-9}\end{bmatrix}$

6. $A=\begin{bmatrix}-3 & 1\\ 4 & 2\end{bmatrix}$, $f(x)=x^2+5x-2 \Rightarrow \mathbf{f(A)=?}$

Yanıt / The Answer is: $\begin{bmatrix}\mathbf{-4} & \mathbf{4}\\ \mathbf{16} & \mathbf{16}\end{bmatrix}$

7. $A=\begin{bmatrix} 3 & 0 \\ -1 & \dfrac{1}{3} \end{bmatrix} \Rightarrow \mathbf{A^{-1}=?}$

Yanıt / The Answer is: $\begin{bmatrix} \mathbf{1/3} & \mathbf{0} \\ \mathbf{1} & \mathbf{3} \end{bmatrix}$

8. $A=\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \Rightarrow \mathbf{A^{-1}=?}$

Yanıt / The Answer is: $\begin{bmatrix} \mathbf{2} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{3} \end{bmatrix}$

9. $\begin{vmatrix} x & 2 \\ x-3 & 4 \end{vmatrix} = -2 \Rightarrow \mathbf{x=?}$

Yanıt / The Answer is: **–4**

10. $A=\begin{bmatrix} -1 & 3 & 2 \\ 4 & 7 & 1 \\ 2 & 2 & -2 \end{bmatrix} \Rightarrow \mathbf{|A|=?}$

Yanıt / The Answer is: **34**

11. $A=\begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} \Rightarrow \mathbf{|A^2|=?}$

Yanıt / The Answer is: **64**

12. $A=\begin{bmatrix} 1001 & 1000 \\ 1000 & 1001 \end{bmatrix} \Rightarrow \mathbf{\det A=?}$

Yanıt / The Answer is: **2001**

1. $\begin{bmatrix} -2 & 1 & 3 \\ -1 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix} = ?$

A) $\begin{bmatrix} 13 & -6 \\ 0 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 12 & -5 \\ 1 & -1 \end{bmatrix}$ C) $\begin{bmatrix} 14 & -3 \\ -3 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 15 & -2 \\ -4 & 5 \end{bmatrix}$ E) $\begin{bmatrix} 12 & -4 \\ -2 & 6 \end{bmatrix}$

2. $A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 9 \\ -3 \end{bmatrix}$

$A \cdot B = C \Rightarrow$ **$x - y = ?$**

A) −7 B) −5 C) 1

D) 8 E) 12

3. $A = \begin{bmatrix} -3 & x \\ 4 & y \end{bmatrix}$, $A^2 = \begin{bmatrix} 5 & 1 \\ -4 & 0 \end{bmatrix} \Rightarrow$ **$x + y = ?$**

A) −3 B) −2 C) −1

D) 0 E) 1

4. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \Rightarrow$ **$(B \cdot A)^{-1} = ?$**

A) $\begin{bmatrix} 0 & \frac{1}{4} \\ -\frac{1}{4} & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 & \frac{1}{16} \\ \frac{1}{16} & 0 \end{bmatrix}$ C) $\begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

D) $\begin{bmatrix} 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 \end{bmatrix}$ E) $\begin{bmatrix} 0 & -\frac{1}{4} \\ \frac{1}{4} & 0 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 0 & 2 \end{bmatrix} = 4x \Rightarrow$ **$x = ?$**

A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ C) $2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

D) $2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

6. $A = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$, $B = [1 \; -1 \;\; 3] \Rightarrow$ **$A \cdot B^T = ?$**

A) $\begin{bmatrix} 10 \\ -1 \end{bmatrix}$ B) $\begin{bmatrix} 10 \\ -2 \end{bmatrix}$ C) $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$

D) $\begin{bmatrix} -10 \\ 2 \end{bmatrix}$ E) $\begin{bmatrix} -10 \\ -2 \end{bmatrix}$

7. $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}-1 & 1 & 2\\ 3 & 4 & 5\end{bmatrix}$, $C=\begin{bmatrix}0 & 1 & 2\\ -1 & 1 & 3\end{bmatrix}$

$\Rightarrow$ **$A\cdot B + C = ?$**

A) $\begin{bmatrix}8 & 10 & 14\\ 5 & 20 & 19\end{bmatrix}$

B) $\begin{bmatrix}5 & 8 & 14\\ 8 & 20 & 29\end{bmatrix}$

C) $\begin{bmatrix}5 & 10 & 14\\ 8 & 20 & 29\end{bmatrix}$

D) $\begin{bmatrix}8 & 20 & 29\\ 5 & 10 & 14\end{bmatrix}$

E) $\begin{bmatrix}5 & 14 & 10\\ 8 & 20 & 29\end{bmatrix}$

8. $A=\begin{bmatrix}-2 & 3 & 0\\ -1 & 4 & 5\end{bmatrix}$, $C=\begin{bmatrix}-5 & 4 & 1\\ 7 & -1 & 5\end{bmatrix}$

$2A - B = 3C \Rightarrow$ **$B = ?$**

A) $\begin{bmatrix}12 & 14 & -16\\ 13 & -7 & 8\end{bmatrix}$

B) $\begin{bmatrix}13 & -7 & -5\\ 15 & 12 & -7\end{bmatrix}$

C) $\begin{bmatrix}12 & -5 & 4\\ -12 & 16 & 7\end{bmatrix}$

D) $\begin{bmatrix}11 & -6 & -3\\ -23 & 11 & -5\end{bmatrix}$

E) $\begin{bmatrix}13 & 7 & -2\\ -12 & 15 & 3\end{bmatrix}$

9. $\begin{bmatrix}x+1 & -1\\ 3y & 2\end{bmatrix}\cdot\begin{bmatrix}-2\\ 1\end{bmatrix}=\begin{bmatrix}x\\ 8\end{bmatrix} \Rightarrow$ **$x+y=?$**

A) −2
B) −3
C) −4
D) −5
E) −6

10. $A=\begin{bmatrix}-2 & 1\\ 3 & 0\end{bmatrix}$, $B=\begin{bmatrix}x & -2\\ 4 & 1\end{bmatrix}$

$|A| + |B| = 10 \Rightarrow$ **$x = ?$**

A) 5
B) 6
C) 7
D) 8
E) 9

11. $A=\begin{bmatrix}-2 & 3\\ 4 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 4\\ 5 & 2\end{bmatrix} \Rightarrow$ **$|A\cdot B| = ?$**

A) 250
B) 252
C) 254
D) 256
E) 258

12. $x \in (0, \pi)$,

$$\begin{vmatrix}\cos 2x & \sin 2x\\ \sin x & \cos x\end{vmatrix} = -\frac{1}{2} \Rightarrow x = ?$$

A) 30°
B) 40°
C) 60°
D) 120°
E) 150°

YANITLAR / ANSWERS

1. A	**2. B**	**3. E**	**4. A**	**5. B**	**6. A**	**7. C**	**8. D**
9. A	**10. A**	**11. B**	**12. B**				

1. $A=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 4\end{bmatrix},\quad B=\begin{bmatrix}-1 & -2 & 1\\ 2 & -3 & 4\end{bmatrix}$

$\Rightarrow \mathbf{A+B=?}$

A) $\begin{bmatrix}0 & 0 & 2\\ 1 & 0 & 3\end{bmatrix}$

B) $\begin{bmatrix}0 & 0 & 4\\ 1 & -3 & 8\end{bmatrix}$

C) $\begin{bmatrix}0 & 0 & 1\\ 1 & 1 & 8\end{bmatrix}$

D) $\begin{bmatrix}0 & -4 & 3\\ -1 & -9 & 16\end{bmatrix}$

E) $\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 4\end{bmatrix}$

2. $A=\begin{bmatrix}1 & 2 & 3\\ 3 & -1 & 2\end{bmatrix}\Rightarrow \mathbf{A^T=?}$

A) $\begin{bmatrix}1 & 3\\ 2 & -1\\ 3 & 2\end{bmatrix}$

B) $\begin{bmatrix}1 & 3\\ 2 & -1\\ 2 & 3\end{bmatrix}$

C) $\begin{bmatrix}3 & -1 & 2\\ 1 & 2 & 3\end{bmatrix}$

D) $\begin{bmatrix}1 & 3\\ 2 & 1\\ 3 & 2\end{bmatrix}$

E) $\begin{bmatrix}3 & 1\\ -1 & 2\\ 2 & 3\end{bmatrix}$

3. $A=\begin{bmatrix}5 & 4\\ 6 & 5\end{bmatrix},\quad A^T\cdot B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\Rightarrow \mathbf{B=?}$

A) $\begin{bmatrix}5 & -6\\ -4 & 5\end{bmatrix}$

B) $\begin{bmatrix}5 & 6\\ 4 & 5\end{bmatrix}$

C) $\begin{bmatrix}5 & -4\\ -6 & 5\end{bmatrix}$

D) $\begin{bmatrix}5 & 4\\ 6 & 5\end{bmatrix}$

E) $\begin{bmatrix}-5 & -6\\ -4 & 5\end{bmatrix}$

4. $A-3\cdot\begin{bmatrix}4 & 2\\ -1 & 3\end{bmatrix}+4=0\Rightarrow \mathbf{A=?}$

A) $\begin{bmatrix}16 & 6\\ -3 & 13\end{bmatrix}$

B) $\begin{bmatrix}8 & 6\\ -3 & 5\end{bmatrix}$

C) $\begin{bmatrix}-3 & 6\\ 13 & 16\end{bmatrix}$

D) $\begin{bmatrix}13 & 16\\ -3 & 6\end{bmatrix}$

E) $\begin{bmatrix}-3 & 16\\ 13 & 6\end{bmatrix}$

5. $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\Rightarrow \mathbf{|A|=?}$

A) $\cos 2\alpha$

B) $\sin 2\alpha$

C) 1

D) $-\cos 2\alpha$

E) $-\sin 2\alpha$

6. $A=\begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix}\quad B=\begin{bmatrix}-2 & 1\\ 4 & 5\end{bmatrix}$

$A\cdot x=B\Rightarrow \mathbf{x=?}$

A) $\begin{bmatrix}-12 & -1\\ 34 & 4\end{bmatrix}$

B) $\begin{bmatrix}-17 & 3\\ -13 & 5\end{bmatrix}$

C) $\begin{bmatrix}21 & -2\\ 15 & 8\end{bmatrix}$

D) $\begin{bmatrix}-12 & 4\\ 8 & -6\end{bmatrix}$

E) $\begin{bmatrix}-21 & 7\\ 33 & 13\end{bmatrix}$

7. $A=\begin{bmatrix}-4 & 2\\ 4 & 1\end{bmatrix},\ B=\begin{bmatrix}-10\\ 13\end{bmatrix},\ A\cdot x=B \Rightarrow$ **x = ?**

A) $\begin{bmatrix}5\\2\end{bmatrix}$ B) $\begin{bmatrix}3\\1\end{bmatrix}$ C) $\begin{bmatrix}4\\-2\end{bmatrix}$

D) $\begin{bmatrix}7\\-3\end{bmatrix}$ E) $\begin{bmatrix}-3\\2\end{bmatrix}$

---

8. $\begin{vmatrix}1 & -2 & 3\\ 4 & 2 & 0\\ -1 & 1 & 3\end{vmatrix}=?$

A) 36 B) 38 C) 40

D) 46 E) 48

---

9. $A=\begin{bmatrix}-1 & 1\\ -3 & -1\end{bmatrix} \Rightarrow$ **A^{99} = ?**

A) $2^{99}\cdot I_2$ B) $3^{99}\cdot I_2$ C) $4^{99}\cdot I_2$

D) $5^{99}\cdot I_2$ E) $6^{99}\cdot I_2$

---

10. $\begin{vmatrix}3 & 5\\ -2 & |x-1|\end{vmatrix}=19 \Rightarrow$ **$\sum x$ = ?**

A) 1 B) 2 C) 4

D) 8 E) 10

---

11. $\begin{bmatrix}-2 & 1 & 3\\ 4 & -1 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & 0\\ -1 & 1\\ 2 & 2\end{bmatrix}\cdot\begin{bmatrix}-1 & 3\\ 1 & 4\end{bmatrix}=?$

A) $\begin{bmatrix}-8 & -9\\ 10 & -7\end{bmatrix}$ B) $\begin{bmatrix}-7 & -12\\ -5 & 10\end{bmatrix}$ C) $\begin{bmatrix}4 & 37\\ -6 & 11\end{bmatrix}$

D) $\begin{bmatrix}-6 & -12\\ -14 & 8\end{bmatrix}$ E) $\begin{bmatrix}-10 & -6\\ -12 & 14\end{bmatrix}$

---

12. $A=\begin{bmatrix}-\sin\alpha & \cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix} \Rightarrow$ **A^{-1} = ?**

A) $\begin{bmatrix}-\sin\alpha & \cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$ B) $\begin{bmatrix}-\sin\alpha & -\cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$

C) $\begin{bmatrix}\sin\alpha & \cos\alpha\\ \cos\alpha & -\sin\alpha\end{bmatrix}$ D) $\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

E) $\begin{bmatrix}-\cos\alpha & \sin\alpha\\ -\sin\alpha & -\cos\alpha\end{bmatrix}$

YANITLAR / ANSWERS

1. B 2. A 3. A 4. B 5. C 6. A 7. B 8. E
9. A 10. B 11. C 12. A

1. $A=\begin{bmatrix}2 & 3 & -4\\ 1 & 2 & 3\end{bmatrix},\ B=\begin{bmatrix}-1 & 2 & 5\\ 0 & 1 & -4\end{bmatrix}$

$\Rightarrow$ **A + B = ?**

A) $\begin{bmatrix}3 & 1 & -9\\ 1 & 1 & 7\end{bmatrix}$ B) $\begin{bmatrix}1 & 5 & 1\\ 1 & 3 & -1\end{bmatrix}$ C) $\begin{bmatrix}-2 & 6 & -20\\ 0 & 2 & -12\end{bmatrix}$

D) $\begin{bmatrix}-2 & -\frac{3}{2} & 1\\ 1 & 2 & -\frac{4}{3}\end{bmatrix}$ E) $\begin{bmatrix}1 & 4 & 1\\ 1 & 2 & 1\end{bmatrix}$

2. $A=\begin{bmatrix}2 & 3x-1\\ -4 & 5\end{bmatrix}\ B=\begin{bmatrix}4 & -6\\ 1 & 4-y\end{bmatrix}\ C=\begin{bmatrix}6 & 2\\ z+2 & 3\end{bmatrix}$

$A+B=C \Rightarrow$ **x + y + z = ?**

A) 10

B) 9

C) 8

D) 5

E) 4

3. $A=\begin{bmatrix}2 & 1 & -2\\ 3 & -1 & 4\end{bmatrix},\ B=\begin{bmatrix}1 & 0 & 4\\ 2 & -1 & 1\\ 3 & 2 & 3\end{bmatrix}$

$\Rightarrow$ **A · B = ?**

A) $\begin{bmatrix}-2 & -5 & 3\\ 13 & 9 & 23\end{bmatrix}$ B) $\begin{bmatrix}-2 & 13\\ -5 & 9\\ 5 & 23\end{bmatrix}$ C) $\begin{bmatrix}-1 & 5 & 6\\ 9 & 7 & 12\end{bmatrix}$

D) $\begin{bmatrix}2 & 4 & 5\\ 7 & 6 & -3\end{bmatrix}$ E) $\begin{bmatrix}-2 & 5 & -5\\ 9 & -13 & 20\end{bmatrix}$

4. $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \Rightarrow$ **A^{21} = ?**

A) $2^{20}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ B) $-2^{20}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ C) $\begin{bmatrix}1 & 18\\ 0 & 1\end{bmatrix}$

D) $\begin{bmatrix}1 & 21\\ 0 & 1\end{bmatrix}$ E) $\begin{bmatrix}1 & 1\\ 0 & 21\end{bmatrix}$

5. $A=\begin{bmatrix}2 & 1\\ 2 & 3\end{bmatrix} \Rightarrow$ **A^{-1} = ?**

A) $\frac{1}{2}\begin{bmatrix}-3 & 1\\ 2 & -2\end{bmatrix}$ B) $\frac{1}{4}\begin{bmatrix}3 & 2\\ 1 & 2\end{bmatrix}$ C) $\frac{1}{4}\begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}$

D) $\begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}$ E) $\frac{1}{8}\begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}$

6. $\begin{bmatrix}2 & -1 & 3\\ -1 & 3 & 2\\ 1 & 2 & -1\end{bmatrix}-2A=\begin{bmatrix}2 & 3 & 1\\ 3 & -1 & 2\\ 1 & -2 & 1\end{bmatrix} \Rightarrow$ **A = ?**

A) $\begin{bmatrix}0 & -2 & 1\\ -2 & 2 & 0\\ 0 & 2 & -1\end{bmatrix}$ B) $\begin{bmatrix}0 & 2 & -1\\ -1 & 1 & -2\\ 0 & -2 & 1\end{bmatrix}$ C) $\begin{bmatrix}1 & 2 & 0\\ -2 & 2 & 0\\ 0 & 2 & -1\end{bmatrix}$

D) $\begin{bmatrix}0 & -2 & 0\\ -2 & 2 & 2\\ 1 & 0 & -1\end{bmatrix}$ E) $\begin{bmatrix}1 & -2 & -1\\ -1 & 3 & 1\\ 0 & 2 & -1\end{bmatrix}$

7. $A=\begin{bmatrix}1 & 3\\ 5 & 3\end{bmatrix}$

$f(x)=2x^2-4x+3 \Rightarrow$ **$f(A)=?$**

A) $\begin{bmatrix}18 & 13\\ 10 & 29\end{bmatrix}$ B) $\begin{bmatrix}31 & 12\\ 20 & 39\end{bmatrix}$ C) $\begin{bmatrix}14 & 8\\ 12 & 16\end{bmatrix}$

D) $\begin{bmatrix}30 & 17\\ 10 & 13\end{bmatrix}$ E) $2\begin{bmatrix}31 & 12\\ 20 & 39\end{bmatrix}$

8. $A=\begin{bmatrix}2 & -1 & 3\\ 2 & -1 & 1\\ 1 & 3 & -2\end{bmatrix} \Rightarrow$ **$|A|=?$**

A) 12
B) 13
C) 14
D) 16
E) 18

9. $\begin{vmatrix}\mathbf{3915} & \mathbf{3916}\\ \mathbf{3917} & \mathbf{3915}\end{vmatrix}=?$

A) −11739
B) −11741
C) −11743
D) −11745
E) −11747

10. $\begin{vmatrix}x & 1 & 0\\ 2 & -1 & 3\\ 4 & -1 & -2\end{vmatrix}=31 \Rightarrow$ **$x=?$**

A) 7
B) 6
C) 5
D) 4
E) 3

11. $A=\begin{bmatrix}2 & 4\\ x & 7\end{bmatrix},\ A^{-1}=\begin{bmatrix}\frac{7}{2} & -2\\ -\frac{3}{2} & 1\end{bmatrix} \Rightarrow$ **$x=?$**

A) $\frac{7}{2}$ B) 3 C) $\frac{5}{2}$

D) 2 E) $\frac{3}{2}$

12. $A=\begin{bmatrix}2 & 2\\ 3 & 4\end{bmatrix},\ B=\begin{bmatrix}4 & -2\\ -3 & 2\end{bmatrix} \Rightarrow$ **$(A\cdot B)^{12}=?$**

A) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ B) $\frac{1}{64}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ C) $2^8\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

D) $2^{12}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ E) $2^{16}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

YANITLAR / ANSWERS

1. B	**2. E**	**3. A**	**4. D**	**5. C**	**6. A**	**7. B**	**8. C**
9. E	**10. E**	**11. B**	**12. D**				

ÖZEL TANIMLI FONKSİYONLAR
THE SPECIAL DOMAIN FUNCTIONS

BÖLÜM
CHAPTER
4

FONKSİYONLARIN EN GENİŞ TANIM KÜMESİ
THE LARGEST DOMAIN OF FUNCTIONS

Tanım: y = f(x) biçiminde verilen f fonksiyonu için görüntüleri bir gerçel sayı olan tüm x değerlerinin kümesine, f fonksiyonunun **en geniş tanım kümesi** denir.

Definition: *The set that consists of all the values of x whose images are real numbers for the function f, as y = f(x), is called as the largest domain of the function f.*

1. $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0 \Rightarrow$ ÇK(SS) = R , (E = R)
2. $f(x) = \dfrac{g(x)}{h(x)}$, $h(x) \neq 0 \Rightarrow$ ÇK(SS) = R , (E = R)
3. $n \in N^+$, $\sqrt[2n]{f(x)} \Rightarrow E = f(x) \geq 0$
4. $f(x) = \log_a g(x) \Rightarrow E = \{g(x) > 0,\ a > 0,\ a \neq 1\}$

ÖRNEK / EXAMPLE 1

$f(x) = \dfrac{x^4 - 2x^2 + 3}{x^2 - 3x + 2}$ **fonksiyonunun en geniş tanım kümesini bulunuz.**

Find the largest domain of the function $f(x) = \dfrac{x^4 - 2x^2 + 3}{x^2 - 3x + 2}$?

A) R
B) ∅
C) {–1, –2}
D) R – {–1, –2}
E) R – {1, 2}

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x^2 - 3x + 2 \neq 0$

$\swarrow \searrow$

$-2 \quad -1$

$(x - 2) \cdot (x - 1) \neq 0$

$x \neq 2,\quad x \neq 1 \Rightarrow R - \{1, 2\}$

ÖRNEK / EXAMPLE 2

$f(x) = \sqrt{3 - |2 - x|}$ **fonksiyonunun en geniş tanım kümesini bulunuz.**

Find the largest domain of the function $f(x) = \sqrt{3 - |2 - x|}$.

A) [–2, –1]
B) [–1, 3]
C) [–1, 5]
D) [0, 3]
E) [0, 5]

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$0 \leq 3 - |2 - x| \Rightarrow |2 - x| \leq 3$

$-3 \leq 2 - x \leq 3$

$-5 \leq -x \leq 1$

$-1 \leq x \leq 5 \Rightarrow E = [-1, 5]$

NOT / NOTE

$f : [-\pi, \pi] \to [-1, 1]$, $f(x) = \sin x$ tek fonksiyondur.

$f : [-\pi, \pi] \to [-1, 1]$, $f(x) = \sin x$ is an odd function.

$f : [-\pi, \pi] \to [-1, 1]$, $f(x) = \cos x$ çift fonksiyondur.

$f : [-\pi, \pi] \to [-1, 1]$, $f(x) = \cos x$ is an even function.

TEK VE ÇİFT FONKSİYONLAR
ODD and EVEN FUNCTIONS

Tanım / *Definition*: $f: A \to B$, $y = f(x)$

1. $\forall x \in A$ için $f(-x) = -f(x)$ ise f fonksiyonu tek fonksiyondur.
   For all the values of $x \in A$, if $f(-x) = -f(x)$, then f is an odd function.
2. $\forall x \in A$ için $f(-x) = f(x)$ ise f fonksiyonu çift fonksiyondur.
   For all the values of $x \in A$, if $f(-x) = f(x)$, then f is an even function.

   Örneğin / For example;

   $R \to R$, $f(x) = -x^4 + 5x^2 - 1$

   $f(-x) = -(-x)^4 + 5(-x)^2 - 1$

   $= -x^4 + 5x^2 - 1 \Rightarrow \mathbf{f(-x) = f(x)}$ olduğundan f fonksiyonu çift fonksiyondur.

   since $f(-x) = f(x)$, f is an even function.

ÖRNEK / EXAMPLE 3

f tek fonksiyon, g çift fonksiyondur. / *f is an odd function, g is an even function.*

$$h(x) = \frac{f(x)+g(x)}{f(x)}, \quad f(-2) = 4, \quad g(-2) = 1 \Rightarrow \mathbf{h(2) = ?}$$

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{5}{4}$ D) 1 E) $\frac{9}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$h(2) = \frac{f(2)+g(2)}{f(2)} = \frac{-4+1}{-4} = \frac{3}{4}$$

PARÇALI FONKSİYON / PIECEWISE FUNCTION

Tanım / Definition: $A, B, C, D \subset R$,

$f : A \to B$, $g: C \to D$,

$h : (A \cup B) \to (B \cup D)$

$$h(x) = \begin{cases} f(x) & x \in A \\ g(x) & x \in C \end{cases}$$

biçiminde tanımlanan fonksiyona **parçalı fonksiyon** denir. / *is called **piecewise function**, when a function is defined as above.*

ÖRNEK / EXAMPLE 4

$$f: R \to R \quad f(x) = \begin{cases} \log_3 x & x > 0 \\ \dfrac{3x+1}{x-1} & x \le 0 \end{cases} \Rightarrow \mathbf{f(9) + f(-3) = ?}$$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$f(9) = \log_3 9 = 2$

$f(-3) = \dfrac{-9+1}{-3-1} = \dfrac{-8}{-4} = 2$

$f(9) + f(-3) = 4$

ÖRNEK / EXAMPLE 5

$$f: R \to R \quad f(x) = \begin{cases} x^2 & x < -1 \text{ ise} \\ 1 & -1 \le x \le 1 \text{ ise} \\ x & 1 < x \text{ ise} \end{cases}$$ **fonksiyonunun grafiğini çiziniz.**

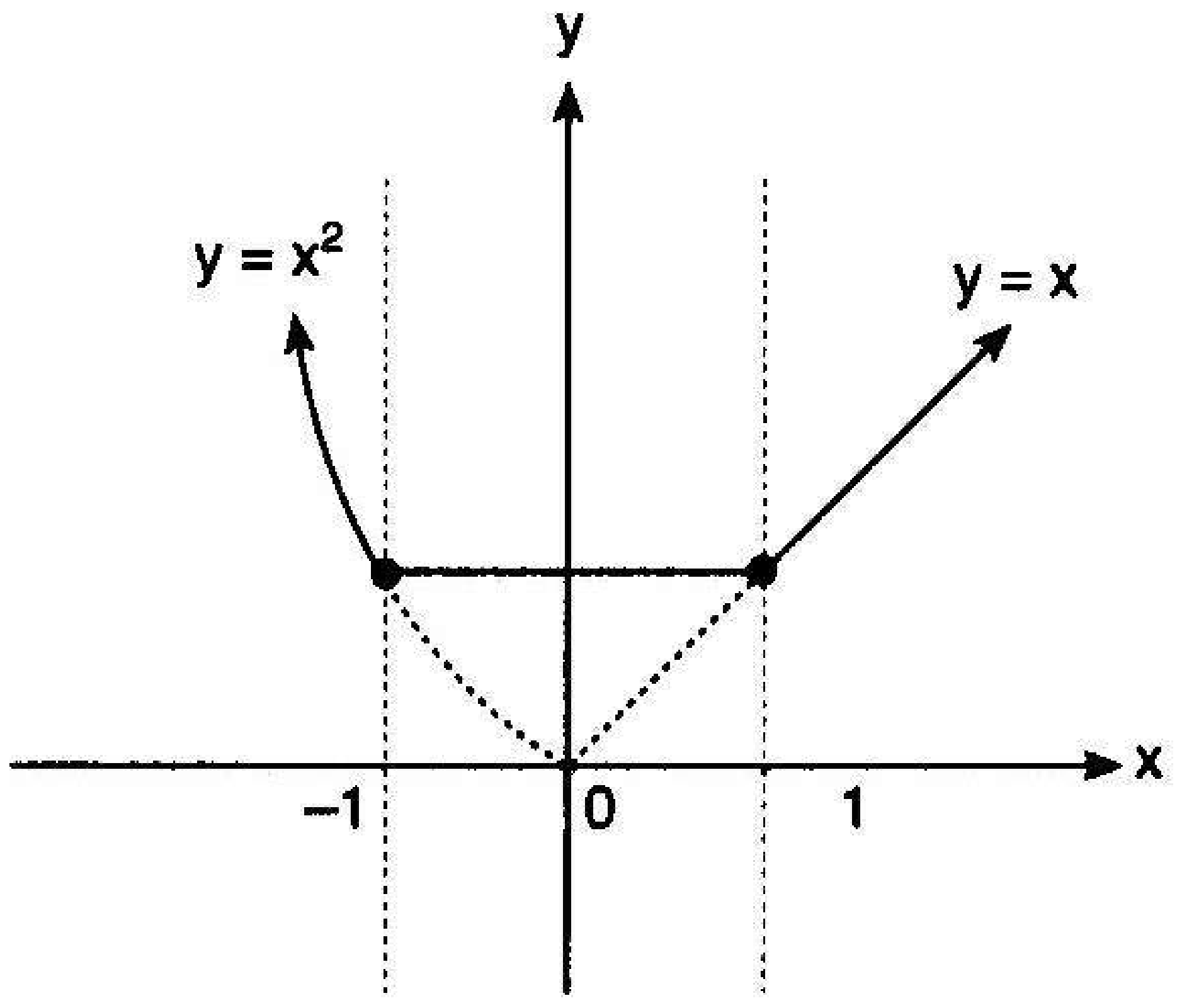
If $f: R \to R \quad f(x) = \begin{cases} x^2 & x < -1 \\ 1 & -1 \le x \le 1 \\ x & 1 < x \end{cases}$ *, then plot the graph of the function.*

ÇÖZÜM / SOLUTION

$x < -1 \Rightarrow y = x^2$

$-1 \le x \le 1 \Rightarrow y = 1$

$1 < x \Rightarrow y = x$



MUTLAK DEĞER FONKSİYONU / ABSOLUTE VALUE FUNCTION

Tanım: $x \in R$ olmak üzere, f(x) in mutlak değeri $|f(x)|$ biçiminde gösterilir.

Definition: *If $x \in R$, the absolute value of f(x), denoted by $|f(x)|$ is as follows:*

$$|f(x)| = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$$

MUTLAK DEĞER FONKSİYONUN ÖZELLİKLERİ
PROPERTIES OF THE ABSOLUTE VALUE FUNCTION

1. $\forall x \in R$ için(*for*) $y = |f(x)| \geq 0$
2. $|f(x)| = a$, $(a \in R^+)$ $\Rightarrow$ $f(x) = \mp a$
3. $|f(x)| \leq a$, $(a \in R^+)$ $-a \leq f(x) \leq a$
4. $|f(x)| \geq a$, $(a \in R^+)$ $\Rightarrow$ $f(x) \geq a$, $f(x) \leq -a$
5. $|f(x) \cdot g(x)| = |f(x)| \cdot |g(x)|$
6. $\left|\dfrac{f(x)}{g(x)}\right| = \dfrac{|f(x)|}{|g(x)|}$ $(g(x) \neq 0)$
7. $a < b$ ve(*and*) $a, b \in R^+$

   $$a < |f(x)| < b \Rightarrow \begin{cases} -b < f(x) < -a \\ a < f(x) < b \end{cases}$$

PEKİŞTİRME / REINFORCE

$f(x) = |x + 3| = -2x + 9 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : {2}

ÖRNEK / EXAMPLE 6

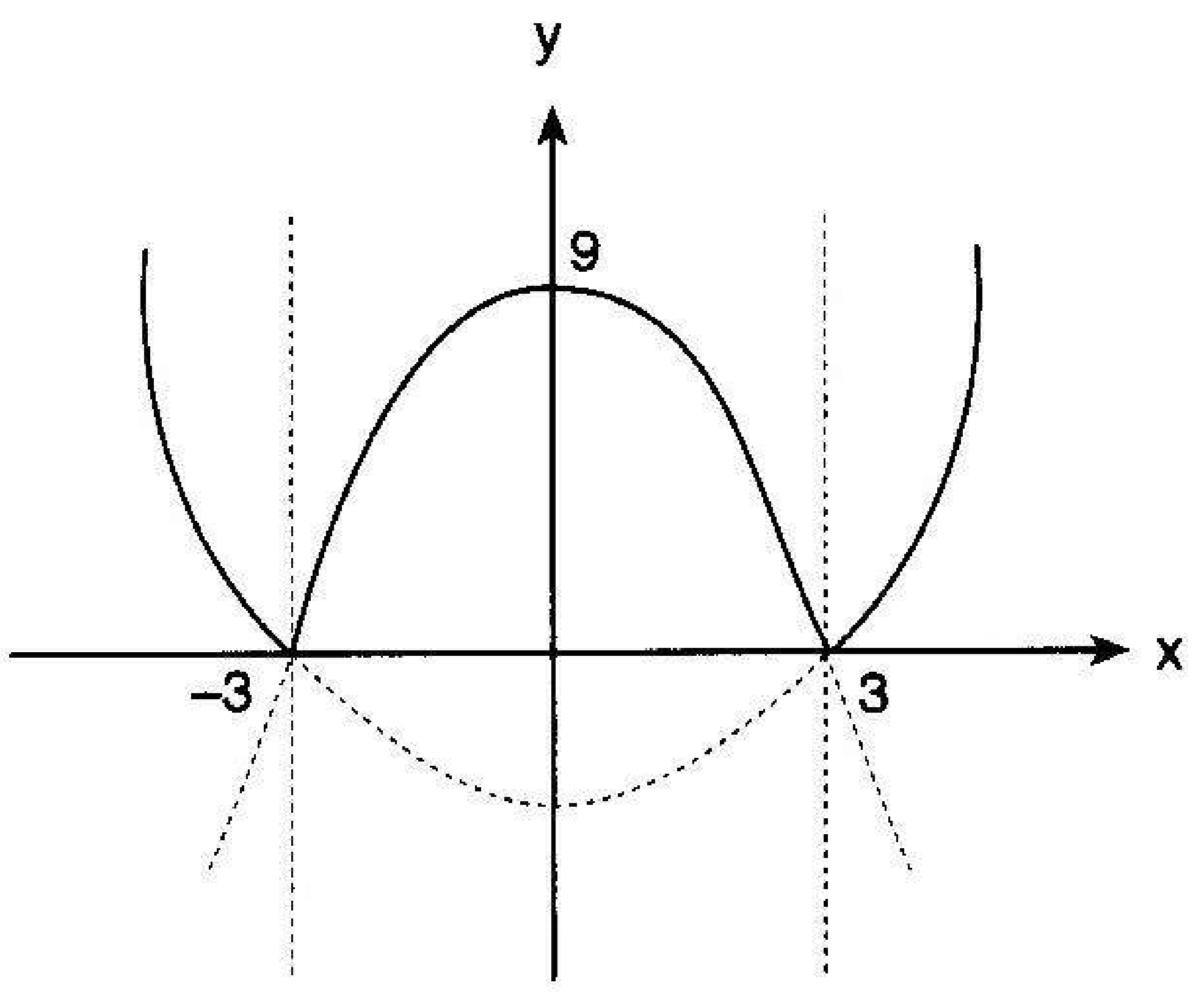
$f(x) = |x^2 - 9|$ fonksiyonunun grafiğini çiziniz.

Plot the graph of the function of $|x^2 - 9|$.

ÇÖZÜM / SOLUTION

x	−∞		−3		3		+∞
		+	●	−	●	+	
f(x)		$x^2 - 9$		$9 - x^2$		$x^2 - 9$	

$x^2 - 9 = 0 \Rightarrow x = \mp 3$



TAM DEĞER FONKSİYONU / GREATEST INTEGER FUNCTION

Tanım: $\forall x \in R$ için x tamsayı ise kendisine, tamsayı değilse kendisinden küçük en büyük tamsayıya x **in tam değeri** denir ve $[|x|]$ biçiminde gösterilir.

Definition: *The greatest integer value of x is defined as, for all values of $x \in R$, itself when x is an integer, the greatest integer which is smaller than x when x is not an integer. It is shown as $[|x|]$.*

$$[|x|] = a \ (a \in Z) \Rightarrow a \le x < a + 1$$

ÖRNEK / EXAMPLE 7

$f : [0, 4] \to R$

$f(x) = [|x|]$ **fonksiyonunun grafiğini çiziniz.**

Plot the graph of the $f(x) = [|x|]$.

ÇÖZÜM / SOLUTION

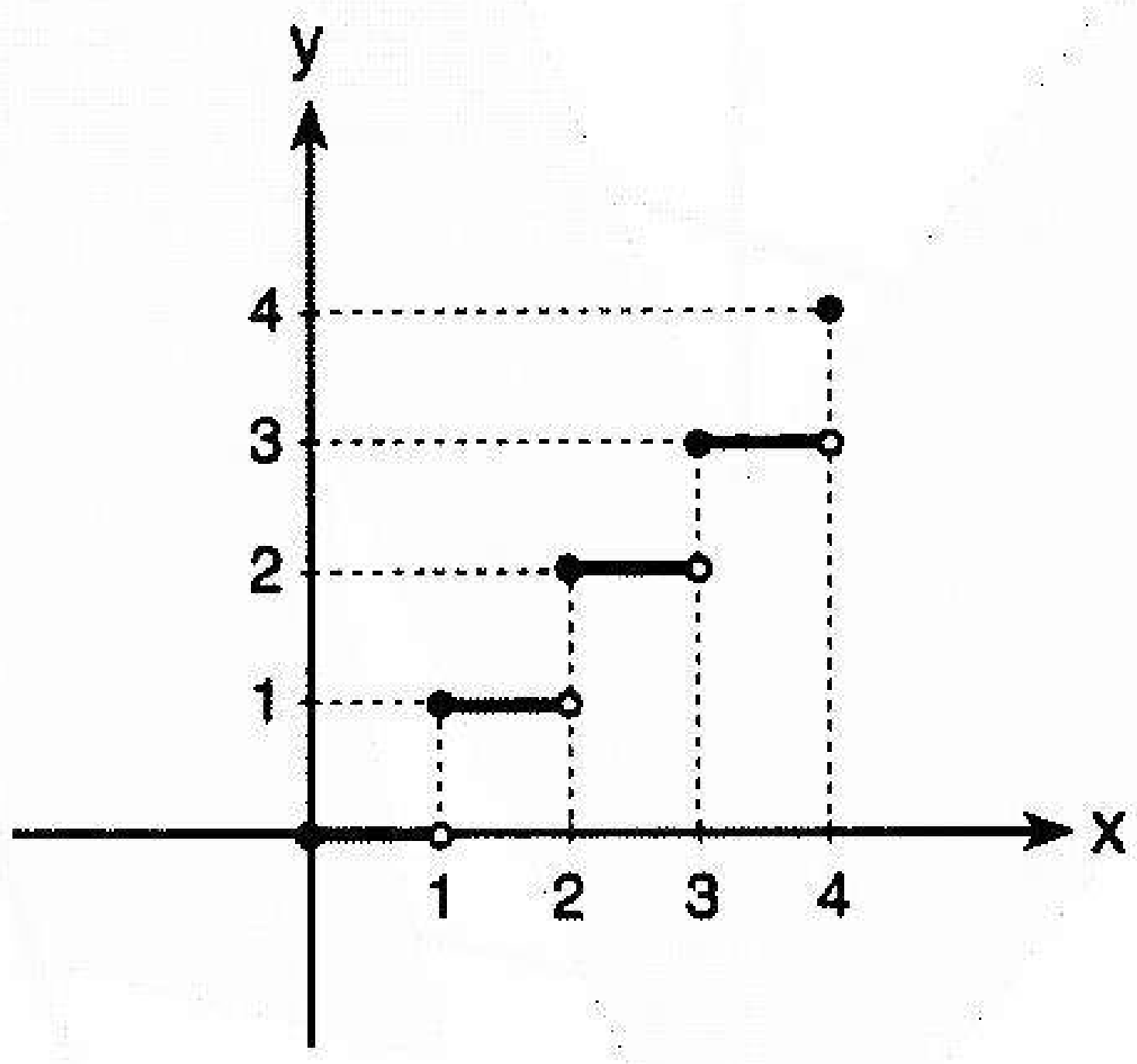
$x \in [0, 1) \Rightarrow f(x) = 0$

$x \in [1, 2) \Rightarrow f(x) = 1$

$x \in [2, 3) \Rightarrow f(x) = 2$

$x \in [3, 4) \Rightarrow f(x) = 3$

$x = 4 \Rightarrow f(x) = 4$



İŞARET FONKSİYONU / SIGNUM FUNCTION

Tanım : $A \subset R$ olmak üzere, $f: A \to R$, $f: x \to y = f(x)$ fonksiyonu için,

Definition: *If $A \subset R$ and $f: A \to R$, $f: x \to y = f(x)$ the function, $g: A \to R$ defined as*

$$g(x) = \text{sgn}(f(x)) = \begin{cases} -1, & f(x) < 0 \\ 0, & f(x) = 0 \\ 1, & f(x) > 0 \end{cases}$$

şeklinde tanımlanır. Burada, $g: A \to R$, fonksiyonuna f nin **işaret fonksiyonu** denir.

*and is called **signum function**.*

ÖRNEK / EXAMPLE 8

$f(x) = |x + 2| - (x - 1)\text{sgn}(4 - x)$, $g(x) = 5 + \text{sgn}(x^2 + 1) \Rightarrow$ **fog(x) = ?**

A) 3 B) 5 C) 6 D) 10 E) 13

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$fog(x) = f(g(x))$, $g(x) = 5 + \underbrace{\text{sgn}(x^2 + 1)}_{1} = 6$

$f(6) = |6 + 2| - (6 - 1)\underbrace{\text{sgn}(4 - 6)}_{-1} \Rightarrow f(6) = 8 + 5 = 13$

ÖZELLİKLER / PROPERTIES

1. $\forall x \in R$ için(*for*) $y = [|x|] \le x < [|x|] + 1$
2. $x \in R, a \in Z \Rightarrow [|x|] = a \Rightarrow a \le x < a + 1$
3. $x \in R, a \in Z \Rightarrow [|x + a|] = [|x|] + a$
4. $x, y \in R \Rightarrow [|x + y|] \ge [|x|] + [|y|]$
5. $x, y \in R \Rightarrow [|x + [|y|]|] \ge [|x|] + [|y|]$

PEKİŞTİRME / REINFORCE

$[|x + [|x|]|] = 6 \Rightarrow$ ÇK(SS) = ?

YANIT / ANSWER : [3, 4)

PEKİŞTİRME / REINFORCE

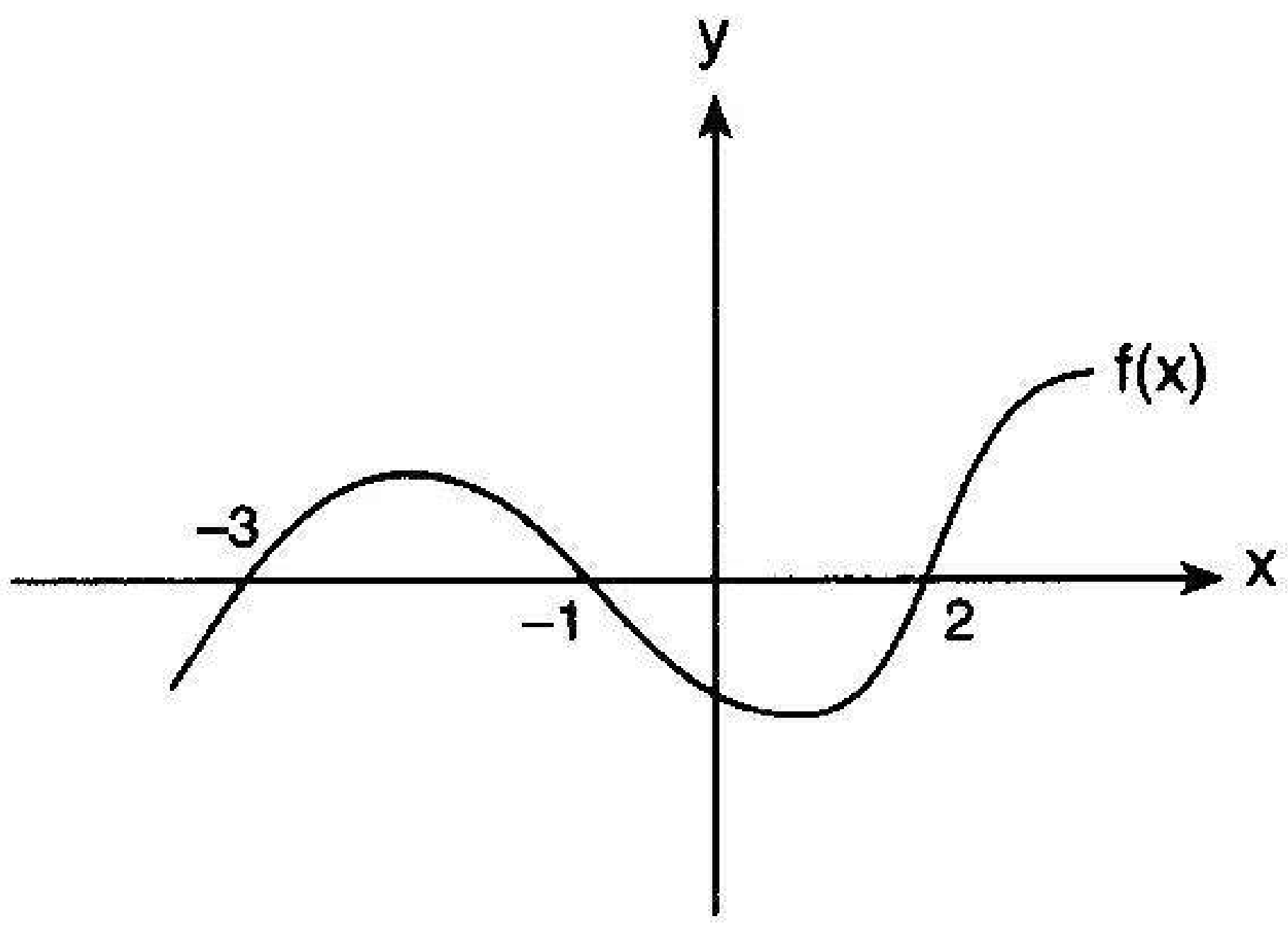
$\text{sgn}[\log_3(x - 2)] = -1 \Rightarrow$ ÇK(SS) = ?

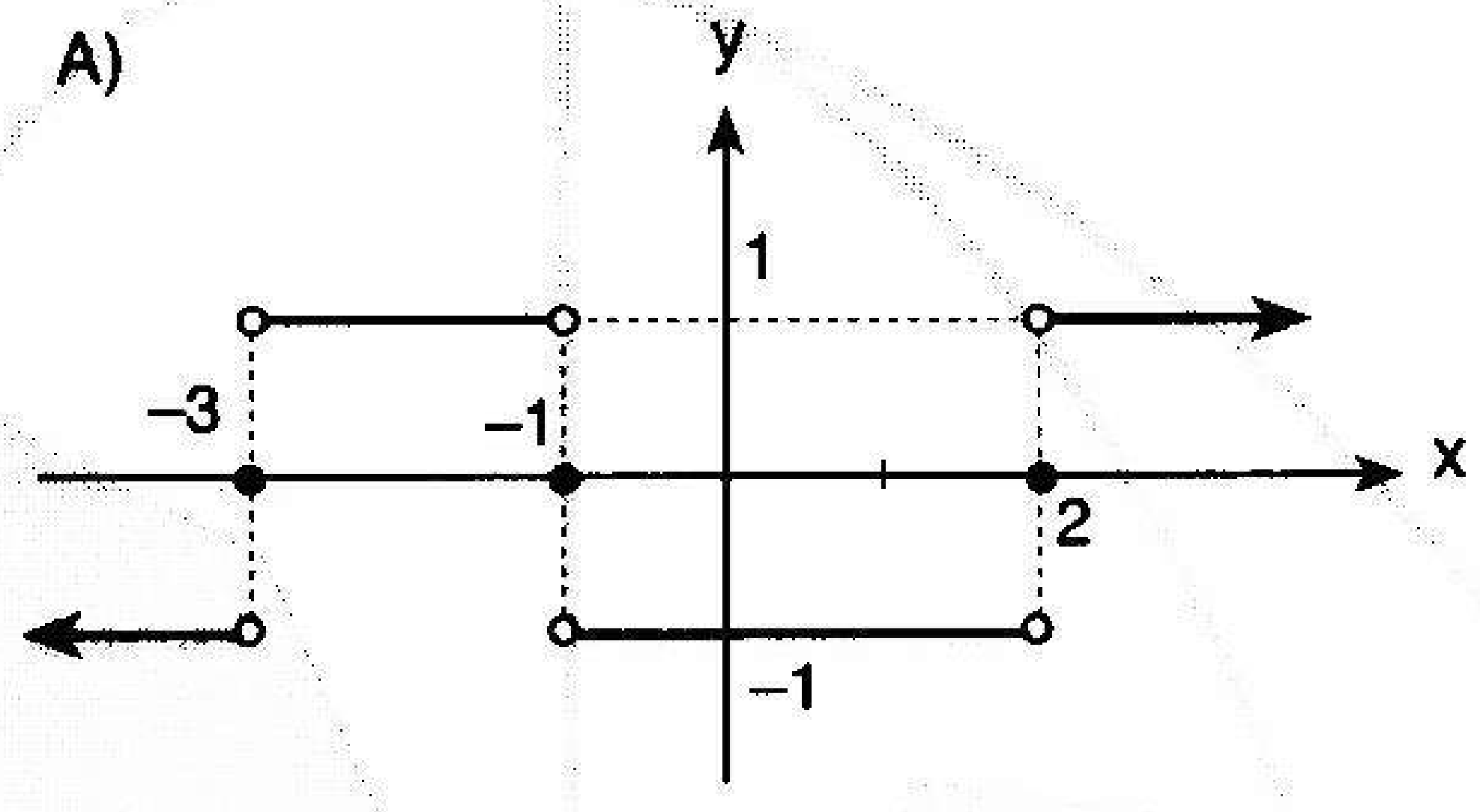
YANIT / ANSWER : (2, 3)

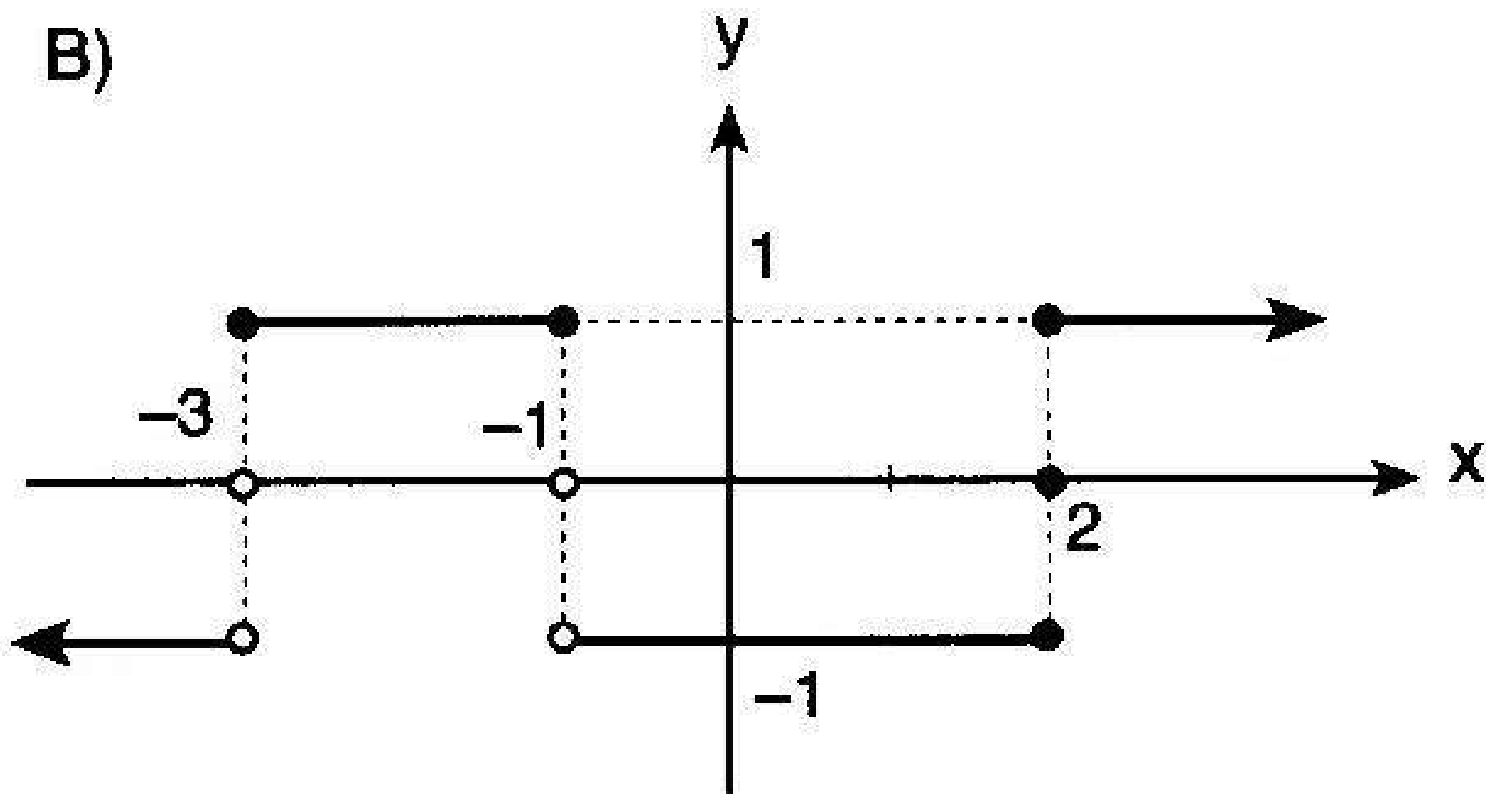
ÖRNEK / EXAMPLE 9

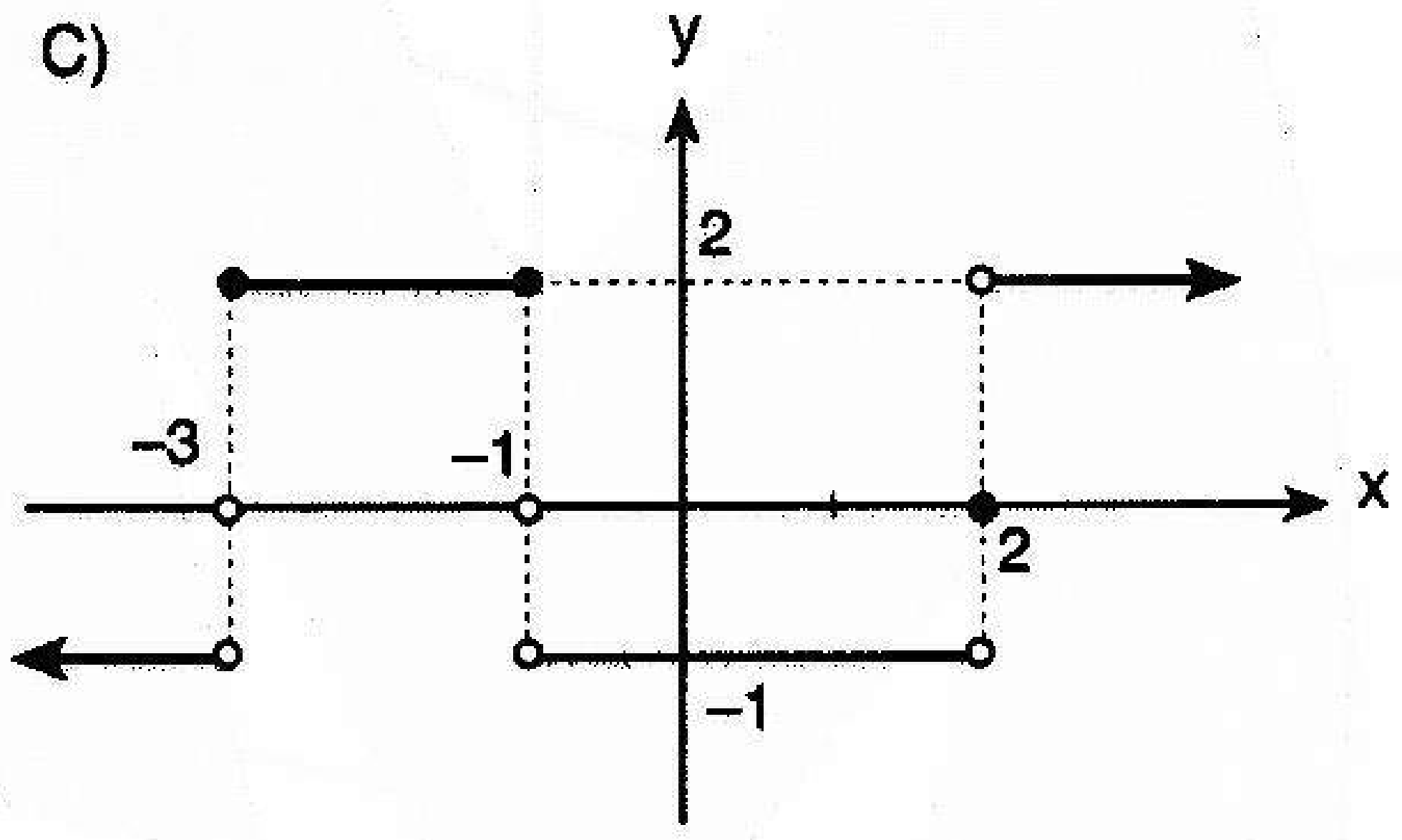
Şekilde f(x) fonksiyonunu grafiği verilmiştir. Buna göre sgn(f(x)) fonksiyonunun grafiği aşağıdakilerden hangisidir?

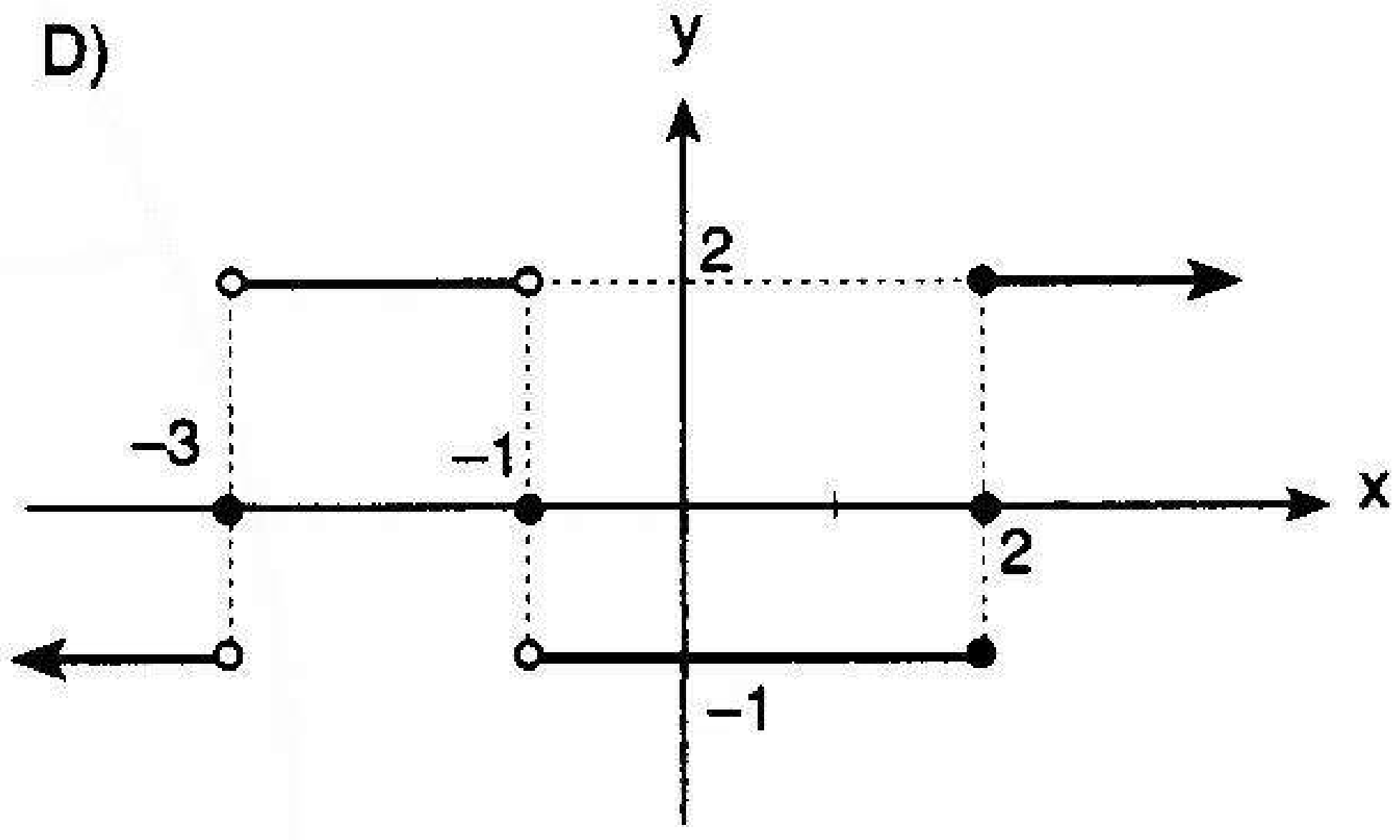
The graph of the function f(x) is given in the figure. According to that, which of the following is the graph of the function sgn(f(x)) ?

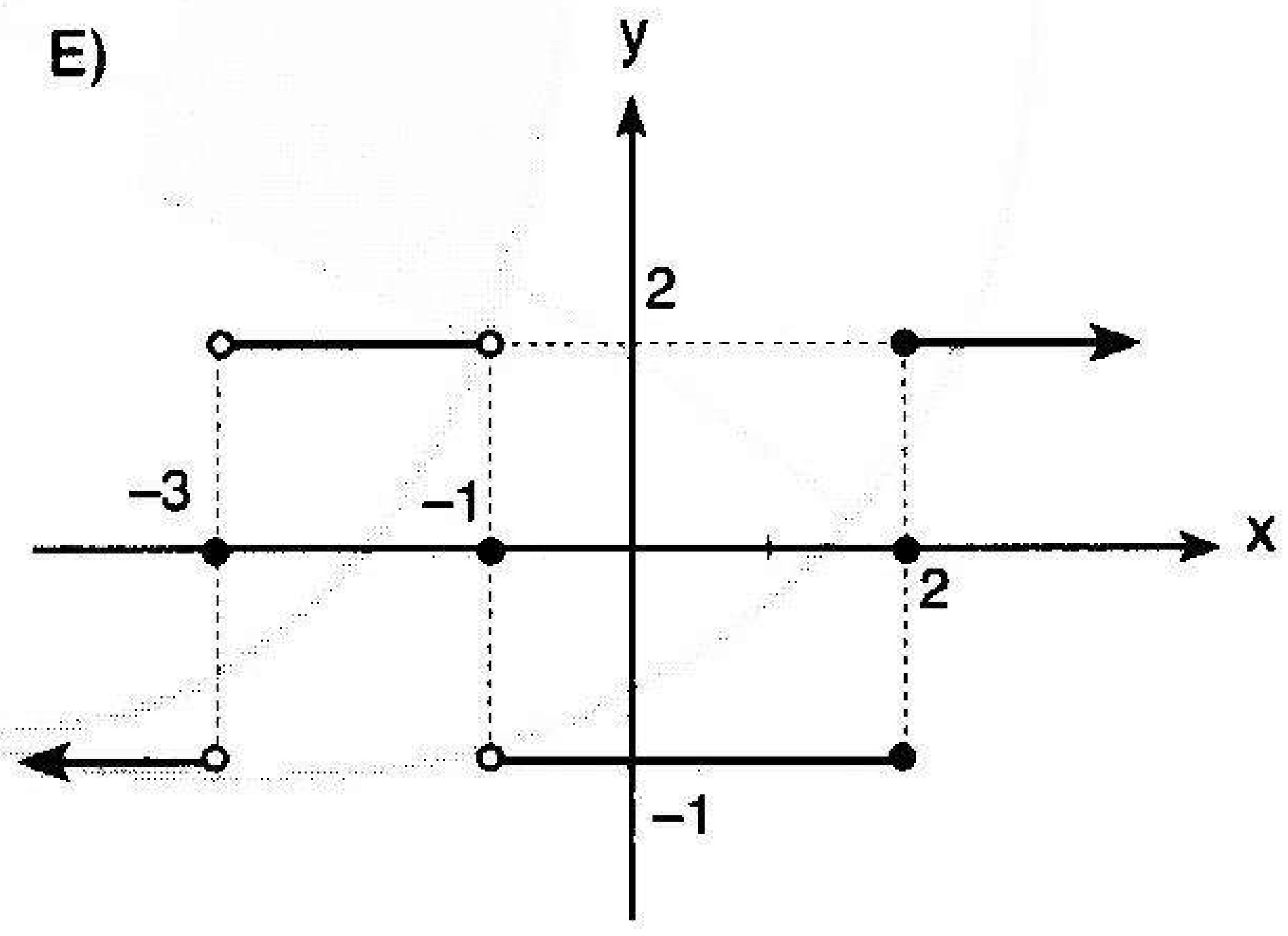


A)


B)


C)


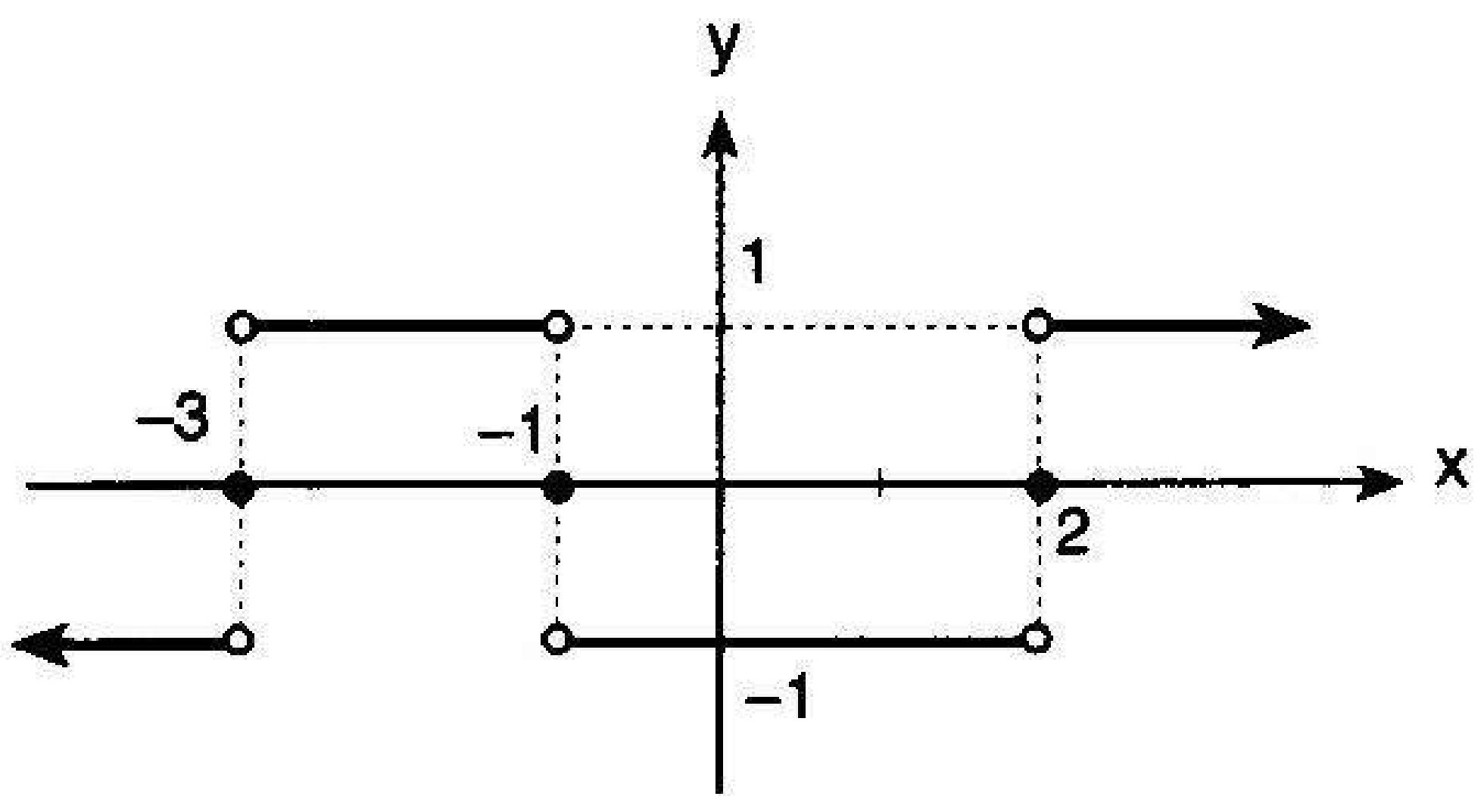
D)


E)


ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

x	−∞		−3		−1		2		+∞
f(x)		−	●	+	●	−	●	+	
sgn(f(x))		−1	○	1	○	−1	○	1	



1. $x \cdot \mathrm{sgn}x = x^2 - 6 \Rightarrow$ **ÇK(SS) = ?**

A) $\{0, 3\}$ B) $\{-3, 3\}$ C) $[-3, 3]$

D) $\{-3, 0\}$ E) $\varnothing$

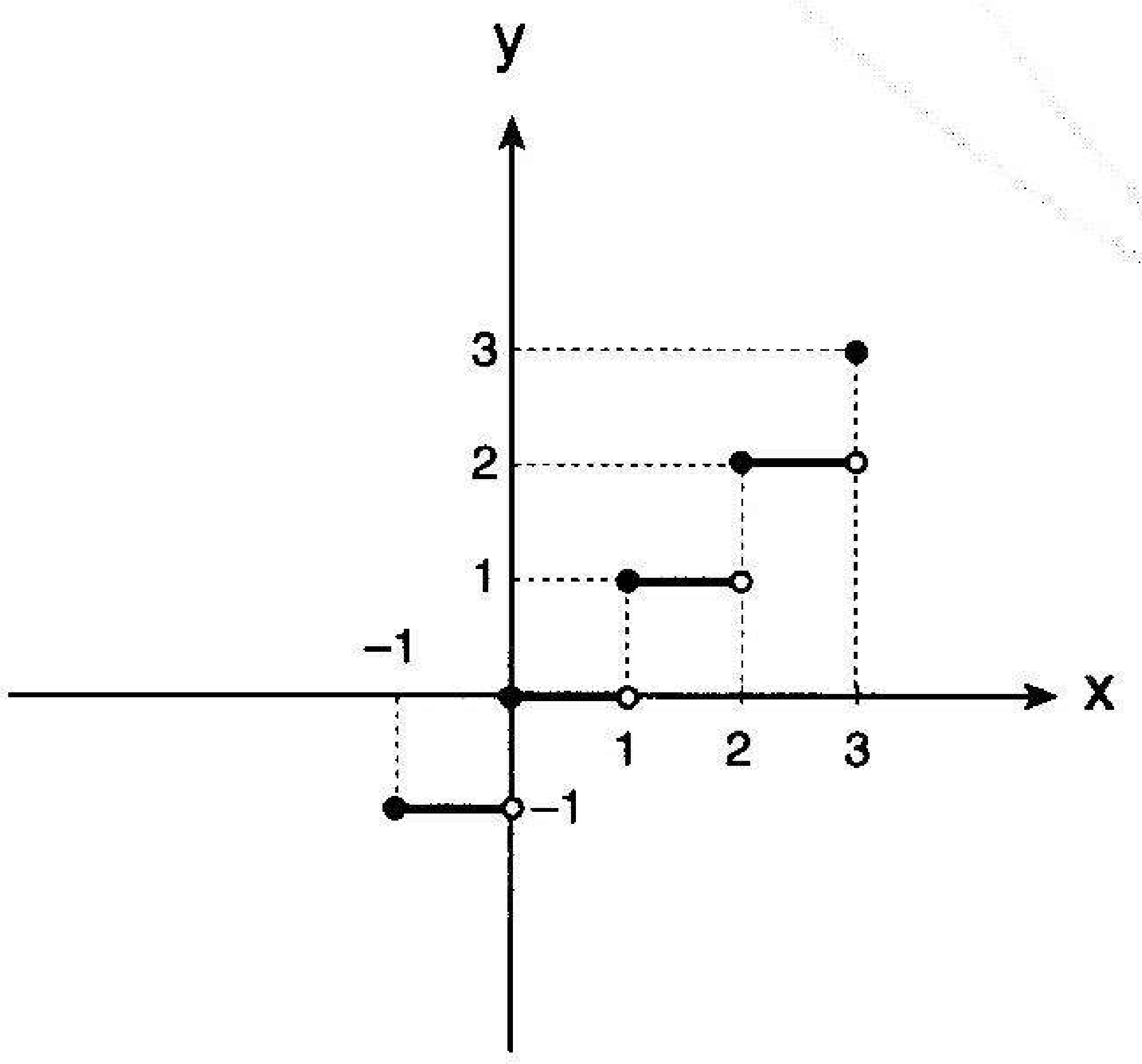
2. $x \in Z$,

$\mathrm{sgn}(x^2 + 4) = [|x + 2|] \Rightarrow$ **x = ?**

A) -2 B) -1 C) 0

D) 1 E) 2

3.



f(x) = ?

A) $f : [-1, 3] \to R,\ f(x) = [|x|]$

B) $f : (-1, 3) \to R,\ f(x) = [|x|]$

C) $f : [-1, 3] \to R,\ f(x) = |x|$

D) $f : (-1, 3) \to R,\ f(x) = \mathrm{sgn}(x)$

E) $f : [-1, 3] \to R,\ f(x) = [|x + 1|]$

4. $[|3x - 6|] \le 2 \Rightarrow$ **ÇK(SS) = ?**

A) $(-2, 2)$ B) $(-\infty, 2]$ C) $(-\infty, 3)$

D) $(-3, 3)$ E) $(-\infty, 3]$

5. $[|2x - 8|] = 4 \Rightarrow$ **ÇK(SS) = ?**

A) $\{0, 1, 2, 3, 4\}$

B) $(-4, 4]$

C) $[6, 13/2)$

D) $(6, 7)$

E) $(-\infty, 4]$

6. $f(x) = \dfrac{6}{|2x-1|}$ ise, $f(x) > \dfrac{2}{3}$ **eşitsizliğini sağlayan tamsayıların toplamı kaçtır?**

If $f(x) = \dfrac{6}{|2x-1|}$, which of the following is the sum of the *integers that satisfies the inequality* $f(x) > \dfrac{2}{3}$*?*

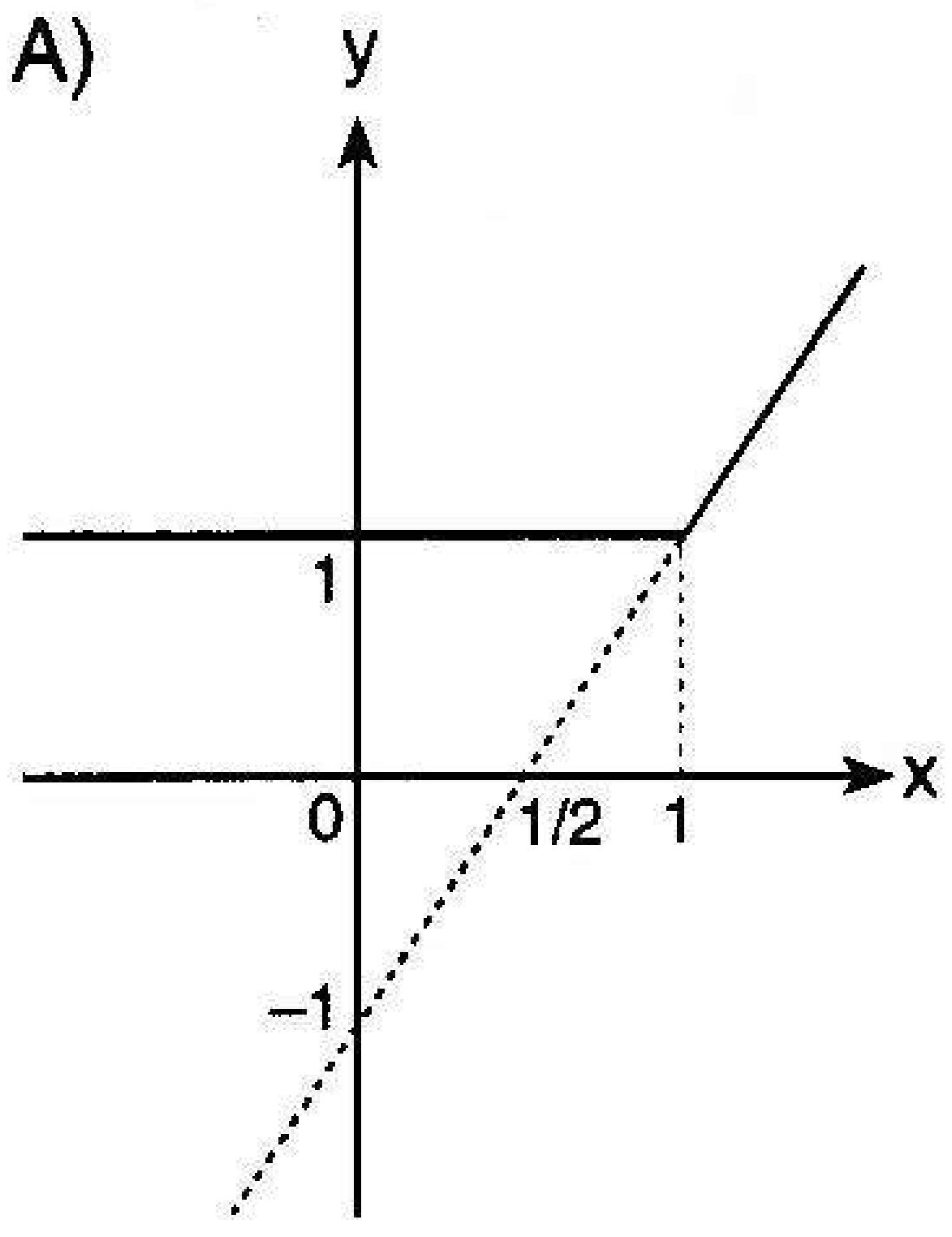
A) -1 B) 0 C) 2

D) 4 E) 6

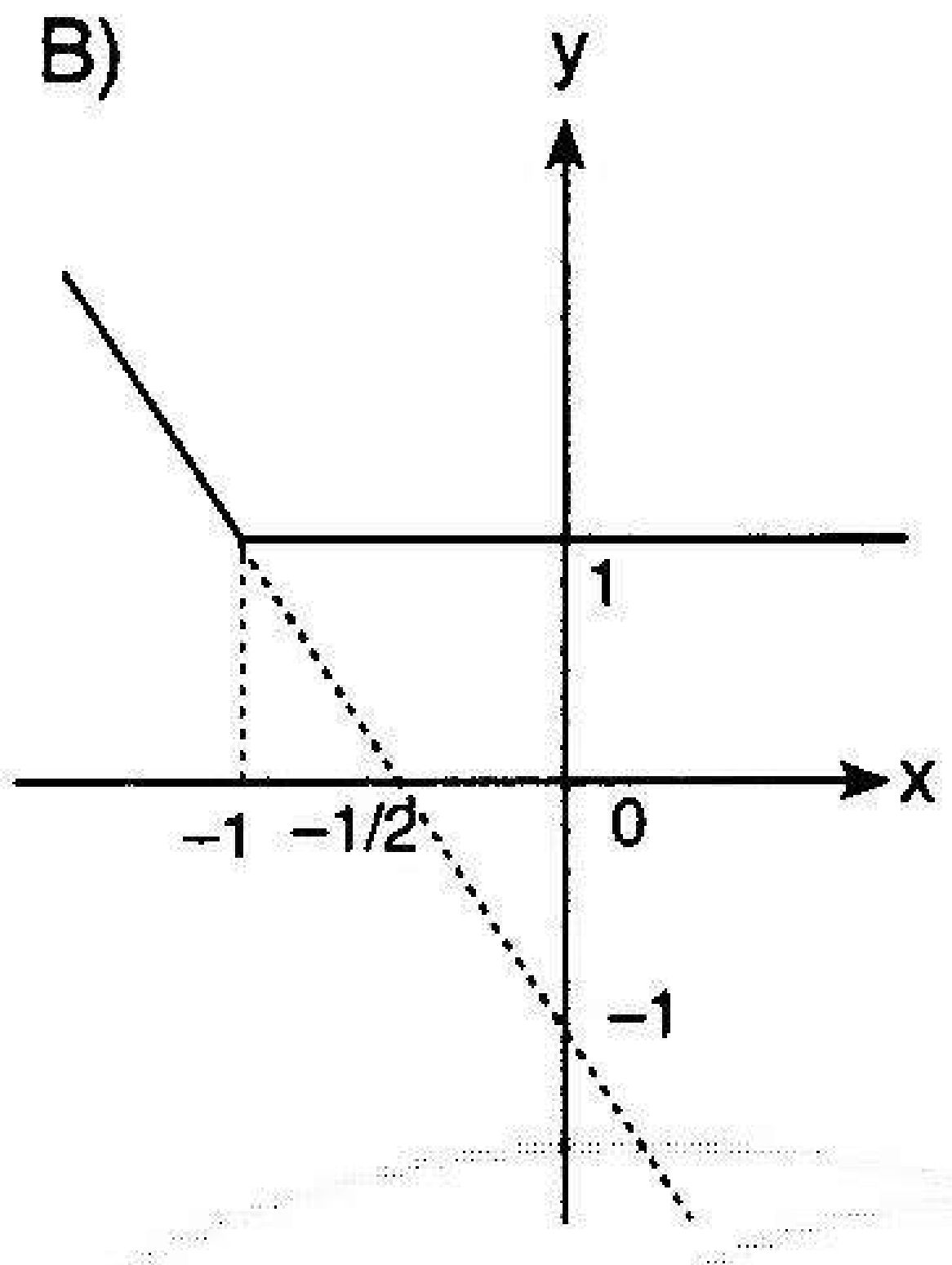
7. **$f(x) = x + |x - 1|$ fonksiyonunun grafiği aşağıdakilerden hangisidir?**

Which of the following is the graph of the function *$f(x) = x + |x - 1|$?*

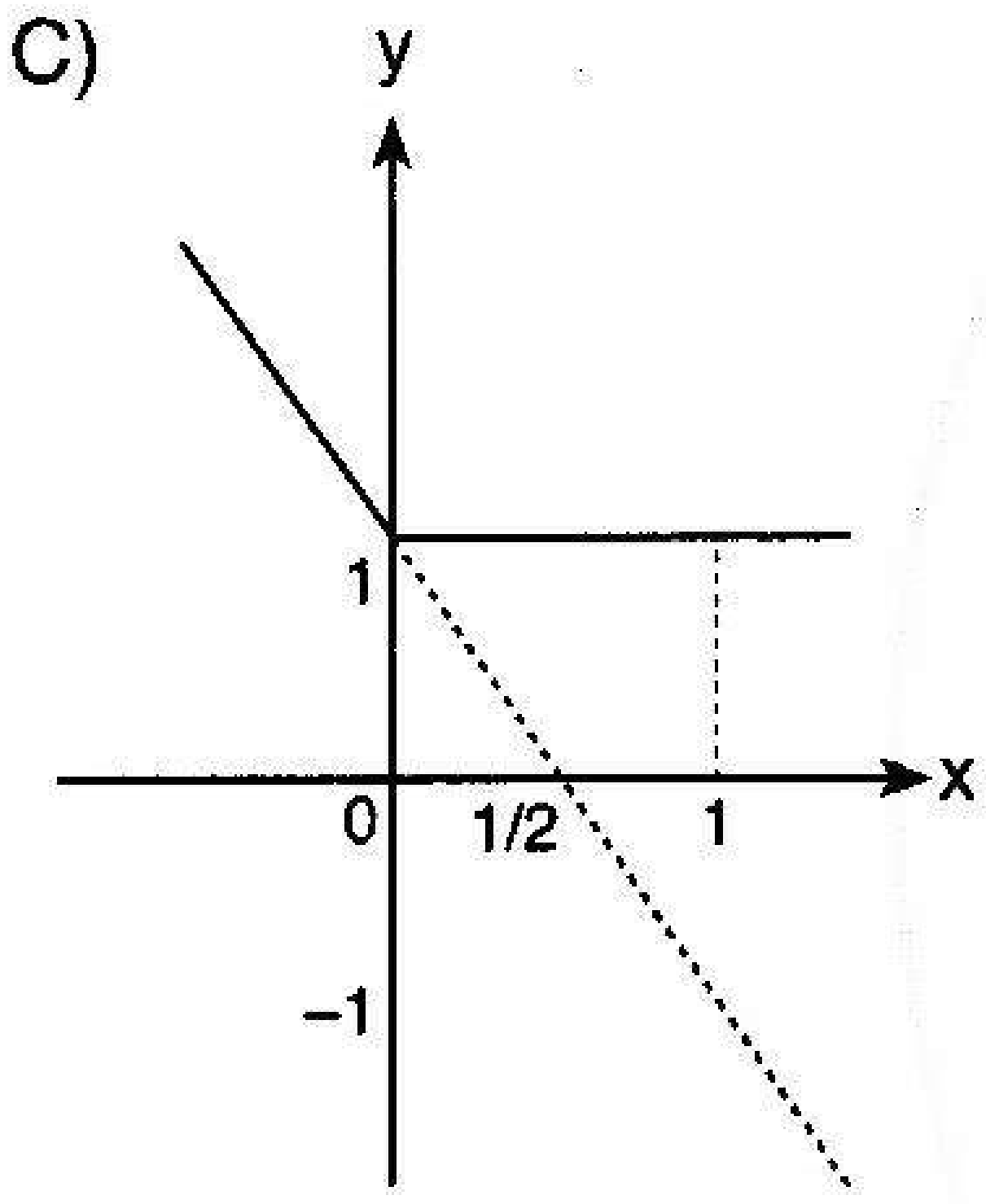
A)



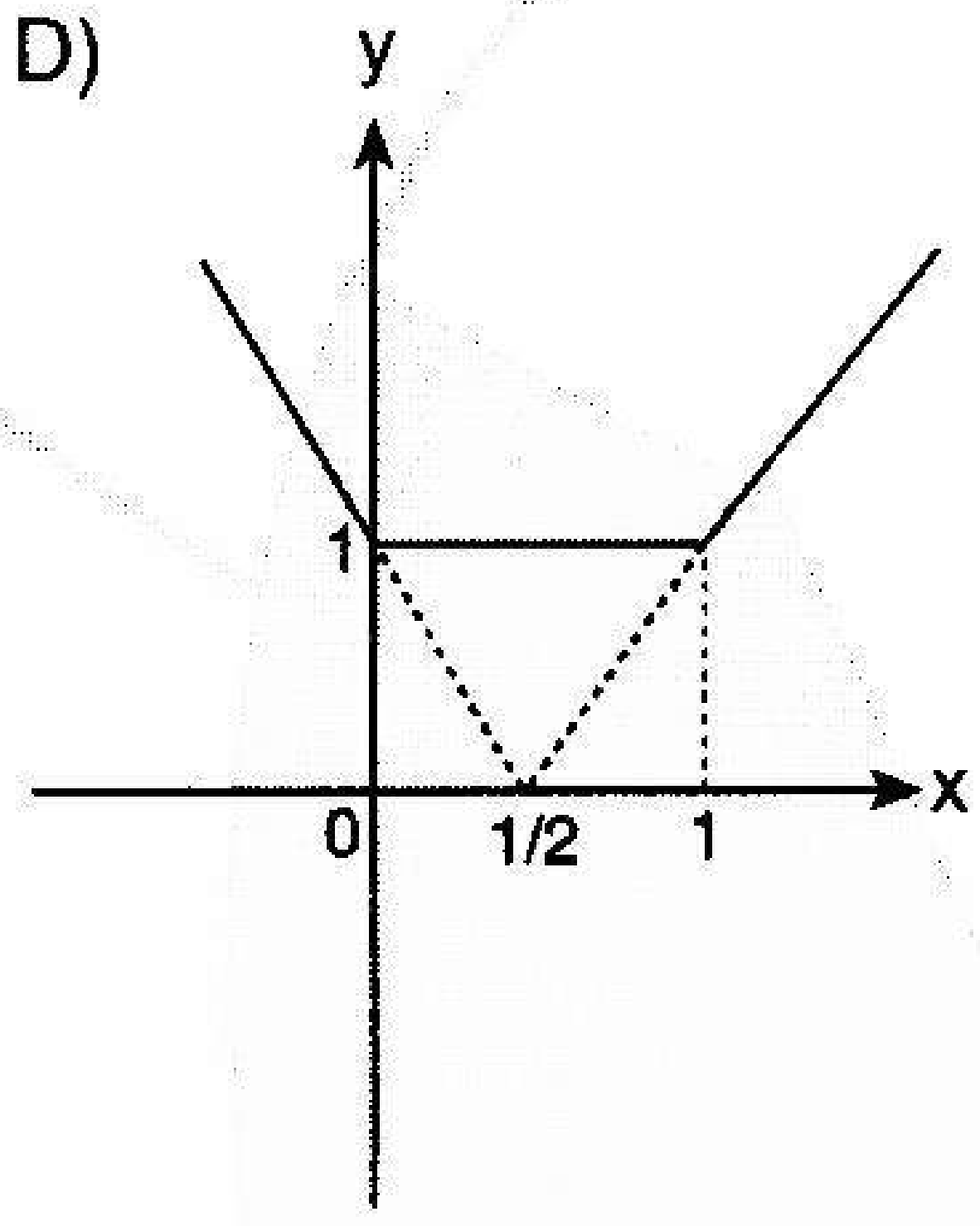
B)



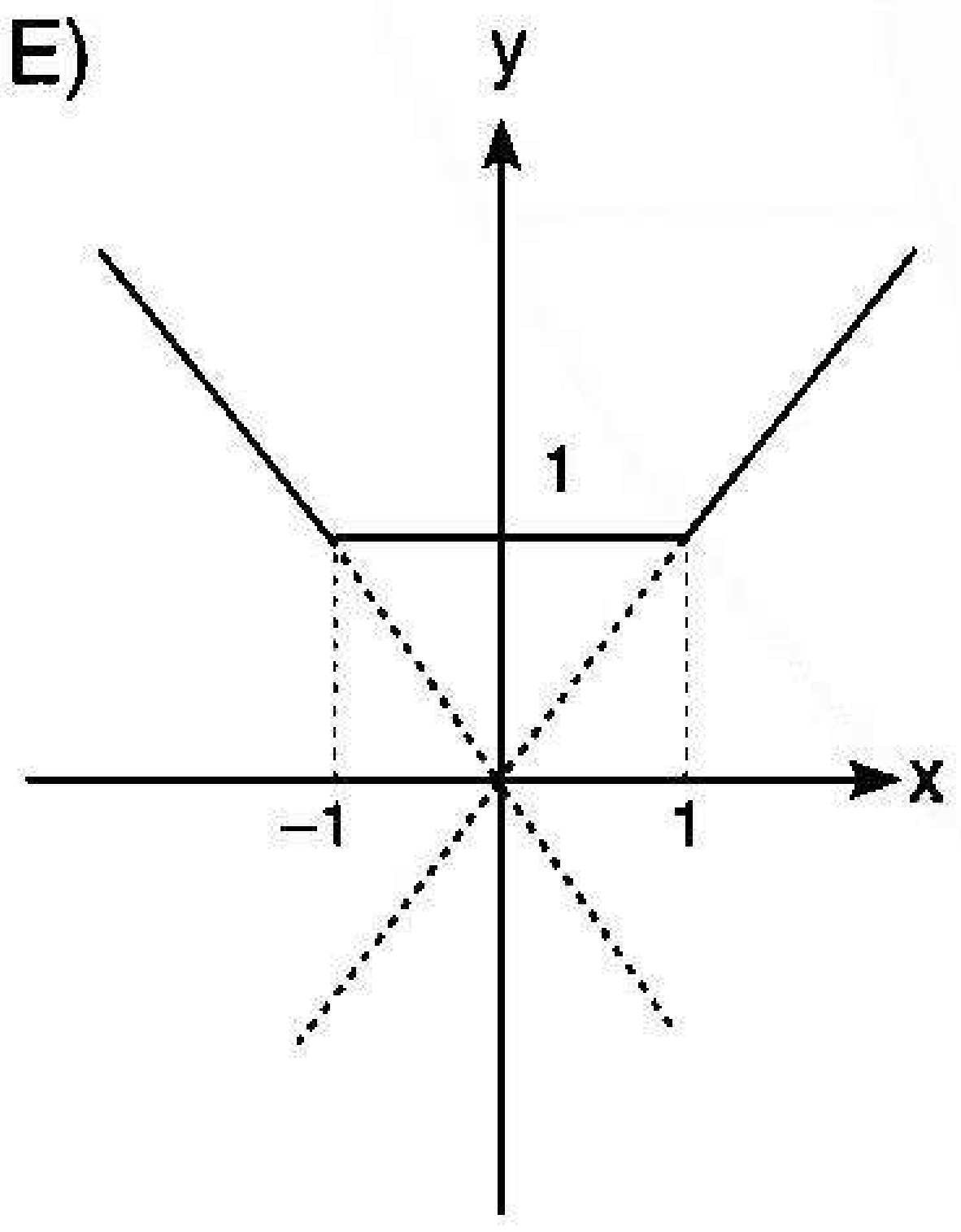
C)



D)



E)



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8. $|5x - 3| < 17 \Rightarrow \sum x = ?$

A) 3
B) 2
C) 0
D) −2
E) −3

---

9. $f(x) = [\,|4x - 7|\,],\ f(x) = 6 \Rightarrow$ **ÇK(SS) = ?**

A) $\left[3, \frac{13}{4}\right)$
B) $[5, 6)$
C) $(5, 6)$
D) $\left(-\infty, \frac{3}{4}\right)$
E) $[3, 4)$

---

10. $f(x) = \big|\,[\,|x + 1|\,]\,\big|$,

$f(x) = 2 \Rightarrow$ **ÇK(SS) = ?**

A) $[2, 3) \cup [4, 6)$
B) $[-3, -2) \cup [1, 2)$
C) $[-3, -2) \cup [-1, 0)$
D) $[-2, -1) \cup [0, 1)$
E) $[2, 3) \cup [4, 5)$

---

11. $x^2 = \text{sgn}(x) \Rightarrow \sum x = ?$

A) $\{-1, 0, 1\}$
B) $\{-1, 0\}$
C) $\{0, 1\}$
D) $\{0\}$
E) $\varnothing$

---

12. $f(x): A \to \{0, 1\}$

$f(x) = \text{sgn}(x - 2) \Rightarrow$ **A = ?**

A) $\{0\} \cup (2, \infty)$
B) $\{2\}$
C) $(2, \infty)$
D) $[2, \infty)$
E) R

YANITLAR / ANSWERS

1. B	**2. B**	**3. A**	**4. C**	**5. C**	**6. D**	**7. A**	**8. A**
9. A	**10. B**	**11. C**	**12. D**				

1. **i.** $x < 0 \Rightarrow \operatorname{sgn}x = -1$

$x \cdot (-1) = x^2 - 6 \Rightarrow x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3 \vee x = 2$

$x < 0 \Rightarrow x = -3$

ii. $x = 0 \Rightarrow 0 \cdot \operatorname{sgn}0 = 0 - 6$

$0 \neq -6$

iii. $x > 0 \Rightarrow \operatorname{sgn}x = 1$

$x = x^2 - 6 \Rightarrow x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$x = -2 \vee x = 3$

$x > 0 \Rightarrow x = 3$

$\Rightarrow$ ÇK(SS) $= \{-3, 3\}$

Yanıt / The Answer is B

---

2. $x^2 + 4 > 0 \Rightarrow \operatorname{sgn}(x^2 + 4) = 1$

$[\![x + 2]\!] = 1 \Rightarrow 1 \le x + 2 < 2$

$-1 \le x < 0$

$x \in \mathbb{Z} \Rightarrow x = -1$

Yanıt / The Answer is B

---

3. $x \in [-1, 0) \Rightarrow [\![x]\!] = -1$

$x \in [0, 1) \Rightarrow [\![x]\!] = 0$

$x \in [1, 2) \Rightarrow [\![x]\!] = 1$

$x \in [2, 3) \Rightarrow [\![x]\!] = 2$

$x = 3 \Rightarrow [\![x]\!] = 3$

Yanıt / The Answer is A

---

4. $[\![3x - 6]\!] \le 2 \Rightarrow 3x - 6 < 3$

$3x < 9$

$x < 3$

Yanıt / The Answer is C

---

5. $[\![2x - 8]\!] = 4 \Rightarrow 4 \le 2x - 8 < 5$

$12 \le 2x < 13$

$6 \le x < \dfrac{13}{2}$

Yanıt / The Answer is C

---

6. $\dfrac{6}{|2x-1|} > \dfrac{2}{3} \Rightarrow \dfrac{|2x-1|}{6} < \dfrac{3}{2}$

$\Rightarrow |2x-1| < 9$

$\Rightarrow -9 < 2x - 1 < 9$

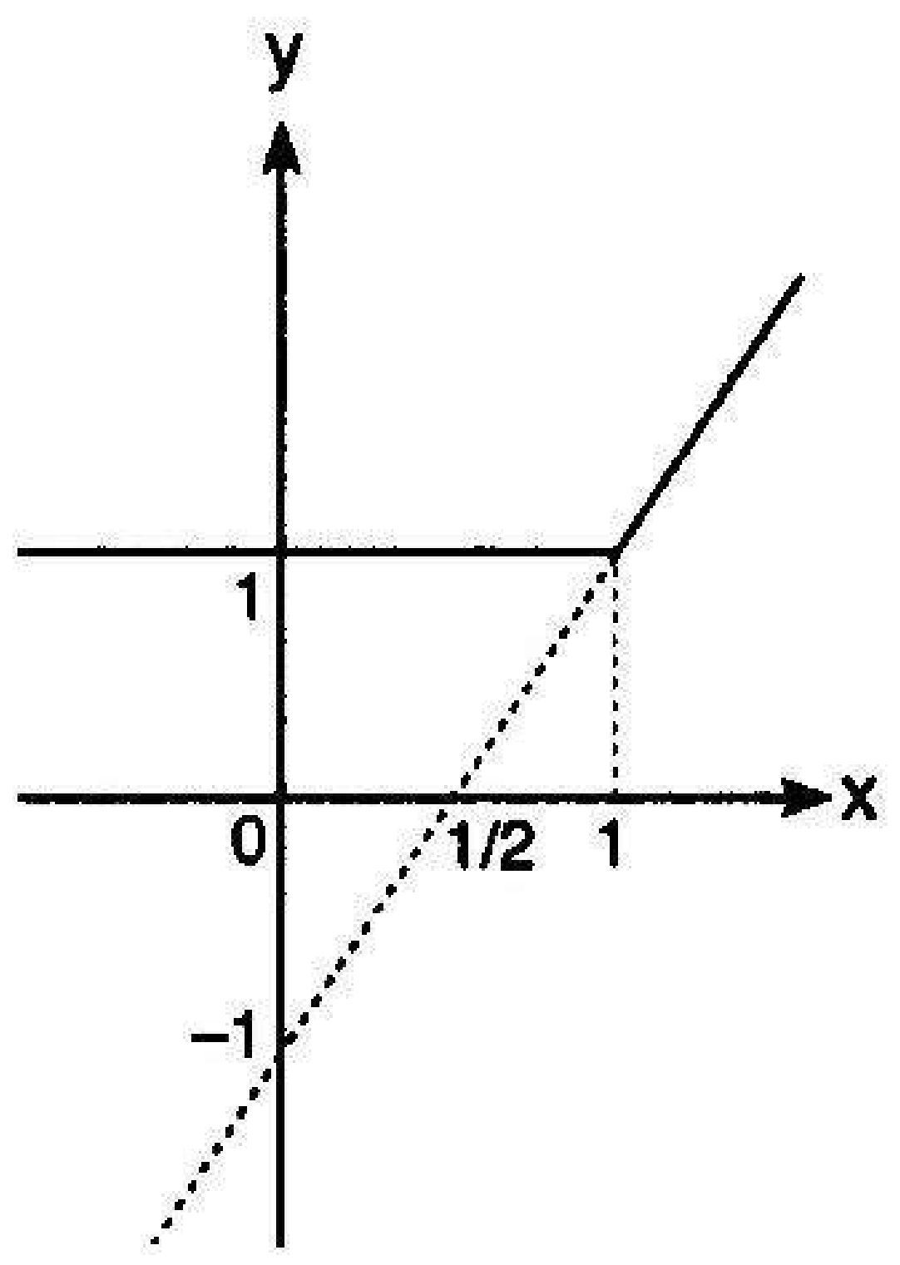
$\Rightarrow -8 < 2x < 10$

$\Rightarrow -4 < x < 5$

$\sum x = (-3) + (-2) + (-1) + 0 + 1 + 2 + 3 + 4 = 4$

Yanıt / The Answer is D

7. $f(x) = \begin{cases} |2x-1|, & x > 1 \\ 1, & x \le 1 \end{cases} \Rightarrow$



Yanıt / The Answer is A

8. $|5x-3| < 17 \Rightarrow -17 < 5x-3 < 17$

$-14 < 5x < 20$

$-\dfrac{14}{5} < x < 4$

$\sum x = (-2) + (-1) + 0 + 1 + 2 + 3 = 3$

Yanıt / The Answer is A

9. $5 \le 4x - 7 < 6$

$12 \le 4x < 13$

$3 \le x < \dfrac{13}{4}$

$\Rightarrow \text{ÇK(SS)} = \left[3, \dfrac{13}{4}\right)$

Yanıt / The Answer is A

10. $f(x) = \left| [\![x+1]\!] \right| = 2$

$[\![x+1]\!] = 2 \quad \vee \quad [\![x+1]\!] = -2$

$2 \le x+1 < 3 \qquad -2 \le x+1 < -1$

$1 \le x < 2 \qquad -3 \le x < -2$

$\text{ÇK(SS)} = [-3, -2) \cup [1, 2)$

Yanıt / The Answer is B

11. $x > 0 \Rightarrow x^2 = 1$

$x = 1$

$x = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

$x < 0 \Rightarrow x^2 \ne -1$

$\Rightarrow \text{ÇK(SS)} = \{0, 1\}$

Yanıt / The Answer is C

12. $f(x): A \to \{0, 1\} \Rightarrow \text{sgn}(x-2) = 0 \quad \wedge \quad \text{sgn}(x-2) = 1$

$x - 2 = 0 \qquad x - 2 > 0$

$x = 2 \qquad x > 2$

$\Rightarrow A = [2, \infty)$

Yanıt / The Answer is D

1. $f(x)=\dfrac{x^2+2x-4}{|x-4|-3}$ **fonksiyonunun en geniş tanım kümesi nedir?**

What is the largest domain of the function $f(x)=\dfrac{x^2+2x-4}{|x-4|-3}$?

Yanıt / The Answer is $\mathbf{R-\{1,7\}}$

---

2. $f(x)=\sqrt{4-\log_2(x-1)}$ **fonksiyonunun en geniş tanım kümesi nedir?**

What is the largest domain of the function

$f(x)=\sqrt{4-\log_2(x-1)}$?

Yanıt / The Answer is $\mathbf{(1,17]}$

---

3. f(x) çift fonksiyondur. / *f(x) is an even function.*

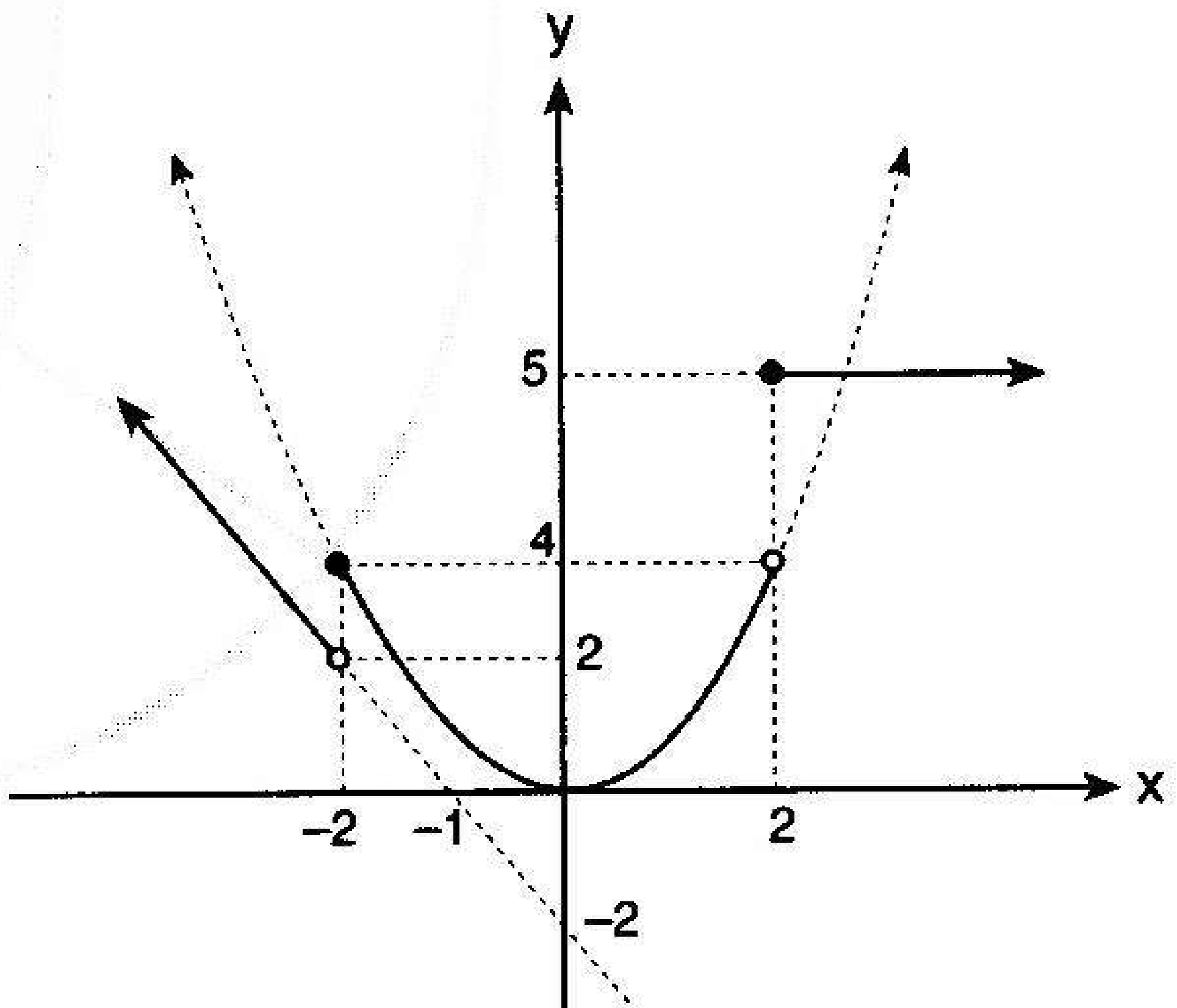
$f(x)-2f(-x)=3x^2-6 \Rightarrow \mathbf{f(x^2)=?}$

Yanıt / The Answer is $\mathbf{x^4-2}$

4. $f(x)=\begin{cases}2x-3, & x>1\\ x^2-4, & x\le 1\end{cases}$, $g(x)=\begin{cases}x+3, & x<4\\ 4, & x\ge 4\end{cases} \Rightarrow (f+g)(x)=?$

Yanıt / The Answer is $\mathbf{(f+g)(x)=\begin{cases}x^2+x-1 & x\le 1\\ 3x & 1<x<4\\ 2x+1 & x\ge 4\end{cases}}$

5.



$\Rightarrow \mathbf{f(x)=?}$

Yanıt / The Answer is $\mathbf{f(x)=\begin{cases}-2x-2 & x<-2\\ x^2 & -2\le x<2\\ 5 & 2\le x\end{cases}}$

6. $f(x) = |2 - x| + \left[\left| -\frac{x}{5} \right|\right] + \operatorname{sgn}\left(\frac{4 - x^2}{x}\right) \Rightarrow$ **$f(-6) = ?$**

Yanıt / The Answer is **10**

7. $2\,[|\,x + 3|] + [|x - 7|] - [|x - 9|] = 20 \Rightarrow$ **ÇK(SS) = ?**

Yanıt / The Answer is **[6, 7)**

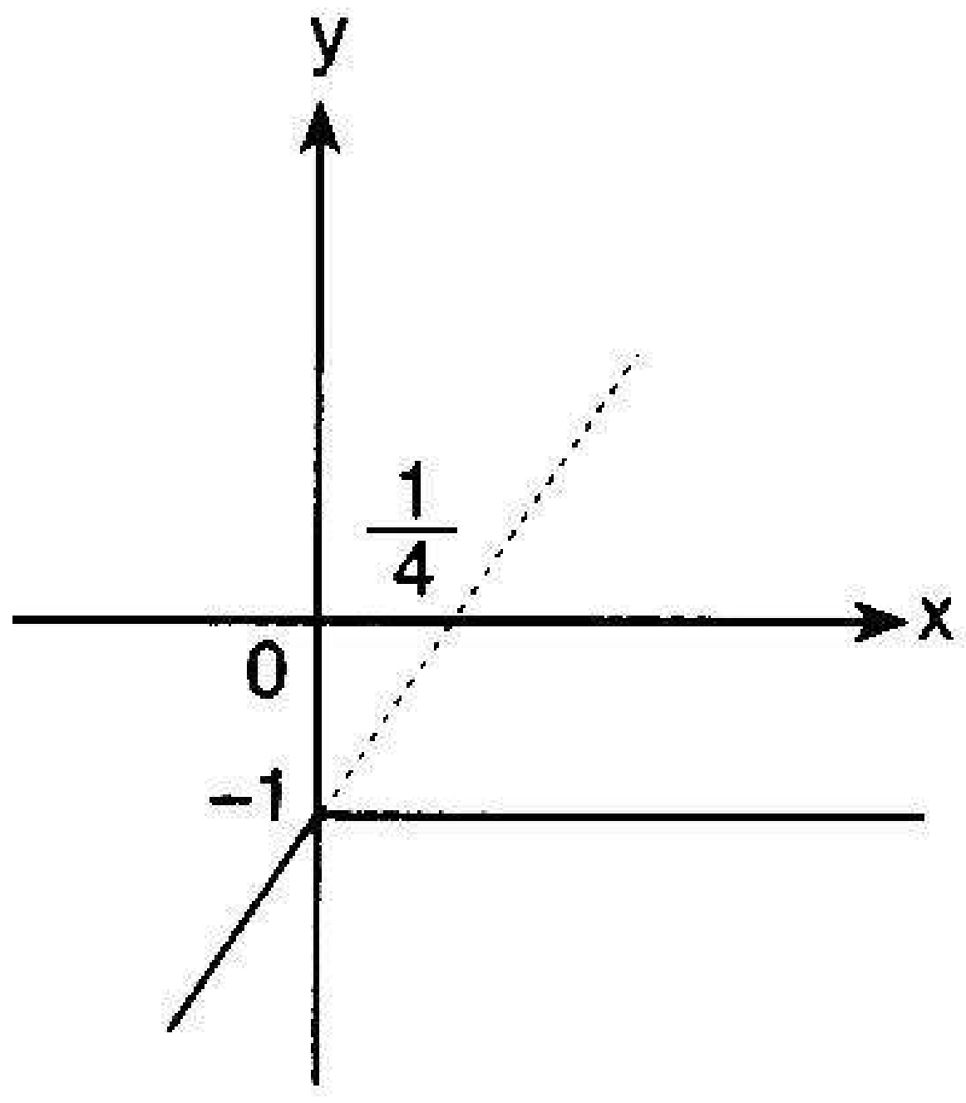
8. $f: R \to R,\ g: R \to R,$

$f: x \to x - |x|,$

$g: x \to 2x - 1$

$\Rightarrow$ **$(gof)\,(x) = ?$**

Yanıt / The Answer is



9. $f(x) = |x - 6| + |3 - x| + 2 + x \Rightarrow$ **$f(5) = ?$**

Yanıt / The Answer is **10**

10. $x \in Z,$

$4 \leq [|3x - 5|] < 13 \Rightarrow$ **$\sum x = ?$**

Yanıt / The Answer is **12**

11. $x \in Z,$

$|x^2 - 7x + 15| = 3 \Rightarrow$ **$\sum x = ?$**

Yanıt / The Answer is **7**

1. $f(x)=\dfrac{5}{[|1-\log x|]}$ **fonksiyonunun en geniş tanım kümesi nedir?**

Which of the following is the largest domain of the function $f(x)=\dfrac{5}{[|1-\log x|]}$?

A) ∅ B) R C) R – (0,10]

D) R – [0,10) E) (0,10]

2. $[|3x+1|]<10 \Rightarrow$ **ÇK(SS) = ?**

A) (–∞,3) B) (–∞,0) C) (0, 3)

D) (3, 9) E) (3, ∞)

3. $\text{sgn}(x^2-2x-15)=-1 \Rightarrow$ **ÇK(SS) = ?**

A) (–∞,–3)

B) (–∞,5)

C) (–∞,–3) ∪ (5, ∞)

D) (–3, 5)

E) (–5, 3)

4. $f(x)=9-\left[\left|\dfrac{x}{3}\right|\right] \Rightarrow$ **f(10) = ?**

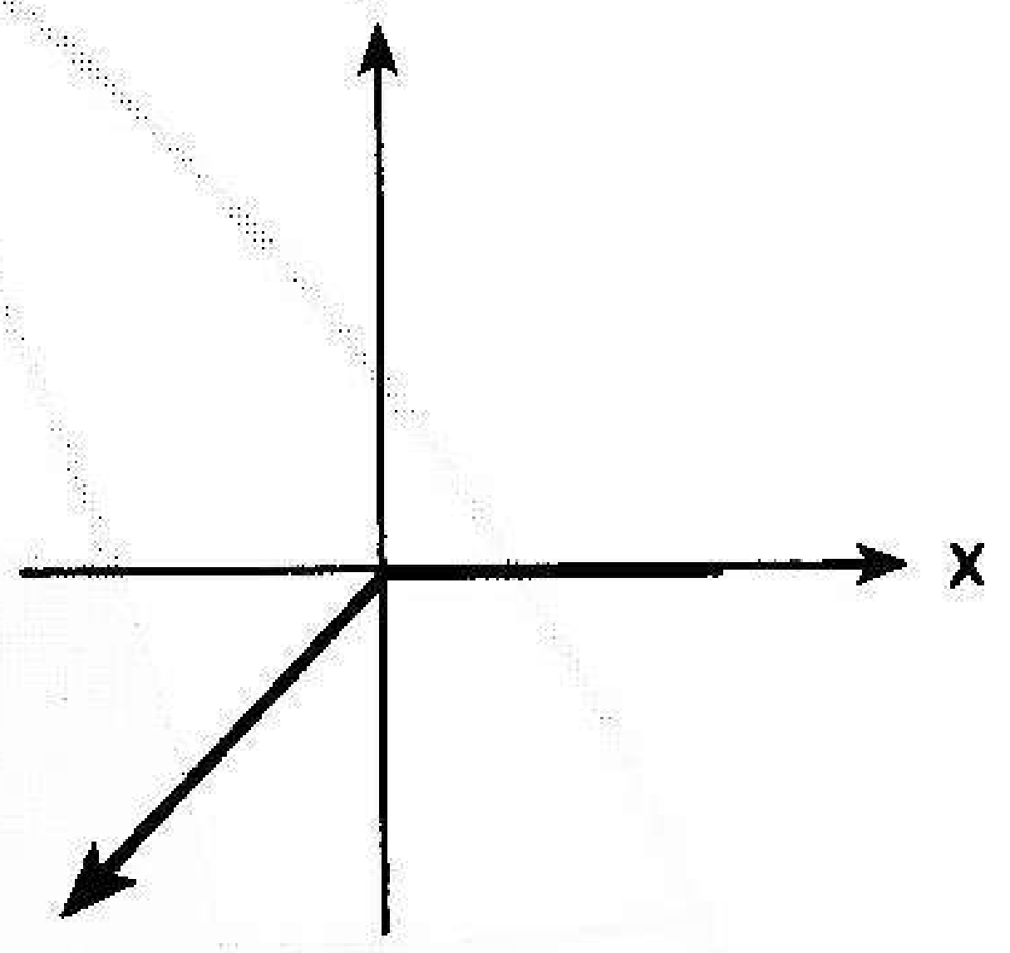
A) 9 B) 8 C) 7

D) 6 E) 5

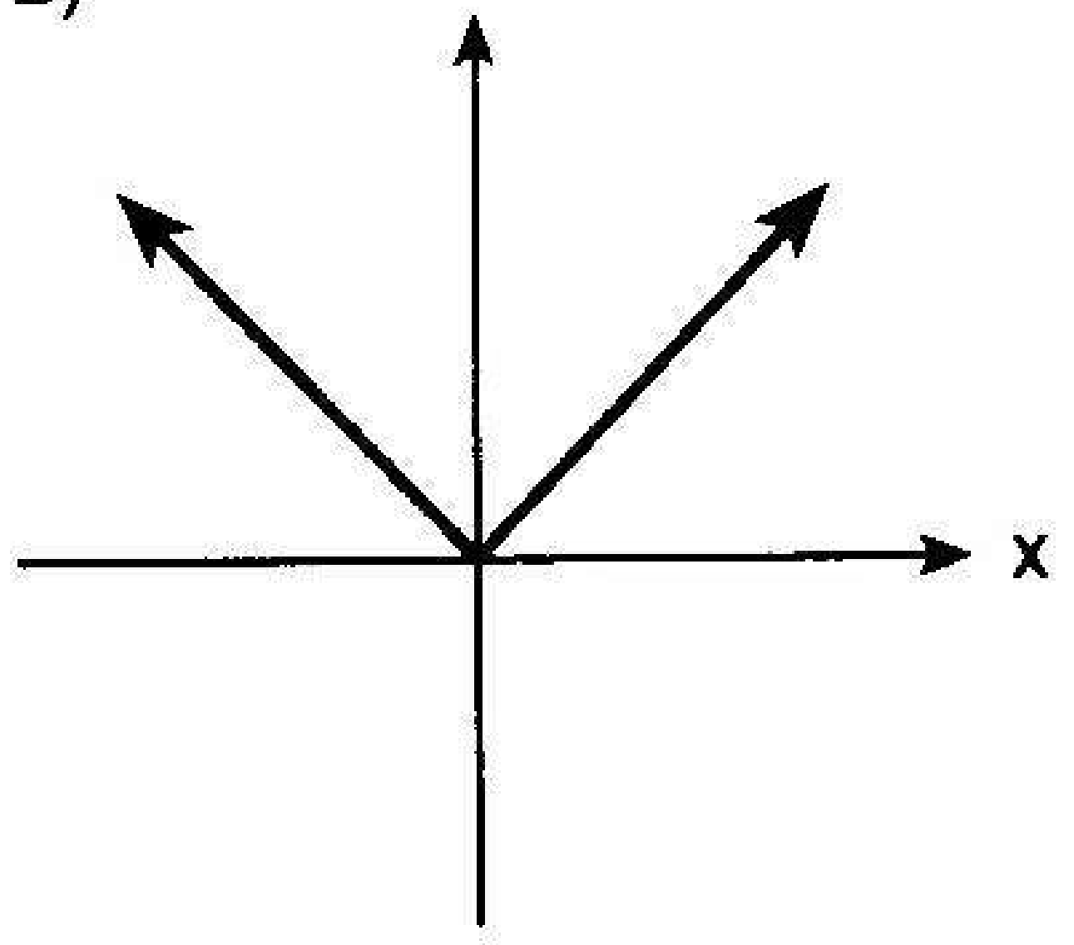
5. **2·f(x) = x + |x| fonksiyonunun grafiği aşağıdakilerden hangisidir?**

Which of the following is the graph of the function $2\cdot f(x)=x+|x|$ = ?

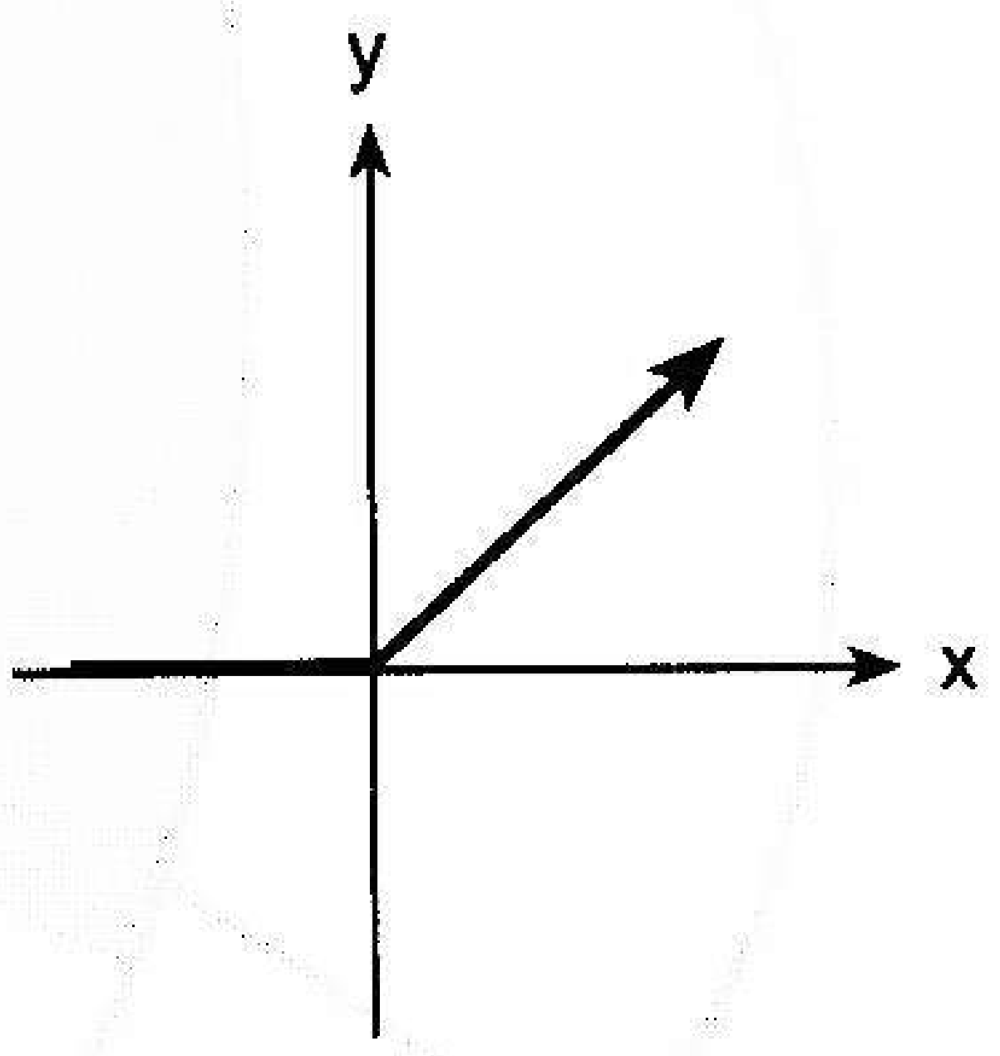
A)



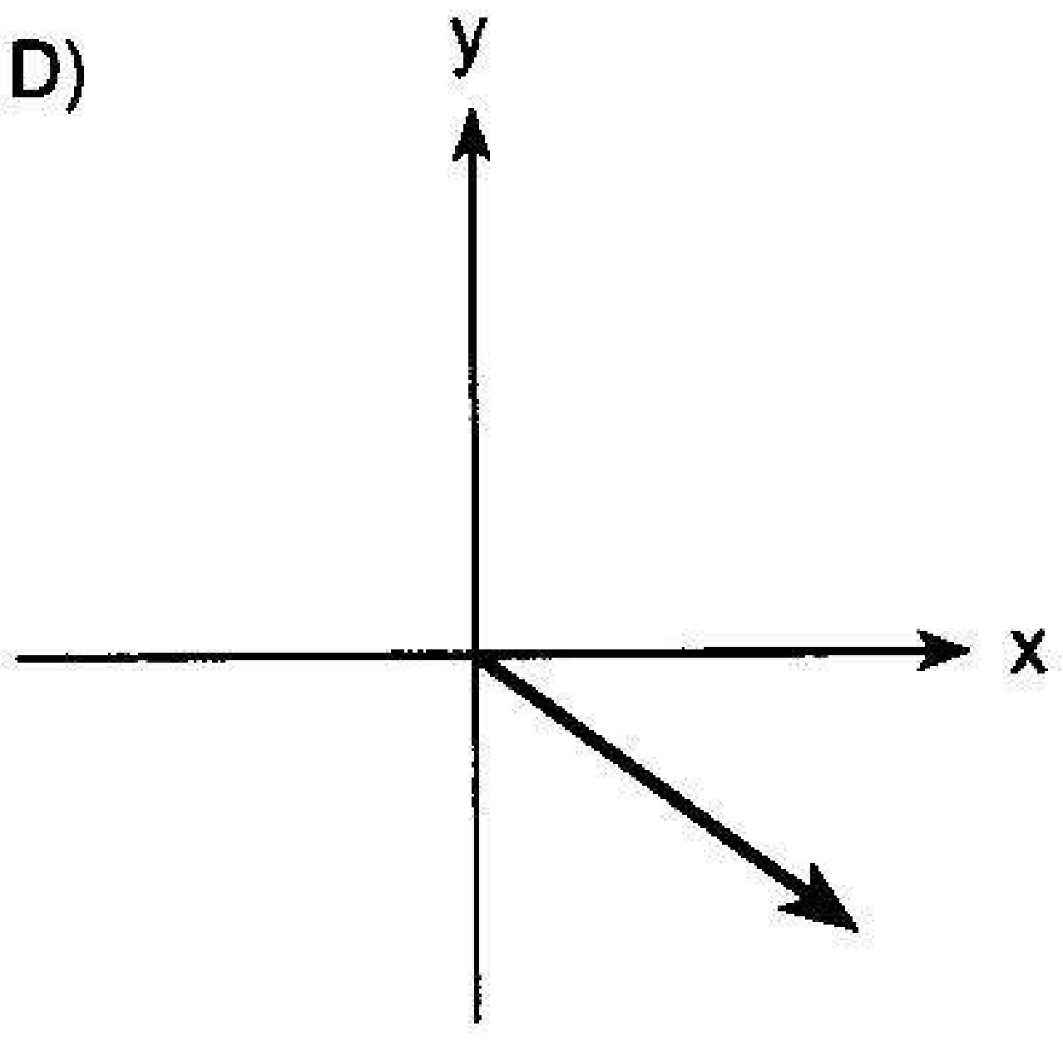
B)



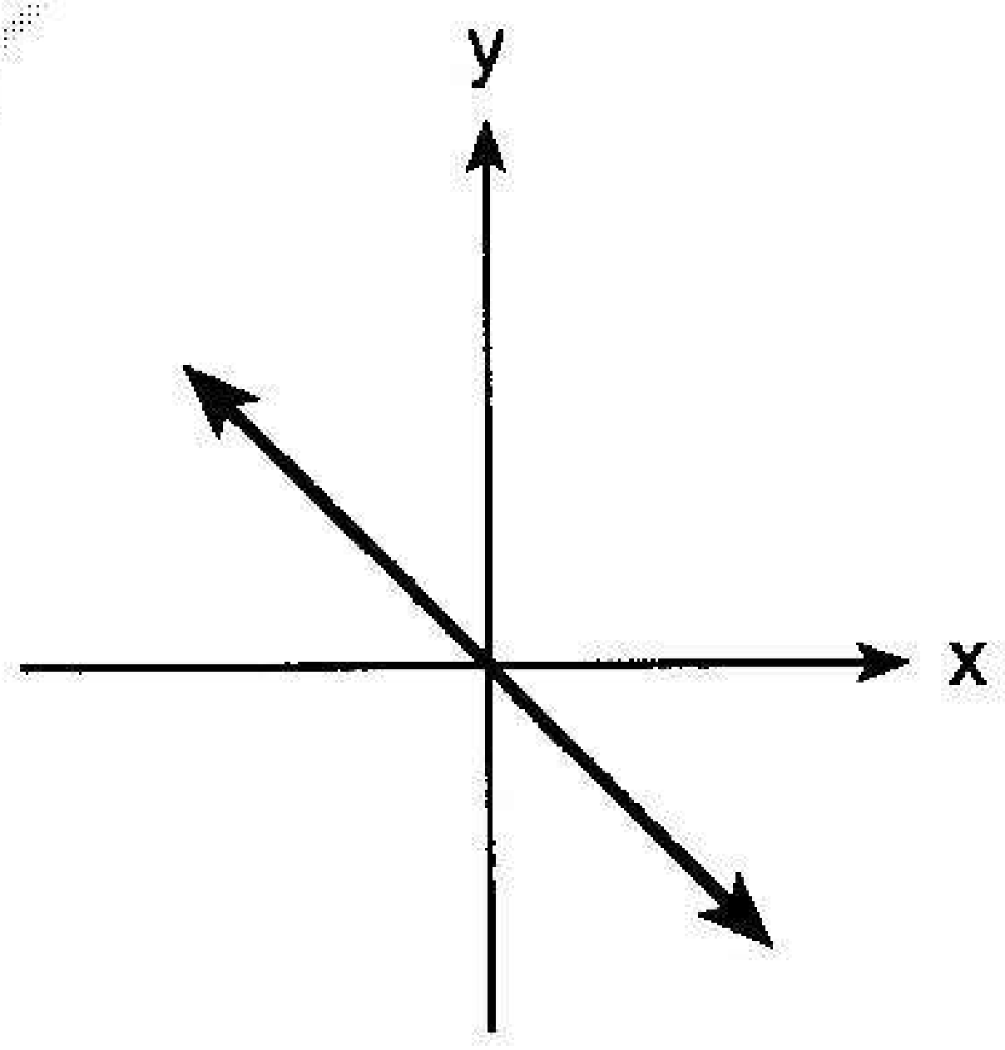
C)



D)



E)



6. e doğal logaritma tabandır.

e is the canoncial basis of a logarithm.

$f(x)=[|\,|x|\,|]-|\,[|x|]\,| \Rightarrow$ **f(–e) = ?**

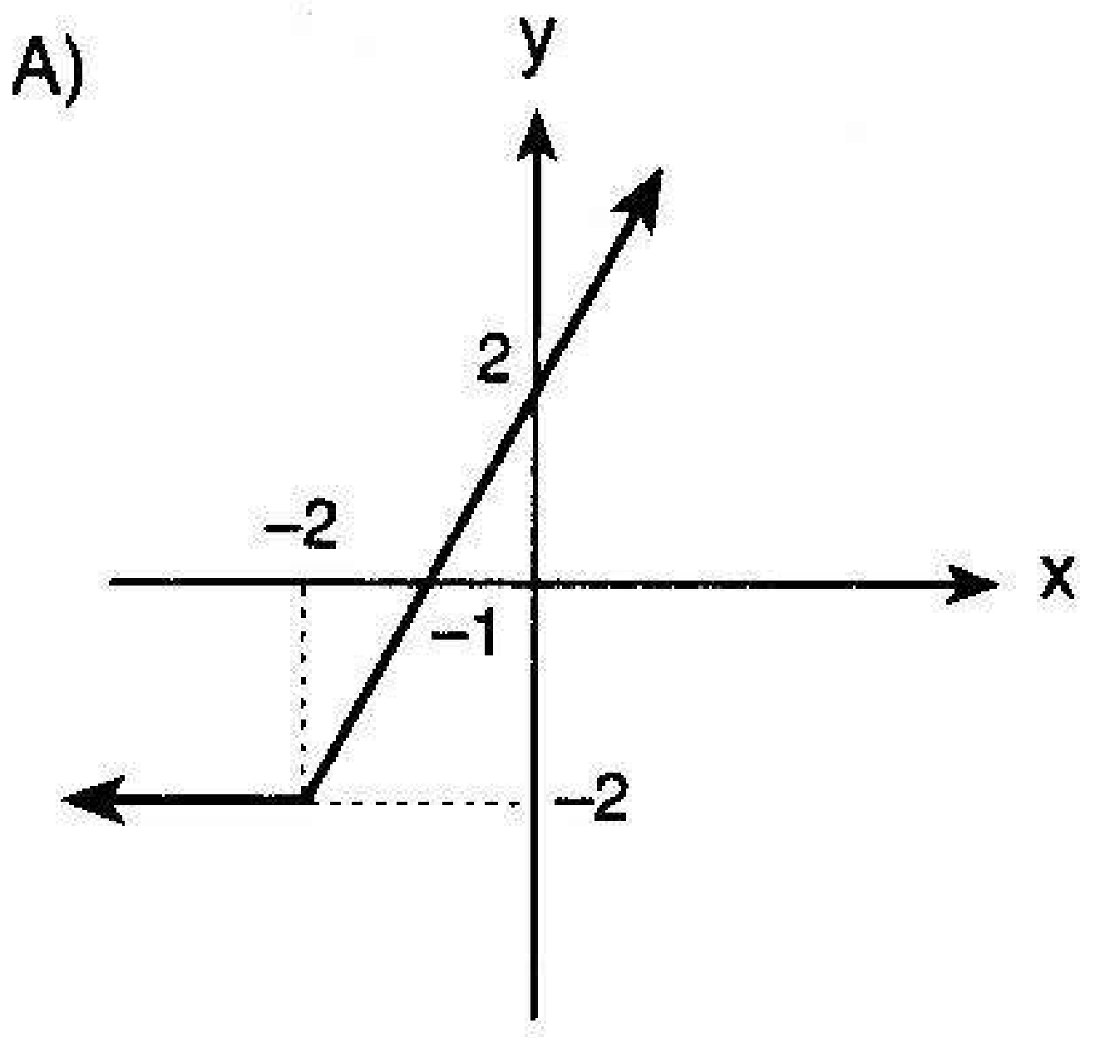
A) –2 B) –1 C) 0

D) 1 E) 2

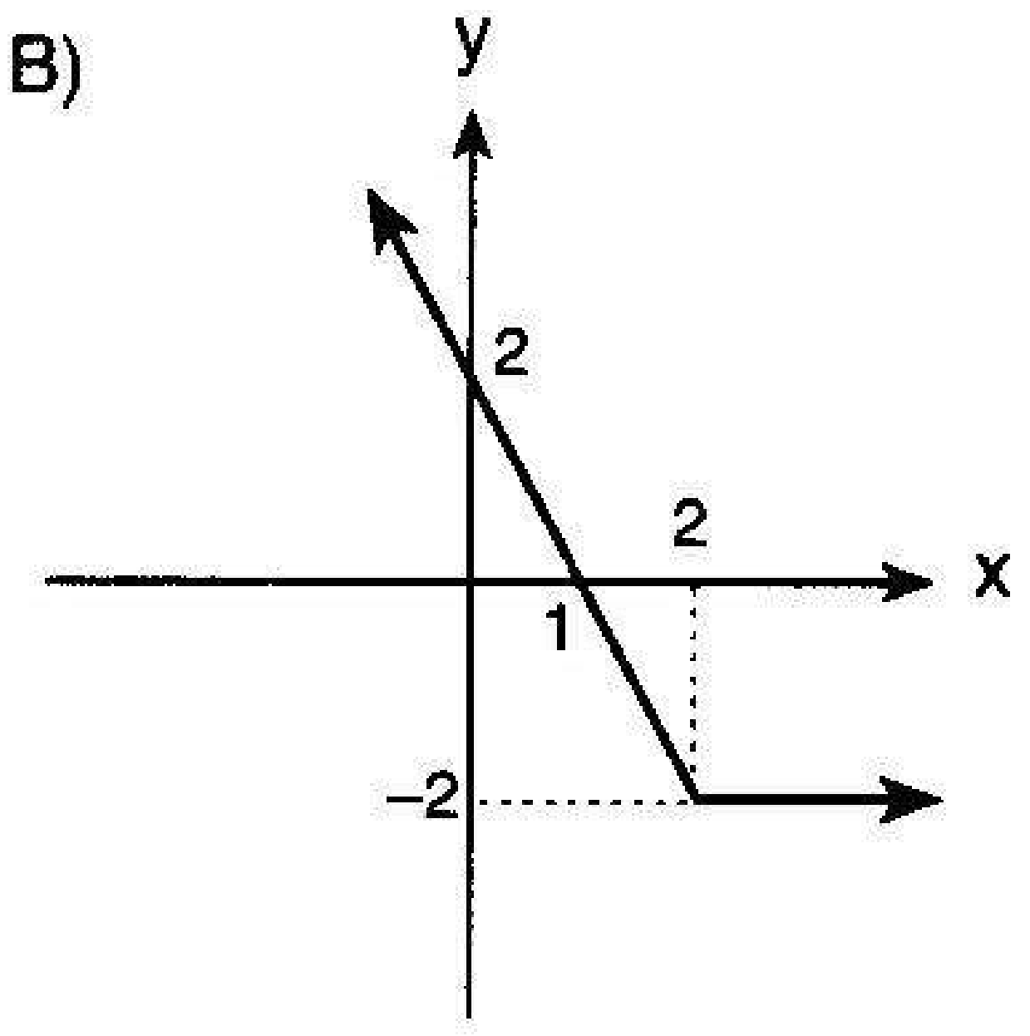
7. **$f(x) = |2 - x| - x$ fonksiyonunun grafiği aşağıdakilerden hangisidir?**

Which of the followings is the graph of the function $f(x) = |2 - x| - x$?

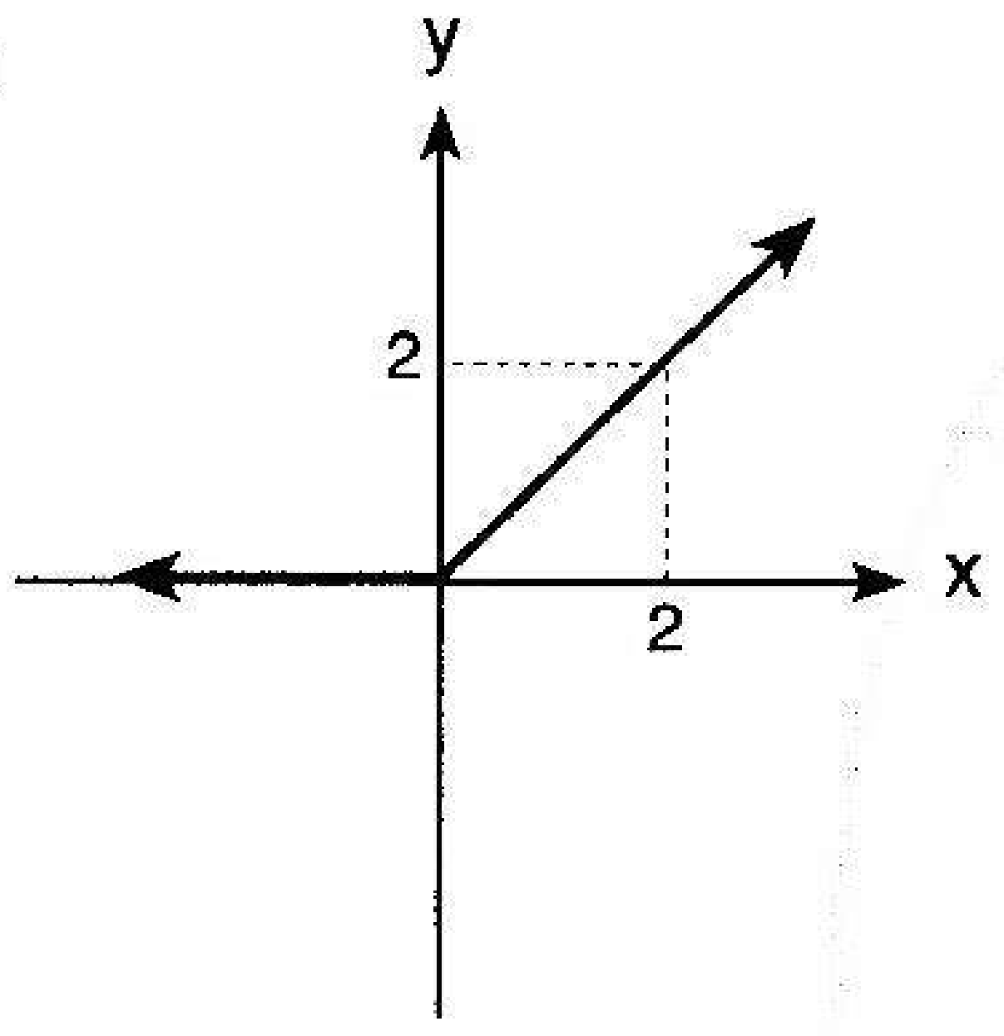
A)



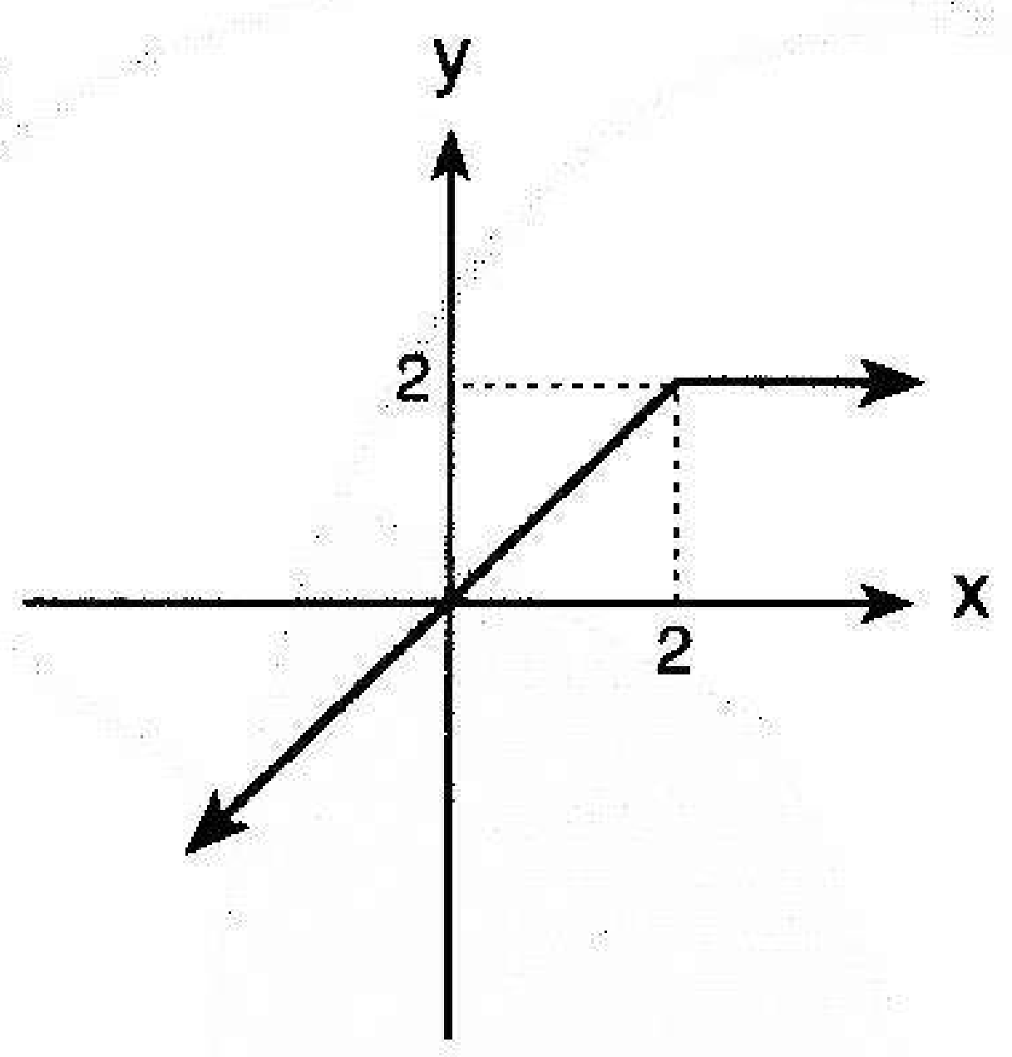
B)



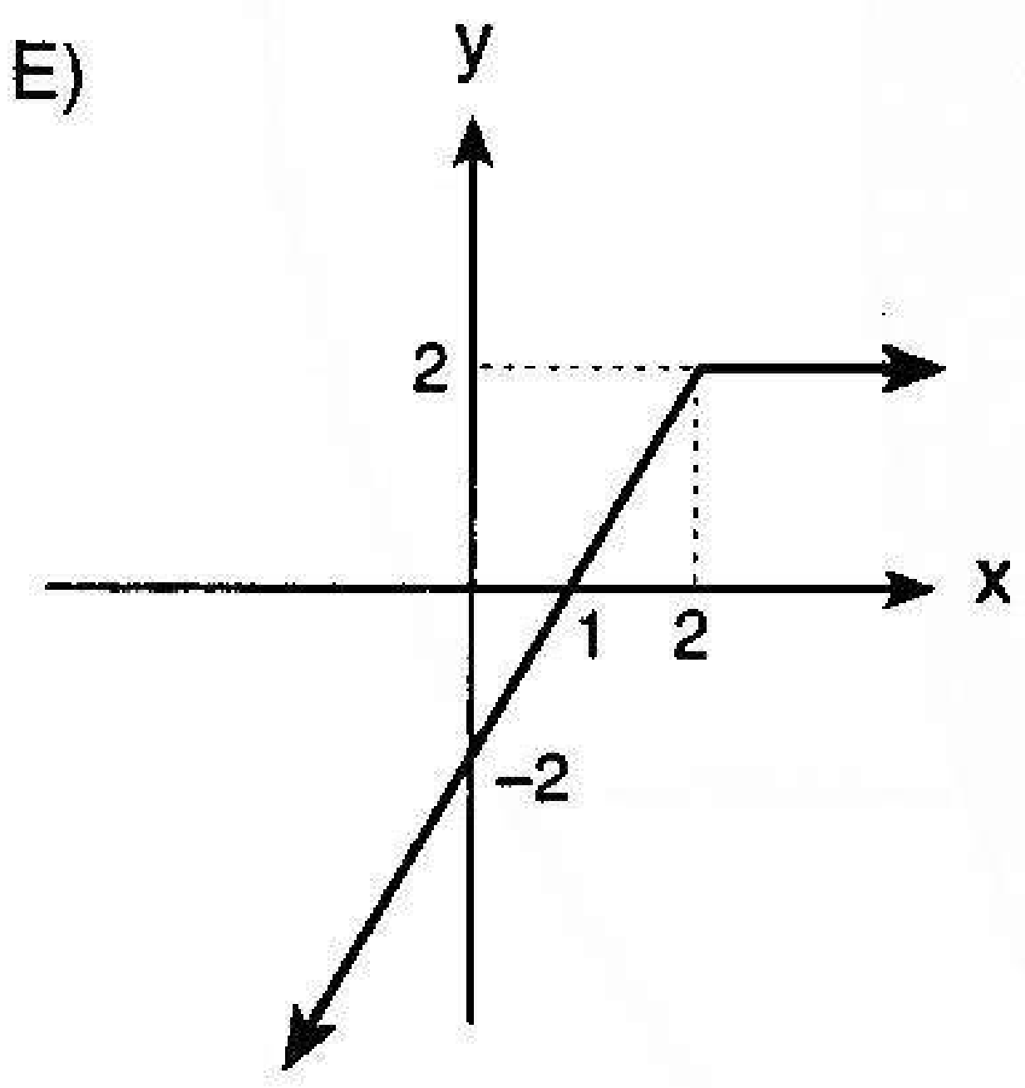
C)



D)



E)



8. $f(x) = \dfrac{3\sqrt{x-5}}{1-\operatorname{sgn}(x^2-9x+14)}$ **fonksiyonunun en geniş tanım kümesi nedir?**

Which of the following is the largest domain of the function $f(x) = \dfrac{3\sqrt{x-5}}{1-\operatorname{sgn}(x^2-9x+14)}$?

A) [1, 5] B) (1, 6) C) [5, 7]
D) [3, 8] E) (3, 8)

9. $f(x) = \left[\left|\dfrac{x}{3}\right|\right] - x \cdot \operatorname{sgn}(1-x) \Rightarrow$ **f(7) + f(–2) = ?**

A) 10 B) 9 C) 8
D) –4 E) –2

10.

$$f(x) = \begin{cases} x^2+1 & x<0 \\ x-\operatorname{sgn}(x) & x\geq 0 \end{cases}$$

$g(x) = x - |x-2| \Rightarrow$ **$f\circ g(0) + g\circ f(3) = ?$**

A) 0 B) 3 C) 5
D) 6 E) 7

11. $\displaystyle\sum_{k=1}^{1962} \operatorname{sgn}(k^2+7k-30) = ?$

A) 1955 B) 1957 C) 1959
D) 1962 E) 1972

12. $\left[\left|\dfrac{x}{2}-1\right|\right] = 3$ **denklemini sağlayan $x \in \mathbb{Z}$ kaç tanedir?**

How many $x \in Z$ can be found that satisfies the equation $\left[\left|\dfrac{x}{2}-1\right|\right] = 3$?

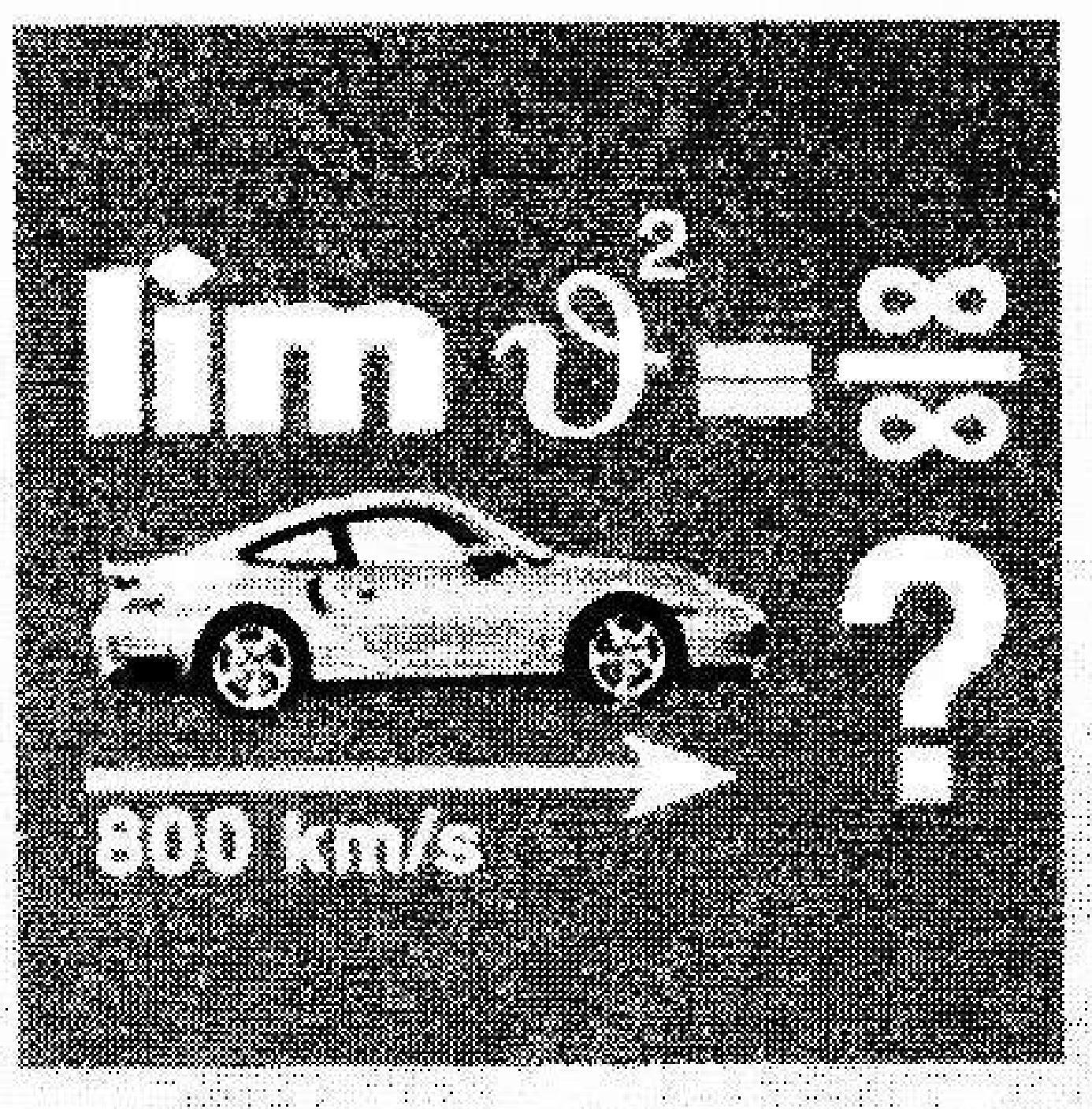
A) 1 B) 2 C) 3
D) 4 E) 5

YANITLAR / ANSWERS

1. C	**2. A**	**3. D**	**4. D**	**5. C**	**6. B**	**7. B**	**8. C**
9. A	**10. E**	**11. B**	**12. B**				

ÜNİTE 9

UNIT

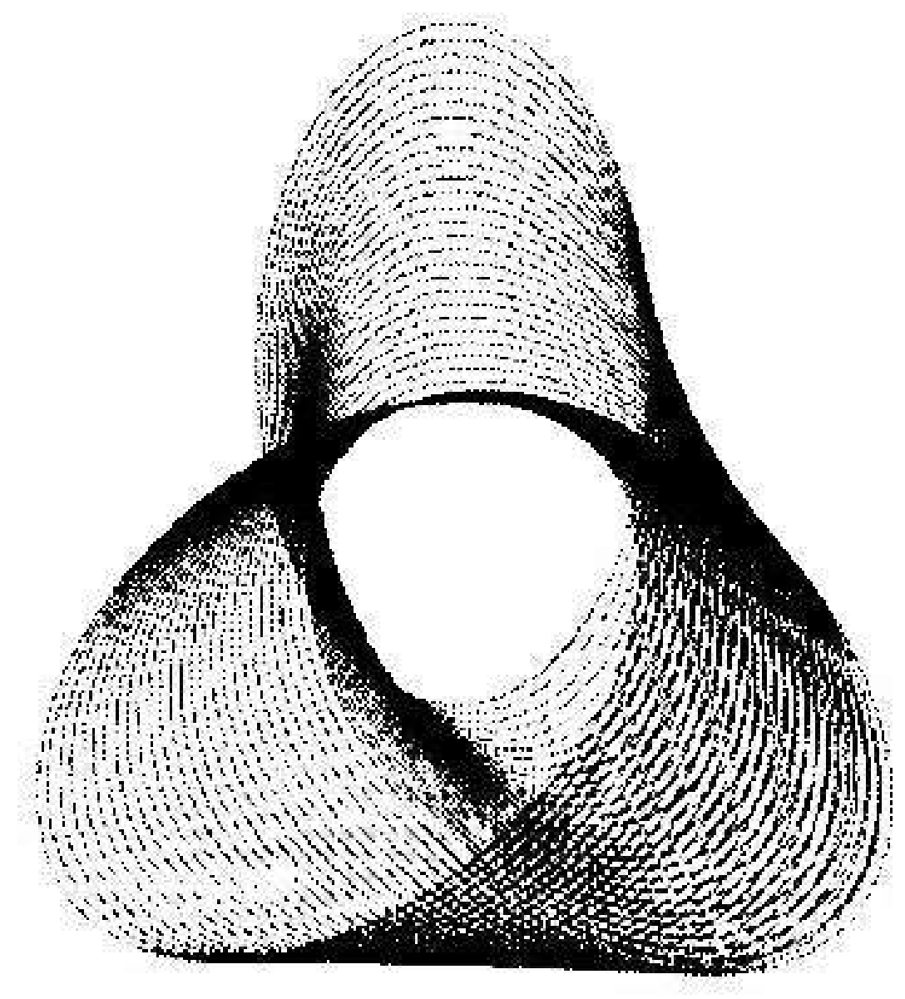


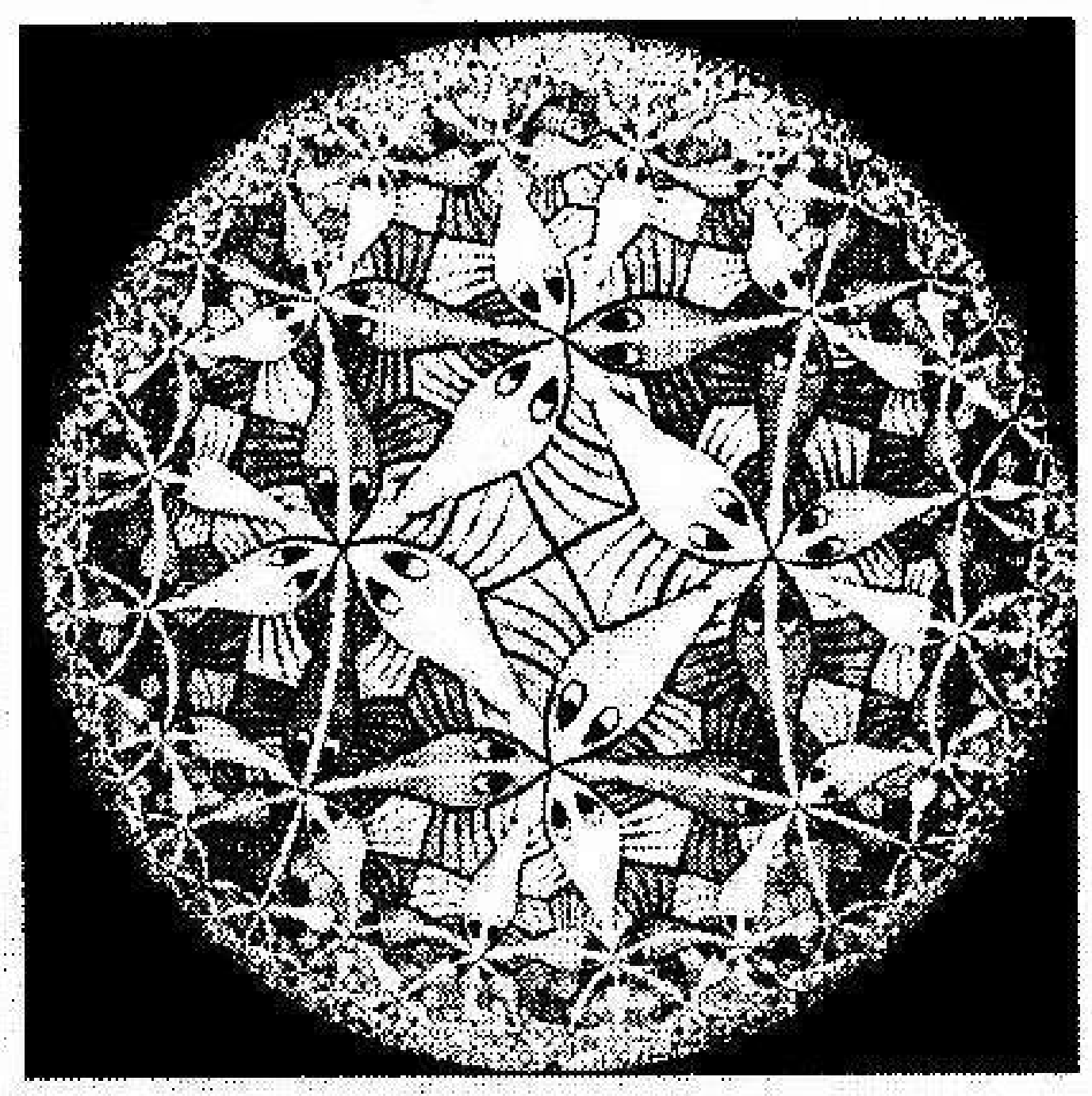
Bu ünitede, limit ve türev konuları ayrıntılı olarak anlatılmıştır. Ilk olarak, limit ve belirsizlikler anlatılmış, daha sonra fonksiyonların türevi açıklanmıştır. Son olarak, limitin türeve uygulanışı açıklanmıştır.

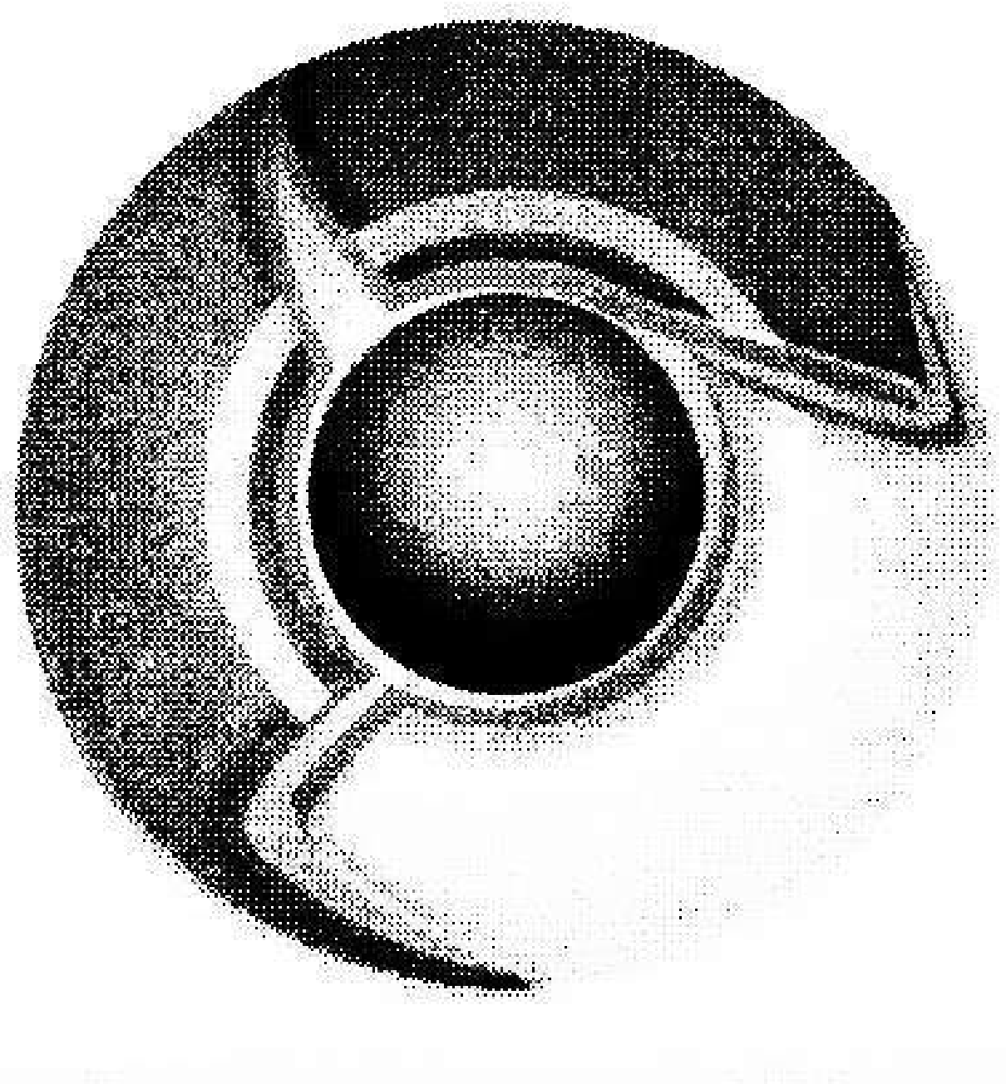
Bu ünite; 132 çözümlü örnek, 48 alıştırma ve 144 test sorusundan oluşmaktadır.

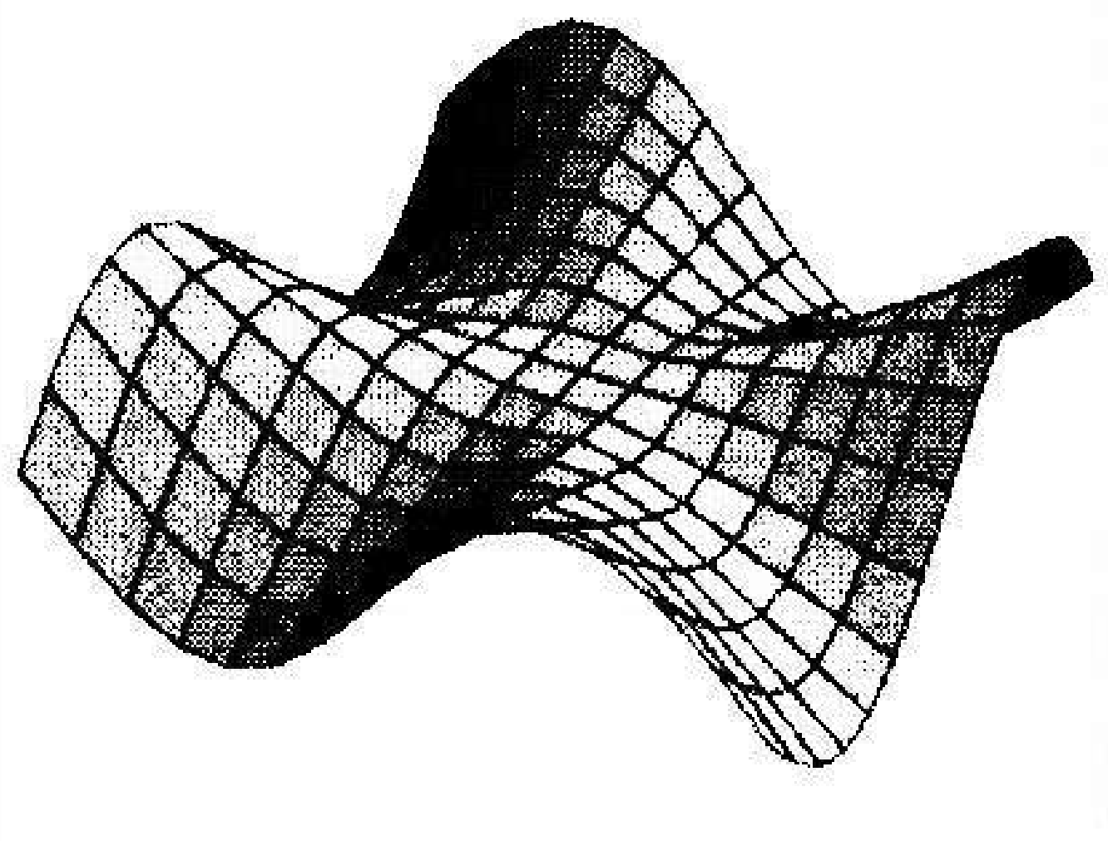
In this unit, limits and derivative are explained in detailed. Firstly, limit and indeterminate forms are explained. Later, derivative of functions is explained. Subsequently, apply the derivative of limits is explained.

This unit includes; 132 examples, 48 exercises, and 144 test questions.





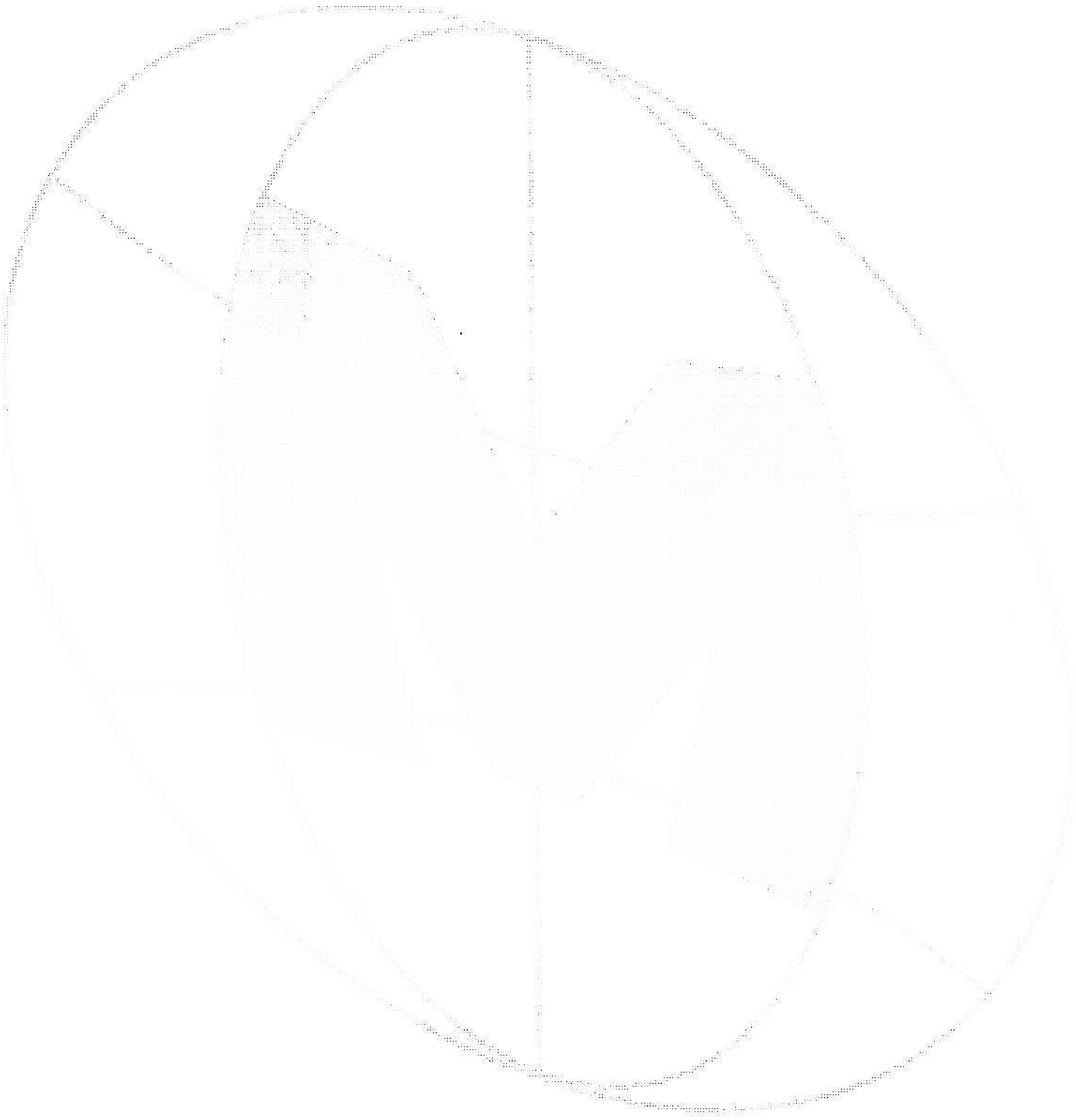




◆ **LİMİT / *LIMITS*** .... 643 – 682

◆ **TÜREV / *DERIVATIVE*** .... 683 – 738

BU SAYFADA KONU MATERYALİ YOKTUR.



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LİMİT
LIMITS

BÖLÜM 1
CHAPTER

$A \subset R,\ a \in A,\ f: A \rightarrow R$ ya da $f: A - \{a\} \rightarrow R$

Let $A \subset R,\ a \in A,\ f: A \rightarrow R$ *or* $f: A - \{a\} \rightarrow R$

$y = f(x)$ şeklindeki fonksiyonlarda x değerleri a ya soldan artan değerlerle yaklaşıyorsa, f fonksiyonunun değeri L_1 e yaklaşır. Buna fonksiyonun *soldan limiti* denir.

If x approaches a from values less than a, the value of function approaches L_1.

That is limit from the left of the function.

$$\lim_{x \to a^-} f(x) = L_1$$

x değerleri a ya sağdan azalan değerlerle yaklaşırsa fonksiyonunun değeri L_2 ye yaklaşır. Buna fonksiyonun *sağdan limiti* denir.

If x approaches a from values greater than a, the value of function approaches L_2.

That is limit from the right of the function.

$$\lim_{x \to a^+} f(x) = L_2$$

Fonksiyonun x = a noktasında limiti varsa sağdan ve soldan limitleri eşittir.

If function has a limit at point x = a, limit from the right and left is equal.

$$\left.\begin{array}{l} \lim\limits_{x \to a^-} f(x) = L_1 \\ \lim\limits_{x \to a^+} f(x) = L_2 \end{array}\right\} \begin{array}{l} L_1 = L_2 \Rightarrow \lim\limits_{x \to a} f(x) = L \\ L_1 \neq L_2 \Rightarrow \lim\limits_{x \to a} f(x) \text{ limit yoktur./ } \textit{There are not limit.} \end{array}$$

ÖRNEK / EXAMPLE 1

$$\lim_{x \to -2^-} f(x) + \lim_{x \to 1^+} f(x) + \lim_{x \to 0} f(x) = ?$$

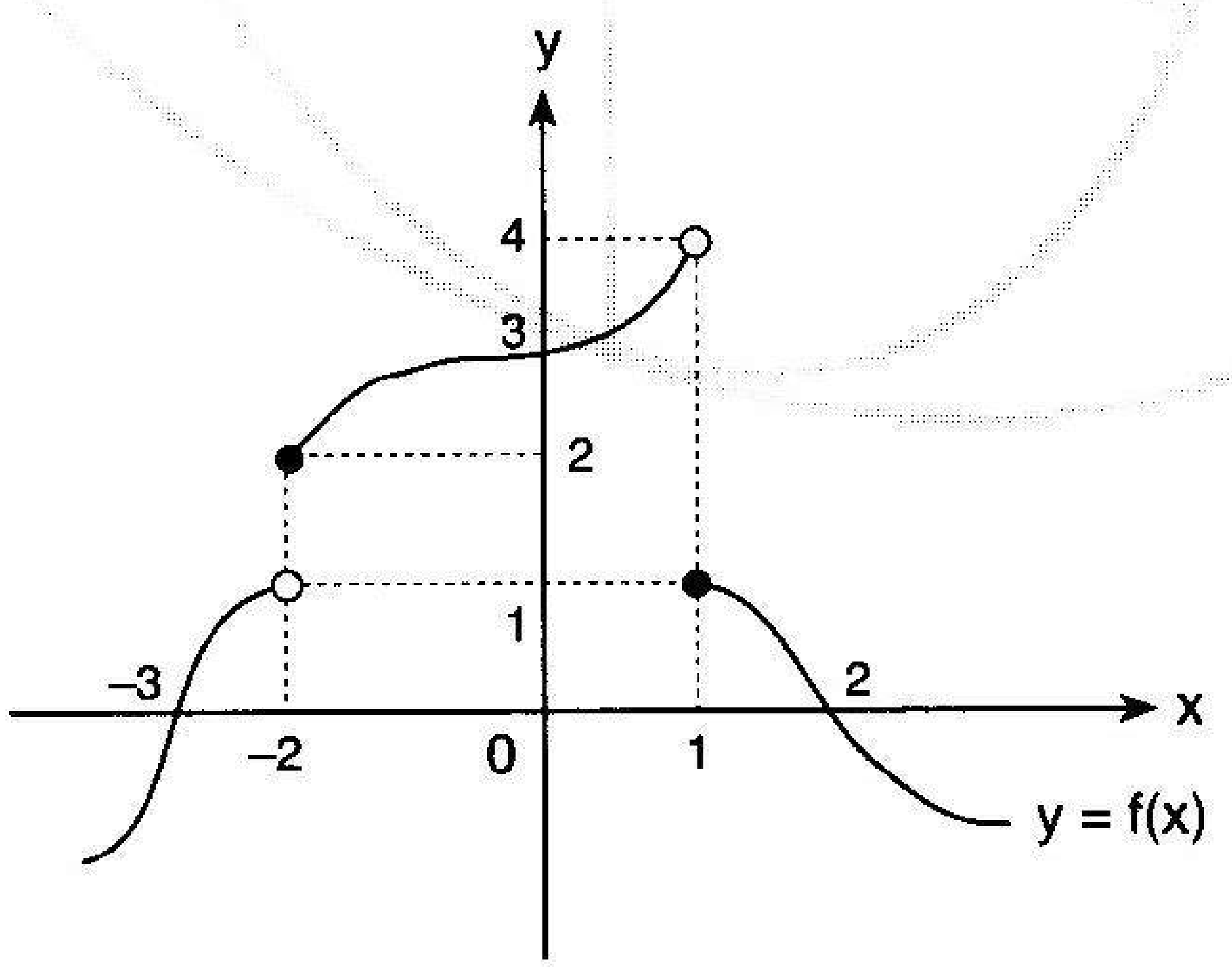
A) –3

B) –2

C) 3

D) 5

E) 7



ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\left.\begin{array}{r} \lim\limits_{x \to -2^-} f(x) = 1 \\ \lim\limits_{x \to 1^+} f(x) = 1 \\ \lim\limits_{x \to 0} f(x) = 3 \end{array}\right\} \Rightarrow \begin{array}{l} \lim\limits_{x \to -2^-} f(x) + \lim\limits_{x \to 1^+} f(x) + \lim\limits_{x \to 0} f(x \\ = 1 + 1 + 3 = 5 \end{array}$$

LİMİTİN ÖZELLİKLERİ / PROPERTIES of LIMITS

1. $\lim\limits_{x\to a} k = k \ (k \in R)$

2. $\lim\limits_{x\to a} [(f\ (x) \mp g(x)] = \lim\limits_{x\to a} f(x) \mp \lim\limits_{x\to a} g(x)$

3. $\lim\limits_{x\to a} [f(x) \cdot g(x)] = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x)$

4. $k \in R,\ \lim\limits_{x\to a} \left[k \cdot f(x)\right] = k \cdot \lim\limits_{x\to a} f(x)$

5. $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} \ \left(\lim\limits_{x\to a} g(x) \neq 0\right)$

6. $\lim\limits_{x\to a} \left[(f(x))^n\right] = \left[\lim\limits_{x\to a} f(x)\right]^n$

7. $\lim\limits_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to a} f(x)}$

f(x) > 0 ve n tek doğal sayı / *f(x) > 0 and n is an odd natural number*

8. $\lim\limits_{x\to a} \left[k^{f(x)}\right] = k^{\lim\limits_{x\to a} f(x)}, \ (k \in R)$

9. $\lim\limits_{x\to a} \left[\log_a f(x)\right] = \log_a \left[\lim\limits_{x\to a} f(x)\right]$

▼ f : R → R, $\lim\limits_{x \to a} f(x)$ bulunurken / *To find the limit of a function f(x) at a point x = a;*

1. x = a noktası kritik nokta ise, sağdan ve soldan limit incelenir.
   Examine the left and the right limits if x = a is a critical number.
2. x = a noktası kritik nokta değilse, $\lim\limits_{x \to a} f(x) = f(a)$ dır.
   $\lim\limits_{x \to a} f(x) = f(a)$ if x = a is not a critical number.

ÖRNEK / EXAMPLE 2

$f(x) = x^3 + 2x^2 - 3x + 2 \Rightarrow \lim\limits_{x \to 2} f(x) = ?$

A) 8 B) 10 C) 12 D) 14 E) 16

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\lim\limits_{x \to 2} f(x) = f(2) = 2^3 + 2 \cdot 2^2 - 3 \cdot 2 + 2$

$\lim\limits_{x \to 2} f(x) = 8 + 8 - 6 + 2 = 12$

PEKİŞTİRME / REINFORCE

$f(x) = x^2 - 4x + 5 \Rightarrow \lim\limits_{x \to 1} f(x) = ?$

YANIT / ANSWER : 2

ÖRNEK / EXAMPLE 3

$\lim\limits_{x \to -3} \dfrac{x^3 + 8}{x + 2} = ?$

A) 15 B) 19 C) 21 D) 27 E) 30

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\lim_{x \to -3} \frac{x^3 + 8}{x + 2} = \frac{(-3)^3 + 8}{-3 + 2} = \frac{-27 + 8}{-1} = \frac{-19}{-1} = 19$$

ÖRNEK / EXAMPLE 4

$$f(x)=\begin{cases}3x+a & x<3\\ 2x+b & x\geq 3\end{cases}$$

$\lim\limits_{x\to 3} f(x)=7 \Rightarrow \mathbf{a+b=?}$

A) –3 B) –1 C) 0 D) 1 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\lim_{x\to 3} f(x)=7 \Rightarrow \lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=7$$

$$\lim_{x\to 3^-} f(x)=3\cdot 3+a=7$$

$$9+a=7$$

$$a=-2$$

$$\lim_{x\to 3^+} f(x)=2\cdot 3+b=7$$

$$6+b=7$$

$$b=1$$

$$a+b=-2+1=-1$$

ÖRNEK / EXAMPLE 5

$f:\mathbb{R}\to\mathbb{R}$

$$f(x)=\begin{cases}2x+1 & x>1\\ 3 & x=1\\ 4x-1 & x<1\end{cases}$$

$\Rightarrow \lim\limits_{x\to 1} f(x)=?$

A) –3 B) –1 C) 0 D) 1 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+} 2x+1=3$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-} 4x-1=3$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^-} f(x)=\lim_{x\to 1} f(x)=3$$

GENİŞLETİLMİŞ REEL SAYI ARALIKLARI
EXTENDING REAL NUMBER INTERVALS

Tanım: R gerçel sayılar kümesi ile $-\infty$ ve $+\infty$ un birleşmesiyle elde edilen kümeye genişletilmiş reel sayı aralıkları denir.

Definition: *Extending real number intervals are a set that consist of the union real number set (R), $-\infty$, and $+\infty$.*

$$R \cup \{-\infty, +\infty\}$$

Sonuçlar / Conclusions

1. $a \in R^+ \Rightarrow a \cdot (+\infty) = \infty \quad , \quad a \cdot (-\infty) = -\infty$

$a \in R^- \Rightarrow a \cdot (+\infty) = -\infty \quad , \quad a \cdot (-\infty) = +\infty$

2. $a \in R^+ \Rightarrow \dfrac{+\infty}{a} = +\infty \quad , \quad \dfrac{-\infty}{a} = -\infty$

$a \in R^- \Rightarrow \dfrac{+\infty}{a} = -\infty \quad , \quad \dfrac{-\infty}{a} = +\infty$

3. $a \in R \Rightarrow \dfrac{a}{\pm\infty} = 0 \ , \ a \neq 0$

4. $a \in R^+ \Rightarrow \dfrac{a}{0^+} = +\infty \quad , \quad \dfrac{a}{0^-} = -\infty$

$a \in R^- \Rightarrow \dfrac{a}{0^+} = -\infty \quad , \quad \dfrac{a}{0^-} = +\infty$

5. $n \in Z$

$\sqrt[2n-1]{\pm\infty} = \pm\infty$

$\sqrt[2n]{+\infty} = \infty$, $\sqrt[2n]{-\infty}$ = tanımsız / *indefinite*

6. $n \in Z$

$(+\infty)^n = +\infty$

$(-\infty)^{2n} = +\infty$

$(-\infty)^{2n-1} = -\infty$

7. $a > 1 \Rightarrow a^{+\infty} = \infty$

$a^{-\infty} = 0$

$0 < a < 1 \Rightarrow a^{+\infty} = 0$

$a^{-\infty} = +\infty$

PEKİŞTİRME / REINFORCE

$\lim\limits_{x\to\infty}\left(\frac{1}{3}\right)^x = ?$

YANIT / ANSWER : 0

PEKİŞTİRME / REINFORCE

$\lim\limits_{x\to-\infty} e^{x^2+3x} = ?$

YANIT / ANSWER : ∞

ÖRNEK / EXAMPLE 6

$\lim\limits_{x\to-\infty} 4^x = ?$

A) $-\infty$ B) -4 C) 0 D) 4 E) ∞

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\lim_{x\to-\infty} 4^x = 4^{-\infty} = \frac{1}{4^{\infty}} = \frac{1}{\infty} = 0$$

ÖRNEK / EXAMPLE 7

$\lim\limits_{x\to-\infty} 3^{2x^2-3x} = ?$

A) 0 B) $\frac{1}{3}$ C) 3 D) 9 E) ∞

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\lim_{x\to-\infty} 3^{2x^2-3x} = 3^{\lim\limits_{x\to-\infty} 2x^2-3x}$$

$$= 3^{\infty}$$

$$= \infty$$

ÖRNEK / EXAMPLE 8

$\lim\limits_{x\to 2} \dfrac{4x+3}{2x-4} = ?$

A) $-\infty$ B) -12 C) 0

D) ∞ E) Limit yoktur. / *There is not limit*

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\lim\limits_{x\to 2^-} \dfrac{4x+3}{2x-4} = \dfrac{11}{0^-} = -\infty$

$\lim\limits_{x\to 2^+} \dfrac{4x+3}{2x-4} = \dfrac{11}{0^+} = +\infty$

$\lim\limits_{x\to 2^-} f(x) \neq \lim\limits_{x\to 2^+} f(x) \Rightarrow \lim\limits_{x\to 2} \dfrac{4x+3}{2x-4}$ yoktur.

There is not $\lim\limits_{x\to 2} \dfrac{4x+3}{2x-4}$.

▼ $x \to +\infty, -\infty$ için polinom fonksiyonlarında limit bulunurken derecesi en büyük olan terimin limitine bakılması yeterlidir.

To find the limit of the polynomial function for $x \to +\infty, -\infty$, only we can see term with maximum degree.

$\lim\limits_{x\to\infty} f(x) = 4x^3 + 5x^2 + 7x + 1 = \lim\limits_{x\to\infty} 4x^3 = 4\cdot(\infty) = \infty$

Mutlak Değer Fonksiyonunun Limiti / Limit of the Absolute Value Function

$\lim\limits_{x\to a} |f(x)| = |f(a)|$

ÖRNEK / EXAMPLE 9

$\lim\limits_{x\to 2} (|2x+5| + |x^2 - 3x|) = ?$

A) 7 B) 9 C) 11 D) 13 E) 15

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$\lim\limits_{x\to 2} (|2x+5| + |x^2 - 3x|) = |2\cdot 2 + 5| + |2^2 - 3\cdot 2|$

$= 9 + 2 = 11$

PEKİŞTİRME / REINFORCE

$\lim\limits_{x\to -1} \left(|x-2| + |x+1|\right) = ?$

YANIT / ANSWER : 3

LİMİTTE BELİRSİZLİK DURUMU / INDETERMINATE FORMS

$\lim\limits_{x \to a} f(x)$ limitinin sonucu $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^{∞} , 0^0 , ∞^0 olabilir.

Bu sonuçların herbirine *belirsizlik* denir.

Result of $\lim\limits_{x \to a} f(x)$ *can be* $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^{∞}, 0^0, ∞^0. *Each of the results is called indeterminate.*

1. $\frac{0}{0}$ Belirsizliği / Indeterminate Form $\frac{0}{0}$

$\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ limitinin sonucu $\frac{0}{0}$ belirsizliği oluyorsa, f(x) ve g(x) fonksiyonlarında (x – a) çarpanı vardır.

To find $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$, when you apply direct substitution and obtain $\frac{0}{0}$ you can conclude that (x – a) must be a common factor to both f(x) and g(x).

$\frac{0}{0}$ belirsizlikleri çözülürken / *While we solve indeterminate form* $\frac{0}{0}$,

$$f(x) = (x - a) \cdot f_1(x) \quad , \quad g(x) = (x - a) \cdot g_1(x)$$

$\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{(x-a) \cdot f_1(x)}{(x-a) \cdot g_1(x)} = \lim\limits_{x \to 0} \frac{f_1(x)}{g_1(x)} = \lim\limits_{x \to a} \frac{f_1(a)}{g_1(a)}$ yolu izlenir. / *is written.*

Eğer belirsizlik kalkmıyorsa bu işleme devam edilir. / *If indeterminate continue, apply the similar process one more time.*

ÖRNEK / EXAMPLE 10

$\lim\limits_{x \to 3} \frac{x^2 - 9}{x - 3} = ?$

A) 0 B) 3 C) 5 D) 6 E) 9

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\lim_{x \to 3} \frac{3^2 - 9}{3 - 3} = \frac{0}{0}$$

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{x-3} = 3 + 3 = 6$$

PEKİŞTİRME / REINFORCE

$\lim\limits_{x \to 1} \frac{x^2 - 1}{x + 1} = ?$

YANIT / ANSWER : 0

ÖRNEK / EXAMPLE 11

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=?$$

A) 1 B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) 4 E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\frac{\sqrt{1}-1}{1-1}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}\lim_{x\to 1}=\frac{\sqrt{x}-1}{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}$$

$$=\lim_{x\to 1}\frac{1}{\sqrt{x}+1}=\frac{1}{1+1}=\frac{1}{2}$$

PEKİŞTİRME / REINFORCE

$$\lim_{x\to 1}\frac{x^2-x+x^4-x^3}{x^2-1}=?$$

YANIT / ANSWER : 1

Trigonometrik Fonksiyonların Limitleri / Limits of Trigonometric Functions

1. $\lim_{x\to a}\sin x=\sin a$

2. $\lim_{x\to a}\cos x=\cos a$

3. $\lim_{x\to 0}\frac{\sin ax}{bx}=\frac{a}{b}$

4. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}=\frac{a}{b}$

5. $\lim_{x\to 0}\frac{\tan ax}{bx}=\frac{a}{b}$

6. $\lim_{x\to 0}\frac{\tan ax}{\tan bx}=\frac{a}{b}$

7. $\lim_{x\to 0}\frac{\sin ax}{\tan bx}=\frac{a}{b}$

8. $\lim_{x\to 0}\frac{\tan ax}{\sin bx}=\frac{a}{b}$

PEKİŞTİRME / REINFORCE

$$\lim_{x \to 3} \frac{\sin(x-3)}{x-3} = ?$$

YANIT / ANSWER : 1

ÖRNEK / EXAMPLE 12

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 5x} = ?$$

A) $-\frac{5}{2}$ B) $-\frac{2}{5}$ C) $\frac{1}{5}$ D) $\frac{2}{5}$ E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b} \Rightarrow \lim_{x \to 0} \frac{\sin 2x}{\sin 5x} = \frac{2}{5}$$

ÖRNEK / EXAMPLE 13

$$\lim_{x \to 2} \frac{\sin(x^2-4)}{x-2} = ?$$

A) – 4 B) – 2 C) 0 D) 2 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\lim_{x \to 2} \frac{\sin(x^2-4)}{x-2} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{(x+2)\sin(x^2-4)}{x^2-4} = \lim_{x \to 2}(x+2) \cdot \lim_{t \to 0} \frac{\sin t}{t} \qquad \left[x^2-4=t \Rightarrow x \to 2 \Rightarrow t \to 0 \right]$$

$$= 4 \cdot 1$$

$$= 4$$

2. $\frac{\infty}{\infty}$ Belirsizliği / Indeterminate Form $\frac{\infty}{\infty}$

m, n ∈ N için / *For m,n ∈ N,*

$$\lim_{x\to\pm\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0}{b_mx^m+b_{m-1}x^{m-1}+\ldots+b_0}=\begin{cases}0\ , & n<m\\ \frac{a_n}{b_m}\ , & n=m\\ \pm\infty\ , & n>m\end{cases}$$

ÖRNEK / EXAMPLE 14

$$\lim_{x\to\infty}\frac{x^2-3}{x^3+2x+1}=?$$

A) 3 B) 2 C) 1 D) 0 E) – 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\lim_{x\to\infty}\frac{x^2-3}{x^3+2x+1}=\lim_{x\to\infty}\frac{x^2}{x^3}=0$$

PEKİŞTİRME / REINFORCE

$$\lim_{x\to-\infty}\frac{x^3-x^2+1}{x-1}=?$$

YANIT / ANSWER : $+\infty$

ÖRNEK / EXAMPLE 15

$$\lim_{x\to\infty} \frac{x^5 - x^4 + 1}{x^3 + 2x} = ?$$

A) $-\infty$ B) $-\frac{5}{3}$ C) 0 D) $\frac{5}{3}$ E) ∞

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\lim_{x\to\infty} \frac{x^5 - x^4 + 1}{x^3 + 2x} = \lim_{x\to\infty} \frac{x^5}{x^3} = \infty$$

ÖRNEK / EXAMPLE 16

$$\lim_{x\to\infty} \frac{4^{x+1} + 3^x + 2^x}{4^{x-1} + 3^x + 2} = ?$$

A) 4 B) 8 C) 12 D) 16 E) 20

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\lim_{x\to\infty} \frac{4^{x+1} + 3^x + 2^x}{4^{x-1} + 3^x + 2} = \lim_{x\to\infty} \frac{4^x\left(4 + \left(\frac{3}{4}\right)^x + \left(\frac{2}{4}\right)^x\right)}{4^x\left(\frac{1}{4} + \left(\frac{3}{4}\right)^x + \frac{2}{4^x}\right)}$$

$$= \lim_{x\to\infty} \frac{4 + 0 + 0}{\frac{1}{4} + 0 + 0} = 16$$

PEKİŞTİRME / REINFORCE

$$\lim_{x\to-\infty} \frac{3^{x+1} + 2^{x-1}}{2^{x-1} + 3^x} = ?$$

YANIT / ANSWER : 1

3. $\infty - \infty$ Belirsizliği / Indeterminate Form $\infty - \infty$

$\lim\limits_{x \to a} (f(x) - g(x))$ limitinin sonucu $\infty - \infty$ belirsizliği oluyorsa, $\frac{0}{0}$ ya da $\frac{\infty}{\infty}$ belirsizliklerine dönüştürülerek çözülür.

An expression such as $\infty - \infty$ is called an indeterminate form becuse we cannot determine the limit. When we encounter this form, we must rewrite it in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

NOT / NOTE

$a > 0$

$$\lim_{x \to \pm\infty} \sqrt{ax^2 + bx + c} = \sqrt{a} \lim_{x \to \pm\infty} \left| x + \frac{b}{2a} \right|$$

ÖRNEK / EXAMPLE 17

$$\lim_{x \to 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right) = ?$$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\lim_{x \to 2} \left(\frac{1}{x-2} - \frac{1}{x^2-4} \right) = \infty - \infty$$

$$\lim_{x \to 2} \frac{x+2-4}{x^2-4} = \lim_{x \to 2} \frac{x-2}{x^2-4}$$

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{x-2}{(x-2) \cdot (x+2)} = \frac{1}{4}$$

ÖRNEK / EXAMPLE 18

$$\lim_{x \to \infty} \left(\sqrt{x^2-1} - \sqrt{x^2+3x+1} \right) = ?$$

A) -2 B) $-\frac{5}{2}$ C) $-\frac{3}{2}$ D) 0 E) -1

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\lim_{x \to \infty} \left(\sqrt{x^2-1} - \sqrt{x^2+3x+1} \right) = \infty - \infty$$

$$\lim_{x \to \infty} \left(\sqrt{x^2-1} - \sqrt{x^2+3x+1} \right) = \left(\sqrt{1} \cdot \lim_{x \to \infty} |x+0| \right) - \left(\sqrt{1} \cdot \lim_{x \to \infty} \left|x + \frac{3}{2}\right| \right)$$

$$= x - \left(x + \frac{3}{2} \right)$$

$$= x - x - \frac{3}{2} = -\frac{3}{2}$$

4. $0 \cdot \infty$ Belirsizliği / Indeterminate Form $0 \cdot \infty$

$\lim\limits_{x \to a} f(x) \cdot g(x) = 0 \cdot \infty$ belirsizliği, $\frac{0}{0}$ veya $\frac{\infty}{\infty}$ belirsizliklerine dönüştürülerek hesaplanır.

The indeterminate $\lim\limits_{x \to a} f(x) \cdot g(x) = 0 \cdot \infty$ *is written in the form* $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$ *and then calculated.*

1. $\lim\limits_{x \to a} f(x) \cdot g(x) = \lim\limits_{x \to a} \dfrac{f(x)}{\dfrac{1}{g(x)}} \quad \left(\dfrac{0}{0}\right)$

2. $\lim\limits_{x \to a} f(x) \cdot g(x) = \lim\limits_{x \to a} \dfrac{g(x)}{\dfrac{1}{f(x)}} \quad \left(\dfrac{\infty}{\infty}\right)$

ÖRNEK / EXAMPLE 19

$\lim\limits_{x \to 0} x \cdot \cot 2x = ?$

A) $\frac{3}{2}$ B) 1 C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$\lim\limits_{x \to 0} x \cdot \cot(2x) \quad (0 \cdot \infty)$

$$\lim_{x \to 0} x \cdot \cot(2x) = \lim_{x \to 0} \frac{x}{\dfrac{1}{\cot(2x)}}$$

$$= \lim_{x \to 0} \frac{x}{\tan 2x}$$

$$= \frac{1}{2}$$

5. 1^{∞} Belirsizliği / Indeterminate Form 1^{∞}

$$\left.\begin{array}{l}\lim\limits_{x\to\infty} f(x)=0\\[2ex] \lim\limits_{x\to\infty} g(x)=+\infty\\[2ex] \lim\limits_{x\to\infty} (f(x)\cdot g(x))=k\end{array}\right\}\Rightarrow \lim_{x\to\infty}\left[1+f(x)\right]^{g(x)}=e^{k}$$

ÖRNEK / EXAMPLE 20

$$\lim_{x\to\infty}\left(\frac{x+7}{x+3}\right)^{x}=?$$

A) $\frac{1}{4}$ B) e^{2} C) e^{3} D) e^{4} E) $\frac{1}{4}e$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\lim_{x\to\infty}\left(\frac{x+7}{x+3}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{4}{x+3}\right)^{x}$$

$$\left.\begin{array}{l}\lim\limits_{x\to\infty}\dfrac{4}{x+3}=0\\[2ex] \lim\limits_{x\to\infty} x=\infty\\[2ex] \lim\limits_{x\to\infty} x\cdot\dfrac{4}{x+3}=4\end{array}\right\}\Rightarrow \lim_{x\to\infty}\left(\frac{x+7}{x+3}\right)^{x}=e^{4}$$

PEKİŞTİRME / REINFORCE

$$\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^{2x}=?$$

YANIT / ANSWER : e^{-4}

6. 0^0 ve ∞^0 Belirsizlikleri / Indeterminate forms 0^0 and ∞^0

$y = (f(x))^{g(x)}$ olmak üzere,

$\lim\limits_{x \to a} (f(x))^{g(x)}$ limiti hesaplandığında 0^0 veya ∞^0 belirsizliklerinin oluşması durumunda her iki tarafında logaritması alınarak limit hesaplanır.

$\lim\limits_{x \to a} \ln y = \lim\limits_{x \to a} g(x) \cdot \ln f(x) = 0 \cdot \infty$ belirsizliğine dönüştürülerek hesaplanır.

$y = (f(x))^{g(x)}$,

If we calculate $\lim\limits_{x \to a} (f(x))^{g(x)}$ and we obtain 0° or ∞^0, we can take logarithm both sides.

$\lim\limits_{x \to a} \ln y = \lim\limits_{x \to a} g(x) \cdot \ln f(x) = 0 \cdot \infty$ is obtained and evaluate.

ÖRNEK / EXAMPLE 21

$\lim\limits_{x \to 0} \sin x^{\sin x} = ?$

A) 0 B) 1 C) $\frac{3}{2}$ D) 2 E) ∞

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$y = \sin x^{\sin x}$

$\ln y = \sin x \cdot \ln \sin x$

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \sin x \cdot \ln \sin x \qquad \left[\frac{1}{\sin x} = t \,,\ x \to 0 \Rightarrow t \to \frac{1}{\sin 0} \to \frac{1}{0} \to \infty \right]$$

$$= -\lim_{t \to \infty} \frac{\ln t}{t} = 0$$

$$\lim_{x \to 0} \ln y = 0 \Rightarrow \ln\left(\lim_{x \to 0} y\right) = 0$$

$$\Rightarrow \lim_{x \to 0} y = e^0 = 1$$

ÖRNEK / EXAMPLE 22

ANKARA ÜNİ. YÖS 2013

$$\lim_{x\to 1}\frac{x\cdot\sqrt{2x-1}}{x-1}=?$$

A) $-\infty$ B) -1 C) 0 D) 1 E) ∞

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\lim_{x\to 1}\frac{x\cdot\sqrt{2x-1}}{x-1}=\frac{1\cdot\sqrt{1}}{0}=\infty$$

ÖRNEK / EXAMPLE 23

İSTANBUL ÜNİ. YÖS 2013

$$\lim_{x\to 3}f(x)=4$$

$$\Rightarrow \lim_{x\to 3}\frac{(f(x)-3)(f^2(x)-4f(x)+1)}{x+3}=?$$

A) 0 B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\lim_{x\to 3}\frac{(4-3)\cdot(4^2-4\cdot 4+1)}{3+3}=\frac{1\cdot 1}{6}$$

$$=\frac{1}{6}$$

ÖRNEK / EXAMPLE 24

OSMAN GAZİ ÜNİV. YÖS 2015

$$\lim_{x\to 2}\left(\frac{x^2-3x+2}{x^2-4}\right)=?$$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) $\frac{1}{6}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\lim_{x\to 2}\left(\frac{x^2-3x+2}{x^2-4}\right)=\frac{0}{0}$$

$$\Rightarrow \lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(x+2)}$$

$$=\lim_{x\to 2}\frac{x-1}{x+2}=\frac{2-1}{2+2}=\frac{1}{4}$$

ÖRNEK / EXAMPLE 25

KARADENİZ TEKNİK ÜNİV. YÖS 2016

$$\lim_{x\to \frac{\pi}{2}}\frac{\sin x-1}{\cos x+\sin x}=?$$

A) $-\infty$ B) -1 C) 0 D) 1 E) ∞

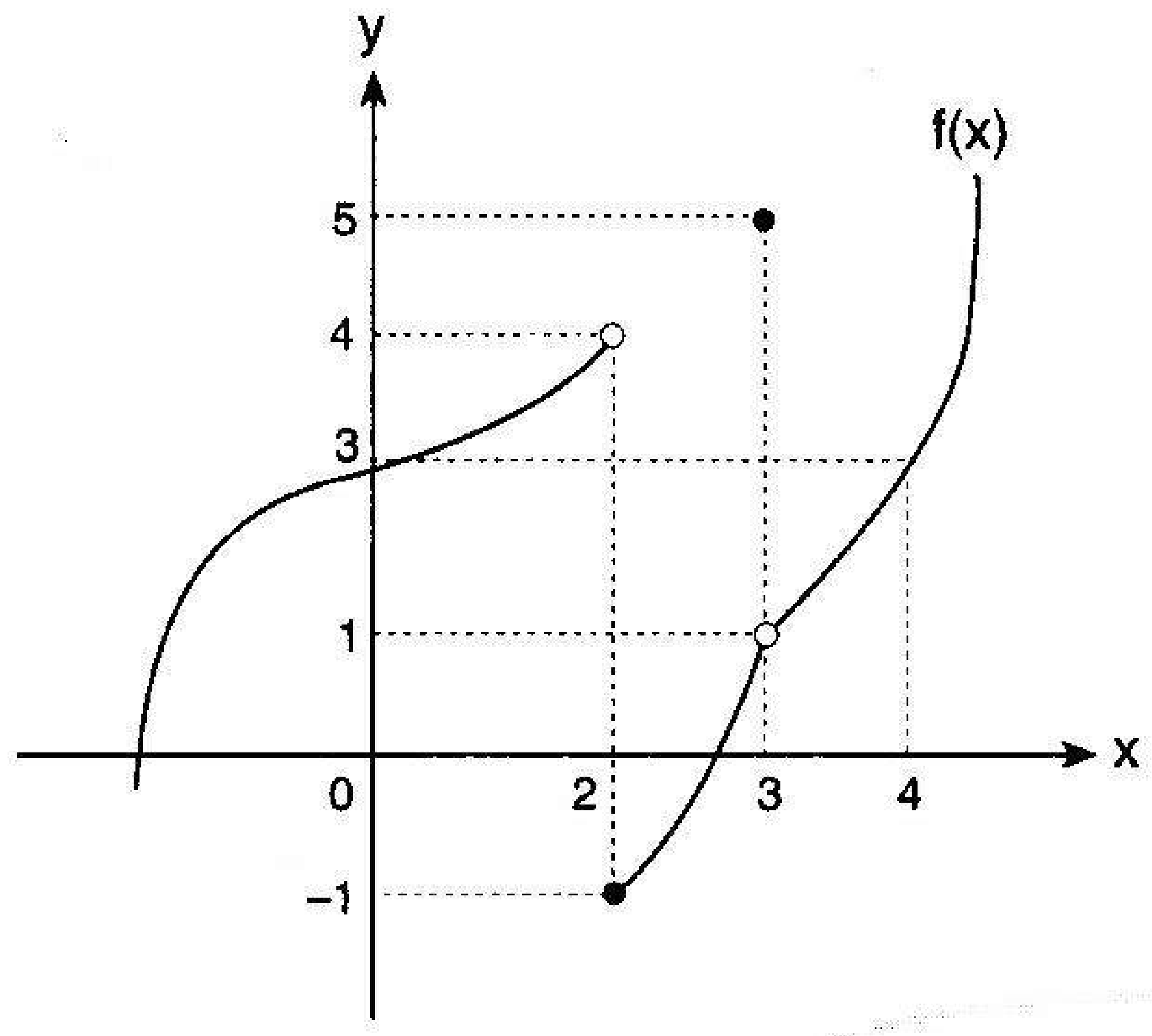
ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\lim_{x\to \frac{\pi}{2}}\frac{\sin x-1}{\cos x+\sin x}$$

$$=\frac{\cos\frac{\pi}{2}-1}{\cos\frac{\pi}{2}+\sin\frac{\pi}{2}}=\frac{1-1}{0+1}=\frac{0}{1}=0$$

1.



$$\lim_{x\to 2^-} f(x) + \lim_{x\to 3} f(x) + \lim_{x\to 4^+} f(x) = ?$$

A) 12
B) 10
C) 9
D) 8
E) 6

---

2. $f(x) = \begin{cases} 3x+a & x<2 \\ x+b & x=2 \\ 5x+c & x>2 \end{cases}$

$\lim\limits_{x\to 2} f(x) = -3 \Rightarrow \mathbf{a+c=?}$

A) –20
B) –22
C) –24
D) –25
E) –27

---

3. $\lim\limits_{x\to\infty} \left(\frac{2}{3}\right)^x + 5^{\frac{3}{x}} = ?$

A) 1
B) $\frac{2}{3}$
C) $\frac{3}{2}$
D) $\frac{4}{9}$
E) $\frac{17}{3}$

---

4. $\lim\limits_{x\to 4^+} \frac{|x^2-3x-4|}{x-4} = ?$

A) –5
B) –3
C) 1
D) 3
E) 5

---

5. $\lim\limits_{x\to -1} \frac{x^3+5x^2-2x}{3x^4-2x^3-4x^2+1} = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

---

6. $\lim\limits_{x\to\frac{\pi}{6}} \frac{\cot x - 2\cos x}{\sin x + \tan x} = ?$

A) – 2
B) – 1
C) 0
D) 1
E) 2

7. $\lim\limits_{x\to 0} \dfrac{\cos x - \cos^3 x}{x^2} = ?$

A) – 2
B) – 1
C) 0
D) 1
E) 2

8. $\lim\limits_{x\to 0} \dfrac{\sin 3x}{x} = ?$

A) – 3
B) – 1
C) 0
D) 1
E) 3

9. $\lim\limits_{x\to 1} \dfrac{\sin(x-1)}{x^2-1} = ?$

A) $\frac{1}{2}$
B) 1
C) $\frac{3}{2}$
D) 2
E) $\frac{5}{2}$

10. $\lim\limits_{x\to 0} \dfrac{\sin^2 x + \tan^2 x}{4x^2} = ?$

A) $\frac{1}{2}$
B) $\frac{1}{3}$
C) $\frac{1}{4}$
D) $\frac{1}{5}$
E) $\frac{1}{6}$

11. $\lim\limits_{x\to 1} \dfrac{ax - \sqrt{x+3}}{x^2-1} = b,\ b \in R \Rightarrow$ **a = ?**

A) 2
B) 3
C) 4
D) 5
E) 6

12. $\lim\limits_{x\to \infty} \dfrac{x^3 - 3}{3x^3 + 2x + 1} = ?$

A) $-\frac{1}{3}$
B) $-\frac{1}{2}$
C) $\frac{1}{3}$
D) $\frac{1}{2}$
E) 1

YANITLAR / ANSWERS

1. D 2. B 3. A 4. E 5. C 6. C 7. D 8. E
9. A 10. A 11. A 12. C

1.

$$\left.\begin{aligned}&\lim_{x\to 2^-} f(x)=4\\&\lim_{x\to 3} f(x)=1\\&\lim_{x\to 4^+} f(x)=3\end{aligned}\right\}\Rightarrow \lim_{x\to 2^-} f(x)+\lim_{x\to 3} f(x)+\lim_{x\to 4^+} f(x)=8$$

Yanıt / The Answer is D

2.

$$\lim_{x\to 2} f(x)=-3 \Rightarrow \lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=-3$$

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(3x+a)=6+a=-3\Rightarrow a=-9$$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}(5x+c)=10+c=-3\Rightarrow c=-13$$

$$\Rightarrow a+c=-9-13=-22$$

Yanıt / The Answer is B

3.

$$\lim_{x\to\infty}\left(\frac{2}{3}\right)^x+5^{\frac{3}{x}}=\left(\frac{2}{3}\right)^{\infty}+5^{\frac{3}{\infty}}$$

$$0<\frac{2}{3}<1\Rightarrow\left(\frac{2}{3}\right)^{\infty}=0$$

$$\frac{3}{\infty}=0\Rightarrow 5^0=1$$

$$\left(\frac{2}{3}\right)^{\infty}+5^{\frac{3}{\infty}}=0+1=1$$

Yanıt / The Answer is A

4.

$$\lim_{x\to 4^+}\frac{|x^2-3x-4|}{x-4}=\lim_{x\to 4^+}\frac{|x-4|\cdot|x+1|}{x-4}$$

$$=\lim_{x\to 4^+}\frac{(x-4)\cdot(x+1)}{x-4}$$

$$\Rightarrow\lim_{x\to 4^+}(x+1)=5$$

Yanıt / The Answer is E

5.

$$\lim_{x\to -1}\frac{x^3+5x^2-2x}{3x^4-2x^3-4x^2+1}$$

$$=\frac{(-1)^3+5\cdot(-1)^2-2\cdot(-1)}{3\cdot(-1)^4-2\cdot(-1)^3-4\cdot(-1)^2+1}$$

$$=\frac{-1+5+2}{3+2-4+1}$$

$$=\frac{6}{2}$$

$$=3$$

Yanıt / The Answer is C

6.

$$\lim_{x\to\frac{\pi}{6}}\frac{\cot x-2\cos x}{\sin x+\tan x}=\frac{\cot\frac{\pi}{6}-2\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}+\tan\frac{\pi}{6}}$$

$$=\frac{\sqrt{3}-2\cdot\frac{\sqrt{3}}{2}}{\frac{1}{2}+\frac{\sqrt{3}}{3}}$$

$$=\frac{\sqrt{3}-\sqrt{3}}{\frac{3+2\sqrt{3}}{6}}=0$$

Yanıt / The Answer is C

7. $$\lim_{x\to 0}\frac{\cos x-\cos^3 x}{x^2}=\lim_{x\to 0}\frac{\cos x\cdot(1-\cos^2 x)}{x^2}$$

$$=\lim_{x\to 0}\cos x\cdot\lim_{x\to 0}\frac{\sin^2 x}{x^2}$$

$$=1\cdot 1=1$$

Yanıt / The Answer is D

---

8. $$\lim_{x\to 0}\frac{\sin 3x}{x}=3\cdot\lim_{3x\to 0}\frac{\sin 3x}{3x}=3$$

Yanıt / The Answer is E

---

9. $$\lim_{x\to 1}\frac{\sin(x-1)}{x^2-1}=\frac{\sin 0}{0}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{\sin(x-1)}{x^2-1}=\lim_{x\to 1}\frac{\sin(x-1)}{(x-1)(x+1)}$$

$$=\lim_{x\to 1}\frac{\sin(x-1)}{x-1}\cdot\lim_{x\to 1}\frac{1}{x+1}\qquad\left[\begin{array}{l} x-1=t\\ x\to 1\Rightarrow t\to 0\end{array}\right]$$

$$=\lim_{t\to 0}\frac{\sin t}{t}\cdot\lim_{t\to 0}\frac{1}{t+2}$$

$$=1\cdot\frac{1}{2}=\frac{1}{2}$$

Yanıt / The Answer is A

---

10. $$\lim_{x\to 0}\frac{\sin^2 x+\tan^2 x}{4x^2}=\lim_{x\to 0}\left(\frac{\sin^2 x}{4x^2}+\frac{\tan^2 x}{4x^2}\right)$$

$$=\frac{1}{4}+\frac{1}{4}$$

$$=\frac{2}{4}=\frac{1}{2}$$

Yanıt / The Answer is A

---

11. $$\lim_{x\to 1}\frac{ax-\sqrt{x+3}}{x^2-1}=\frac{a-\sqrt{4}}{1^2-1}=b$$

$$b\in R\ \Rightarrow\ a-\sqrt{4}=0$$

$$a=2$$

Yanıt / The Answer is A

---

12. $$\lim_{x\to\infty}\frac{x^3-3}{3x^3+2x+1}\quad\lim_{x\to\infty}=\frac{x^3\cdot\left(1-\frac{2}{x^2}+\frac{5}{x^3}\right)}{x^3\cdot\left(3+\frac{1}{x^2}-\frac{2}{x^3}\right)}$$

$$=\frac{1-\frac{2}{\infty^2}+\frac{5}{\infty^3}}{3+\frac{1}{\infty^2}+\frac{2}{\infty^3}}$$

$$=\frac{1-0+0}{3+0-0}$$

$$=\frac{1}{3}$$

Yanıt / The Answer is C

1. $\lim\limits_{x \to \infty} \dfrac{(a-2)x^3 - 3x^2 + x + 1}{bx^2 + 4x + 5} = 3 \Rightarrow a + b = ?$

A) –3

B) –2

C) –1

D) 1

E) 2

2. $\lim\limits_{x \to \infty} \dfrac{3x^2 - 3x + 4}{x^3 - 4x + 5} = ?$

A) –1

B) 0

C) 1

D) 2

E) 3

3. $\lim\limits_{x \to \infty} \dfrac{2^{x+1} + 3^{x+1}}{2^x + 3^x} = ?$

A) $\frac{1}{3}$

B) $\frac{2}{3}$

C) 1

D) $\frac{3}{2}$

E) 3

4. $\lim\limits_{x \to \infty} \left(\dfrac{\sin x}{x} + \dfrac{x^2 + 3x}{2x^2 - 1} \right) = ?$

A) $\frac{3}{2}$

B) $\frac{1}{2}$

C) 1

D) 2

E) ∞

5. $\lim\limits_{x \to -\infty} \dfrac{4}{1 - 3^{\frac{x}{1-x}}} = ?$

A) 4

B) 6

C) 8

D) 10

E) 12

6. $\lim\limits_{x \to -4} \left(\dfrac{1}{x+4} - \dfrac{8}{16 - x^2} \right) = ?$

A) $\frac{1}{2}$

B) $\frac{1}{4}$

C) $\frac{1}{8}$

D) $-\frac{1}{4}$

E) $-\frac{1}{8}$

7. $\lim\limits_{x \to -\infty} \left(\sqrt{x^2 - 4x + 6} - \sqrt{x^2 + ax + 3} \right) = 6 \quad \Rightarrow$ **a = ?**

A) –16

B) –12

C) –8

D) 8

E) 16

8. $\lim\limits_{x \to 0} 2x \cdot \cot 4x = ?$

A) $\frac{1}{2}$

B) 1

C) $\frac{3}{2}$

D) 2

E) 4

9. $\lim\limits_{x \to \infty} \left(\frac{x+5}{x+1} \right)^{3x+2} = ?$

A) e

B) e^3

C) e^4

D) e^6

E) e^{12}

10. $\lim\limits_{x \to \infty} \left(\frac{3x+1}{3x-2} \right)^{x} = ?$

A) e

B) e^2

C) e^3

D) e^4

E) e^5

11. $\lim\limits_{x \to 1} x^{\frac{1}{1-x}} = ?$

A) e^{-1}

B) 1

C) e

D) e^2

E) e^3

12. $\lim\limits_{x \to \infty} \left(e^{\frac{1}{x}} + \frac{2}{x} \right)^{2x} = ?$

A) 0

B) 1

C) e

D) e^2

E) e^4

YANITLAR / ANSWERS

1. D	**2. B**	**3. E**	**4. A**	**5. B**	**6. E**	**7. D**	**8. A**
9. E	**10. A**	**11. A**	**12. E**				

1. $$\lim_{x\to\infty}\frac{(a-2)x^3-3x^2+x+1}{bx^2+4x+5}=3$$

$$\lim_{x\to\infty} f(x)=3 \;\Rightarrow\; a-2=0\,,\; b=-1$$

$$\Rightarrow a+b=2-1=1$$

Yanıt / The Answer is D

2. $$\lim_{x\to\infty}\frac{3x^2-3x+4}{x^3-4x+5}=\lim_{x\to\infty}\frac{3x^2}{x^3}=\lim_{x\to\infty}\frac{3}{x}=\frac{3}{\infty}=0$$

Yanıt / The Answer is B

3. $$\lim_{x\to\infty}\frac{2^{x+1}+3^{x+1}}{2^x+3^x}=\lim_{x\to\infty}\frac{3^{x+1}\left(\left(\frac{2}{3}\right)^{x+1}+1\right)}{3^x\left(\left(\frac{2}{3}\right)^{x}+1\right)}=3$$

Yanıt / The Answer is E

4. $$\lim_{x\to\infty}\left(\frac{\sin x}{x}+\frac{x^2+3x}{2x^2-1}\right)=\lim_{x\to\infty}\frac{\sin x}{x}+\lim_{x\to\infty}\frac{x^2+3x}{2x^2-1}$$

$$=0+\frac{1}{2}=\frac{1}{2}$$

Yanıt / The Answer is B

5. $$\lim_{x\to-\infty}\frac{4}{1-3^{\frac{x}{1-x}}}=\frac{4}{1-3^{\lim\limits_{x\to-\infty}\frac{x}{1-x}}}$$

$$=\frac{4}{1-3^{-1}}$$

$$=\frac{4}{1-\frac{1}{3}}$$

$$=\frac{12}{2}=6$$

Yanıt / The Answer is B

6. $$\lim_{x\to-4}\left(\frac{1}{x+4}-\frac{8}{16-x^2}\right)=\lim_{x\to-4}\frac{x-4+8}{x^2-16}$$

$$=\lim_{x\to-4}\frac{1}{x-4}$$

$$=-\frac{1}{8}$$

Yanıt / The Answer is E

7. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-4x+6}-\sqrt{x^2+ax+3}\right)$

$$=\lim_{x\to-\infty}\sqrt{x^2-4x+6}-\lim_{x\to-\infty}\sqrt{x^2+ax+3}=6$$

$$=\sqrt{1}\lim_{x\to-\infty}\left|x-\frac{4}{2}\right|-\sqrt{1}\lim_{x\to-\infty}\left|x+\frac{a}{2}\right|=6$$

$$=-x+2+x+\frac{a}{2}=6$$

$$2+\frac{a}{2}=6\ \Rightarrow\ a=8$$

Yanıt / The Answer is D

8. $\lim\limits_{x\to0}2x\cdot\cot4x\quad(0\cdot\infty)$

$$\lim_{x\to0}\frac{2x}{\dfrac{1}{\cot4x}}=\lim_{x\to0}\frac{2x}{\tan4x}\qquad\left(\frac{0}{0}\right)$$

$$\Rightarrow\lim_{x\to0}\frac{2x}{\tan4x}=\frac{1}{2}$$

Yanıt / The Answer is A

9. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+1}\right)^{3x+2}=\lim\limits_{x\to\infty}\left(1+\dfrac{4}{x+1}\right)^{3x+2}$

$$\left.\begin{array}{l}\lim\limits_{x\to\infty}\dfrac{4}{x+1}=0\\[2ex]\lim\limits_{x\to\infty}3x+2=\infty\\[2ex]\lim\limits_{x\to\infty}\dfrac{4}{x+1}\cdot(3x+2)=12\end{array}\right\}\Rightarrow\lim_{x\to\infty}\left(1+\frac{4}{x+1}\right)^{3x+2}=e^{12}$$

Yanıt / The Answer is E

10. $\lim\limits_{x\to\infty}\left(\dfrac{3x+1}{3x-2}\right)^{x}=\lim\limits_{x\to\infty}\left(1+\dfrac{3}{3x-2}\right)^{x}$

$$\left.\begin{array}{l}\lim\limits_{x\to\infty}\dfrac{3}{3x-2}=0\\[2ex]\lim\limits_{x\to\infty}x=\infty\\[2ex]\lim\limits_{x\to\infty}\dfrac{3}{3x-2}\cdot x=1\end{array}\right\}\Rightarrow\lim_{x\to\infty}\left(1+\frac{3}{3x-2}\right)^{x}=e$$

Yanıt / The Answer is A

11. $\dfrac{1}{1-x}=t\ \Rightarrow\ x=\dfrac{t-1}{t}$

$$\lim_{x\to1}x^{\frac{1}{1-x}}=\lim_{t\to\infty}\left(1-\frac{1}{t}\right)^{t}=e^{-1}$$

Yanıt / The Answer is A

12. $\lim\limits_{x\to\infty}\left(e^{\frac{1}{x}}+\dfrac{2}{x}\right)^{2x}=\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{2x}$

$$\left.\begin{array}{l}\lim\limits_{x\to\infty}\dfrac{2}{x}=0\\[2ex]\lim\limits_{x\to\infty}2x=\infty\\[2ex]\lim\limits_{x\to\infty}2x\cdot\dfrac{2}{x}=4\end{array}\right\}\Rightarrow\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{2x}=e^{4}$$

Yanıt / The Answer is E

1. $\lim\limits_{x \to 3^-} \dfrac{\sqrt{x^2 - 6x + 9}}{x - 3} = ?$

Yanıt / The Answer is: **−1**

2. $f(x) = \begin{cases} x + 2 & x \geq 2 \\ 2x - 1 & x < 2 \end{cases}$

$\Rightarrow \lim\limits_{x \to 2^-} f(x) + \lim\limits_{x \to 2^+} f(x) + \lim\limits_{x \to 3} f(x) = ?$

Yanıt / The Answer is: **12**

3. $\lim\limits_{\frac{a}{b} \to 1} \dfrac{4a + 3b}{2a + b} = ?$

Yanıt / The Answer is: $\dfrac{7}{3}$

4. $\lim\limits_{x \to 2^-} \left(\dfrac{|x - 2|}{x - 2} \right) + \lim\limits_{x \to 2^+} \left(\dfrac{|x - 2|}{x - 2} \right) = ?$

Yanıt / The Answer is: **0**

5. $\lim\limits_{x \to -\infty} \dfrac{3 \cdot 2^x + 7}{4 - 2^{x+1}} = ?$

Yanıt / The Answer is: $\dfrac{7}{4}$

6. $\lim\limits_{x \to 2} \dfrac{4x^2 - 2x + m - 2}{x^2 - 4} \in R \Rightarrow m = ?$

Yanıt / The Answer is: **−10**

7. $\lim\limits_{x \to 2} \dfrac{\tan(2x-4)}{x-2} = ?$

Yanıt / The Answer is: 2

8. $\lim\limits_{\theta \to x} \dfrac{\tan x - \tan\theta}{\tan(\theta - x)} = ?$

Yanıt / The Answer is: $-\sec^2 x$

9. $\lim\limits_{x \to 3} \dfrac{x^3 - 3x^2 + x - 3}{x^2 - x - 6} = ?$

Yanıt / The Answer is: 2

10. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin x - \cos x}{1 - \tan x} = ?$

Yanıt / The Answer is: $-\dfrac{\sqrt{2}}{2}$

11. $\lim\limits_{x \to 0} \left(\dfrac{\sin 3x}{x} + \dfrac{2x}{\tan x} \right) = ?$

Yanıt / The Answer is: 5

12. $\lim\limits_{x \to 0} \left(\dfrac{\sin^3 x}{8x^3} \right) = ?$

Yanıt / The Answer is: $\dfrac{1}{8}$

13. $\lim\limits_{x\to\frac{\pi}{4}} \frac{\sin^2 x-\cos^2 x}{\sin x-\cos x}=?$

Yanıt / The Answer is: **$\sqrt{2}$**

14. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin\left(x-\frac{\pi}{2}\right)}{\cos x}=k,\quad k\in R$

$\Rightarrow \lim\limits_{x\to k}\left(3^{x+1}+e^{-x}\right)=?$

Yanıt / The Answer is: **1 + e**

15. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}=?$

Yanıt / The Answer is: **1**

16. $\lim\limits_{x\to 9} \frac{2\sqrt{x}-6}{x-9}=?$

Yanıt / The Answer is: $\frac{1}{3}$

17. $\lim\limits_{y\to x} \frac{\sin^2 y-\sin^2 x}{y^2-x^2}=?$

Yanıt / The Answer is: **1**

18. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+7x}-\sqrt{x^2+2x}\right)=?$

Yanıt / The Answer is: $\frac{5}{2}$

19. $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2+6x}-\sqrt{4x^2+9}\right)=?$

Yanıt / The Answer is: $-\frac{3}{2}$

20. $\lim\limits_{x\to5}\left(\frac{1}{x-5}-\frac{3x-8}{x^2-3x-10}\right)=?$

Yanıt / The Answer is: $\frac{-2}{7}$

21. $\lim\limits_{x\to\infty}\left(x\cdot\tan\frac{3}{x}\right)=?$

Yanıt / The Answer is: 3

22. $\lim\limits_{x\to\infty}\left[(x^2-4)\cdot\frac{3}{2x}\right]=?$

Yanıt / The Answer is: ∞

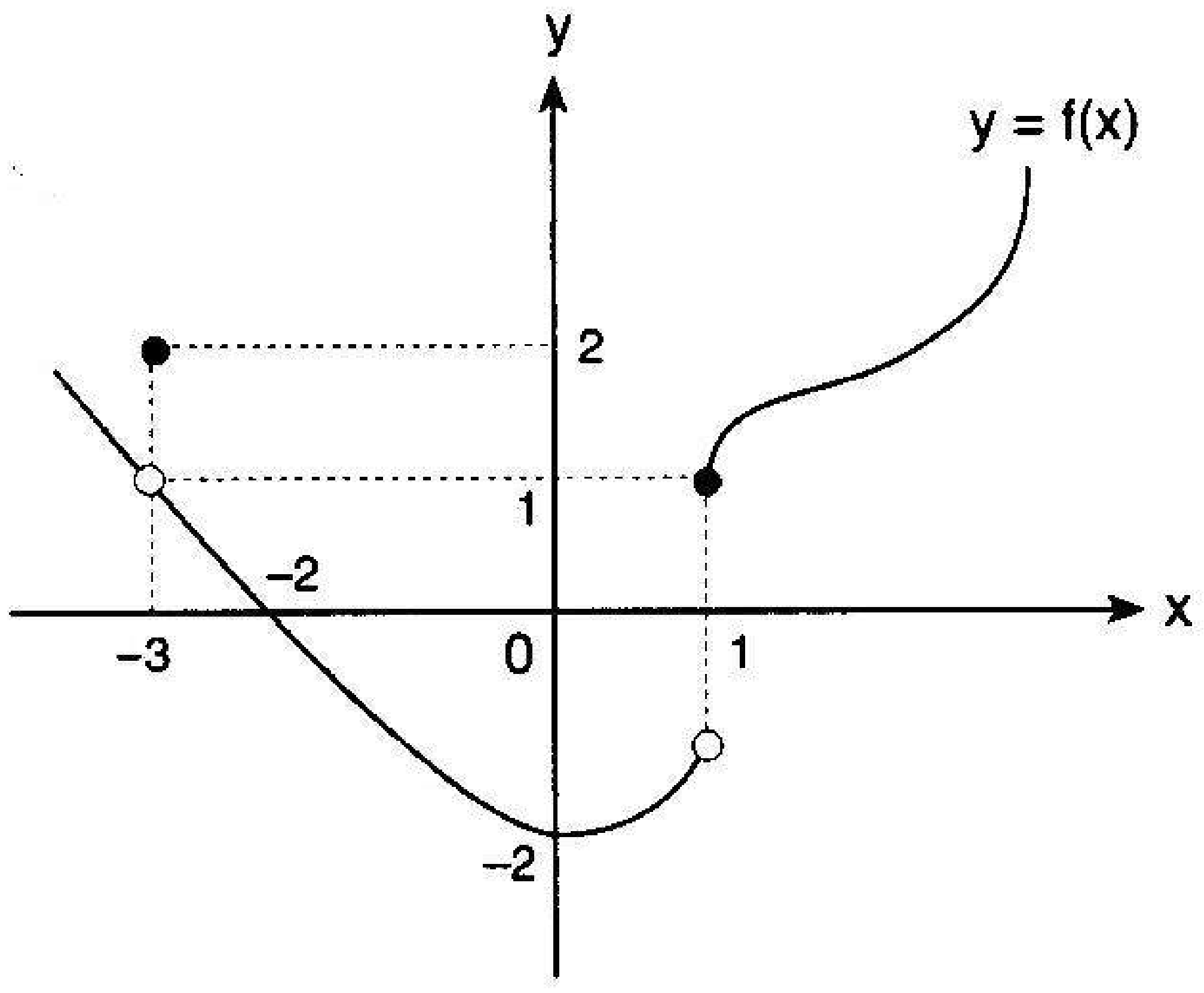
23. $\lim\limits_{x\to\infty}\left(\frac{x-2}{x+2}\right)^{x+2}=?$

Yanıt / The Answer is: e^{-4}

24. $\lim\limits_{x\to0}(1+3x)^{\frac{2}{x}}=?$

Yanıt / The Answer is: e^6

1.



$\lim\limits_{x\to-3} f(x)+\lim\limits_{x\to-2} f(x)+\lim\limits_{x\to1^+} f(x)=?$

A) 1
B) 2
C) 3
D) 4
E) 5

2. $\lim\limits_{x\to3} 2x^2-7x+6=?$

A) 0
B) 2
C) 3
D) 5
E) 8

3. $f(x)=\begin{cases} x^2+x-1 & x\le1 \\ x^2-x & x>1 \end{cases}$

$\Rightarrow \lim\limits_{x\to1^-} f(x)=?$

A) 0
B) 1
C) 2
D) 3
E) 4

4. $\lim\limits_{x\to3} \dfrac{x^2-2x+5}{x-1}=?$

A) 0
B) 2
C) 4
D) 6
E) 8

5. $\lim\limits_{x\to1} \dfrac{|x-3|}{x^2+1}=?$

A) −1
B) 0
C) 1
D) 3
E) 5

6. $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin2x+1}{\cos x-3}=?$

A) $-\dfrac{1}{3}$
B) $-\dfrac{1}{4}$
C) $-\dfrac{1}{5}$
D) $-\dfrac{1}{6}$
E) $-\dfrac{1}{7}$

7. $\lim\limits_{x \to y} \dfrac{3x^2 - y^2}{x^2 + y^2} = ?$

A) –1

B) 0

C) 1

D) 2

E) 3

8. $\lim\limits_{x \to 3^+} \dfrac{|x-3|}{2x + |x-2|} = ?$

A) $\dfrac{1}{4}$

B) $\dfrac{1}{5}$

C) $\dfrac{1}{6}$

D) 0

E) $\dfrac{1}{8}$

9. $f(x) = \begin{cases} 2x^2 - 3 & x \le 1 \\ x^2 + 2 & x > 1 \end{cases}$

$\Rightarrow \lim\limits_{x \to 1^+} f(x) = ?$

A) –3

B) –1

C) 1

D) 3

E) 5

10. $\lim\limits_{x \to -\infty} \left(5^x + \dfrac{4}{x} + 3\right) = ?$

A) 1

B) 3

C) 4

D) 5

E) 6

11. $\lim\limits_{x \to 3^+} \dfrac{|x^2 - 2x - 3|}{x - 3} = ?$

A) –4

B) –2

C) 1

D) 4

E) 5

12. $\lim\limits_{x \to 2^-} \dfrac{|x^2 - 4|}{x - 2} = ?$

A) –6

B) –4

C) 0

D) 2

E) 4

YANITLAR / ANSWERS

1. B 2. C 3. B 4. C 5. C 6. A 7. C 8. D
9. D 10. B 11. D 12. B

1. $\lim\limits_{x \to 3} \dfrac{x^2+2x-15}{x^2-9} = ?$

A) $\frac{3}{2}$

B) $\frac{4}{3}$

C) $\frac{5}{4}$

D) $\frac{6}{5}$

E) $\frac{7}{6}$

2. $\lim\limits_{x \to -2} \dfrac{x^3+2x^2+x+2}{x^2-4} = ?$

A) $-\frac{1}{4}$

B) $-\frac{3}{4}$

C) $-\frac{5}{4}$

D) $-\frac{7}{4}$

E) $-\frac{9}{4}$

3. $\lim\limits_{x \to y} \dfrac{x\sqrt{x}-y\sqrt{y}}{\sqrt{x}-\sqrt{y}} = ?$

A) 3y
B) 2y
C) y
D) −y
E) −2y

4. $\lim\limits_{x \to 3} \dfrac{\sqrt{x^2-9}+\sqrt{x-3}}{\sqrt{x-3}} = ?$

A) $\sqrt{2}+1$

B) $\sqrt{3}+1$

C) $\sqrt{6}+1$

D) $\sqrt{3}+2$

E) $\sqrt{6}+2$

5. $\lim\limits_{x \to 0} \dfrac{\sin x}{3x} = ?$

A) 0

B) $\frac{1}{3}$

C) 1

D) 3

E) ∞

6. $\lim\limits_{x \to 4} \dfrac{\sin(x-4)}{x^2-16} = ?$

A) $\frac{1}{4}$

B) $\frac{1}{5}$

C) $\frac{1}{6}$

D) $\frac{1}{7}$

E) $\frac{1}{8}$

7. $\lim\limits_{x \to 0} \dfrac{\sin^3 x + \sin x^3}{x^3} = ?$

A) –2

B) –1

C) 0

D) 1

E) 2

8. $\lim\limits_{x \to -1} \dfrac{3x^2 + ax - 2}{x^2 - 1} = b \in R \Rightarrow$ **a = ?**

A) –1

B) 0

C) 1

D) 2

E) 3

9. $\lim\limits_{x \to 0} \dfrac{\sin x + \tan 2x}{3x} = ?$

A) $\frac{1}{3}$

B) $\frac{2}{3}$

C) 1

D) $\frac{4}{3}$

E) $\frac{5}{3}$

10. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cos x - \sin x}{\cos 2x} = ?$

A) $-\frac{\sqrt{2}}{2}$

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) $\frac{\sqrt{2}}{2}$

11. $\lim\limits_{x \to 3} \dfrac{\sin(3x-9)}{12x-36} = ?$

A) 0

B) 1

C) $\frac{1}{2}$

D) $\frac{1}{3}$

E) $\frac{1}{4}$

12. $\lim\limits_{x \to 0} \dfrac{\cos 2x + \sin x - 1}{x} = ?$

A) 0

B) 1

C) 2

D) 3

E) 4

YANITLAR / ANSWERS

1. B **2. C** **3. A** **4. C** **5. B** **6. E** **7. E** **8. C**
9. C **10. E** **11. E** **12. B**

1. $\lim\limits_{x \to -2} \dfrac{x^2 - 4}{x + 2} = ?$

A) –4 B) –2 C) 1

D) 2 E) 4

2. $\lim\limits_{x \to 0} \dfrac{\sin(\tan x)}{\sin x} = ?$

A) –1 B) 0 C) 1

D) 2 E) $\dfrac{\pi}{2}$

3. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin(\cos x)}{\cos x} = ?$

A) –1 B) 0 C) 1

D) $\dfrac{\pi}{2}$ E) $\dfrac{2}{\pi}$

4. $\lim\limits_{x \to \pi} \dfrac{\cos x + 1}{\sin x} = ?$

A) –2 B) –1 C) 0

D) 1 E) 2

5. $\lim\limits_{x \to 0} \dfrac{\cos^2 x - 1}{\sin x} = ?$

A) –2 B) –1 C) 0

D) 1 E) 2

6. $\lim\limits_{x \to 1} \dfrac{\sin(x-1)}{x^3 - 1} = ?$

A) $\dfrac{1}{2}$ B) $\dfrac{1}{3}$ C) $\dfrac{1}{4}$

D) $\dfrac{1}{5}$ E) $\dfrac{1}{6}$

7. $\lim\limits_{x\to\infty}\dfrac{\sqrt{4x^2-x+5}+x}{3x-2}=?$

A) 1 B) $\frac{4}{3}$ C) $\frac{5}{3}$

D) 2 E) $\frac{7}{3}$

8. $\lim\limits_{x\to\infty}\left(\dfrac{1}{x^2}+\dfrac{2}{x^2}+\dfrac{3}{x^2}+\ldots+\dfrac{x-1}{x^2}\right)=?$

A) 0

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{5}$

E) ∞

9. $\lim\limits_{x\to 3}\left(\log_3(6x-9)-\log_3(2x-3)\right)=?$

A) 1 B) 3 C) 6

D) 9 E) ∞

10. $\lim\limits_{x\to\infty} x\cdot\sin\dfrac{1}{x}=?$

A) 0 B) $\frac{1}{2}$ C) 1

D) 2 E) ∞

11. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{3x}=?$

A) e^2 B) e^4 C) e^6

D) e^8 E) e^{10}

12. $\lim\limits_{x\to\infty}\left(1+\dfrac{x-3}{x^2+4}\right)^{3x}=?$

A) 1 B) e C) e^2

D) e^3 E) e^4

YANITLAR / ANSWERS							
1. A	2. C	3. C	4. C	5. C	6. B	7. A	8. A
9. A	10. C	11. C	12. D				

1. $\lim\limits_{x \to y} \dfrac{x^2 - y^2}{x - y} = ?$

A) $-2y$ B) $-y$ C) 0 D) y E) $2y$

2. $\lim\limits_{x \to 0} \dfrac{9^x - 1}{3^x - 1} = ?$

A) 1 B) 2 C) 4 D) 6 E) 12

3. $\lim\limits_{x \to 3^-} \dfrac{|x^2 - x - 6|}{x - 3} = ?$

A) -7 B) -6 C) -5 D) -4 E) -3

4. $\lim\limits_{x \to 0} \dfrac{\sin 4x \cdot \cos 4x}{2x} = ?$

A) -2 B) -1 C) 0 D) 1 E) 2

5. $\lim\limits_{x \to \pi} \dfrac{\sin x \cdot \sin 3x}{\cos 4x - \cos 2x} = ?$

A) 0 B) $-\dfrac{1}{2}$ C) -1 D) $-\dfrac{3}{2}$ E) -2

6. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 - x + 4} + 3x}{2x - 3} = ?$

A) 1 B) $\dfrac{3}{2}$ C) 2 D) $\dfrac{5}{2}$ E) 3

7. $\lim\limits_{x\to\infty}\left(\dfrac{3x}{|x+3|}+\dfrac{4x+1}{|-2x|}\right)=?$

A) 1
B) 3
C) 5
D) 7
E) 9

8. $\lim\limits_{x\to\infty}\dfrac{3^x-4^{x+1}}{3^x+4^{x+1}}=?$

A) -1
B) $-\dfrac{1}{2}$
C) $-\dfrac{1}{3}$
D) $-\dfrac{1}{4}$
E) $-\dfrac{1}{5}$

9. $\lim\limits_{x\to\infty}\left(\sqrt{x+4}-\sqrt{x+3}\right)=?$

A) –1
B) 0
C) 1
D) 2
E) 3

10. $\lim\limits_{x\to 0}\left[(\cos x-1)\cdot\cot^2 x\right]=?$

A) -2
B) -1
C) $-\dfrac{1}{2}$
D) $-\dfrac{1}{4}$
E) 0

11. $\lim\limits_{x\to\infty}\left(1+\dfrac{4x}{x^2+3}\right)^{2x}=?$

A) e^2
B) e^4
C) e^6
D) e^8
E) e^{10}

12. $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x-5}\right)^{3x}=?$

A) e^{24}
B) e^{20}
C) e^{16}
D) e^{18}
E) e^{12}

YANITLAR / ANSWERS

1. E	2. B	3. C	4. E	5. B	6. A	7. C	8. A
9. B	10. C	11. D	12. A				

1. $\lim\limits_{x \to 1} \dfrac{x^3 - 3x^2 + x + 1}{x^2 + 3x - 4} = ?$

A) $-\dfrac{1}{5}$ B) $-\dfrac{2}{5}$ C) $-\dfrac{3}{5}$ D) $-\dfrac{4}{5}$ E) 1

2. $\lim\limits_{x \to 0} \dfrac{\sin x + \tan 3x}{x} = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

3. $\lim\limits_{x \to 0} \dfrac{\tan^2 2x}{x^2} = ?$

A) 1 B) 2 C) 4 D) 6 E) 8

4. $\lim\limits_{x \to \infty} \left(\dfrac{3^{x+1} - 5^x}{3^{x+2} - 5^{x+1}} \right) = ?$

A) 0 B) $\dfrac{1}{2}$ C) $\dfrac{1}{3}$ D) $\dfrac{1}{4}$ E) $\dfrac{1}{5}$

5. $\lim\limits_{x \to \infty} \dfrac{3x^3 - x^2 + x - 5}{x^4 - x^2 + 1} = ?$

A) 0 B) 1 C) 2 D) 4 E) 5

6. $\lim\limits_{x \to \infty} \dfrac{\sqrt{9x^2 - x + 5} + x}{2x - 3} = ?$

A) 1
B) 2
C) 3
D) 4
E) 5

7. $\lim\limits_{x \to \infty} \left(\log_{10}\sqrt{10x^2+3} - \log_{10}\sqrt{x^2-x+1} \right) = ?$

A) 0

B) 1

C) $\frac{1}{2}$

D) $\sqrt{10}$

E) ∞

8. $\lim\limits_{x \to -\infty} \left(\sqrt{4x^2-2x+3} + x \right) = ?$

A) $-\infty$

B) -2

C) 0

D) 2

E) ∞

9. $\lim\limits_{x \to \frac{\pi}{2}} \left[(1-\sin x) \cdot \tan^2 x \right] = ?$

A) $\frac{1}{2}$

B) 1

C) $\frac{3}{2}$

D) 2

E) 4

10. $\lim\limits_{x \to \infty} \left(\frac{x^2+7}{x^2+3} \right)^{2x^2} = ?$

A) e^7

B) e^8

C) e^{10}

D) e^{14}

E) e^{16}

11. $\lim\limits_{x \to \infty} \left(1 + \frac{3x}{x^2-3} \right)^x = ?$

A) e

B) e^2

C) e^3

D) e^4

E) e^5

12. $\lim\limits_{x \to 0} x^x = ?$

A) Limit yoktur. / *There is not limit.*

B) 3

C) 2

D) 1

E) 0

YANITLAR / ANSWERS							
1. B	2. D	3. C	4. E	5. A	6. B	7. C	8. E
9. A	10. B	11. C	12. D				

TÜREV
DERIVATIVE

BÖLÜM CHAPTER 2

Tanım: $f : A \to R$, $y = f(x)$ ve $x_0 \in A$ da sürekli olmak üzere,

$\lim\limits_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$ limitine f(x) in x_0 noktasındaki türevi denir.

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

f(x) in x_0 noktasındaki türevi, $f'(x_0)$, $\frac{df(x_0)}{dx}$ simgelerinden biri ile gösterilir.

***Definition:** Let $f: A \to R$, $y = f(x)$. For any $x_0 \in A$*

if $\lim\limits_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$ exists, this limit is called the derivative of f at the point x_0.

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

The derivative of a function f at a point x_0 is denoted by $f'(x_0)$, $\frac{df(x_0)}{dx}$.

ÖRNEK / EXAMPLE 1

$f : R \to R$,

$f'(x) = x^3 - 4x^2 + 2x - 3 \Rightarrow \mathbf{f'(2) = ?}$

A) –4 B) –3 C) –2 D) –1 E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{x^3 - 4x^2 + 2x - 3 + 7}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^3 - 4x^2 + 2x + 4}{x - 2}$$

$$= \lim_{x \to 2} \frac{(x - 2) \cdot (x^2 - 2x - 2)}{x - 2}$$

$$= \lim_{x \to 2} (x^2 - 2x - 2) = 4 - 4 - 2 = -2$$

NOT / NOTE

$x - x_0 = h \Rightarrow x \to x_0 \Leftrightarrow h \to 0$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

NOT / NOTE

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

ÖRNEK / EXAMPLE 2

f: R → R,

$f(x) = x^2 - 4x + 4 \Rightarrow f'(x) = \dfrac{df(x)}{dx} = ?$

A) $x - 2$
B) $2x - 2$
C) $2x - 4$
D) $x - 4$
E) $4x - 2$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 4x - 4h + 4 - x^2 + 4x - 4}{h}$$

$$= \lim_{h \to 0} \frac{h(2x - 4 + h)}{h}$$

$$= 2x - 4$$

TÜREV ALMA KURALLARI / RULES for TAKING DERIVATIVE

1. $c \in R$

$f(x) = c \Rightarrow f'(x) = 0$

2. $n \in R$

$f(x) = x^n \Rightarrow f'(x) = n \cdot x^{n-1}$

3. $a, n \in R$

$f(x) = a \cdot x^n \Rightarrow f'(x) = a \cdot n \cdot x^{n-1}$

4. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

5. $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$

6. $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{(g(x))^2}$ $[\, g(x) \neq 0 \,]$

7. $(f^m(x))' = m \cdot f^{m-1}(x) \cdot f'(x)$

8. $f(x) = \sqrt{g(x)} \Rightarrow f'(x) = \dfrac{g'(x)}{2\cdot\sqrt{g(x)}}$

$f(x) = \sqrt[m]{g^n(x)} \Rightarrow f'(x) = \dfrac{n\cdot f'(x)}{m\cdot\sqrt[m]{f(x)^{m-n}}}$

9. $m, n \in R$,

$f(x) = g(mx+n)$

$\Rightarrow f'(x) = (mx+n)'\cdot g'(mx+n) = mg'(mx+n)$

10. $f(x) = |g(x)|$

$$f'(x) = \begin{cases} g'(x) & , \quad g(x) \geq 0 \\ -g'(x) & , \quad g(x) < 0 \end{cases}$$

ÖRNEK / EXAMPLE 3

$f(x) = 2x^3 - 5x^2 + 4x - 7 \quad \Rightarrow \quad$ **$f'(x) = ?$**

A) $3x^2 - 5x - 4$

B) $6x^2 - 10x + 4$

C) $6x^2 - 10x - 7$

D) $3x^2 - 5x + 4$

E) $x^3 - x^2 + 4$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$f(x) = 2x^3 - 5x^2 + 4x - 7$

$\Rightarrow f'(x) = 6x^2 - 10x + 4$

PEKİŞTİRME / REINFORCE

$f(x) = 3x^2 - x^3 + 1 \Rightarrow f'(1) = ?$

YANIT / ANSWER : 3

PEKİŞTİRME / REINFORCE

$f(x) = (3x^2 + 1)\cdot(2x - 1) \Rightarrow f'(0) = ?$

YANIT / ANSWER : 2

ÖRNEK / EXAMPLE 4

$f(x) = (x^2 - 1) \cdot (2x^2 - 3x + 1) \Rightarrow f'(2) = ?$

A) 21 B) 24 C) 27 D) 30 E) 33

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$f'(x) = (x^2 - 1)' \cdot (2x^2 - 3x + 1) + (2x^2 - 3x + 1)' \cdot (x^2 - 1)$$

$$= 2x \cdot (2x^2 - 3x + 1) + (4x - 3) \cdot (x^2 - 1)$$

$$f'(2) = 2 \cdot 2\,(2 \cdot 2^2 - 3 \cdot 2 + 1) + (4 \cdot 2 - 3) \cdot (2^2 - 1)$$

$$= 4 \cdot (8 - 6 + 1) + (8 - 3) \cdot (4 - 1)$$

$$= 4 \cdot 3 + 5 \cdot 3$$

$$= 12 + 15 = 27$$

ÖRNEK / EXAMPLE 5

$f(x) = \dfrac{x^2 - 3}{3x + 1} \Rightarrow f'(-1) = ?$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$f'(x) = \frac{(x^2 - 3)' \cdot (3x + 1) - (3x + 1)' \cdot (x^2 - 3)}{(3x + 1)^2}$$

$$= \frac{2 \cdot x(3x + 1) - 3 \cdot (x^2 - 3)}{(3x + 1)^2}$$

$$= \frac{6x^2 + 2x - 3x^2 + 9}{(3x + 1)^2} = \frac{3x^2 + 2x + 9}{(3x + 1)^2}$$

$$f'(-1) = \frac{3 \cdot (-1)^2 + 2 \cdot (-1) + 9}{(3 \cdot (-1) + 1)^2}$$

$$= \frac{3 - 2 + 9}{(-2)^2}$$

$$= \frac{10}{4} = \frac{5}{2}$$

ÖRNEK / EXAMPLE 6

$f(x) = (x^4 - 2x)^5 \Rightarrow$ **$f'(1) = ?$**

A) 10 B) 12 C) 14 D) 16 E) 18

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$f(x) = (x^4 - 2x)^5$

$\Rightarrow f'(x) = 5 \cdot (x^4 - 2x)^4 \cdot (x^4 - 2x)'$

$= 5 \cdot (x^4 - 2x)^4 \cdot (4x^3 - 2)$

$f'(1) = 5 \cdot (1^4 - 2 \cdot 1)^4 \cdot (4 \cdot 1^3 - 2)$

$= 5 \cdot (-1)^4 \cdot 2$

$= 5 \cdot 1 \cdot 2 = 10$

ÖRNEK / EXAMPLE 7

$f(x) = \sqrt[3]{x^2 - 3} \Rightarrow$ **$f'(2) = ?$**

A) $\frac{1}{5}$ B) $\frac{4}{3}$ C) $\frac{1}{2}$ D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$f(x) = \sqrt[3]{x^2 - 3} = \left(x^2 - 3\right)^{\frac{1}{3}}$

$\Rightarrow f'(x) = \frac{1}{3} \cdot (x^2 - 3)^{-\frac{2}{3}} \cdot (x^2 - 3)' = \frac{1}{3} \cdot (x^2 - 3)^{-\frac{2}{3}} \cdot 2x$

$f'(2) = \frac{1}{3} \cdot (2^2 - 3)^{-\frac{2}{3}} \cdot 2 \cdot 2$

$= \frac{1}{3} \cdot 1 \cdot 2 \cdot 2$

$= \frac{4}{3}$

PEKİŞTİRME / REINFORCE

$f(x) = \sqrt{x^2 + 4} \Rightarrow f'(0) = ?$

YANIT / ANSWER : 0

ÖRNEK / EXAMPLE 8

$f(x) = (2x^2 - 6)^5 \Rightarrow$ **$f'(2) = ?$**

A) 640 B) 608 C) 576 D) 544 E) 520

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$f(x) = (2x^2 - 6)^5 \Rightarrow f'(x) = 5\cdot(2x^2 - 6)^4 \cdot (2x^2 - 6)'$$
$$= 5\cdot(2x^2 - 6)^4 \cdot 4x$$

$$\Rightarrow f'(2) = 5\cdot 2^4 \cdot 8 = 640$$

BİLEŞKE FONKSİYONUN TÜREVİ DERIVATIVE of a COMPOSITE FUNCTION

$$\left.\begin{array}{l} y = f(t) \\ t = g(x) \end{array}\right\} \Rightarrow y = f(g(x)) = (fog)(x)$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = f'(t)\cdot g'(x) \Rightarrow (fog)'(x) = f'(g(x))\cdot g'(x)$$

ÖRNEK / EXAMPLE 9

$f(x) = x^2 - 3x,\ g(x) = 5x + 1 \Rightarrow$ **$(fog)'(-2) = ?$**

A) –85 B) –90 C) –95 D) –100 E) –105

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$f(x) = x^2 - 3x \Rightarrow f'(x) = 2x - 3,$

$g(x) = 5x + 1 \Rightarrow g'(x) = 5$

$$(fog)'(x) = f'(g(x)) \cdot g'(x)$$
$$= (2\cdot(5x + 1) - 3)\cdot 5$$

$$(fog)'(-2) = (2\cdot(-9) - 3)\cdot 5 = -105$$

TRİGONOMETRİK FONKSİYONLARIN TÜREVİ
DERIVATIVE of TRIGONOMETRIC FUNCTIONS

A. Sinüs Fonksiyonunun Türevi / The Derivative of The Sine Function

1. $f(x) = \sin x \Rightarrow f'(x) = \cos x$
2. $f(x) = \sin(u(x)) \Rightarrow f'(x) = u'(x) \cdot \cos(u(x))$
3. $f(x) = \sin^n x \Rightarrow f'(x) = n \cdot \sin^{n-1} x \cdot \cos x$
4. $f(x) = \sin^n(u(x)) \Rightarrow f'(x) = n \cdot \sin^{n-1}(u(x)) \cdot \cos(u(x)) \cdot u'(x)$

ÖRNEK / EXAMPLE 10

$f(x) = \sin(3x + 2) \Rightarrow$ **$f'(x) = ?$**

A) $-2\cos(3x + 2)$
B) $2\cos(3x + 2)$
C) $2\sin(3x + 2)$
D) $3 \cdot \cos(3x + 2)$
E) $-3 \cdot \cos(3x + 2)$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$f(x) = \sin(3x + 2)$

$\Rightarrow f'(x) = \cos(3x + 2) \cdot 3 = 3 \cdot \cos(3x + 2)$

ÖRNEK / EXAMPLE 11

$f(x) = \sin^3 2x \Rightarrow$ **$f'\left(\frac{\pi}{4}\right) = ?$**

A) $-\frac{\sqrt{3}}{2}$
B) $-\frac{1}{2}$
C) 0
D) $\frac{1}{2}$
E) $\frac{\sqrt{3}}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$f(x) = \sin^3 2x$

$\Rightarrow f'(x) = 3 \cdot \sin^2 2x \cdot \cos 2x \cdot 2$

$f'\left(\frac{\pi}{4}\right) = 3 \cdot \sin^2 \frac{\pi}{2} \cdot \cos \frac{\pi}{2} \cdot 2 = 0$

PEKİŞTİRME / REINFORCE

$f(x) = \cos^2 x \Rightarrow f'(\frac{\pi}{2}) = ?$

YANIT / ANSWER : 0

B. Kosinüs Fonksiyonunun Türevi / The Derivative of The Cosine Function

1. $f(x) = \cos x \Rightarrow f'(x) = -\sin x$
2. $f(x) = \cos(u(x)) \Rightarrow f'(x) = -u'(x) \cdot \sin(u(x))$
3. $f(x) = \cos^n x \Rightarrow f'(x) = -n \cdot \cos^{n-1} x \cdot \sin x$
4. $f(x) = \cos^n(u(x)) \Rightarrow f'(x) = -n \cdot \cos^{n-1}(u(x)) \cdot \sin(u(x)) \cdot u'(x)$

PEKİŞTİRME / REINFORCE

$f(x) = \cos\frac{\pi}{2} \Rightarrow f'(-1) = ?$

YANIT / ANSWER : 0

ÖRNEK / EXAMPLE 12

$f(x) = x \cdot \cos x \Rightarrow$ **$f'(\pi) = ?$**

A) -1 B) 0 C) $\pi - 1$ D) $\pi + 1$ E) 2π

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$f'(x) = (x)' \cdot \cos x + (\cos x)' \cdot x$$
$$= \cos x - \sin x \cdot x$$
$$f'(\pi) = \cos\pi - \sin\pi \cdot \pi$$
$$= -1 - 0 = -1$$

ÖRNEK / EXAMPLE 13

$f(x) = \cos^2 3x \Rightarrow$ **$f'\left(\frac{\pi}{6}\right) = ?$**

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{1}{4}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$f(x) = \cos^2 3x$$
$$f'(x) = 2\cos 3x \cdot (\cos 3x)'$$
$$= 2 \cdot \cos 3x \cdot (-\sin(3x) \cdot 3)$$
$$= -6 \cdot \cos 3x \cdot \sin 3x$$
$$= -3 \cdot 2 \cdot \sin 3x \cdot \cos 3x = -3\sin 6x$$

$$f'\left(\frac{\pi}{6}\right) = -3 \cdot \sin\left(6 \cdot \frac{\pi}{6}\right) = -3 \cdot \sin\pi = -3 \cdot 0 = 0$$

C. Tanjant Fonksiyonunun Türevi / The Derivative of The Tangent Function

1. $f(x) = \tan x \Rightarrow f'(x) = 1 + \tan^2 x = \dfrac{1}{\cos^2 x} = \sec^2 x$
2. $f(x) = \tan(u(x)) \Rightarrow f'(x) = u'(x) \cdot (1 + \tan^2(u(x)))$
3. $f(x) = \tan^n x \Rightarrow f'(x) = n \cdot \tan^{n-1} x \cdot (1 + \tan^2 x)$
4. $f(x) = \tan^n (u(x)) \Rightarrow f'(x) = n \cdot \tan^{n-1}(u(x)) \cdot (1 + \tan^2(u(x))) \cdot u'(x)$

ÖRNEK / EXAMPLE 14

$f(x) = \tan(\sin x) \Rightarrow \mathbf{f'(\pi) = ?}$

A) –1 B) 0 C) 1 D) 2 E) 3

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$f(x) = \tan(\sin x)$

$\Rightarrow f'(x) = (\tan(\sin x))' = (\sin x)' \cdot (1 + \tan^2(\sin x))$

$= \cos x \cdot (1 + \tan^2(\sin x))$

$f'(\pi) = \cos\pi \cdot (1 + \tan^2(\sin\pi)) = (-1) \cdot (1 + 0) = -1$

ÖRNEK / EXAMPLE 15

$f(x) = \tan^3 x \Rightarrow \mathbf{f'\left(\frac{\pi}{4}\right) = ?}$

A) 1 B) 2 C) 3 D) 6 E) 12

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$f(x) = \tan^3 x \Rightarrow f'(x) = 3 \cdot \tan^2 x \cdot (1 + \tan^2 x)$

$f'\left(\frac{\pi}{4}\right) = 3 \cdot \tan^2\left(\frac{\pi}{4}\right) \cdot \left(1 + \tan^2 \frac{\pi}{4}\right)$

$= 3 \cdot 1 \cdot (1 + 1)$

$= 6$

D. Kotanjant Fonksiyonunun Türevi / The Derivative of The Cotangent Function

1. $f(x) = \cot x \Rightarrow f'(x) = -(1+\cot^2 x) = -\dfrac{1}{\sin^2 x} = -\mathrm{cosec}^2 x$

2. $f(x) = \cot(u(x)) \Rightarrow f'(x) = -u'(x)\cdot(1+\cot^2(u(x)))$

3. $f(x) = \cot^n x \Rightarrow f'(x) = -n\cdot\cot^{n-1}x\cdot(1+\cot^2 x)$

4. $f(x) = \cot^n(u(x)) \Rightarrow f'(x) = -n\cdot\cot^{n-1}(u(x))\cdot(1+\cot^2(u(x)))\cdot u'(x)$

ÖRNEK / EXAMPLE 16

$f(x) = \tan 4x + \cot 2x \Rightarrow f'\left(\dfrac{2\pi}{3}\right) = ?$

A) $\dfrac{32}{3}$ B) 11 C) $\dfrac{35}{3}$ D) 13 E) $\dfrac{40}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$f(x) = \tan 4x + \cot 2x$$

$$f'(x) = (1+\tan^2 4x)\cdot 4 - (1+\cot^2 2x)\cdot 2$$

$$f'\left(\frac{2\pi}{3}\right) = \left(1+\tan^2\frac{8\pi}{3}\right)\cdot 4 - \left(1+\cot^2\frac{4\pi}{3}\right)\cdot 2$$

$$= \left(1+\tan^2\frac{2\pi}{3}\right)\cdot 4 - \left(1+\cot^2\frac{4\pi}{3}\right)\cdot 2$$

$$= \left(1+(-\sqrt{3})^2\right)\cdot 4 - \left(1+\left(\frac{1}{\sqrt{3}}\right)^2\right)\cdot 2$$

$$= (1+3)\cdot 4 - \left(1+\frac{1}{3}\right)\cdot 2$$

$$= 16 - \frac{8}{3}$$

$$= \frac{40}{3}$$

TERS TRİGONOMETRİK FONKSİYONLARIN TÜREVİ
DERIVATIVE of INVERSE TRIGONOMETRIC FUNCTIONS

1. $f(x)=\arcsin x \quad \Rightarrow \quad f'(x)=\dfrac{1}{\sqrt{1-x^2}}$

$f(x)=\arcsin(u(x)) \quad \Rightarrow \quad f'(x)=\dfrac{u'(x)}{\sqrt{1-(u(x))^2}}$

2. $f(x)=\arccos x \quad \Rightarrow f'(x)=\dfrac{-1}{\sqrt{1-x^2}}$

$f(x)=\arccos(u(x)) \Rightarrow f'(x)=\dfrac{-u'(x)}{\sqrt{1-(u(x))^2}}$

3. $f(x)=\arctan x \quad \Rightarrow \quad f'(x)=\dfrac{1}{1+x^2}$

$f(x)=\arctan(u(x)) \quad \Rightarrow f'(x)=\dfrac{u'(x)}{1+(u(x))^2}$

4. $f(x)=\operatorname{arccot} x \quad \Rightarrow \quad f'(x)=\dfrac{-1}{1+x^2}$

$f(x)=\operatorname{arccot}(u(x)) \Rightarrow f'(x)=\dfrac{-u'(x)}{1+(u(x))^2}$

ÖRNEK / EXAMPLE 17

$f(x) = \arctan(x^2 - 1) \Rightarrow$ **$f'(2) = ?$**

A) $\frac{2}{5}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) 1 E) $\frac{6}{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$f'(x)=\frac{(x^2-1)'}{1+(x^2-1)^2}$$

$$=\frac{2x}{1+(x^2-1)^2}$$

$$f'(2)=\frac{4}{1+(2^2-1)^2}$$

$$=\frac{4}{10}=\frac{2}{5}$$

PEKİŞTİRME / REINFORCE

$f(x) = \arcsin 2x \Rightarrow f'(0) = ?$

YANIT / ANSWER : 2

LOGARİTMİK BİR FONKSİYONUN TÜREVİ
DERIVATIVE of a LOGARITHMIC FUNCTION

1 $f(x)=\log_a x \Rightarrow f'(x)=\frac{1}{x\cdot \ln a}=\frac{1}{x}\cdot \log_a e$

$f(x)=\log_a(u(x)) \Rightarrow f'(x)=\frac{u'(x)}{u(x)\cdot \ln a}=\frac{u'(x)}{u(x)}\cdot \log_a e$

ÖRNEK / EXAMPLE 18

$f(x)=\log_5(x^2-2) \Rightarrow f'(5)=?$

A) $\frac{13}{5\cdot \ln 5}$ B) $\frac{23}{2\cdot \ln 5}$ C) $\frac{13}{\ln 5}$ D) $\frac{2}{5\cdot \ln 5}$ E) $\frac{10}{23\cdot \ln 5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$f(x)=\log_5(x^2-2)$

$$f'(x)=\frac{2x}{(x^2-2)\cdot \ln 5} \Rightarrow f'(5)=\frac{2\cdot 5}{(5^2-2)\cdot \ln 5}=\frac{10}{23\cdot \ln 5}$$

2 $f(x)=\ln x \Rightarrow f'(x)=\frac{1}{x}$

$f(x)=\ln(u(x)) \Rightarrow f'(x)=\frac{u'(x)}{u(x)}$

ÖRNEK / EXAMPLE 19

$f(x)=\ln(x^2+3x) \Rightarrow f'(x)=?$

A) $\frac{2x+2}{x^2+2x}$ B) $\frac{2}{x+3x^2}$ C) $\frac{2x+3}{x^2+3x}$ D) $\frac{2x+3}{x+3}$ E) $\frac{2x}{x^2+3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$f(x)=\ln(x^2+3x) \Rightarrow f'(x)=\frac{2x+3}{x^2+3x}$$

PEKİŞTİRME / REINFORCE

$f(x)=\ln(x^2-2x) \Rightarrow f'(1)=?$

YANIT / ANSWER : 0

ÜSTEL FONKSİYONLARIN TÜREVİ
DERIVATIVE of EXPONENTIAL FUNCTIONS

1 $f(x) = e^{x} \Rightarrow f'(x) = e^{x}$

$f(x) = e^{u(x)} \Rightarrow f'(x) = u'(x) \cdot e^{u(x)}$

ÖRNEK / EXAMPLE 20

$f(x) = e^{x^2+1} \Rightarrow f'(1) = ?$

A) $\frac{e^2}{2}$ B) e^2 C) $2 \cdot e^2$ D) $4 \cdot e^2$ E) $2 \cdot e^3$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$f(x) = e^{x^2+1} \Rightarrow f'(x) = e^{x^2+1} \cdot 2x$

$f'(1) = e^2 \cdot 2$

$= 2 \cdot e^2$

2 $f(x) = a^{x} \Rightarrow f'(x) = a^{x} \cdot \ln a = \frac{1}{\log_a e} \cdot a^{x}$

$f(x) = a^{u(x)} \Rightarrow f'(x) = u'(x) \cdot a^{u(x)} \cdot \ln a = \frac{u'(x)}{\log_a e} a^{u(x)}$

ÖRNEK / EXAMPLE 21

$f(x) = 5^{\cos x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

A) $\ln\frac{1}{5}$ B) $\ln 5$ C) $\ln\frac{2}{5}$ D) $\ln 25$ E) $\ln\frac{3}{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$f(x) = 5^{\cos x} \Rightarrow f'(x) = 5^{\cos x} \cdot (-\sin x) \cdot \ln 5$

$f'\left(\frac{\pi}{2}\right) = 5^{\cos\frac{\pi}{2}} \cdot \left(-\sin\frac{\pi}{2}\right) \cdot \ln 5$

$= 5^{0} \cdot (-1) \cdot \ln 5$

$= -\ln 5 = \ln\frac{1}{5}$

TERS FONKSİYONUN TÜREVİ / DERIVATIVE of an INVERSE FUNCTION

A, B $\subset$ R ve f : A $\rightarrow$ B fonksiyonu birebir ve örtendir.

Let f : A $\rightarrow$ B and A, B $\subset$ R be one to one and onto function.

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$$

ÖRNEK / EXAMPLE 22

$f : R^+ \rightarrow R^+$,

$f(x) = \sqrt{8x}$

$\Rightarrow$ **$(f^{-1})'(4) = ?$**

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{8}$ E) $\frac{1}{16}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$$

$$(f^{-1})'(y_0) = \frac{1}{\frac{8}{2\sqrt{8x}}} = \frac{\sqrt{2x}}{2}$$

$$\left.\begin{array}{l}\sqrt{8x} = 4\\ 8x = 16\\ \Rightarrow x = 2\end{array}\right\} \Rightarrow (f^{-1})'(4) = \frac{1}{f'(2)} = \frac{\sqrt{2}\cdot 2}{2} = 1$$

YÜKSEK BASAMAKTAN TÜREVLER / HIGHER ORDER DERIVATIVES

$y' = f'(x)$ 1. basamaktan türev / *1st order derivative*

$y'' = f''(x)$ 2. basamaktan türev / *2nd order derivative*

$y''' = f'''(x)$ 3. basamaktan türev / *3rd order derivative*

.

.

.

$y^{(n)} = f^{(n)}(x)$: n. basamaktan türev / *n th order derivative*

ÖRNEK / EXAMPLE 23

$f(x) = x^2 + 3x - 5 \Rightarrow f''(x) = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$f(x) = x^2 + 3x - 5$

$f'(x) = 2x + 3$

$f''(x) = 2$

ÖRNEK / EXAMPLE 24

$f(x) = \cos x \Rightarrow f^{(46)}(x) = ?$

A) sinx B) cosx C) 0 D) –sinx E) –cosx

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f^{(4)}(x) = \cos x \Rightarrow f^{(46)}(x) = f''(x) = -\cos x$

$$\begin{array}{r|l} 46 & 4 \\ \hline -\,44 & 11 \\ \hline ② & \end{array}$$

PARAMETRİK FONKSİYONLARIN TÜREVİ
DERIVATIVE of PARAMETRIC FUNCTIONS

$$\left.\begin{array}{l} y = f(x) \\ x = u(t) \\ y = v(t) \end{array}\right\} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{v'(t)}{u'(t)}$$

NOT / NOTE

$x = f(t),\ y = g(t) \Rightarrow$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{y''(t)\cdot x'(t) - x''(t)\cdot y'(t)}{(x'(t))^3}$$

ÖRNEK / EXAMPLE 25

$$\left.\begin{aligned} x &= 6t-3t^2 \\ y &= 4t^3+3t^2 \end{aligned}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=2} = ?$$

A) −14 B) −10 C) 0 D) 10 E) 14

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$$

$$= (12t^2+6t)\cdot\left(\frac{1}{6-6t}\right)$$

$$= 6\cdot(2t^2+t)\cdot\frac{1}{6(1-t)} = \frac{2t^2+t}{1-t}$$

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{2\cdot 2^2+1}{1-2} = \frac{10}{-1} = -10$$

ÖRNEK / EXAMPLE 26

$$\left.\begin{aligned} y &= \sin t \\ x &= \cos t \end{aligned}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{2}} = ?$$

A) −1 B) $-\frac{\sqrt{2}}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) 0 E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\frac{d^2y}{dx^2} = \frac{y''(t)\cdot x'(t) - x''(t)\cdot y'(t)}{(x'(t))^3}$$

$$= \frac{(-\sin t)-(\sin t) - (-\cos t)\cdot\cos t}{(-\sin t)^3}$$

$$= \frac{\sin^2 t+\cos^2 t}{(-\sin t)^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{2}} = \frac{1}{-1} = -1$$

KAPALI FONKSİYONLARIN TÜREVİ / DERIVATIVE of IMPLICIT FUNCTIONS

F(x,y) = 0 biçiminde tanımlanan fonksiyona *kapalı fonksiyon* denir.

Implicit function is a function that is formed by F(x,y) = 0.

$$F(x,y)=0 \Rightarrow \frac{dy}{dx}=-\frac{F'_x(x,y)}{F'_y(x,y)}$$

F'_x : x' e göre türev, $y \in R$

F'_y : y' ye göre türev, $x \in R$

F'_x : derivative of F with respect to x

F'_y : derivative of F with respect to y

ÖRNEK / EXAMPLE 27

$$x^2+2xy-y^2=0 \Rightarrow \frac{dy}{dx}=?$$

A) $\frac{x-y}{y}$ B) $\frac{y-x}{x+y}$ C) $\frac{x+y}{y-x}$ D) $\frac{x+y}{x-y}$ E) $\frac{x-y}{x+y}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$F(x,y)=x^2+2xy-y^2=0$$

$$\frac{dy}{dx}=-\frac{2x+2y}{2x-2y}$$

$$=-\frac{2\cdot(x+y)}{2\cdot(x-y)}$$

$$=\frac{x+y}{y-x}$$

L HOSPITAL KURALI / L' HOSPITAL'S RULE

$\lim\limits_{x \to x_0} \dfrac{f(x)}{g(x)}$ limiti bulunurken $\dfrac{0}{0}$, $\dfrac{\infty}{\infty}$ şeklinde belirsizlik oluşuyorsa limit, pay ve paydanın ayrı ayrı türevleri alınarak bulunabilir.

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x_0)}{g'(x_0)}$$

Tekrar belirsizlik devam ediyorsa kural yeniden uygulanır.

If lim $\dfrac{f(x)}{g(x)}$ $_{x \to x_0}$ is equal to 0/0 or ∞/∞ then the limit is found by taking the derivatives of the numerator and denominator seperately.

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x_0)}{g'(x_0)}.$$

If the indeterminate forms continue, then apply the same rule one more time.

ÖRNEK / EXAMPLE 28

$$f(x) = \frac{3x^2 + 4x - 5}{6x^2 + x + 3} \Rightarrow \lim_{x \to \infty} f(x) = ?$$

A) $\dfrac{4}{3}$ B) 1 C) $\dfrac{3}{4}$ D) $\dfrac{1}{2}$ E) $\dfrac{1}{4}$

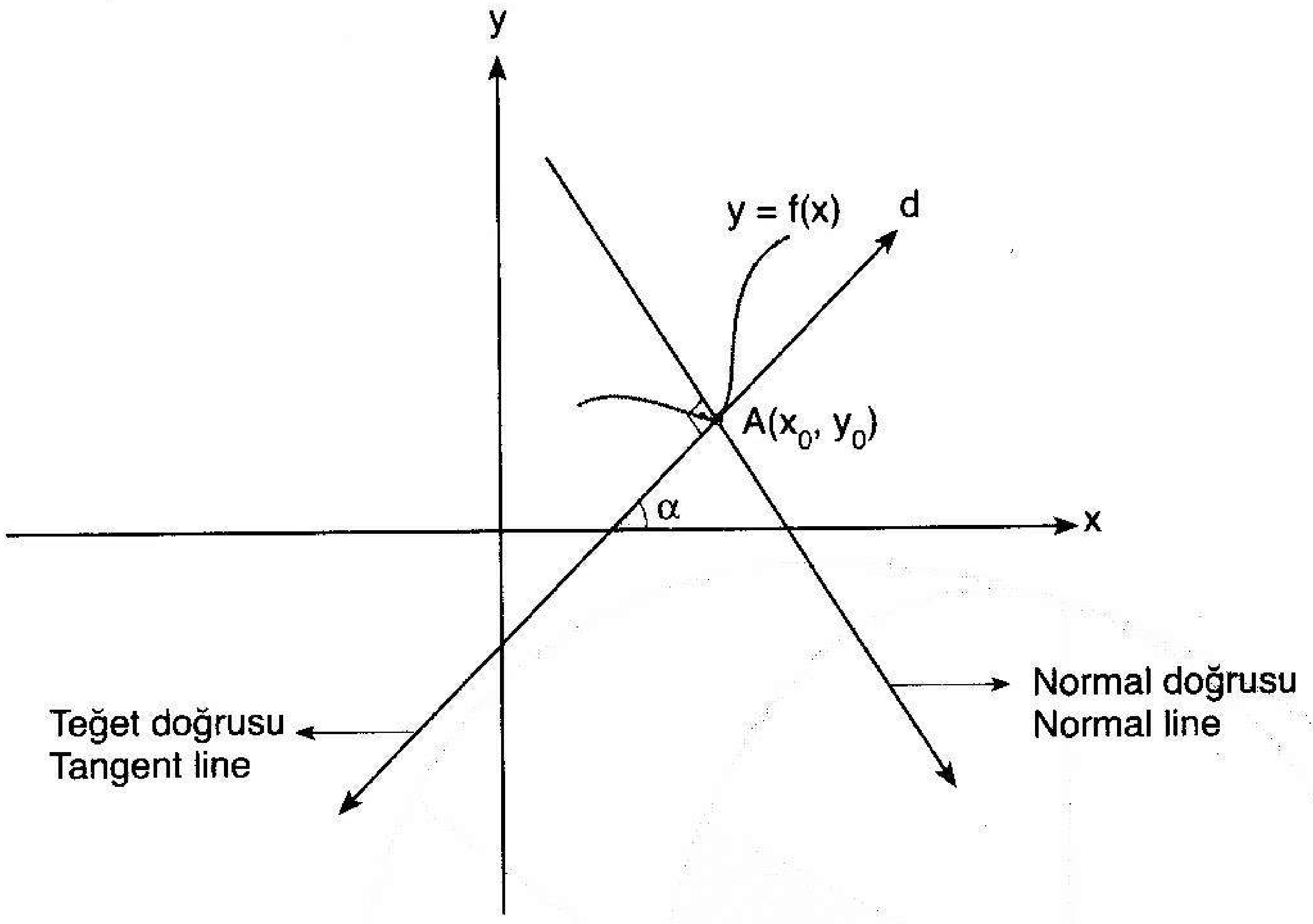
ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\lim_{x \to \infty} \frac{3x^2 + 4x - 5}{6x^2 + x + 3} \quad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x \to \infty} \frac{6x + 4}{12x + 1} = \lim_{x \to \infty} \frac{6}{12} = \frac{1}{2}$$

TÜREVİN GEOMETRİK ANLAMI / GEOMETRIC MEANING of DERIVATIVE


y
y = f(x)
d
A(x0, y0)
α
x
Teğet doğrusu
Tangent line
Normal doğrusu
Normal line

y = f(x) fonksiyonuna A (x_0, y_0) noktasından çizilen teğetin eğimi aşağıdaki gibidir.

The function y = f(x) has a tangent at A (x_0, y_0) whose slope is

$$\left.\begin{array}{l} m_d = f'(x_0) \\ m_d = \tan\alpha \end{array}\right\} \Rightarrow m_d = f'(x_0) = \tan\alpha$$

ÖRNEK / EXAMPLE 29

$f'(2) = ?$

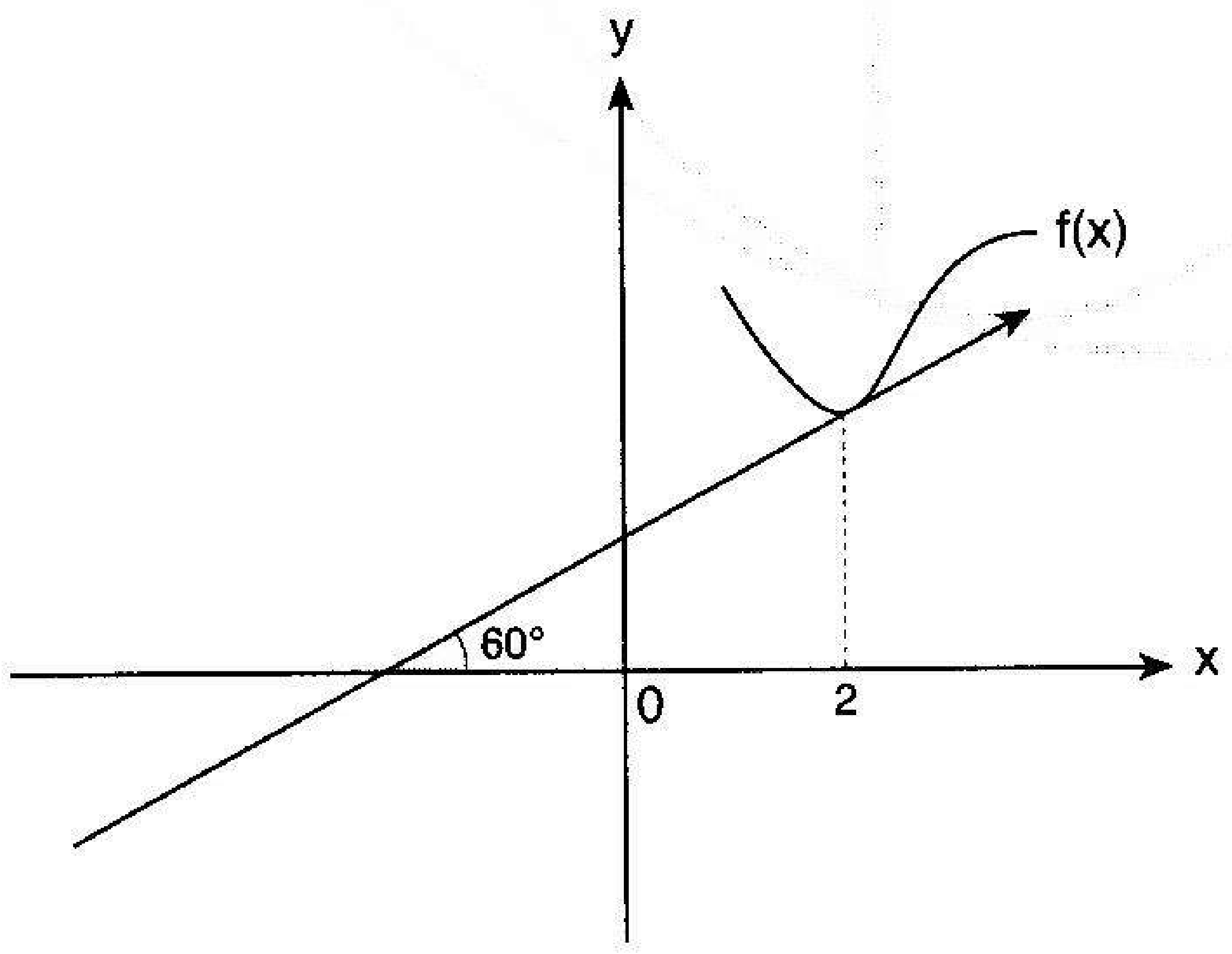
A) $\frac{\sqrt{3}}{3}$

B) $\frac{\sqrt{3}}{2}$

C) 1

D) $\frac{\sqrt{2}}{2}$

E) $\sqrt{3}$



ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\left.\begin{array}{l} f'(2) = m_d \\ m_d = \tan 60^\circ \end{array}\right\} \Rightarrow f'(2) = \tan 60^\circ = \sqrt{3}$$

ÖRNEK / EXAMPLE 30

YILDIZ TEKNİK ÜNİ. YÖS 2013

$f(x) = \ln^2(xe^x)$

$\Rightarrow$ **$f'(1) = ?$**

A) $-2e$ B) $3e$ C) 4 D) e^2 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$f(x) = \ln^2(xe^x)$

$$\Rightarrow f'(x) = \frac{2\ln(xe^x)\cdot(e^x + xe^x)}{xe^x}$$

$$\Rightarrow f'(1) = \frac{2\ln(e)\cdot(2e)}{e} = 4$$

ÖRNEK / EXAMPLE 31

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$y = \frac{\sin x}{x^2} \Rightarrow y'\left(\frac{\pi}{2}\right) = ?$$

A) $2^5\pi^2$ B) $2^4\pi^3$ C) $-2^4\pi^2$ D) $-2^4\pi^{-3}$ E) $-2^4\pi^4$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$y = \frac{\sin x}{x^2} \Rightarrow y' = \frac{\cos x\cdot x^2 - 2x\sin x}{x^4}$$

$$y'\left(\frac{\pi}{2}\right) = \frac{\cos\frac{\pi}{2}\cdot\left(\frac{\pi}{2}\right)^2 - 2\cdot\frac{\pi}{2}\sin\frac{\pi}{2}}{\left(\frac{\pi}{2}\right)^4}$$

$$= \frac{-\pi}{\frac{\pi^4}{16}} = -\frac{16}{\pi^3} = -2^4\pi^{-3}$$

ÖRNEK / EXAMPLE 32

MARMARA ÜNİ. YÖS 2013

$f(x)=\frac{x}{\ln x} \Rightarrow f'(e)=?$

A) $-e$ B) $\frac{1}{e}$ C) 0 D) 1 E) e

ÇÖZÜM / SOLUTION

Yanıt / Answer C

$$f(x)=\frac{x}{\ln x} \Rightarrow f'(x)=\frac{x'\cdot\ln x-(\ln x)'\cdot x}{(\ln x)^2}$$

$$=\frac{\ln x-\frac{1}{x}\cdot x}{(\ln x)^2}$$

$$\Rightarrow f'(e)=\frac{\ln e-1}{(\ln e)^2}=\frac{1-1}{1}=\frac{0}{1}=0$$

ÖRNEK / EXAMPLE 33

MARMARA ÜNİ. YÖS 2013

$f(x)=\frac{mx+4}{x-1}$, $f'(2)=-5 \Rightarrow m=?$

A) -3 B) 0 C) 1 D) 2 E) 4

ÇÖZÜM / SOLUTION

Yanıt / Answer C

$$f(x)=\frac{mx+4}{x-1},\ f'(2)=-5$$

$$f'(x)=\frac{m\cdot(x-1)-(mx+4)}{(x-1)^2}$$

$$f'(2)=\frac{m-2m-4}{1}$$

$$-m-4=-5$$

$$-m=-1$$

$$m=1$$

ÖRNEK / EXAMPLE 34

ANKARA ÜNİ. YÖS 2015

$f(x) = x^2|x^3 - 10| \Rightarrow$ **$f'(2) = ?$**

A) –82 B) –40 C) –20 D) 20 E) 40

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$f(x) = x^2 \cdot |x^3 - 10|$

$f(x) = x^2 \cdot (-x^3 + 10)$

$f(x) = -x^5 + 10x^2$

$f'(x) = -5x^4 + 20x$

$f'(2) = -5 \cdot 2^4 + 20 \cdot 2$

$f'(2) = -80 + 40 = -40$

ÖRNEK / EXAMPLE 35

ULUDAĞ ÜNİ. YÖS 2016

$$x^2 + 2xy - 3y^2 - 4x + 11 = 0 \Rightarrow \left.\frac{dy}{dx}\right|_{(2,\,-1)} = ?$$

A) -1 B) $-\frac{3}{5}$ C) $-\frac{1}{5}$ D) $\frac{1}{5}$ E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\frac{dy}{dx} = -\frac{Fx}{Fy} = -\frac{2x+2y-4}{2x-6y}$$

$$(2,\,-1) \Rightarrow -\frac{-2}{10} = \frac{1}{5}$$

1. $f(x) = ax^2 - 3x + 1$

$\frac{d}{dx} f(x)\Big|_{x=1} = 5 \Rightarrow \mathbf{a = ?}$

A) 6 B) 5 C) 4

D) 3 E) 2

---

2. $\left.\begin{array}{l} f(x) = x^3 - ax^2 + bx + 3 \\ f(1) = 12 \\ f'(2) = 8 \end{array}\right\} \Rightarrow \mathbf{b = ?}$

A)4 B) 6 C) 8

D) 12 E) 16

---

3. $f(x) = \frac{x^2 - 3x}{3x + 1} \Rightarrow \mathbf{f'(1) = ?}$

A) $-\frac{3}{16}$ B) $\frac{1}{8}$ C) $\frac{1}{16}$

D) $\frac{3}{16}$ E) $\frac{5}{16}$

---

4. $f(x) = (x+1)^2 \cdot (3x+1)^2 \Rightarrow \frac{\mathbf{df(x)}}{\mathbf{dx}}\Big|_{x=1} \mathbf{= ?}$

A) 96 B) 120 C) 144

D) 156 E) 160

---

5. $f(x) = \frac{\sqrt{x}}{1 + \sqrt{x}} \Rightarrow \frac{\mathbf{df(x)}}{\mathbf{dx}}\Big|_{x=4} \mathbf{= ?}$

A) 12 B) 1 C) $\frac{1}{36}$

D) $\frac{1}{16}$ E) $\frac{1}{12}$

---

6. $f(3x) = x^2 .\ g(x - 2),$

$f'(3) = 2,$

$g(-1) = 4 \Rightarrow \mathbf{g'(-1) = ?}$

A) 3 B) 2 C) –2

D) –3 E) –4

7. $f(x) = (x^2 - x)^{-2} \Rightarrow$ $\mathbf{f'(2) = ?}$

A) -4 B) $-\frac{8}{3}$ C) $-\frac{3}{4}$

D) $\frac{1}{4}$ E) $\frac{1}{2}$

8. $f(x) = x^3 - 6x + g(4x - 7)$

$g'(5) = -3$

$\Rightarrow$ $\mathbf{f'(3) = ?}$

A) -6 B) -3 C) 0

D) 6 E) 9

9. $f(x) = x \cdot \sqrt{x^3 + 8} \Rightarrow$ $\mathbf{f'(2) = ?}$

A) $\frac{4}{9}$ B) $\frac{5}{2}$ C) 5

D) $\frac{13}{2}$ E) 7

10. $f(4) = -3$,

$f'(4) = 2$,

$g'(-3) = -5 \Rightarrow$ $\mathbf{(gof)'(4) = ?}$

A) -16 B) -14 C) -12

D) -10 E) -8

11. $f(x) = \frac{x^3 - 1}{x} \Rightarrow$ $\mathbf{f'(x) = ?}$

A) $3x + \frac{1}{x}$ B) $2x + \frac{1}{x^2}$ C) $3x - \frac{1}{x^2}$

D) $4x + \frac{1}{x^2}$ E) $3x + \frac{1}{x^2}$

12. $a < 0$,

$f(x) = \frac{1}{3}x^3 + x^2 - 3x + 7$

$\frac{d}{dx} f(x)\Big|_{x=a} = 0 \Rightarrow$ $\mathbf{a = ?}$

A) -4 B) -3 C) -2

D) 1 E) 6

YANITLAR / ANSWERS

1. C	**2. D**	**3. B**	**4. E**	**5. C**	**6. C**	**7. C**	**8. E**
9. E	**10. D**	**11. B**	**12. B**				

1. $f(x) = ax^2 - 3x + 1$

$f'(x) = 2ax - 3$

$f'(1) = 2a \cdot 1 - 3 = 5$

$5 = 2a - 3$

$2a = 8$

$a = 4$

Yanıt / The Answer is C

2. $f(x) = x^3 - ax^2 + bx + 3$

$f(1) = 1 - a + b + 3 = 12 \Rightarrow b - a = 8$

$f'(x) = 3x^2 - 2ax + b$

$f'(2) = 12 - 4a + b = 8 \Rightarrow 4a - b = 4$

$$\left.\begin{array}{r} b - a = 8 \\ 4a - b = 4 \end{array}\right\} \Rightarrow b = 12$$

Yanıt / The Answer is D

3. $f(x) = \dfrac{x^2 - 3x}{3x + 1}$

$$\Rightarrow f'(x) = \frac{(x^2 - 3x)' \cdot (3x + 1) - (3x + 1)' \cdot (x^2 - 3x)}{(3x + 1)^2}$$

$$= \frac{(2x - 3) \cdot (3x + 1) - 3 \cdot (x^2 - 3x)}{(3x + 1)^2}$$

$$f'(1) = \frac{(2 \cdot 1 - 3)(3 \cdot 1 + 1) - 3 \cdot (1 - 3)}{(3 \cdot 1 + 1)^2}$$

$$= \frac{-1 \cdot 4 + 6}{16} = \frac{1}{8}$$

Yanıt / The Answer is B

4. $f(x) = (x + 1)^2 \cdot (3x + 1)^2$

$$\frac{df(x)}{dx} = \left((x + 1)^2\right)' \cdot (3x + 1)^2 + \left((3x + 1)^2\right)' \cdot (x + 1)^2$$

$$= (2x + 2) \cdot (3x + 1)^2 + (18x + 6) \cdot (x + 1)^2$$

$$\left.\frac{df(x)}{dx}\right|_{x=1} = (2 + 2) \cdot (3 + 1)^2 + (18 + 6) \cdot (1 + 1)^2$$

$$= 4 \cdot 16 + 24 \cdot 4 = 160$$

Yanıt / The Answer is E

5. $f(x) = \dfrac{\sqrt{x}}{1 + \sqrt{x}}$

$$f'(x) = \frac{(\sqrt{x})' \cdot (1 + \sqrt{x}) - (1 + \sqrt{x})' \cdot \sqrt{x}}{(1 + \sqrt{x})^2}$$

$$= \frac{\frac{1}{2\sqrt{x}} \cdot (1 + \sqrt{x}) - \frac{1}{2\sqrt{x}} \cdot \sqrt{x}}{(1 + \sqrt{x})^2}$$

$$f'(4) = \frac{\frac{1}{4} \cdot 3 - \frac{1}{4} \cdot 2}{9}$$

$$= \frac{\frac{3}{4} - \frac{1}{2}}{9} = \frac{1}{36}$$

Yanıt / The Answer is C

6. $f(3x) = x^2 \cdot g(x - 2) \Rightarrow$

$(3x)' \cdot f'(3x) = (x^2)' \cdot g(x - 2) + g'(x - 2) \cdot x^2$

$3 \cdot f'(3x) = 2x \cdot g(x - 2) + g'(x - 2) \cdot x^2$

$3 \cdot f'(3) = 2 \cdot 1 \cdot g(1 - 2) + g'(1 - 2) \cdot 1^2$

$$\left.\begin{array}{r} g(-1) = 4 \\ f'(3) = 2 \end{array}\right\} \Rightarrow 3 \cdot 2 = 2 \cdot 4 + g'(-1) \cdot 1$$

$$6 = 8 + g'(-1) \cdot 1 \Rightarrow g'(-1) = -2$$

Yanıt / The Answer is C

7. $f(x) = (x^2 - x)^{-2}$

$f'(x) = -2\cdot(x^2 - x)'\cdot(x^2 - x)^{-3}$

$= -2\cdot(2x - 1)\cdot(x^2 - x)^{-3}$

$f'(2) = -2\cdot3 \cdot 2^{-3}$

$= -\dfrac{3}{4}$

Yanıt / The Answer is C

---

8. $f(x) = x^3 - 6x + g(4x - 7)$

$f'(x) = 3x^2 - 6 + g'(4x - 7)\cdot(4x - 7)'$

$= 3x^2 - 6 + 4 \cdot g'(4x - 7)$

$g'(5) = -3 \Rightarrow f'(3) = 3\cdot3^2 - 6 + 4 \cdot g'(5)$

$= 21 + 4\cdot(-3) = 9$

Yanıt / The Answer is E

---

9. $f(x) = x\cdot\sqrt{x^3 + 8}$

$f'(x) = (x)'\cdot\sqrt{x^3+8} + x\cdot(\sqrt{x^3+8})'$

$= \sqrt{x^3+8} + x\cdot\dfrac{3x^2}{2\cdot\sqrt{x^3+8}}$

$f'(2) = \sqrt{2^3+8} + 2\cdot\dfrac{3\cdot2^2}{2\cdot\sqrt{2^3+8}}$

$= 4 + 2\cdot\dfrac{12}{8} = 7$

Yanıt / The Answer is E

---

10. $(gof)'(x) = g'(f(x)) \cdot f'(x)$

$(gof)'(4) = g'(f(4)) \cdot f'(4)$

$= g'(-3) \cdot 2$

$= (-5) \cdot 2$

$= -10$

Yanıt / The Answer is D

---

11. $f(x) = \dfrac{x^3 - 1}{x}$

$f(x) = \dfrac{x^3}{x} - \dfrac{1}{x}$

$f(x) = x^2 - x^{-1}$

$f'(x) = 2x + x^{-2}$

$f'(x) = 2x + \dfrac{1}{x^2}$

Yanıt / The Answer is B

---

12. $f(x) = \dfrac{1}{3}x^3 + x^2 - 3x + 7$

$\dfrac{df(x)}{dx} = \dfrac{1}{3}\cdot3x^2 + 2x - 3$

$= x^2 + 2x - 3$

$\left.\dfrac{df(x)}{dx}\right|_{x=a} = a^2 + 2a - 3 = 0$

$(a+3)\cdot(a-1) = 0$

$a < 0 \Rightarrow a = -3$

Yanıt / The Answer is B

1. $f(x)=\cos 2x \Rightarrow \lim\limits_{h\to 0}\dfrac{f\left(\frac{\pi}{6}+h\right)-f\left(\frac{\pi}{6}\right)}{h}=?$

A) $-\sqrt{3}$

B) $-\dfrac{1}{\sqrt{3}}$

C) 1

D) 2

E) $\sqrt{3}$

2. $f(x)=\sin^2\sqrt{x} \Rightarrow \dfrac{df(x)}{dx}=f'(x)=?$

A) $\dfrac{1}{2\sqrt{x}}\sin 2\sqrt{x}$

B) $\dfrac{1}{\sqrt{x}}\sin 2\sqrt{x}$

C) $\dfrac{1}{2\sqrt{x}}\sin\sqrt{x}$

D) $\dfrac{1}{2\sqrt{x}}\sin 2\sqrt{x}\cdot\cos\sqrt{x}$

E) $\dfrac{1}{4\sqrt{x}}\sin 2\sqrt{x}$

3. $f(x) = 3\sin(3x+5) \Rightarrow f'(7°) = ?$

A) 3 cos 26°

B) 9 cos 26°

C) 15 cos 26°

D) 3 sin 26°

E) –sin 26°

4. $f(x)=\dfrac{x\cdot\sin x}{1+\tan x} \Rightarrow f'\left(\dfrac{\pi}{4}\right)=?$

A) $\sqrt{2}$

B) $\dfrac{\sqrt{2}}{2}$

C) $\dfrac{\sqrt{2}}{4}$

D) $-\sqrt{2}$

E) -2

5. $f(x)=x\cdot\sin x \Rightarrow \left.\dfrac{d^2f(x)}{dx^2}\right|_{x=\frac{\pi}{2}}=?$

A) -1

B) 0

C) 1

D) $\dfrac{\pi}{2}$

E) $-\dfrac{\pi}{2}$

6. $f(x)=\dfrac{1-\cos 2x}{1+\cos 2x} \Rightarrow f'\left(\dfrac{\pi}{3}\right)=?$

A) $8\sqrt{3}$

B) $4\sqrt{3}$

C) $\dfrac{2\sqrt{3}}{3}$

D) $\dfrac{\sqrt{3}}{3}$

E) 6

7. $f(x)=\tan\left(\frac{\pi}{2}\cos x\right) \Rightarrow \mathbf{f'}\left(\frac{\pi}{3}\right)=?$

A) $-\frac{\pi\sqrt{3}}{2}$

B) $-\frac{\pi}{2}$

C) 0

D) π

E) $\frac{\pi\sqrt{3}}{2}$

8. $f(x)=x\cdot\sin\frac{1}{x} \Rightarrow \left.\frac{\mathbf{df(x)}}{\mathbf{dx}}\right|_{x=\frac{2}{\pi}}=?$

A) -1

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) 1

9. $f(x)=\arcsin\frac{x-1}{3} \Rightarrow \mathbf{f'(1)=?}$

A) $\frac{1}{\sqrt{3}}$

B) $\frac{1}{3}$

C) $\frac{1}{3\sqrt{3}}$

D) $\frac{\sqrt{3}}{6}$

E) $\frac{1}{9}$

10. f(x) = arctan (sinx)

$\cos a=\frac{2}{5} \Rightarrow \mathbf{f'(a)=?}$

A) $\frac{7}{24}$

B) $\frac{5}{23}$

C) $\frac{3}{20}$

D) $\frac{2}{9}$

E) $\frac{1}{25}$

11. $f(x)=\arccos\sqrt{1-4x} \Rightarrow \mathbf{f'(x)=?}$

A) $-2\sqrt{4x^2}$

B) $-x\sqrt{x+4x^2}$

C) $\frac{-2}{\sqrt{x-4x^2}}$

D) $\frac{1}{\sqrt{x-4x^2}}$

E) $-\sqrt{x+4x^2}$

12. $f(x)=\text{arc}\cot(\cot(x)) \Rightarrow \mathbf{f'(1)=?}$

A) -1

B) 0

C) 1

D) $\frac{3}{4}$

E) $\frac{9}{4}$

YANITLAR / ANSWERS

1. A	**2. A**	**3. B**	**4. C**	**5. E**	**6. A**	**7. A**	**8. E**
9. B	**10. B**	**11. D**	**12. C**				

1. $$\lim_{h\to 0}\frac{f\left(\frac{\pi}{6}+h\right)-f\left(\frac{\pi}{6}\right)}{h}=f'\left(\frac{\pi}{6}\right)$$

$$f(x)=\cos 2x \Rightarrow f'(x)=-2\cdot\sin 2x$$

$$f'\left(\frac{\pi}{6}\right)=-2\cdot\sin\left(2\cdot\frac{\pi}{6}\right)$$

$$=-2\cdot\frac{\sqrt{3}}{2}$$

$$=-\sqrt{3}$$

Yanıt / The Answer is A

---

2. $$f(x)=\sin^2\sqrt{x}$$

$$f'(x)=2\cdot\sin\sqrt{x}\cdot\cos\sqrt{x}\cdot(\sqrt{x})'$$

$$=2\cdot\sin\sqrt{x}\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$$

$$=\sin 2\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$$

Yanıt / The Answer is A

---

3. $f(x) = 3\cdot\sin(3x + 5)$

$f'(x) = 3\cdot(\sin(3x + 5))'$

$= 3\cdot(3x + 5)'\cdot\cos(3x + 5)$

$= 9\cdot\cos(3x + 5)$

$\Rightarrow f'(7°) = 9\cdot\cos 26°$

Yanıt / The Answer is B

---

4. $$f(x)=\frac{x\cdot\sin x}{1+\tan x}$$

$$f'(x)=\frac{(x\cdot\sin x)'\cdot(1+\tan x)-(1+\tan x)'\cdot x\cdot\sin x}{(1+\tan x)^2}$$

$$f'\left(\frac{\pi}{4}\right)=\frac{\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{\pi}{4}\right)\cdot 2-2\cdot\left(\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}\right)}{4}$$

$$=\frac{\sqrt{2}+\sqrt{2}\cdot\frac{\pi}{4}-\sqrt{2}\cdot\frac{\pi}{4}}{4}=\frac{\sqrt{2}}{4}$$

Yanıt / The Answer is C

---

5. $f(x) = x\cdot\sin x$

$f'(x) = (x)'\cdot\sin x + (\sin x)'\cdot x = \sin x + \cos x\cdot x$

$f''(x) = \cos x + (\cos x)'\cdot x + (x)'\cdot\cos x$

$= \cos x + (-\sin x)\cdot x + \cos x$

$$f''\left(\frac{\pi}{2}\right)=0-\frac{\pi}{2}+0=-\frac{\pi}{2}$$

Yanıt / The Answer is E

---

6. $$f(x)=\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}$$

$$=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$$

$$f(x)=\tan^2 x \Rightarrow f'(x)=2\cdot\tan x\cdot(1+\tan^2 x)$$

$$f'\left(\frac{\pi}{3}\right)=2\cdot\tan\frac{\pi}{3}\cdot\left(1+\tan^2\frac{\pi}{3}\right)$$

$$=2\cdot\sqrt{3}\cdot(1+3)=8\sqrt{3}$$

Yanıt / The Answer is A

7. $f(x)=\tan\left(\frac{\pi}{2}\cos x\right)$

$$f'(x)=\left(\frac{\pi}{2}\cos x\right)'\cdot\left(1+\tan^2\left(\frac{\pi}{2}\cos x\right)\right)$$

$$=-\frac{\pi}{2}\sin x\cdot\left(1+\tan^2\left(\frac{\pi}{2}\cos x\right)\right)$$

$$f'(\frac{\pi}{3})=-\frac{\pi}{2}\cdot\frac{\sqrt{3}}{2}\cdot\left(1+\tan^2\left(\frac{\pi}{2}\cdot\frac{1}{2}\right)\right)$$

$$=\frac{-\sqrt{3}\,\pi}{4}\cdot(1+1)=\frac{-\sqrt{3}\,\pi}{2}$$

Yanıt / The Answer is A

---

8. $f(x)=x\cdot\sin\frac{1}{x}$

$$f'(x)=(x)'\cdot\sin\frac{1}{x}+\left(\sin\frac{1}{x}\right)'\cdot x$$

$$=\sin\frac{1}{x}-\frac{1}{x^2}\cos\frac{1}{x}x$$

$$=\sin\frac{1}{x}-\frac{1}{x}\cos\frac{1}{x}$$

$$f'\left(\frac{2}{\pi}\right)=\sin\frac{\pi}{2}-\frac{2}{\pi}\cdot\cos\frac{\pi}{2}=1-0=1$$

Yanıt / The Answer is E

---

9. $f(x)=\arcsin\frac{x-1}{3}$

$$f'(x)=\frac{\left(\frac{x-1}{3}\right)'}{\sqrt{1-\left(\frac{x-1}{3}\right)^2}}=\frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x-1}{3}\right)^2}}$$

$$f'(1)=\frac{\frac{1}{3}}{\sqrt{1-\left(\frac{1-1}{3}\right)^2}}=\frac{1}{3}$$

Yanıt / The Answer is B

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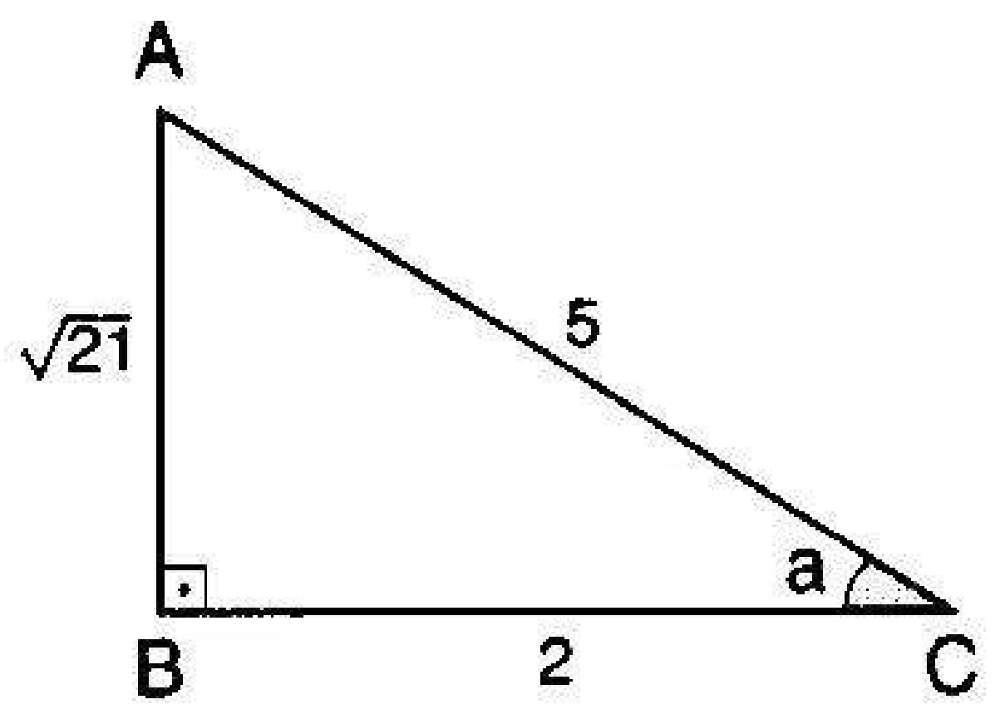
10. $f(x)=\arctan(\sin x)$



$$f'(x)=\frac{(\sin x)'}{1+\sin^2 x}=\frac{\cos x}{1+\sin^2 x}$$

$$\cos a=\frac{2}{5}\Rightarrow\sin a=\frac{\sqrt{21}}{5}$$

$$f(a)=\frac{\cos a}{1+\sin^2 a}=\frac{\frac{2}{5}}{1+\frac{21}{25}}=\frac{\frac{2}{5}}{\frac{46}{25}}$$

$$f(a)=\frac{5}{23}$$

Yanıt / The Answer is B

---

11. $f(x)=\arccos\sqrt{1-4x}$

$$f'(x)=\frac{(\sqrt{1-4x})'}{\sqrt{1-(\sqrt{1-4x})^2}}=-\frac{-\frac{4}{2\cdot\sqrt{1-4x}}}{\sqrt{1-1+4x}}$$

$$=\frac{4}{2\cdot\sqrt{4x-16x^2}}$$

$$=\frac{2}{\sqrt{4x-16x^2}}=\frac{1}{\sqrt{x-4x^2}}$$

Yanıt / The Answer is D

---

12. $f(x)=\operatorname{arccot}(\cot(x))=x\Rightarrow f'(x)=1$

Yanıt / The Answer is C

1. $f(x) = \ln\cos x \Rightarrow$ **$f'(x) = ?$**

A) $\sin x$
B) $\cot x$
C) $-\cot x$
D) $-\tan x$
E) $\tan x$

---

2. $f(x) = (x^2)^{\sin x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

A) 0
B) 1
C) π
D) 2π
E) 6π

---

3. $y = \sqrt{4 + \ln x} \Rightarrow \frac{dy}{dx} = ?$

A) $\frac{1}{2x\sqrt{4+\ln x}}$

B) $\frac{1}{2x\sqrt{4-\ln x}}$

C) $\frac{1}{4x\sqrt{4+\ln x}}$

D) $\frac{1}{4x\sqrt{4-\ln x}}$

E) $\frac{1}{x\sqrt{4+\ln x}}$

---

4. $f(x) = 3^{\cos x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

A) $-\ln 6$
B) $-\ln 3$
C) $\ln 2$
D) $\ln 3$
E) $\ln 6$

---

5. $y = \log_2 x^2 \Rightarrow \frac{dy}{dx} = ?$

A) $\frac{2+x}{\ln 2}$

B) $\frac{2}{x \cdot \ln 2}$

C) $\frac{1}{x \cdot \ln 2}$

D) $\frac{x \cdot \ln 2}{x + \ln 2}$

E) $\frac{\ln 2}{x + \ln 2}$

---

6. $f(x) = 5^x - 8^x \Rightarrow$ **$f'(0) = ?$**

A) $\ln\frac{3}{8}$

B) $\ln\frac{1}{2}$

C) $\ln\frac{5}{8}$

D) $\ln\frac{3}{4}$

E) $\ln\frac{7}{8}$

7. $f(x) = (2x-1)^3 \cdot \ln x \Rightarrow \mathbf{f'(1) = ?}$

A) -2

B) -1

C) 0

D) 1

E) 2

8. $f(x) = \sqrt{e^x} \cdot \ln x^2 \Rightarrow \mathbf{f'(1) = ?}$

A) $\frac{2}{\sqrt{e}}$

B) $\frac{4}{\sqrt{e}}$

C) $2\sqrt{e}$

D) $3\sqrt{e}$

E) $4\sqrt{e}$

9. $f(x) = \ln(\arctan x) \Rightarrow \mathbf{f'(1) = ?}$

A) $\frac{\pi}{3}$

B) $\frac{2}{\pi}$

C) $\frac{\pi}{4}$

D) $\frac{1}{2\pi}$

E) π

10. $f(x) = \log x^3 \Rightarrow \mathbf{f'\left(\frac{1}{\ln 10}\right) = ?}$

A) 3

B) 1

C) 0

D $3 \cdot \ln 10$

E) $(\ln 10)^2$

11. $f(x) = 5^{3x-3}$

$f'(a) = \ln 5^{375} \Rightarrow \mathbf{a = ?}$

A) 4

B) 3

C) 0

D) 1

E) 2

12. $f(x) = \log_a (x^2 + 3x + 1)$

$\frac{d}{d(x)} f(x)\Big|_{x=1} = \frac{1}{6} \Rightarrow \mathbf{a = ?}$

A) e^3

B) e^4

C) e^5

D) e^6

E) e^{30}

YANITLAR / ANSWERS

1. D	2. C	3. A	4. B	5. B	6. C	7. D	8. C
9. B	10. A	11. E	12. D				

1. $f(x) = \ln\cos x$

$$f'(x) = \frac{-\sin x}{\cos x}$$

$$f'(x) = -\tan x$$

Yanıt / The Answer is D

---

2. $f(x) = (x^2)^{\sin x} = x^{2\sin x}$

$$y = x^{2\sin x}$$

$$\ln y = \ln x^{2\sin x} = 2\sin x \cdot \ln x$$

$$(\ln y)' = (2\sin x \cdot \ln x)'$$

$$\frac{y'}{y} = 2\cdot\cos x\cdot\ln x + \frac{1}{x}\cdot 2\sin x$$

$$\frac{y'}{x^{2\sin x}} = \left(2\cdot\cos x\cdot\ln x + \frac{1}{x}\cdot 2\sin x\right)$$

$$f'(x) = y' = x^{2\sin x}\cdot\left(2\cdot\cos x\cdot\ln x + \frac{1}{x}\cdot 2\sin x\right)$$

$$f'\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^2\cdot\left(0 + \frac{2}{\pi}\cdot 2\right)$$

$$= \frac{\pi^2}{4}\cdot\frac{4}{\pi} = \pi$$

Yanıt / The Answer is C

---

3. $y = \sqrt{4+\ln x}$

$$\frac{dy}{dx} = \frac{(4+\ln x)'}{2\sqrt{4+\ln x}}$$

$$= \frac{\frac{1}{x}}{2\sqrt{4+\ln x}} = \frac{1}{2x\sqrt{4+\ln x}}$$

Yanıt / The Answer is A

---

4. $f(x) = 3^{\cos x}$

$$f'(x) = (\cos x)'\cdot 3^{\cos x}\cdot\ln 3$$

$$= -\sin x\cdot 3^{\cos x}\cdot\ln 3$$

$$f'\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2}\cdot 3^{\cos\frac{\pi}{2}}\cdot\ln 3$$

$$= -1\cdot 1\cdot\ln 3 = -\ln 3$$

Yanıt / The Answer is B

---

5. $y = \log_2 x^2$

$$\frac{dy}{dx} = \frac{(x^2)'}{x^2}\cdot\log_2 e$$

$$= \frac{2x}{x^2\cdot\ln 2}$$

$$= \frac{2}{x\cdot\ln 2}$$

Yanıt / The Answer is B

---

6. $f(x) = 5^x - 8^x$

$$f'(x) = 5^x\cdot\ln 5 - 8^x\cdot\ln 8$$

$$f'(0) = \ln 5 - \ln 8$$

$$= \ln\frac{5}{8}$$

Yanıt / The Answer is C

7. $f(x) = (2x-1)^3 \cdot \ln x$

$$f'(x) = \left((2x-1)^3\right)' \ln x + (\ln x)' \cdot (2x-1)^3$$

$$= 3 \cdot (2x-1)^2 \cdot 2 \cdot \ln x + \frac{1}{x} \cdot (2x-1)^3$$

$$f'(1) = 3 \cdot 1^2 \cdot 2 \cdot \ln 1 + \frac{1}{1} \cdot 1^2$$

$$= 0 + 1 = 1$$

Yanıt / The Answer is D

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8. $f(x) = \sqrt{e^x} \cdot \ln x^2$

$$f'(x) = \left(\sqrt{e^x}\right)' \ln x^2 + (\ln(x^2))' \cdot \sqrt{e^x}$$

$$= \frac{e^x}{2\sqrt{e^x}} \cdot \ln x^2 + \frac{2x}{x^2} \cdot \sqrt{e^x}$$

$$= \frac{e^x}{2\sqrt{e^x}} \cdot \ln x^2 + \frac{2}{x} \cdot \sqrt{e^x}$$

$$f'(1) = \frac{e}{2\sqrt{e}} \cdot \ln 1^2 + \frac{2}{1} \cdot \sqrt{e}$$

$$= 0 + 2\sqrt{e} = 2\sqrt{e}$$

Yanıt / The Answer is C

---

9. $f(x) = \ln(\arctan x)$

$$f'(x) = \frac{(\arctan x)'}{\arctan x} = \frac{\frac{1}{1+x^2}}{\arctan x}$$

$$f'(1) = \frac{\frac{1}{2}}{\frac{\pi}{4}} = \frac{2}{\pi}$$

Yanıt / The Answer is B

---

10. $f(x) = \log x^3$

$$f'(x) = \frac{(x^3)'}{x^3} \cdot \log_{10} e = \frac{3x^2}{x^3 \cdot \ln 10} = \frac{3}{x \cdot \ln 10}$$

$$f'\left(\frac{1}{\ln 10}\right) = \frac{3}{\frac{1}{\ln 10} \cdot \ln 10} = 3$$

Yanıt / The Answer is A

---

11. $f(x) = 5^{3x-3}$

$$f'(x) = (3x-3)' \cdot 5^{3x-3} \cdot \ln 5$$

$$= 3 \cdot 5^{3x-3} \cdot \ln 5$$

$$f'(a) = 3 \cdot 5^{3a-3} \cdot \ln 5 = 5^{3a-3} \cdot \ln 5^3$$

$$f'(a) = \ln(5)^{3 \cdot 5^{3a-3}} = \ln 5^{375}$$

$$3 \cdot 5^{3a-3} = 375$$

$$5^{3a-3} = 125 \Rightarrow a = 2$$

Yanıt / The Answer is E

---

12. $f(x) = \log_a (x^2 + 3x + 1)$

$$f'(x) = \frac{(x^2 + 3x + 1)'}{(x^2 + 3x + 1) \cdot \ln a}$$

$$= \frac{2x + 3}{(x^2 + 3x + 1) \cdot \ln a}$$

$$f'(1) = \frac{5}{5 \cdot \ln a} = \frac{1}{6} \Rightarrow a = e^6$$

Yanıt / The Answer is D

1. $\left.\begin{matrix} y = 3x^2 - 1 \\ z = 2y^3 + 4 \end{matrix}\right\} \Rightarrow \dfrac{dz}{dx} = ?$

A) $36x\,(3x^2 - 1)^2$

B) $6x \cdot (3x^2 - 1)^2$

C) $36 \cdot (x^2 - 1)^2$

D) $x\,(3x^2 - 1)^2$

E) $18x\,(3x^2 - 1)$

---

2. $\left.\begin{matrix} x = 4t - t^2 \\ y = 2t^2 + t \end{matrix}\right\} \Rightarrow \dfrac{dy}{dx} = ?$

A) $\dfrac{2t+4}{4t-2}$

B) $\dfrac{4t-1}{2t+4}$

C) $\dfrac{4t+1}{4-2t}$

D) $\dfrac{2t-4}{4t+1}$

E) $\dfrac{t^2-1}{4+2t}$

---

3. $\left.\begin{matrix} y = t^3 + t \\ x = t^3 + 1 \end{matrix}\right\} \Rightarrow \left.\dfrac{d^2y}{dx^2}\right|_{x=2} = ?$

A) $-\dfrac{1}{9}$

B) $-\dfrac{2}{9}$

C) $-\dfrac{4}{9}$

D) $-\dfrac{7}{9}$

E) -1

---

4. $\left.\begin{matrix} x = \ln t \\ y = t^2 \end{matrix}\right\} \Rightarrow \dfrac{d^2y}{dx^2} = ?$

A) t

B) t^2

C) $2t^2$

D) $4t^2$

E) $8t^2$

---

5. $\left.\begin{matrix} y = 3x + 2 \\ x = 2t^2 + 1 \\ t = 3u \end{matrix}\right\} \Rightarrow \left.\dfrac{dy}{du}\right|_{u=\frac{1}{3}} = ?$

A) 3

B) 9

C) 15

D) 27

E) 36

---

6. $f(x) = \sin x \Rightarrow f^{(57)}(x) = ?$

A) sinx

B) cosx

C) –sinx

D) –cosx

E) tanx

7. $f(x)=\frac{1}{x} \Rightarrow f^{(15)}(1)=?$

A) 15!

B) 16!

C) −14!

D) −15!

E) −16!

8. $2x^2+3xy-4y^2=0 \Rightarrow \frac{dy}{dx}=?$

A) $\frac{4x-3y}{3x+8y}$

B) $\frac{-4x-3y}{3x-8y}$

C) $\frac{4x+3y}{3x+8y}$

D) $\frac{2x-3y}{3x+4y}$

E) $\frac{2x+3y}{3x-8y}$

9. $x^y+y^x+3=0 \Rightarrow \left.\frac{dy}{dx}\right|_{(1,\,1)}=?$

A) −3

B) −1

C) 0

D) 1

E) 3

10. $\lim\limits_{x\to\pi}\frac{1+\cos x}{\sin^2 x}=?$

A) $-\frac{1}{4}$

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) $\frac{1}{4}$

11. $\lim\limits_{x\to 1}\frac{x^3-2x^2+x}{2x-2}=?$

A) −2

B) −1

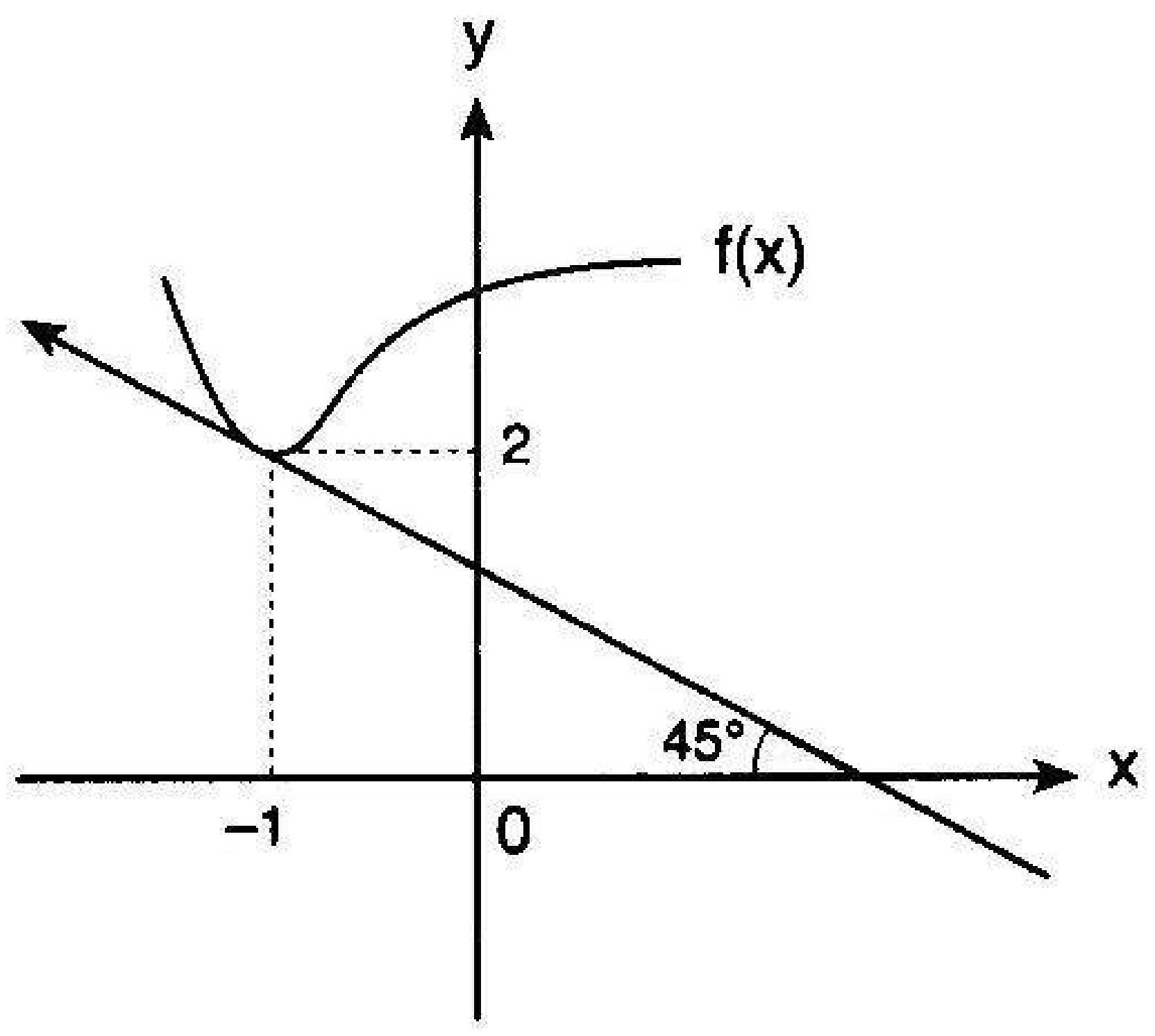
C) 0

D) 1

E) 2

12. $g(x)=3x\cdot f(x)$

$\Rightarrow g'(-1)=?$



A) 7

B) 8

C) 9

D) 10

E) 11

YANITLAR / ANSWERS

1. A　2. C　3. B　4. D　5. E　6. B　7. D　8. B

9. B　10. D　11. C　12. C

1. $\dfrac{dz}{dx}=\dfrac{dz}{dy}\cdot\dfrac{dy}{dx}$

$=6y^2\cdot 6x$

$=36\cdot y^2\cdot x$

$=36\cdot(3x^2-1)^2\cdot x$

$=36x\cdot(3x^2-1)^2$

Yanıt / The Answer is A

---

2. $\dfrac{dy}{dx}=\dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}}=\dfrac{4t+1}{4-2t}$

Yanıt / The Answer is C

---

3. $\dfrac{d^2y}{dx^2}=\dfrac{y''(t)\cdot x'(t)-x''(t)\cdot y'(t)}{(x'(t))^3}$

$$\left.\begin{array}{l} y'(t)=3t^2+1 \\ y''(t)=6t \\ x'(t)=3t^2 \\ x''(t)=6t \end{array}\right\} \Rightarrow \quad \frac{d^2y}{dx^2}=\frac{6t\cdot 3t^2-6t\cdot(3t^2+1)}{(3t^2)^3}$$

$$=\frac{18t^3-18t^3-6t}{27t^6}=\frac{-2}{9t^5}$$

$$x=2 \Rightarrow t=1 \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=2}=-\frac{2}{9}$$

Yanıt / The Answer is B

---

4. $\dfrac{d^2y}{dx^2}=\dfrac{y''(t)\cdot x'(t)-x''(t)\cdot y'(t)}{(x'(t))^3}$

$$\left.\begin{array}{l} y'(t)=2t \\ y''(t)=2 \\ x'(t)=\dfrac{1}{t} \\ x''(t)=-\dfrac{1}{t^2} \end{array}\right\} \Rightarrow \frac{d^2y}{dx^2}=\frac{2\cdot\dfrac{1}{t}+\dfrac{1}{t^2}\cdot 2t}{\left(\dfrac{1}{t}\right)^3}$$

$$=\frac{\dfrac{4}{t}}{\dfrac{1}{t^3}}=4t^2$$

Yanıt / The Answer is D

---

5. $\dfrac{dy}{du}=\dfrac{dy}{dx}\cdot\dfrac{dx}{dt}\cdot\dfrac{dt}{dt}$

$=3\cdot 4t\cdot 3$

$=36t \quad \left[u=\dfrac{1}{3}\Rightarrow t=1\right]$

$=36$

Yanıt / The Answer is E

---

6. $f(x)=\sin x$

$f'(x)=\cos x$

$f''(x)=-\sin x$

$f'''(x)=-\cos x$

$f^{(4)}(x)=\sin x$

$f^{(4)}(x)=f(x) \Rightarrow f^{(57)}(x)=\cos x$

$$\begin{array}{r|l} 57 & \underline{4} \\ -\underline{56} & 14 \\ \textcircled{1} & \end{array}$$

Yanıt / The Answer is B

7. $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1 \cdot x^{2} = -1! \cdot x^{2}$

$f''(x) = -1 \cdot -2 \cdot x^{-3} = 2! \cdot x^{3}$

$f'''(x) = -1 \cdot -2 \cdot -3 \cdot x^{-4} = -3! \cdot x^{4}$

$\vdots$

$f^{(15)}(x) = -15! \cdot x^{-16}$

$f^{(15)}(1) = -(15) \cdot (1)^{-16} = -15!$

Yanıt / The Answer is D

8. $2x^2 + 3xy - 4y^2 = 0$

$$\frac{dy}{dx} = -\frac{f'_x}{f'_y} = -\frac{4x+3y}{3x-8y}$$

$$= \frac{-4x-3y}{3x-8y}$$

Yanıt / The Answer is B

9. $x^y + y^x + 3 = 0$

$$\frac{dy}{dx} = -\frac{f'_x}{f'_y} = -\frac{y \cdot x^{y-1} + y^x \cdot \ln y}{x^y \cdot \ln x + x \cdot y^{x-1}}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = -\frac{1 \cdot x^0 + 1^1 \cdot \ln 1}{1^1 \cdot \ln 1 + 1 \cdot 1^0} = -\frac{1}{1} = -1$$

Yanıt / The Answer is B

10. $\lim\limits_{x \to \pi} \frac{1+\cos x}{\sin^2 x} \quad \left(\frac{0}{0}\right)$

$$\lim_{x \to \pi} \frac{-\sin x}{2 \cdot \sin x \cdot \cos x} = \lim_{x \to \pi} \frac{-1}{2\cos x} = \frac{1}{2}$$

Yanıt / The Answer is D

11. $\lim\limits_{x \to 1} \frac{x^3 - 2x^2 + x}{2x-2} \quad \left(\frac{0}{0}\right)$

$$\Rightarrow \lim_{x \to 1} \frac{3x^2 - 4x + 1}{2} = 0$$

Yanıt / The Answer is C

12. $g(x) = 3x \cdot f(x)$

$g'(x) = 3 \cdot f(x) + 3x \cdot f'(x)$

$g'(-1) = 3 \cdot f(-1) + (-3) \cdot f'(-1)$

$$\left.\begin{matrix} f(-1) = 2 \\ f'(-1) = \tan 135° = -1 \end{matrix}\right\} \Rightarrow g'(-1) = 3 \cdot 2 - 3 \cdot (-1) = 6 + 3 = 9$$

Yanıt / The Answer is C

1. $f(x) = x^2 + \sqrt{x} \Rightarrow \mathbf{f'(9) = ?}$

Yanıt / The Answer is: $\mathbf{\frac{109}{6}}$

2. $f(x) = \frac{x}{x^2 - 1} \Rightarrow \mathbf{f'(2) = ?}$

Yanıt / The Answer is: $\mathbf{-\frac{5}{9}}$

3. $f\,(x) = (3 - 2x)^6 \Rightarrow \mathbf{f'\,(1) = ?}$

Yanıt / The Answer is: **–12**

4. $f(3x - 2) = x^3 - 3x + 1 \Rightarrow \mathbf{f'\,(4) = ?}$

Yanıt / The Answer is: **3**

5. $\left.\begin{aligned} f(x) &= \sqrt{x} \\ g(x) &= x^2 + 3 \end{aligned}\right\} \Rightarrow \mathbf{(fog)'(1) = ?}$

Yanıt / The Answer is: $\mathbf{\frac{1}{2}}$

6. $f(x) = x^4 - 3x^2 + 6x + 3 \Rightarrow \mathbf{\lim\limits_{x \to 1} \frac{f(x) - f(1)}{x - 1} = ?}$

Yanıt / The Answer is: **4**

7. $f(x)=\sqrt{1+x^3} \Rightarrow \mathbf{f'(2)=?}$

Yanıt / The Answer is: **2**

8. $y=\sin^3 x \Rightarrow \mathbf{\frac{dy}{dx}=?}$

Yanıt / The Answer is: **3·sin²x·cosx**

9. $f(x)=\cos(\sin x) \Rightarrow \mathbf{f'\left(\frac{\pi}{2}\right)=?}$

Yanıt / The Answer is: **0**

10. $f(x)=e^{\sin x} \Rightarrow \mathbf{f'(0)=?}$

Yanıt / The Answer is: **1**

11. $f(x)=x\cdot e^x \Rightarrow \mathbf{f'(2)+f''(2)=?}$

Yanıt / The Answer is: $\mathbf{7e^2}$

12. $f(x)=\arccos(2x+1) \Rightarrow \mathbf{f'\left(-\frac{1}{2}\right)=?}$

Yanıt / The Answer is: **–2**

13. $f(x) = (x^2 + 1) \cdot g(2x + 1)$

$g(5) = 6, \ g'(5) = 3$

$\Rightarrow \mathbf{f'(2) = ?}$

Yanıt / The Answer is: **54**

14. $x = f(t) = t^3 - 1$

$y = g(t) = 2t^2 + 2t \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=1} = ?$

Yanıt / The Answer is: **2**

15. $f(x) = \dfrac{\ln x}{x^2+1} \Rightarrow \left.\dfrac{df(x)}{dx}\right|_{x=1} = f'(1) = ?$

Yanıt / The Answer is: $\dfrac{1}{2}$

16. $f(x) = x(2 - \ln x) \Rightarrow \left.\dfrac{df(x)}{dx}\right|_{x=e} = ?$

Yanıt / The Answer is: **0**

17. $f(x) = \sin 6x \cdot \cos 6x \Rightarrow f'\left(\dfrac{\pi}{12}\right) = ?$

Yanıt / The Answer is: **–6**

18. $x^3 + 4xy^2 - y^4 - 27 = 0 \Rightarrow \dfrac{dy}{dx} = ?$

Yanıt / The Answer is: $\dfrac{3x^2 + 4y^2}{4y^3 - 8xy}$

19. $y = x^x \Rightarrow \dfrac{dy}{dx} = ?$

Yanıt / The Answer is: $x^x \cdot \ln(ex)$

20. $f(x) = (x^2+1)\cdot\ln(2x+1) \Rightarrow f'(1) = ?$

Yanıt / The Answer is: $\dfrac{2\ln(27\cdot e^2)}{3}$

21. $\left.\begin{aligned} x &= t^3+3t \\ y &= t^3-3t \end{aligned}\right\} \Rightarrow \left.\dfrac{d^2y}{dx^2}\right|_{t=1} = ?$

Yanıt / The Answer is: $\dfrac{1}{6}$

22. $x \neq -3, 4,$

$f(x) = x^3 - x^2 - 12x + 7$

$\Rightarrow (f^{-1})'(7) = ?$

Yanıt / The Answer is: $-\dfrac{1}{12}$

23. $y = \log_3(3x+1) \Rightarrow \dfrac{dy}{dx} = ?$

Yanıt / The Answer is: $\dfrac{3}{(3x+1)\cdot\ln 3}$

24. $\left.\begin{aligned} x &= t-t^3 \\ y &= 1-t \end{aligned}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=1} = ?$

Yanıt / The Answer is: $\dfrac{1}{2}$

1. $f(x) = x^3 - 2x^2 + 3x - 5 \Rightarrow$

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = ?$$

A) 2 B) 4 C) 6 D) 7 E) 8

2. $f(x) = 4x^2 - 7x + 5 \Rightarrow$

$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = ?$$

A) 0 B) 5 C) 10 D) 14 E) 17

3. $f(x) = \begin{cases} ax^2 - 3 & x \le 2 \\ 3x^2 + 6x & x > 2 \end{cases}$

$f'(1) + f'(3) = 30 \Rightarrow$ **$a = ?$**

A) –2 B) 2 C) 3 D) 6 E) 8

4. $f(x) = \sqrt{x^2 + 3} \Rightarrow$ **$f'(1) = ?$**

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

5. $f(x) = (x^2 - 2x) \cdot (x^3 + 3)$

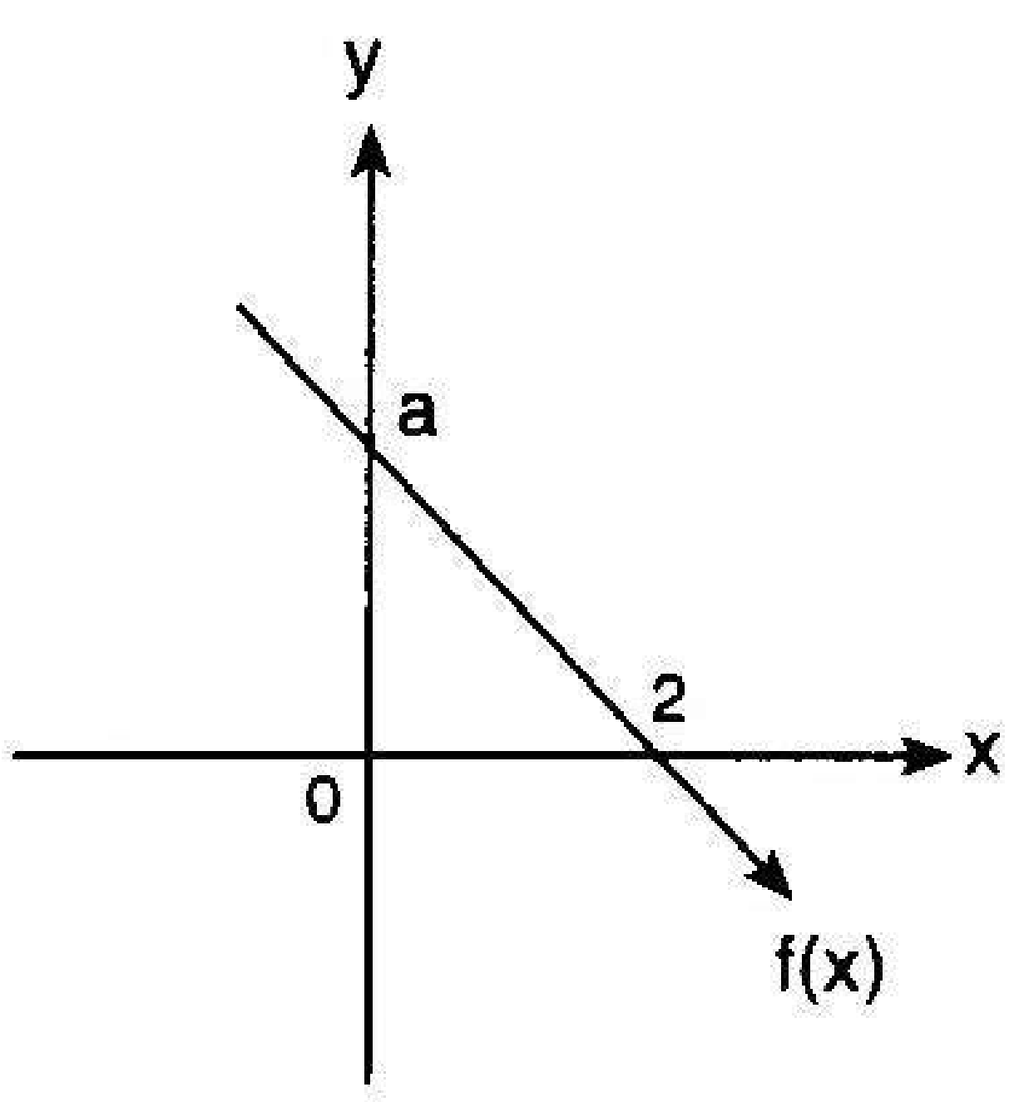
$\Rightarrow$ **$f'(-1) = ?$**

A) –3 B) 0 C) 1 D) 3 E) 5

6. $g(x) = x^2 \cdot f(x)$

$g'(4) = -48$

$\Rightarrow$ **$a = ?$**



A) 2 B) 3 C) 4 D) 5 E) 6

7. $f(x)=\dfrac{2x^2-5}{\sqrt[3]{(x^2-3)^2}} \Rightarrow \mathbf{f'(2)=?}$

A) –2 B) –1 C) 0 D) 1 E) 2

8. $f(x) = |3x^2 + 4x - 8| + 5x \Rightarrow \mathbf{f'(1) = ?}$

A) –5 B) –4 C) –3 D) –2 E) –1

9. $f(3x - 2) = x^2 - 4x + 7 \Rightarrow \mathbf{f'(13) = ?}$

A) 2 B) $\frac{5}{2}$ C) 3 D) $\frac{7}{2}$ E) 4

10. $f(2x + 1) = x^2 \cdot g(x + 1)$,
$g(2) = -1$,
$g'(2) = 3 \Rightarrow \mathbf{f'(3) = ?}$

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

11. $f(2x - 1) = (x^2 + 2) \cdot g(x)$,
$g(1) = -2$,
$g'(1) = 4 \Rightarrow \mathbf{f'(1) = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

12. $f(x) = x^{20} + x^{19} + \ldots + x^0 \Rightarrow \mathbf{f'(-1) = ?}$

A) –20 B) –10 C) 10 D) 20 E) 25

YANITLAR / ANSWERS

1. D	**2. E**	**3. C**	**4. A**	**5. C**	**6. B**	**7. C**	**8. A**
9. A	**10. B**	**11. D**	**12. B**				

1. $f(x) = \sin^2(2x - 3) \Rightarrow \mathbf{f'(x) = ?}$

A) $2\sin(2x - 3)$

B) $2\sin(4x - 6)$

C) $2\sin(3 - 2x)$

D) $2\sin(2x - 3)\cdot\cos x$

E) $2\cos(2x - 3)\cdot\sin(4x - 6)$

2. $f(x) = \tan(\sin x) \Rightarrow \mathbf{f'(\pi) = ?}$

A) -1

B) 0

C) 1

D) 2

E) 3

3. $f(x) = \sin 2x\cdot\cos 3x \Rightarrow \mathbf{f'\left(\frac{\pi}{2}\right) = ?}$

A) -2

B) -1

C) 0

D) 1

E) 2

4. $f(x) = \sin^3 3x \Rightarrow \mathbf{f'\left(\frac{\pi}{4}\right) = ?}$

A) $\frac{-11\sqrt{2}}{4}$

B) $\frac{-9\sqrt{2}}{4}$

C) $\frac{-7\sqrt{2}}{4}$

D) $\frac{-5\sqrt{2}}{4}$

E) $\frac{-3\sqrt{2}}{4}$

5. $f(x) = 4\sin^2 3x\cdot\cos^2 3x \Rightarrow \mathbf{f'\left(\frac{\pi}{18}\right) = ?}$

A) $-3\sqrt{3}$

B) -3

C) 0

D) 3

E) $3\sqrt{3}$

6. $f(x) = \sin^2(\cos x) \Rightarrow \mathbf{f'\left(\frac{3\pi}{2}\right) = ?}$

A) $-\frac{\sqrt{2}}{2}$

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) $\frac{\sqrt{2}}{2}$

7. $f(x) = \sin 2x \cdot \cos 6x \Rightarrow f'\left(\frac{\pi}{6}\right) = ?$

A) -1

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) 1

8. $f(x) = \frac{\sin x}{\sqrt{\cos x}} \Rightarrow f'(0) = ?$

A) $-\frac{1}{2}$

B) 0

C) $\frac{1}{2}$

D) 1

E) $\frac{3}{2}$

9. $f(x) = \arccos(\cos x) \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

A) -2

B) -1

C) 0

D) 1

E) 2

10. $f(x) = x \cdot \arctan x \Rightarrow f'(1) = ?$

A) $\frac{\pi+1}{3}$

B) $\frac{\pi+1}{4}$

C) $\frac{\pi+2}{4}$

D) $\frac{\pi+3}{4}$

E) $\frac{\pi+5}{5}$

11. $f(x) = \operatorname{arccot}\frac{x}{2} \Rightarrow f'(2) = ?$

A) $-\frac{1}{8}$

B) $-\frac{1}{6}$

C) $-\frac{1}{5}$

D) $-\frac{1}{4}$

E) $-\frac{1}{2}$

12. $f(x) = x \cdot \arcsin x + \sqrt{1-x^2} \Rightarrow f'\left(\frac{1}{2}\right) = ?$

A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$

D) $\frac{\pi}{4}$ E) $\frac{\pi}{2}$

YANITLAR / ANSWERS

1. B	**2. A**	**3. C**	**4. B**	**5. E**	**6. C**	**7. A**	**8. D**
9. D	**10. C**	**11. D**	**12. B**				

1. $f(x) = \log_2(8x^2 + 4) \Rightarrow \mathbf{f'(2) = ?}$

A) $\dfrac{8}{9\ln 2}$ B) $\dfrac{10}{9\ln 2}$ C) $\dfrac{16}{9\ln 2}$

D) $\dfrac{25}{9\ln 2}$ E) $\dfrac{28}{9\ln 2}$

2. $x \in Z$,

$f(3x^2 - x + 1) = \log_4(5x^2 - 4) \Rightarrow \mathbf{f'(3) = ?}$

A) $\log_4 e$ B) $2\log_4 e$ C) $3\log_4 e$

D) $4\log_4 e$ E) $5\log_4 e$

3. $f(x) = \cos(\ln(2x^2 - 1)) \Rightarrow \mathbf{f'(1) = ?}$

A) 0 B) 1 C) 2

D) 3 E) 4

4. $f(x) = \ln(\ln(x^2 + 1)) \Rightarrow \mathbf{f'(1) = ?}$

A) $\log_5 e$ B) $\log_4 e$ C) $\log_3 e$

D) $\log_2 e$ E) 1

5. $\left.\begin{array}{l} x = \ln t \\ y = \cos t \end{array}\right\} \Rightarrow \left.\dfrac{\mathbf{dy}}{\mathbf{dx}}\right|_{t=\frac{\pi}{2}} \mathbf{= ?}$

A) $-\dfrac{\pi}{2}$ B) 0 C) $\dfrac{\pi}{4}$

D) $\dfrac{\pi}{3}$ E) $\dfrac{\pi}{2}$

6. $f(x) = \log_3(4x^3 - x^2) \Rightarrow \mathbf{f'(1) = ?}$

A) $\dfrac{5}{3\ln 3}$ B) $\dfrac{7}{3\ln 3}$ C) $\dfrac{8}{3\ln 3}$

D) $\dfrac{10}{3\ln 3}$ E) $\dfrac{11}{3\ln 3}$

7. $f(x) = \ln[(3x^2 - 2x + 1)^4 \cdot (2x^2 - x)] \Rightarrow \mathbf{f'(1) = ?}$

A) 8

B) 11

C) 12

D) 15

E) 20

8. $f(x) = \log_5(x^2 - 3x)^2 \Rightarrow \mathbf{f'(1) = ?}$

A) $\frac{1}{\ln 5}$

B) $\frac{1}{2 \cdot \ln 3}$

C) $\frac{1}{3 \cdot \ln 5}$

D) $\frac{1}{4 \cdot \ln 6}$

E) $\frac{1}{5 \cdot \ln 5}$

9.

$f(x) = x \cdot \ln x \Rightarrow \left.\frac{\mathbf{df(x)}}{\mathbf{dx}}\right|_{x=e} = \mathbf{?}$

A) $\frac{1}{2}$

B) 1

C) 2

D) $\frac{3}{2}$

E) 3

10. $y = \ln x^3 + \ln^3 x \Rightarrow \frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{?}$

A) $\frac{3(1+\ln x)}{x}$

B) $\frac{3(1+\ln^2 x)}{x}$

C) $\frac{1+\ln^2 x}{x^2}$

D) $\frac{3\ln x}{x}$

E) $\frac{3(x+\ln x)}{x}$

11.

$f(x) = x^2 \cdot \ln(x^2+1) \Rightarrow \left.\frac{\mathbf{df(x)}}{\mathbf{dx}}\right|_{x=1} = \mathbf{?}$

A) ln 2e

B) $\ln\frac{2}{e}$

C) ln 4e

D) $\ln\frac{4}{e}$

E) ln 3e

12. $f(x) = \log_{2x} 4 \Rightarrow \mathbf{f'(2) = ?}$

A) $\log_4 \sqrt{e}$

B) $\log_4 e$

C) $-\log_4 \sqrt{e}$

D) $-\log_4 \sqrt[3]{e}$

E) $-\log_4 \sqrt[3]{e^2}$

YANITLAR / ANSWERS							
1. A	**2. B**	**3. A**	**4. D**	**5. A**	**6. D**	**7. B**	**8. A**
9. C	**10. B**	**11. C**	**12. C**				

1. $f(x) = e^{x^2+2x} \cdot \sin x \Rightarrow \mathbf{f'(0) = ?}$

A) 0

B) 1

C) e^2

D) $2e^2$

E) e

2. $x \in Z$,

$f(6x^2 + x) = e^{x^3+10x} \Rightarrow \mathbf{f'(7) = ?}$

A) e^3

B) e^5

C) e^6

D) e^9

E) e^{11}

3. $f(x) = 5^{\cos^2 x} \Rightarrow \mathbf{f'\left(\frac{\pi}{4}\right) = ?}$

A) $-5\sqrt{5}\ln 5$

B) $-\sqrt{5}\ln 5$

C) $\ln 5$

D) $5\ln 5$

E) $25\ln 5$

4. $f(x) = 4^{(2^{x^2})} \Rightarrow \mathbf{f'(1) = ?}$

A) $64 \cdot \ln 2 \cdot \ln 4$

B) $60 \cdot \ln 2 \cdot \ln 4$

C) $48 \cdot \ln 8$

D) $36 \cdot \ln 6$

E) $64 \cdot \ln 4$

5. $f(x) = x^2 \cdot 2^{3x^2} \Rightarrow \mathbf{f'(1) = ?}$

A) $14 \cdot (2 + \ln 3)$

B) $16 \cdot (1 + \ln 2)$

C) $18 \cdot (1 + \ln 2)$

D) $12 \cdot (1 + \ln 4)$

E) $16 \cdot (1 + \ln 8)$

6. $y = (\sin x)^{x^2} \Rightarrow \mathbf{\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = ?}$

A) -1

B) 0

C) 1

D) 2

E) 4

7. $f(x) = e^x \cdot \ln x \Rightarrow$ **$f'(x) = ?$**

A) $e^x (\ln x + \frac{1}{x})$

B) $e^x \left(1 + \frac{1}{x}\right)$

C) $\ln x (e^x + 1)$

D) $\frac{e^x}{x}$

E) $e^x + \ln x$

8. $f(x) = (e^x)^{e^x} \Rightarrow$ **$f'(\ln 2) = ?$**

A) 24

B) 12

C) 4 + ln64

D) 8 + ln 256

E) 64 ln 2

9. $f(x) = x^{\sin x} \Rightarrow$ **$f'(\frac{\pi}{2}) = ?$**

A) $\frac{-\pi}{4}$

B) $\frac{-\pi}{3}$

C) $\frac{-\pi}{2}$

D) 1

E) $\frac{3\pi}{2}$

10. $y = e^x \cdot e^{\ln x} \Rightarrow$ **$\frac{dy}{dx} = ?$**

A) $e^x (x + 1)$

B) $\frac{e^x}{x}$

C) $\frac{e^x \ln x}{x}$

D) $e^x (x + 2)$

E) $e^x \left(1 + \frac{1}{x}\right)$

11. $f(x) = 2^{\ln x^3} \Rightarrow$ **$f'(1) = ?$**

A) ln2

B) ln4

C) ln6

D) ln8

E) ln16

12. $f(x) = x^x + 2^x \Rightarrow$ **$f'(1) = ?$**

A) 1 + ln4

B) 2 + ln4

C) 3 + ln4

D) 4 + ln4

E) 5 + ln4

YANITLAR / ANSWERS

1. B	**2. E**	**3. B**	**4. A**	**5. E**	**6. B**	**7. A**	**8. D**
9. D	**10. A**	**11. D**	**12. A**				

1. $f(x) = (8x-5)^7 \Rightarrow f^{(7)}(x) = ?$

A) $7! \cdot 8^6$

B) $7! \cdot 8^7$

C) $7! \cdot 8^8$

D) $8! \cdot 8^7$

E) $8! \cdot 8^8$

2. $f(x) = \sin(5x+4) \Rightarrow f^{(21)}(x) = ?$

A) $-5^{21} \cdot \sin(5x+4)$

B) $5^{21} \cdot \sin(5x+4)$

C) $-5^{21} \cdot \cos(5x+4)$

D) $5^{21} \cdot \cos(5x+4)$

E) $5^{20} \cdot \cos(5x+4)$

3. $f(x) = \frac{1}{3x+2} \Rightarrow f^{(18)}(-1) = ?$

A) $17! \cdot 3^{17}$

B) $-17! \cdot 3^{17}$

C) $18! \cdot 3^{18}$

D) $-18! \cdot 3^{18}$

E) $-18! \cdot 3^{17}$

4. $f(x) = x^n \Rightarrow f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \ldots + \frac{f^{(n)}(1)}{n!} = ?$

A) 2^n

B) n

C) 1

D) 0

E) 2^{-n}

5. $f(x) = 4x^3 + 12x^2 + 21 \Rightarrow f'''(x) = ?$

A) 8

B) 12

C) 16

D) 20

E) 24

6. $y = (3x-1)^5$, $\frac{d^3y}{dx^3} = a \cdot (3x-1)^2 \Rightarrow a = ?$

A) 1620

B) 1640

C) 1660

D) 1680

E) 1690

7. $f : R^+ \to R,\ f(x) = x^2 - 4x + 3 \Rightarrow \mathbf{(f^{-1})'(8) = ?}$

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{5}$

E) $\frac{1}{6}$

8. $f : R^- \to R,\ f(x) = x^2 - 5x - 4 \Rightarrow \mathbf{(f^{-1})'(2) = ?}$

A) $-\frac{1}{8}$

B) $-\frac{1}{7}$

C) $-\frac{1}{6}$

D) $-\frac{1}{5}$

E) $-\frac{1}{4}$

9. $f(x) = x^5 - 2x^4 + x^3 - 2x^2 + 4x - 4$

$\Rightarrow \mathbf{(f^{-1})'(4) = ?}$

A) $\frac{1}{6}$

B) $\frac{1}{8}$

C) $\frac{1}{12}$

D) $\frac{1}{16}$

E) $\frac{1}{24}$

10. $f(x) = \frac{(x-2)^3}{2} \Rightarrow \mathbf{f^{-1}(4) + (f^{-1})'(4) = ?}$

A) $\frac{16}{17}$

B) $\frac{1}{25}$

C) $\frac{5}{3}$

D) $\frac{25}{6}$

E) 25

11. $f : [1, \infty] \to R,$

$f(x) = x^2 - 3x + 5 \Rightarrow \mathbf{(f^{-1})'(9) = ?}$

A) $\frac{1}{3}$

B) $\frac{1}{5}$

C) $\frac{1}{7}$

D) $\frac{1}{9}$

E) $\frac{1}{11}$

12. $f(x) = x^3 + 5x \Rightarrow \mathbf{(f^{-1})'(6) = ?}$

A) $\frac{1}{8}$ B) $\frac{1}{9}$ C) $\frac{2}{15}$

D) $\frac{3}{13}$ E) $\frac{3}{14}$

YANITLAR / ANSWERS

1. B	**2. D**	**3. D**	**4. A**	**5. E**	**6. A**	**7. E**	**8. B**
9. E	**10. D**	**11. B**	**12. A**				

1. $$\left.\begin{array}{l} y = t^2 - 2 \\ t = u^3 - 3u^2 \\ u = 2x + 1 \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = ?$$

A) 20
B) 22
C) 24
D) 27
E) 30

2. $$\left.\begin{array}{l} y = 3u^2 - 5 \\ u = 2t + 1 \\ t = x^3 \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = ?$$

A) 108
B) 110
C) 114
D) 120
E) 124

3. $$\left.\begin{array}{l} x = e^{2t^2} \\ y = 3t^2 - 5t \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = ?$$

A) 1
B) $\frac{1}{e^2}$
C) $\frac{1}{2e^2}$
D) $\frac{1}{3e^2}$
E) $\frac{1}{4e^2}$

4. $$\left.\begin{array}{l} x = 4t^2 - 3t \\ y = t^2 + 5t \end{array}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=0} = ?$$

A) $\frac{34}{27}$
B) $\frac{40}{27}$
C) $\frac{44}{27}$
D) $\frac{46}{27}$
E) $\frac{49}{27}$

5. $$\left.\begin{array}{l} x = \cos t \\ y = (\sin t)^2 \end{array}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=\pi/4} = ?$$

A) –2
B) –3
C) –4
D) –5
E) –6

6. $$\left.\begin{array}{l} x = e^{t^2+3} \\ y = e^{3t^2-2} \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = ?$$

A) $3e^{-3}$
B) $3e^{-5}$
C) $4e^{-7}$
D) $8e^{-12}$
E) $9e^{-16}$

7. $F(x, y) = x^2 - 5xy + 6x = 0 \Rightarrow \left.\dfrac{dy}{dx}\right|_{(-1,\,1)} = ?$

A) 1 B) $\dfrac{4}{5}$ C) $\dfrac{3}{5}$

D) $\dfrac{2}{5}$ E) $\dfrac{1}{5}$

8. $x^2y = \ln x + 2 \Rightarrow \left.\dfrac{dy}{dx}\right|_{(1,\,2)} = ?$

A) -6 B) -5 C) -4

D) -3 E) -2

9. $F(x, y) = x^2 - \ln(x^2 \cdot y) + 3xy - 4 = 0$

$\Rightarrow \left.\dfrac{dy}{dx}\right|_{(1,\,1)} = ?$

A) $-\dfrac{3}{2}$ B) $-\dfrac{5}{2}$ C) $-\dfrac{7}{2}$

D) $-\dfrac{9}{2}$ E) $-\dfrac{11}{2}$

10. $e^{x+y} = y^x \Rightarrow \dfrac{dy}{dx} = ?$

A) $\dfrac{y}{x \cdot (x+y)}$ B) $\dfrac{y^2}{x \cdot (y-x)}$ C) $\dfrac{y^3}{x \cdot (y-x)}$

D) $\dfrac{y^3}{x^2 - xy}$ E) $\dfrac{y^2}{x^2 + y}$

11. $x^3 + y^3 = 1 \Rightarrow \dfrac{d^2y}{dx^2} = ?$

A) $-\dfrac{x}{y^3}$ B) $-\dfrac{2x}{y^5}$ C) $\dfrac{x}{y^4}$

D) $\dfrac{4x}{y^5}$ E) $-\dfrac{4x}{y^5}$

12. $F(x, y) = 4x^3y - x^2y^2 + 3xy^2 - 2xy = 0$

$\Rightarrow \left.\dfrac{dy}{dx}\right|_{(1,\,-1)} = ?$

A) $-\dfrac{11}{2}$ B) $-\dfrac{9}{2}$ C) $-\dfrac{7}{2}$

D) $-\dfrac{5}{2}$ E) $-\dfrac{3}{2}$

YANITLAR / ANSWERS

1. C	**2. A**	**3. E**	**4. D**	**5. A**	**6. A**	**7. E**	**8. D**
9. A	**10. B**	**11. B**	**12. B**				

1. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1-\sin x}{\cos x} = ?$

A) 0

B) 1

C) 2

D) 3

E) 4

2. $\lim\limits_{x \to \pi} \dfrac{\cos\frac{x}{2}}{x-\pi} = ?$

A) $-\frac{1}{8}$

B) $-\frac{1}{6}$

C) $-\frac{1}{5}$

D) $-\frac{1}{4}$

E) $-\frac{1}{2}$

3. $\lim\limits_{x \to 0} \dfrac{4\sin 9x}{3x} = ?$

A) 6

B) 9

C) 12

D) 15

E) 18

4. $\lim\limits_{x \to 0} \dfrac{\sin x - x}{x^3} = ?$

A) $-\frac{1}{12}$

B) $-\frac{1}{6}$

C) $-\frac{1}{3}$

D) $-\frac{1}{2}$

E) $\frac{1}{3}$

5. $\lim\limits_{x \to 2} \dfrac{\ln x - \ln 2}{x^2 - 4} = ?$

A) $-\frac{1}{8}$

B) $-\frac{1}{6}$

C) 1

D) $\frac{1}{6}$

E) $\frac{1}{8}$

6. $\lim\limits_{x \to 0^+} \dfrac{\ln \sin x}{\ln \tan x} = ?$

A) -1

B) $-\infty$

C) 0

D) $+\infty$

E) 1

7. $a > 2 \Rightarrow \lim\limits_{x \to 0} \dfrac{e^{ax} - e^{x} - x}{x^{2}} = ?$

A) $+\infty$

B) 1

C) 0

D) -1

E) $-\infty$

8. $\lim\limits_{x \to \infty} \dfrac{x + \ln x}{x \cdot \ln x} = ?$

A) -4

B) -2

C) 0

D) 2

E) 4

9. $f'(a) = ?$

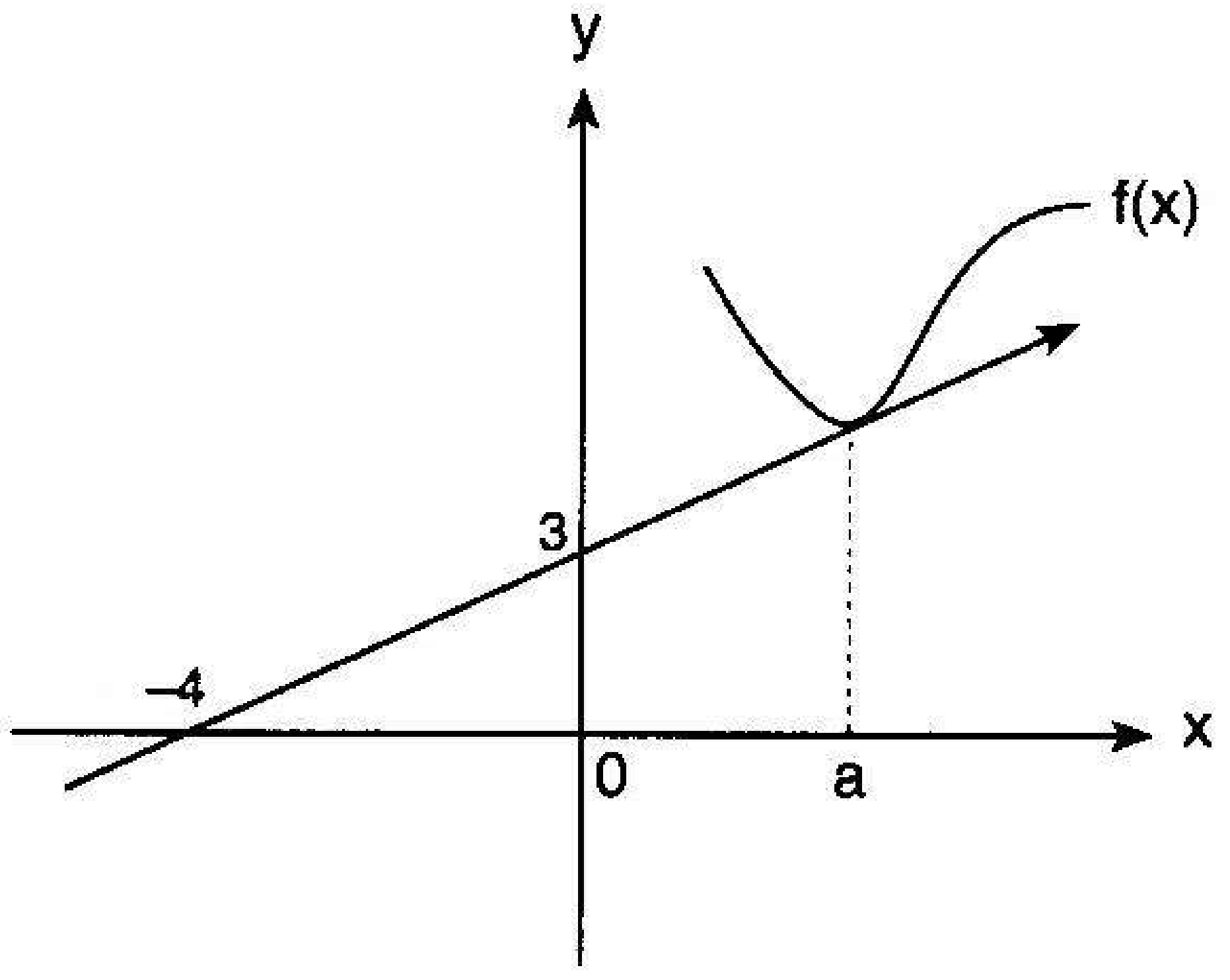
A) $\dfrac{4}{3}$

B) $\dfrac{3}{4}$

C) $\dfrac{3}{5}$

D) $-\dfrac{3}{4}$

E) $-\dfrac{4}{3}$



10. $h(x) = x^2 \cdot f(2x + 1)$

$\Rightarrow h'(-3) = ?$

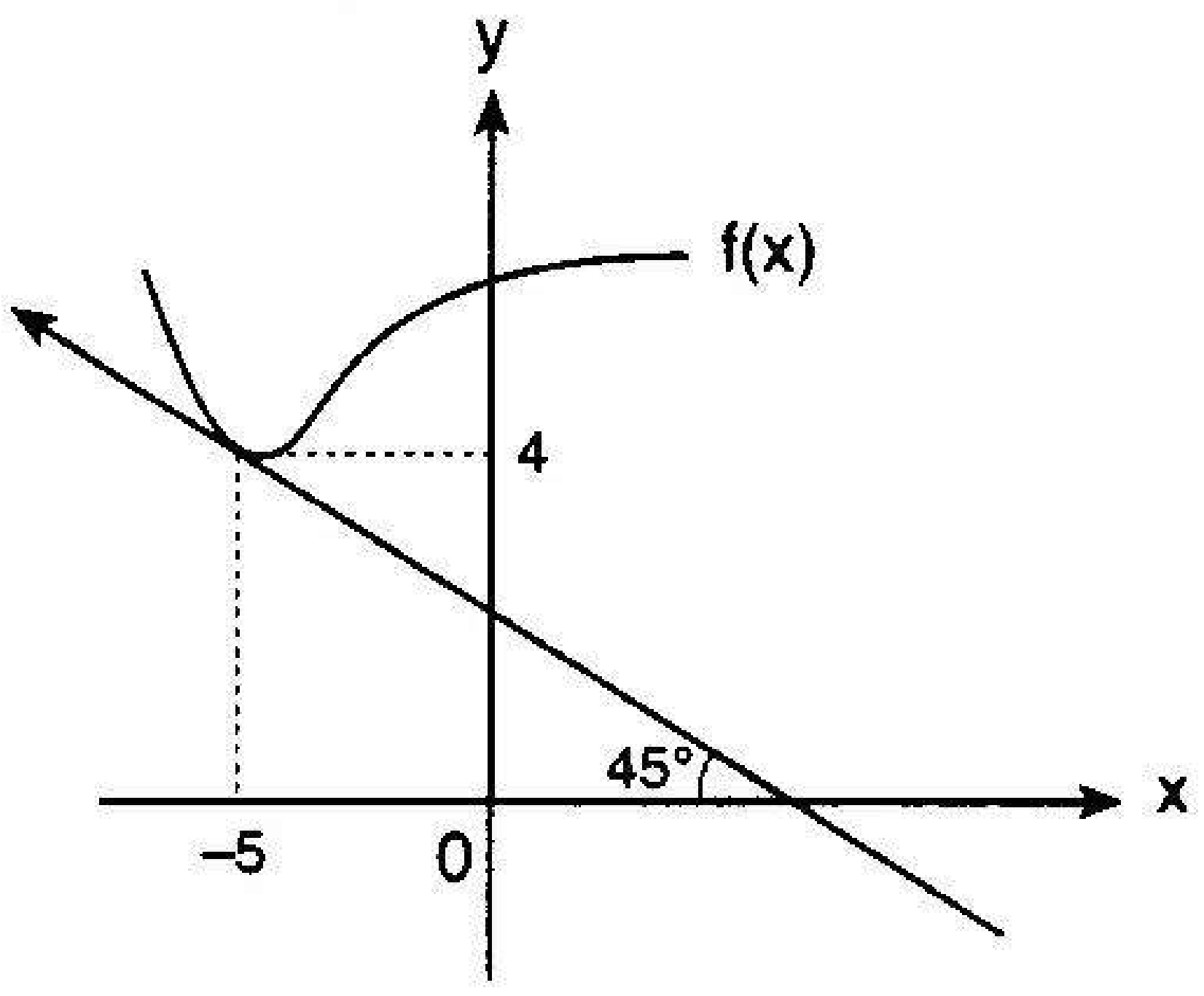
A) -6

B) -12

C) -18

D) -24

E) -42



11. $d : y = 3x + 1,$

$h(x) = \dfrac{f(x)}{x} \Rightarrow h'(2) = ?$

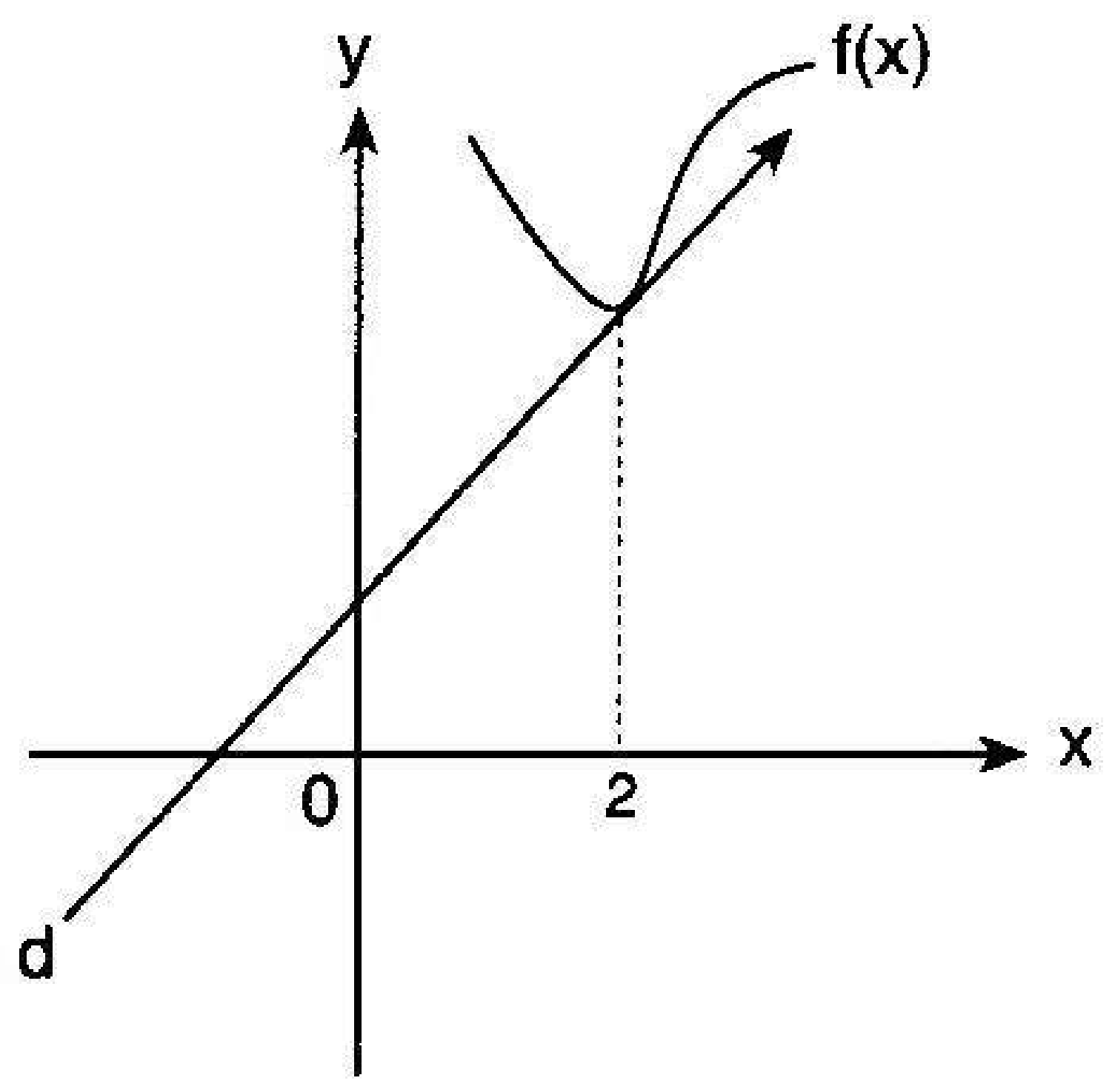
A) $-\dfrac{1}{4}$

B) $-\dfrac{1}{2}$

C) $-\dfrac{3}{4}$

D) $-\dfrac{4}{3}$

E) -2



12. **b = ?**

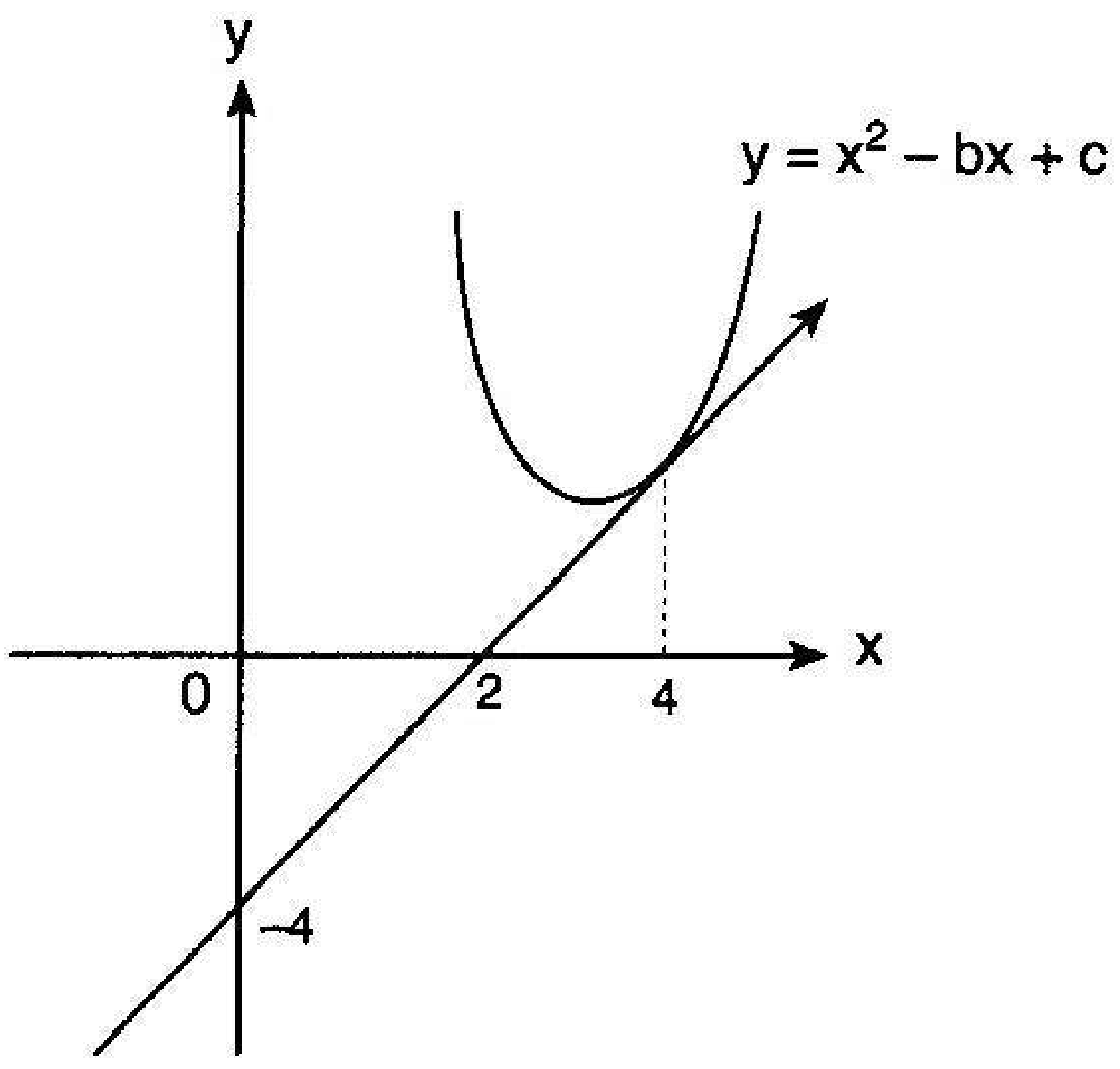
A) 2

B) 4

C) 6

D) 8

E) 10



YANITLAR / ANSWERS

1. A 2. E 3. C 4. B 5. E 6. D 7. A 8. C

9. B 10. E 11. A 12. C

ÜNİTE 10

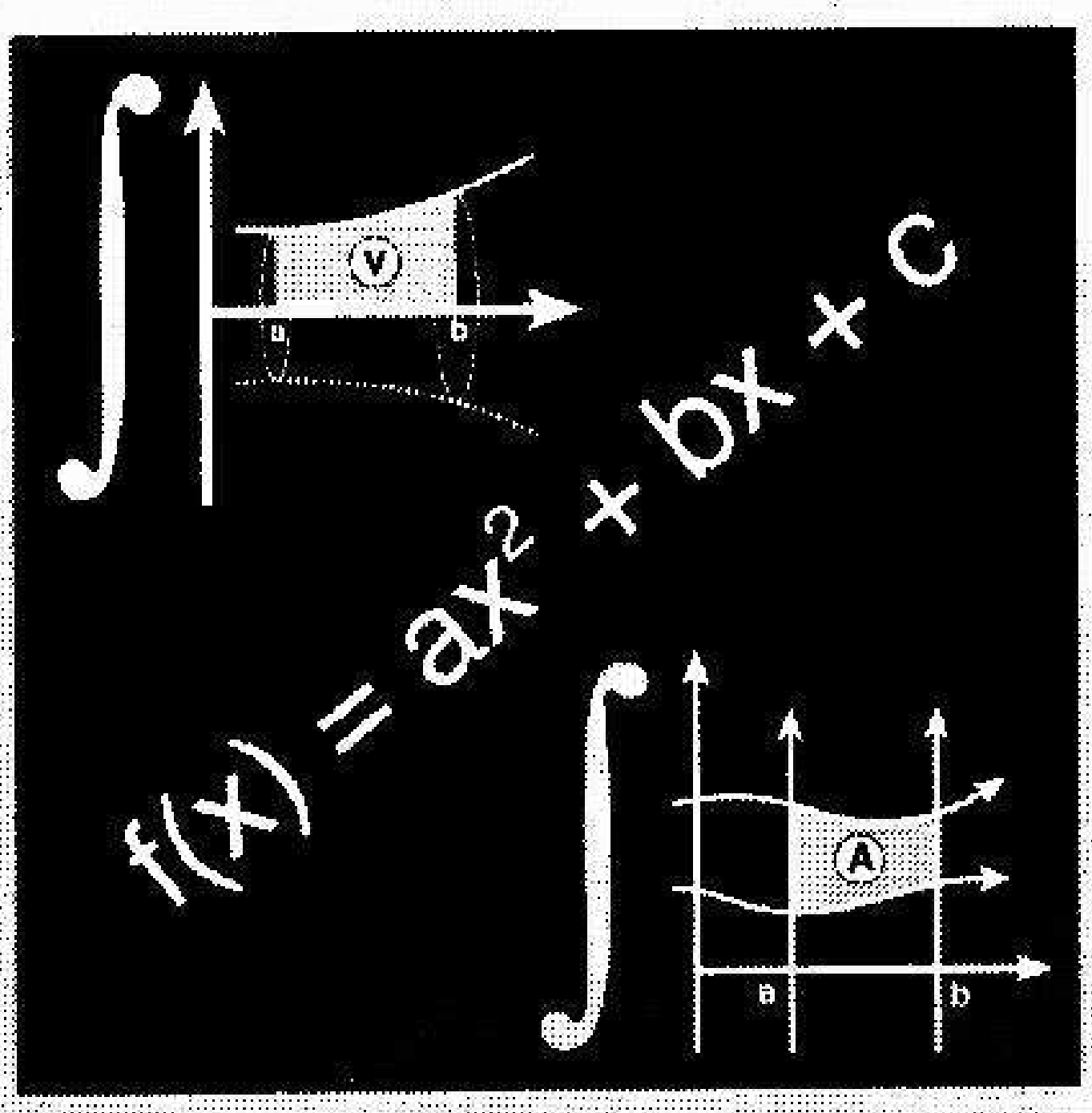
UNIT

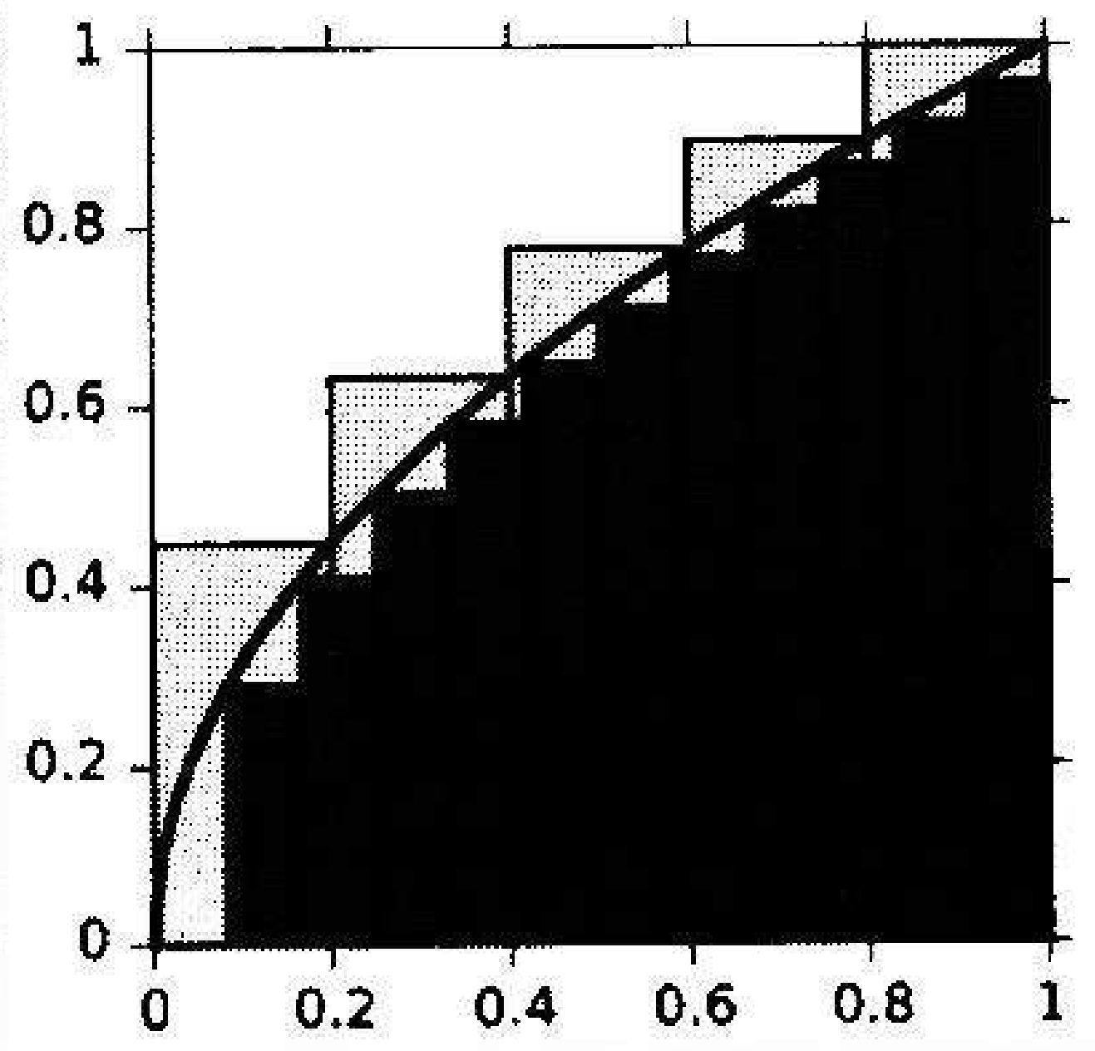
Bu ünitede, belirsiz integral, belirli integral ve integral uygulamaları ayrıntılı anlatılmıştır. Ilk olarak, belirsiz integral ve integral alma yöntemleri , daha sonra belirli integral konusu anlatılmıştır. Son olarak, integral uygulamaları anlatılmıştır.

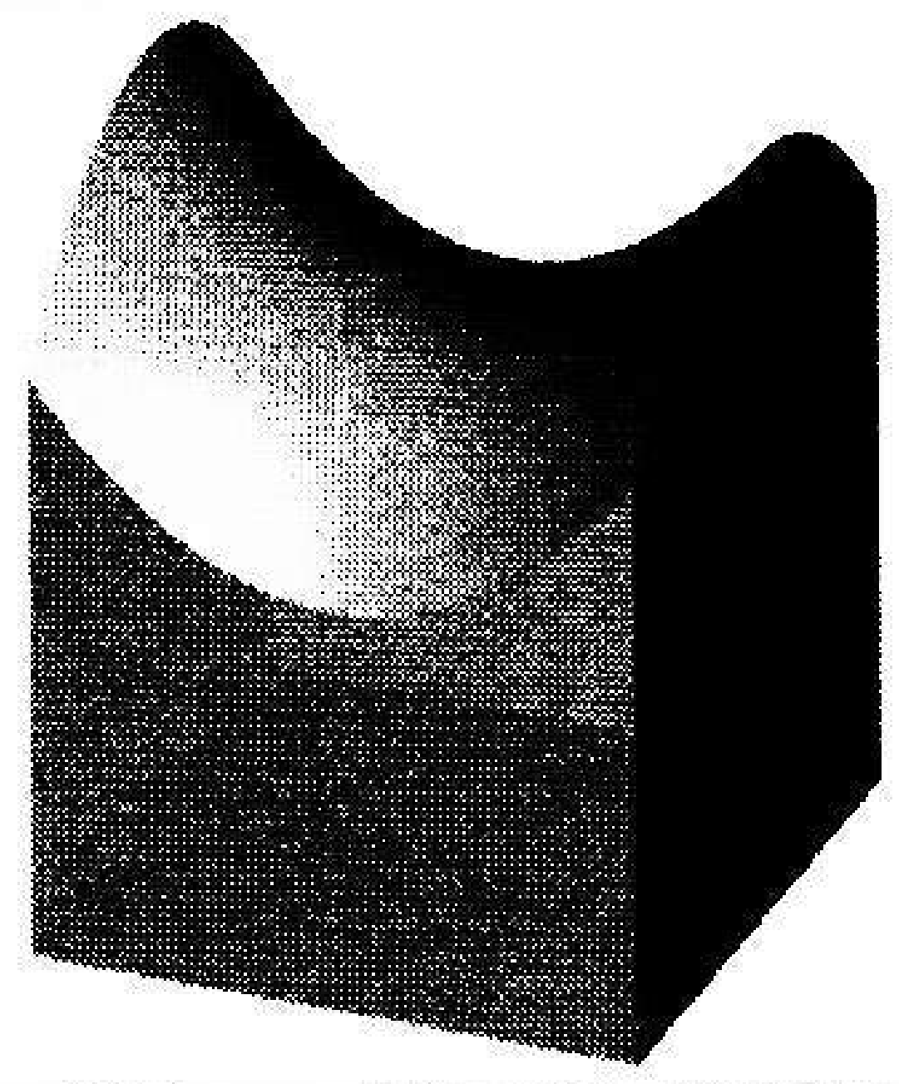
Bu ünite; 127 çözümlü örnek, 60 alıştırma ve 84 test sorusundan oluşmaktadır.

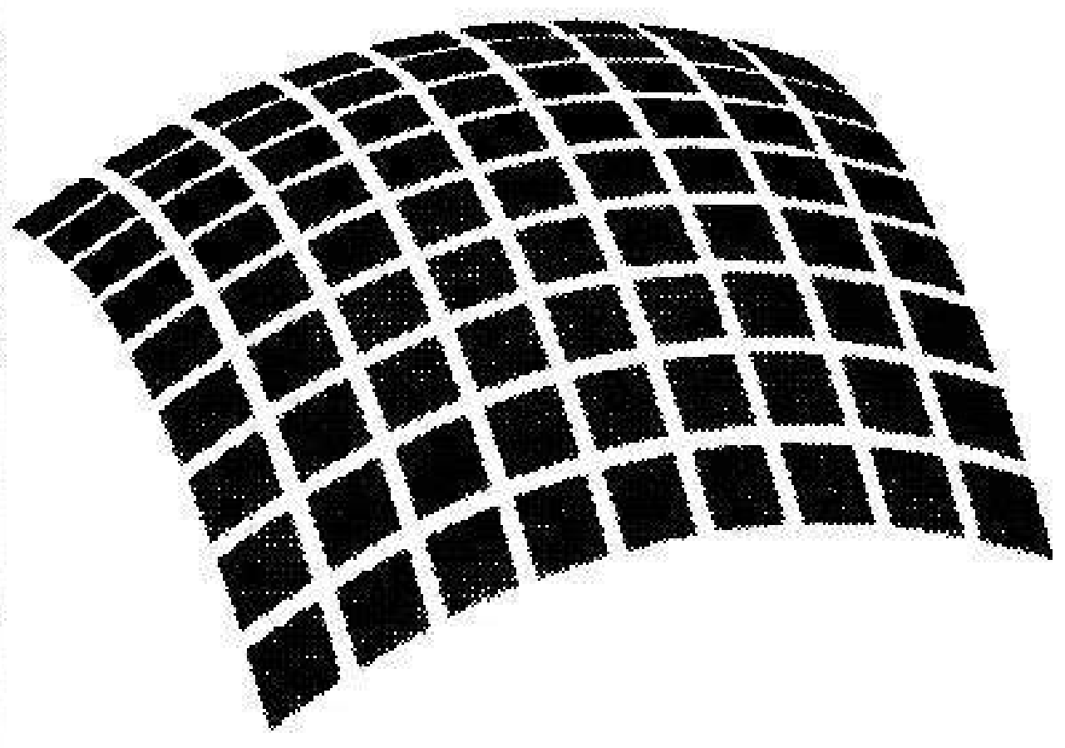
In this unit, indefinite integral, definite integral and applications of the integral are explained in detailed. Firstly, indefinite integral and integral methods are explained. Later, definite integral is explained. Subsequently, applications of integral are explained.

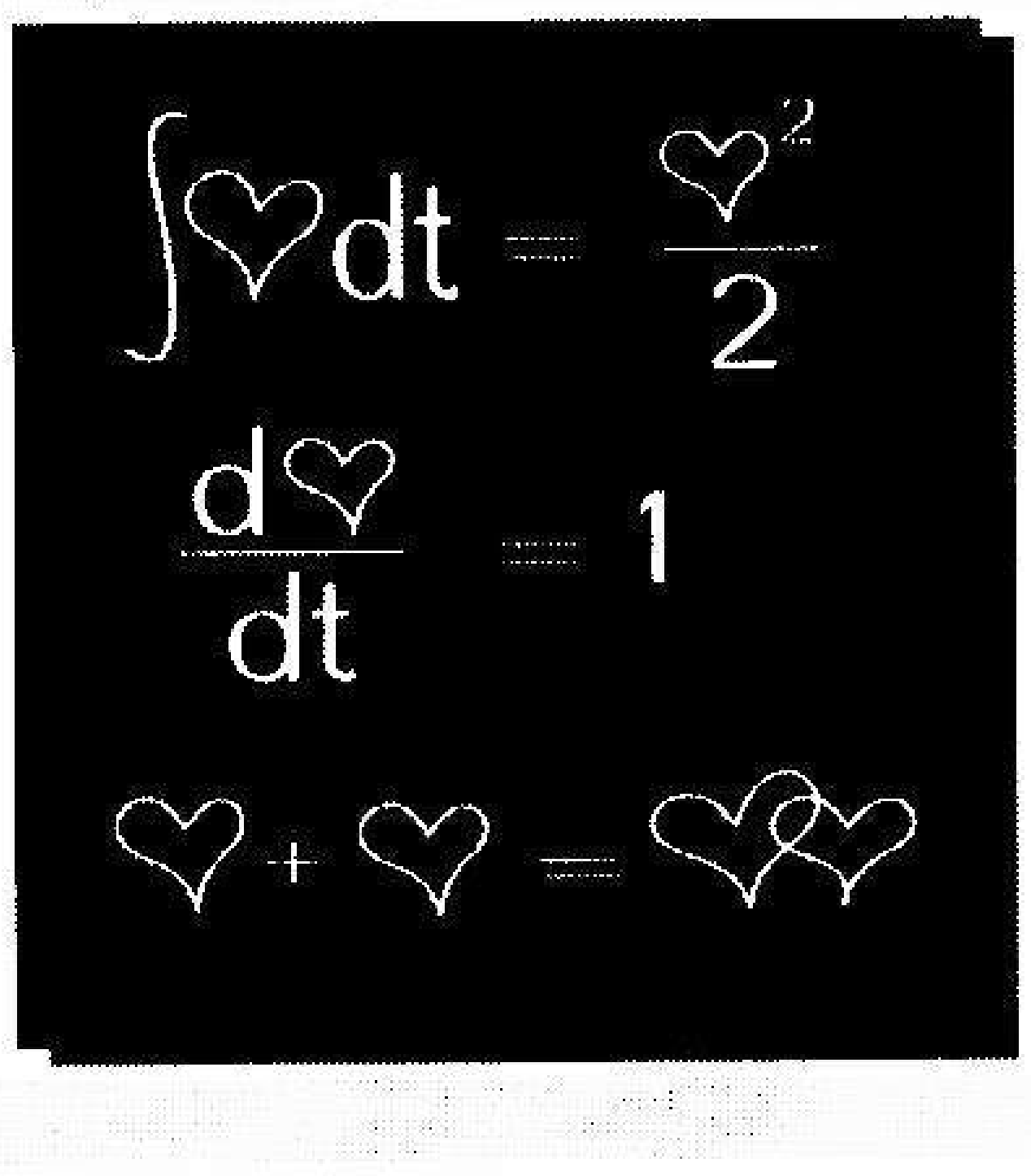
This unit includes; 127 examples, 60 exercises, and 84 test questions.







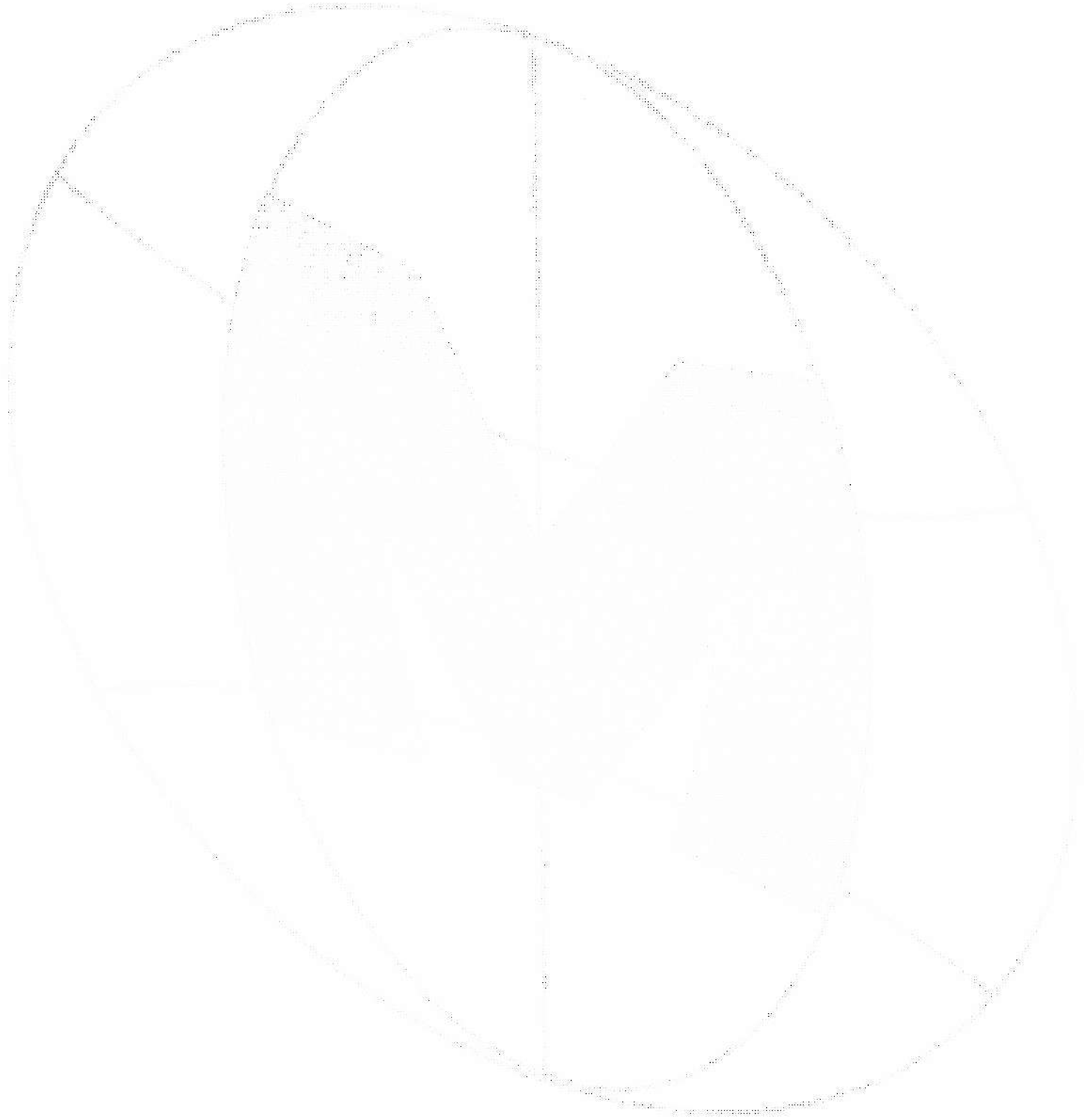




◆ **BELİRSİZ İNTEGRAL / *INDEFINITE INTEGRAL*** .... 741 – 784

◆ **BELİRLİ İNTEGRAL / *DEFINITE INTEGRAL*** .... 785 – 826

BU SAYFADA KONU MATERYALİ YOKTUR.



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BELİRSİZ İNTEGRAL
INDEFINITE INTEGRAL

BÖLÜM
CHAPTER
1

Tanım: F(x) fonksiyonunun türevi f(x) ve diferansiyeli d(F(x)) = f(x) dx olmak üzere, f(x) + C yi bulma işlemine belirsiz integral alma işlemi denir.

Definition: *Let the derivative of F(x) is f(x) and the differantion is dF(x) = f(x)dx. The operation of finding all solutions of f(x) + C is called indefinite integration.*

$$F'(x) = f(x) \Leftrightarrow \int f(x)\,dx = F(x) + C$$

Belirsiz İntegralin Özellikleri / Properties of Indefinite Integral

1. $\frac{d}{dx}\int f(x)\,dx = \left(\int f(x)\,dx\right)' = f(x)$
2. $\int k\cdot f(x)\,dx = k\cdot\int f(x)dx \quad (k\in R)$
3. $\int (f(x)\pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

Temel İntegral Formülleri / Basic Integration Formulas

1. $\int a\,dx = a\int dx = a\cdot x + C \quad (C\in R)$
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (C\in R)$
3. $\int \frac{1}{x}\,dx = \ln|x| + C\ , \quad (C\in R)$
4. $\int e^x\,dx = e^x + C\ , \quad (C\in R)$
5. $\int a^x\,dx = \frac{a^x}{\ln a} + C = a^x \log_a e + C\ , \quad (C\in R)$
6. $\int \sin x\,dx = -\cos x + C\ , \quad (C\in R)$

7. $\int \cos x\,dx = \sin x + c \;,\; (c \in R)$

8. $\int \frac{1}{\sin^2 x} dx = \int \operatorname{cosec}^2 x\,dx = \int (1+\cot^2 x)dx = -\cot x + c \;,\; (c \in R)$

9. $\int \frac{1}{\cos^2 x} dx = \int \sec^2 x\,dx = \int (1+\tan^2 x)dx = \tan x + c \;,\; (c \in R)$

10. $\int \frac{1}{1+x^2} dx = \arctan x + c \;,\; (c \in R)$

11. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c \;,\; (c \in R)$

12. $\int \frac{1}{\sqrt{1+x^2}} dx = \ln\left(x+\sqrt{1+x^2}\right) + c \;,\; (c \in R)$

ÖRNEK / EXAMPLE 1

$\int x \cdot f(x)\,dx = x^3 - x^2 + c \Rightarrow \mathbf{f(2) = ?}$

A) 2 B) 4 C) 6 D) 8 E) 10

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$\int x \cdot f(x)dx = x^3 - x^2 + C$

$\frac{d}{dx}\int x \cdot f(x)dx = (x^3 - x^2 + C)'$

$x \cdot f(x) = 3x^2 - 2x$

$f(x) = \frac{3x^2 - 2x}{x} = 3x - 2$

$\Rightarrow f(2) = 3 \cdot 2 - 2 = 4$

ÖRNEK / EXAMPLE 2

$f'(x) = 3x^2 + 4x - 2,\ f(0) = 3 \Rightarrow \mathbf{f(1) = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$f'(x) = 3x^2 + 4x - 2$$

$$\Rightarrow \int f'(x)dx = \int (3x^2 + 4x - 2)\ dx$$

$$f(x) = \frac{3x^3}{3} + \frac{4x^2}{2} - 2x + C$$

$$= x^3 + 2x^2 - 2x + C$$

$$f(0) = 3 \Rightarrow f(0) = 0^3 + 2\cdot 0^2 - 2\cdot 0 + C = 3$$

$$C = 3$$

$$f(1) = 1^3 + 2\cdot 1^2 - 2\cdot 1 + 3$$

$$= 1 + 2 - 2 + 3 = 4$$

ÖRNEK / EXAMPLE 3

$\int \mathbf{(12x^2 - 6x)\,dx = ?}$

A) $4x^3 - 3x^2 + C$ B) $3x^3 - 4x^2 + C$ C) $6x^3 - 3x^2 + C$ D) $2x^3 - 4x^2 + C$ E) $4x^3 - 4x^2 + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\int (12x^2 - 6x)\ dx = \int 12x^2dx - \int 6x\ dx$$

$$= \frac{12x^3}{3} - \frac{6x^2}{2} + C$$

$$= 4x^3 - 3x^2 + C$$

PEKİŞTİRME / REINFORCE

$\int (6x^2 + 2x)\ dx = ?$

YANIT / ANSWER : $2x^3 + x^2 + C$

ÖRNEK / EXAMPLE 4

$$\int \frac{x\,dx}{\sqrt[3]{x}} = ?$$

A) $\frac{5}{3}\cdot x^{\frac{3}{5}}+C$ B) $15x^{\frac{5}{3}}+C$ C) $\frac{3}{5}\cdot x^{\frac{5}{3}}+C$

D) $15x^{\frac{5}{3}}+C$ E) $\frac{1}{3}x^{\frac{3}{5}}+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int \frac{x\,dx}{\sqrt[3]{x}} = \int \frac{x\,dx}{x^{\frac{1}{3}}}$$

$$= \int x^{1-\frac{1}{3}}\,dx$$

$$= \int x^{\frac{2}{3}}\,dx$$

$$= \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} = \frac{3}{5}\cdot x^{\frac{5}{3}}+C$$

ÖRNEK / EXAMPLE 5

$$\int \left(\sqrt[4]{x}+x^2\right)dx = ?$$

A) $\frac{4x^{\frac{5}{4}}}{5}+\frac{x^3}{3}+C$ B) $4x^{\frac{1}{4}}+\frac{x^3}{3}+C$ C) $x^{\frac{5}{4}}+x^3+C$

D) $x^{\frac{5}{4}}+\frac{x^3}{3}+C$ E) $\frac{4x^{\frac{4}{5}}}{5}+x^3+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\int \left(\sqrt[4]{x}+x^2\right)dx = \int \sqrt[4]{x}\,dx + \int x^2\,dx$$

$$= \int x^{\frac{1}{4}}\,dx + \int x^2\,dx$$

$$= \frac{4x^{\frac{5}{4}}}{5}+\frac{x^3}{3}+C$$

ÖRNEK / EXAMPLE 6

$\int (2^x + x^3)\,dx = ?$

A) $2^x \cdot \ln 2 + \frac{x^4}{4} + C$ B) $2^x \cdot \ln 2 - \frac{x^4}{4} + C$ C) $\frac{2^x}{\ln 2} + \frac{x^4}{4} + C$

D) $\frac{2^x}{\ln 2} + \frac{x^3}{3} + C$ E) $\frac{2^x}{\ln 2} + 3x^2 + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int (2^x + x^3)\,dx = \int 2^x\,dx + \int x^3\,dx$$

$$= \frac{2^x}{\ln 2} + \frac{x^4}{4} + C$$

PEKİŞTİRME / REINFORCE

$\int (e^x + 2x)\,dx = ?$

YANIT / ANSWER : $e^x + x^2 + C$

ÖRNEK / EXAMPLE 7

$\int \left(\sin x + \frac{2}{1+x^2}\right) dx = ?$

A) cosx + arctanx + C

B) –cosx + tanx + C

C) cosx + arctanx + C

D) –cosx + 2arctanx + C

E) –cosx + arctanx + C

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\int \left(\sin x + \frac{2}{1+x^2}\right) dx = \int \sin x\,dx + \int \frac{2}{1+x^2}\,dx$$

$$= -\cos x + 2\arctan x + C$$

ÖRNEK / EXAMPLE 8

$\int \left(\tan^2 x + 3\right) dx = ?$

A) $\tan x + 3x + C$
B) $\dfrac{\tan^3 x}{3} + 3x + C$
C) $\tan^2 x + 3x + C$
D) $\tan^2 x + 2x + C$
E) $\tan x + 2x + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\begin{aligned}\int \left(\tan^2 x + 3\right) dx &= \int \left(\tan^2 x + 1 + 2\right) dx \\ &= \int \left(\tan^2 x + 1\right) dx + \int 2\, dx \\ &= \tan x + 2x + C\end{aligned}$$

İNTEGRAL ALMA YÖNTEMLERİ / INTEGRATION METHODS

1. DEĞİŞKEN DEĞİŞTİRME YÖNTEMİ / CHANGE of VARIABLE

$\int f(x)dx$ integralinde, $x = u(t)$ şeklinde bir değişken değiştirmesi yapıldığında,

$x = u(t) \Rightarrow dx = u'(t)\, dt$ olur.

$\int f(x)dx = \int f(u(t)) \cdot u'(t) \cdot dt$ integrali hesaplandıktan sonra sonuç tekrar x değişkeni cinsinden ifade edilir.

When conversion $x = u(t)$ is applied in $\int f(x)\, dx$, dx is equal to $u'(t) \cdot dt$. Then the resulting integral is as the following:

$\int f(x)\, dx = \int f(u(t)) \cdot u'(t) \cdot dt$ after calculating this integral, we again convert the result to find the answer in terms of x.

NOTLAR / NOTES

- ✓ $\int f^n(x) \cdot f'(x)\, dx = \dfrac{[f(x)]^{n+1}}{n+1} + C$
- ✓ $\int \dfrac{f'(x)\, dx}{f(x)} = \ln|f(x)| + C$
- ✓ $\int \sin(ax+b)\, dx = -\dfrac{1}{a}\cos(ax+b) + C$
- ✓ $\int \cos(ax+b)\, dx = \dfrac{1}{a}\sin(ax+b) + C$
- ✓ $\int \dfrac{dx}{ax+b} = \dfrac{1}{a} \ln|ax+b| + C$
- ✓ $\int e^{ax+b}\, dx = \dfrac{1}{a} \cdot e^{ax+b} + C$

ÖRNEK / EXAMPLE 9

$\int \sin 3x \, dx = ?$

A) $-\frac{1}{3}\cos 3x + C$ B) $\frac{1}{3}\cos 3x + C$ C) $-3 \cdot \cos 3x + C$

D) $3\cos 3x + C$ E) $\sin^3 3x + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\left.\begin{aligned} 3x &= u \\ 3dx &= du \\ dx &= \frac{du}{3} \end{aligned}\right\} \Rightarrow \int \sin 3x \, dx = \int \sin u \, \frac{du}{3}$$

$$= \frac{1}{3} \cdot \int \sin u du$$

$$= -\frac{1}{3} \cdot \cos u + C$$

$$= -\frac{1}{3} \cos 3x + C$$

PEKİŞTİRME / REINFORCE

$\int \cos(5x + 1)\, dx = ?$

YANIT / ANSWER : sin(5x+1) / 5 + C

ÖRNEK / EXAMPLE 10

$\int (x^2 + 1)^3 \, 2x \, dx = ?$

A) $3 \cdot (x^2 + 1)^3 + C$ B) $4 \cdot (x^2 + 1)^4 + C$ C) $\frac{(x^2 + 1)^3}{4} + C$

D) $\frac{(x^2 + 1)^4}{4} + C$ E) $\frac{(x^2 + 1)^4}{3} + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\left.\begin{aligned} x^2 + 1 &= u \\ 2x dx &= du \end{aligned}\right\} \Rightarrow \int (x^2 + 1)^3 \, 2x dx = \int u^3 \cdot du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{(x^2 + 1)^4}{4} + C$$

ÖRNEK / EXAMPLE 11

$\int \tan x \, dx = ?$

A) ln Icosxl + C

B) –ln Icosxl + C

C) –ln Isinxl + C

D) ln Isinxl + C

E) ln Itanxl + C

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\left.\begin{aligned} \cos x &= u \\ -\sin x\,dx &= du \end{aligned}\right\} \Rightarrow \int \tan x\,dx = \int \frac{\sin x}{\cos x}dx$$

$$= \int \frac{-du}{u}$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

ÖRNEK / EXAMPLE 12

$\int \frac{\arctan x}{1+x^2} dx = ?$

A) $\frac{\arctan^2 x}{2} + C$

B) $\arctan^2 x + C$

C) $2\arctan^2 x + C$

D) $\frac{\arctan^3 x}{3} + C$

E) $\arctan x + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\left.\begin{aligned} u &= \arctan x \\ du &= \frac{1}{1+x^2}dx \end{aligned}\right\} \Rightarrow \int \frac{\arctan x}{1+x^2}dx = \int u\,du$$

$$= \frac{u^2}{2} + C = \frac{\arctan^2 x}{2} + C$$

ÖRNEK / EXAMPLE 13

$\int (x^2+3x)^2 \cdot (2x+3)\, dx = ?$

A) $\frac{(x^2+3x)^3}{3}+x^2+C$ B) $(x^2+3x)^3+C$ C) $\frac{(x^2+3x)^3}{3}+C$

D) $\frac{(2x+3)^3}{3}+C$ E) $(4x^2+3x)^3+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\left.\begin{array}{l} x^2+3x=u \\ (2x+3)\,dx=du \end{array}\right\} \Rightarrow \int (x^2+3x)^2 \cdot (2x+3)\,dx = \int u^2 \cdot du$$

$$=\frac{u^3}{3}+C$$

$$=\frac{(x^2+3x)^3}{3}+C$$

ÖRNEK / EXAMPLE 14

$\int \frac{dx}{9x^2+4} = ?$

A) $\arctan\frac{3x}{4}+C$ B) $\frac{1}{3}\arctan\frac{3x}{2}+C$ C) $\frac{1}{6}\arctan\frac{3x}{2}+C$

D) $\frac{1}{4}\arctan\frac{x}{2}+C$ E) $\frac{1}{9}\arctan\frac{2x}{2}+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int \frac{dx}{9x^2+4} = \int \frac{dx}{4\cdot\left(1+\frac{9}{4}x^2\right)} = \int \frac{dx}{4\cdot\left(1+\left(\frac{3x}{2}\right)^2\right)}$$

$$\left.\begin{array}{l} \frac{3x}{2}=u \\ dx=\frac{2}{3}du \end{array}\right\} \Rightarrow \frac{1}{4}\int \frac{dx}{1+\left(\frac{3x}{2}\right)^2} = \frac{1}{4}\cdot\frac{2}{3}\int \frac{du}{1+u^2}$$

$$=\frac{1}{6}\arctan u + C$$

$$=\frac{1}{6}\arctan u + C = \frac{1}{6}\arctan\frac{3x}{2}+C$$

PEKİŞTİRME / REINFORCE

$\int \frac{dx}{x^2+4} = ?$

YANIT / ANSWER : $\frac{1}{2}\arctan\frac{x}{2}+C$

2. BASİT KESİRLERE AYIRMA YÖNTEMİ / DIVIDING into SIMPLE FRACTIONS

$\int \frac{P(x)}{Q(x)}\,dx$ şeklinde verilen integral ifadesi hesaplanırken;

To calculate the integration $\int \frac{P(x)}{Q(x)}\,dx$.

1. der [P(x)] ≥ der [Q(x)] ise, polinom bölmesi yapılarak integral alınır.
   If der [P(x)] ≥ der [Q(x)], we use polynomial division
2. der [P(x)] < der [Q(x)] ise / *If der [P(x)] < der [Q(x)],*
   - **a.** Q(x) çarpanlara ayrılıyorsa ifade basit kesirlere ayrılarak integral alınır.
     If Q(x) is factoring, we use the method dividing into simple fractions.
   - **b.** Q(x) çarpanlarına ayrılmıyorsa Q(x) tam kare ifadeye tamamlanarak integral alınır.
     If Q(x) is not factoring, we use the perfect square expression.

ÖRNEK / EXAMPLE 15

$\int \frac{x}{x+1}\,dx = ?$

A) $\ln|x+1| + C$

B) $x + \ln|x+1| + C$

C) $\frac{1}{2}\ln|x+1| + C$

D) $x \cdot \ln|x+1| + C$

E) $x - \ln|x+1| + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\int \frac{x}{x+1}\,dx = \int \left(1 - \frac{1}{x+1}\right) dx$$

$$= \int dx - \int \frac{dx}{x+1}$$

$$= x - \ln|x+1| + C$$

$$\begin{array}{r|l} x & x+1 \\ \cline{2-2} \underline{-\;x+1} & 1 \\ -1 & \end{array}$$

$$\frac{x}{x+1} = 1 - \frac{1}{x+1}$$

PEKİŞTİRME / REINFORCE

$\int \frac{2x-1}{2x+1}\,dx = ?$

YANIT / ANSWER : $x - \ln|2x+1| + C$

ÖRNEK / EXAMPLE 16

$$\int \frac{dx}{x^2+4x+5} = ?$$

A) $\arctan(x+2)+C$

B) $\arctan x + C$

C) $\ln\left|\frac{x+4}{x+1}\right|+C$

D) $\arctan(x+4)+C$

E) $\arctan(x+1)+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$x^2+4x+5 = x^2+4x+4+1 = (x+2)^2+1$$

$$\int \frac{dx}{x^2+4x+5} = \int \frac{dx}{1+(x+2)^2}$$

$$\left.\begin{matrix} x+2=u \\ dx=du \end{matrix}\right\} \Rightarrow \int \frac{du}{1+u^2} = \arctan u + C$$

$$= \arctan(x+2)+C$$

ÖRNEK / EXAMPLE 17

$$\int \frac{5dx}{x^2-3x-4} = ?$$

A) $5\cdot\ln|x^2-3x-4|+C$

B) $\ln\left|\frac{x-4}{x+1}\right|+C$

C) $\frac{1}{5}\ln\left|\frac{x-4}{x+1}\right|+C$

D) $\ln|x^2-3x-4|+C$

E) $\arctan\left|\frac{x-4}{x+1}\right|+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{5}{x^2-3x-4} = \frac{A}{x-4}+\frac{B}{x+1} = \frac{Ax+A+Bx-4B}{x^2-3x-4}$$

$$\left.\begin{matrix} A+B=0 \\ A-4B=5 \end{matrix}\right\} \Rightarrow A=1,\ B=-1$$

$$\int \frac{5dx}{x^2-3x-4} = \int \frac{dx}{x-4} - \int \frac{dx}{x+1}$$

$$= \ln|x-4| - \ln|x+1| + C$$

$$= \ln\left|\frac{x-4}{x+1}\right|+C$$

NOT / NOTE

Logaritma, **A**rc, **P**olinom, **T**rigonometrik, **Ü**stel fonksiyonlarının herhangi ikisinin çarpımı biçimindeki fonksiyonların integrali alınırken verilen sıra önceliği olan fonksiyona u denir.

In the integration consists of ***L****ogarithmic,* ***A****rc,* ***P****olynomial,* ***T****rigonometric, or* ***E****xponential functions; firstly u is given order expression.*

3. KISMİ İNTEGRASYON YÖNTEMİ / INTEGRATION by PARTS

u ile v, x değişkenine bağlı birer fonksiyon olmak üzere,
u ile dv çarpımının integrali $\int u\cdot dv = u\cdot v - \int v\cdot du$ dur.

Let u and v be a function x .
The integration of u and dv is $\int u\cdot dv = u\cdot v - \int v\cdot du$.

ÖRNEK / EXAMPLE 18

$\int \arctan x \, dx = ?$

A) $\arctan x\cdot x - \frac{1}{2}\ln|1+x^2| + C$

B) $\arctan x\cdot x + \frac{1}{2}\ln|1+x^2| + C$

C) $\arctan x\cdot x - \frac{1}{4}\ln|1+x^2| + C$

D) $\arctan x\cdot x + \frac{1}{4}\ln|1+x^2| + C$

E) $\arctan x\cdot x + 3\ln|1+x^2| + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$\arctan x = u \qquad dx = dv$

$\frac{1}{1+x^2}dx = du \qquad x = v$

$$\int \arctan x \, dx = \arctan x\cdot x - \int \frac{x}{1+x^2}dx \qquad \left[1+x^2 = u \;,\; 2xdx = du \;,\; xdx = \frac{du}{2}\right]$$

$$= \arctan x\cdot x - \frac{1}{2}\int \frac{du}{u}$$

$$= \arctan x\cdot x - \frac{1}{2}\ln|1+x^2| + C$$

ÖRNEK / EXAMPLE 19

$\int x\cdot\sin x\,dx=?$

A) $x^2\cdot\cos x+\sin x+C$ B) $x^2\cdot\cos x+\sin x+C$ C) $-x\cdot\cos x+\sin x+C$

D) $-x\cdot\sin x+\cos x+C$ E) $x\cdot\cos x+\cos x+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

Çözüm / Solution 1

$x=u$ $\sin x\,dx=dv$

$dx=du$ $-\cos x=v$

$$\int x\cdot\sin x\,dx=-x\cdot\cos x+\int\cos x\,dx$$

$$=-x\cdot\cos x+\sin x+C$$

Çözüm / Solution 2

Türev / *Derivative*		İntegral / *Integral*
$+x$	↘	$\sin x$
-1	↘	$-\cos x$
$+0$		$-\sin x$

$$\int x\cdot\sin x\,dx=x\cdot(-\cos x)+\sin x+C$$

ÖRNEK / EXAMPLE 20

$\int 3x\cdot e^{3x}\,dx=?$

A) $x\cdot e^{3x}-\frac{1}{3}e^{3x}+C$ B) $2x\cdot e^{3x}+\frac{1}{3}e^{3x}+C$ C) $2x\cdot e^{3x}-\frac{1}{3}e^{3x}+C$

D) $x\cdot e^{2x}-\frac{1}{3}e^{2x}+C$ E) $x\cdot e^{2x}+\frac{1}{3}e^{2x}+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

Türev / *Derivative*		İntegral / *Integral*
$+3x$	↘	e^{3x}
-3	↘	$\frac{1}{3}e^{3x}$
$+0$		$\frac{1}{9}e^{3x}$

$$\int 3x\cdot e^{3x}\,dx=3x\cdot\frac{1}{3}e^{3x}-3\cdot\frac{1}{9}e^{3x}+C$$

$$=x\cdot e^{3x}-\frac{1}{3}e^{3x}+C$$

NOT / NOTE

$\int f(x)\cdot g(x)\,dx$ integralindeki çarpanlardan f(x) in sonlu sayıdaki türevi sıfır ve g(x) in integrali de kolay alınabiliyorsa integral aşağıdaki gibi hesaplanır.

In integral $\int f(x)\cdot g(x)\,dx$, the finite derivative of f(x) is 0 and the integral of g(x) is taken easily, we calculate as follows.

f(x)		g(x)
$+f(x)$	↘	1. integral / *1st Integral*
$-f'(x)$	↘	2. integral / *2nd Integral*
$+f''(x)$		3. integral / *3rd Integral*
⋮		⋮

ÖRNEK / EXAMPLE 21

$\int x \cdot \cos x \, dx = ?$

A) $-x \cdot \cos x + \sin x + C$
B) $x \cdot \sin x + \cos x + C$
C) $-x \cdot \cos x + \cos x + C$
D) $x \cdot \cos x + \sin x + C$
E) $-x \cdot \cos x - \sin x + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$x = u \quad , \quad \cos x \, dx = dv$

$dx = du \quad , \quad \sin x = v$

$$\int x \cdot \cos x \, dx = x \cdot \sin x - \int \sin x \, dx$$

$$= x \cdot \sin x + \cos x + C$$

ÖRNEK / EXAMPLE 22

$F(x) = \int \ln x \, dx \Rightarrow F(x) = ?$

A) $x \cdot (\ln x - 1) + C$
B) $x \cdot (\ln x + 1) + C$
C) $x \cdot (1 + 2 \cdot \ln x) + C$
D) $x \cdot (1 - 2 \cdot \ln x) + C$
E) $x \cdot (2 + \ln x) + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$u = \ln x \Rightarrow du = \frac{dx}{x}$

$dv = dx \Rightarrow v = x$

$$\int u dv = uv - \int v du$$

$$= x \cdot \ln x - \int x \cdot \frac{dx}{x}$$

$$= x \cdot \ln x - x + C$$

$$= x \cdot (\ln x - 1) + C$$

ÖRNEK / EXAMPLE 23

YÖS / 2008

$\int (x^2+4x)^4 \cdot (2x+4)\,dx = ?$

A) $(x^2+4x)^4+C$ B) $4(x^2+4x)+C$ C) $8(x^2+4x)+C$

D) $\dfrac{(x^2+4x)^4}{4}+C$ E) $\dfrac{(x^2+4x)^5}{5}+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x^2+4x=u \Rightarrow (2x+4)\,dx=du$

$$\int (x^2+4x)^4 \cdot (2x+4)\,dx = \int u^4 \cdot du = \frac{u^5}{5}+C$$

$$= \frac{(x^2+4x)^5}{5}+C$$

ÖRNEK / EXAMPLE 24

YÖS / 2010

$\int \frac{2\ln x}{x}\,dx = ?$

A) $\ln x + C$ B) $\ln^2 x + C$ C) $x\ln x + C$

D) $x^2ex + C$ E) $x^ex + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$\ln x = u \Rightarrow \frac{1}{x}\,dx = du$

$$\int \frac{2\ln x}{x}\,dx = \int 2u\,du$$

$$= u^2 + C$$

$$= \ln^2 x + C$$

ÖRNEK / EXAMPLE 25

YILDIZ TEKNİK ÜNİ. YÖS 2011

$$\frac{3}{2}\int \frac{\sqrt{\ln x}}{x}\,dx = ?$$

A) $\sqrt{\ln^3 x} + C$

B) $\sqrt{\ln x} + C$

C) $\frac{1}{2}x + C$

D) $-\frac{2}{3x} + C$

E) $3x\ln x + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\left.\begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}\right\} \Rightarrow \frac{3}{2}\int \frac{\sqrt{\ln x}}{x}\,dx = \frac{3}{2}\int \sqrt{u}\,du$$

$$= \frac{3}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \sqrt{\ln^3 x} + C$$

ÖRNEK / EXAMPLE 26

YILDIZ TEKNİK ÜNİ. YÖS 2011

$$\int f(x)\,dx = 2x^3 - 5x^2 + C$$

$f(x) = ?$

A) $6x - 5$

B) $6x^2 - 10x$

C) $2x^4 - 5x^3$

D) $6x^2 - 5x + 1$

E) $3x - 1$

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\frac{d}{dx}\left(\int f(x)\,dx\right) = \frac{d}{dx}(2x^3 - 5x^2 + C)$$

$$f(x) = 6x^2 - 10x$$

ÖRNEK / EXAMPLE 27

DOKUZ EYLÜL ÜNİ. YÖS 2011

$\int \ln x \, dx$ **integralinin eşiti aşağıdakilerden hangisidir?**

Which of the following is equal to the integral $\int \ln x \, dx$ *?*

A) $x\ln x - x + C$ B) $x\ln x + x + C$ C) $x\ln x + \frac{x^2}{2} + C$

D) $x\ln x + 2x + C$ E) $x^2\ln x - x + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$$\int \ln x \, dx = \ln x \cdot x - \int x \cdot \frac{1}{x} dx = x\ln x - x + C$$

ÖRNEK / EXAMPLE 28

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$\int \frac{dx}{9+4x^2} = ?$$

A) $\frac{2}{3}\ln x^2 + C$ B) $\sin\frac{2x}{3} + C$ C) $\frac{1}{6}\arctan\frac{2x}{3} + C$

D) $\frac{1}{3}\arcsin\frac{2x}{3} + C$ E) $\frac{1}{3}\arccos\frac{3x}{5} + C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int \frac{dx}{9+4x^2} = \frac{1}{9}\int \frac{dx}{1+\left(\frac{2x}{3}\right)^2}$$

$$\frac{2x}{3} = u \Rightarrow \frac{2dx}{3} = du$$

$$dx = \frac{3du}{2}$$

$$\Rightarrow \frac{1}{9}\int \frac{3du}{2(1+u^2)} = \frac{1}{6}\arctan u = \frac{1}{6}\arctan\frac{2x}{3} + C$$

ÖRNEK / EXAMPLE 29

SÜLEYMAN DEMİREL ÜNİV. YÖS 2015

$\int (e^x+1)^2 \cdot e^x dx = ?$

A) e^x+1+c B) $(e^x+1)^2+c$ C) $(e^x+1)^3+c$

D) $\dfrac{(e^x+1)^2}{2}+c$ E) $\dfrac{(e^x+1)^3}{3}+c$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\int (e^x+1)^2 \cdot e^x dx$

$$\left.\begin{aligned} t &= e^x+1 \\ dt &= e^x dx \end{aligned}\right\} \Rightarrow \int t^2 dt = \frac{t^3}{3}+c$$

$$= \frac{(e^x+1)^3}{3}+c$$

ÖRNEK / EXAMPLE 30

ANKARA ÜNİ. YÖS 2016

$\int \sin x \cdot \cos^2 x \, dx = ?$

A) $\dfrac{\sin^3 x}{3}+C$ B) $\dfrac{-\sin^3 x}{3}+C$ C) $\dfrac{\tan^3 x}{3}+C$

D) $\dfrac{\cos^3 x}{2}+C$ E) $\dfrac{-\cos^3 x}{3}+C$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\cos x = u$

$$-\sin x \, dx = du \Rightarrow \int -u^2 du = \frac{-u^3}{3}+C$$

$$= \frac{-\cos^3 x}{3}+C$$

1. $f'(x) = 12x^3 - 6x + 1$, $f(-1) = 5$, $\mathbf{f(1) = ?}$

A) 5

B) 6

C) 7

D) 8

E) 9

2. $\int (3x-2)\, f(x)\, dx = 2x^3 + \frac{5}{2}x^2 - 6x + C$

$\Rightarrow \mathbf{f(-1) = ?}$

A) – 2

B) –1

C) 0

D) 1

E) 2

3. $\int \frac{e^x - e^{-x}}{2}\, dx = ?$

A) $\frac{1}{2}(e^x + e^{-x}) + C$

B) $(e^x + e^{-x}) + C$

C) $\frac{1}{5}(e^x + e^{-x}) + C$

D) $(e^x - e^{-x}) + C$

E) $(e^{2x} - e^{-2x}) + C$

4. $\int \frac{dx}{\sqrt{x\sqrt{x}}} = ?$

A) $\frac{1}{4}\sqrt[4]{x} + C$

B) $\frac{1}{2}\sqrt[4]{x} + C$

C) $\sqrt[4]{x} + C$

D) $2\sqrt[4]{x} + C$

E) $4\sqrt[4]{x} + C$

5. $\int (x^2y + e^xy + e^y)\, dy = ?$

A) $\frac{x^{3y}}{3} + e^xy + xe^y + C$

B) $\frac{x^2y^2}{2} + e^xy + e^y + C$

C) $\frac{y^2(x^2 + e^x)}{2} + e^y + C$

D) $x^2y + \frac{e^xy^2}{2} + C$

E) $\frac{x^2 + y^2}{2} + \frac{e^xy^2}{2} + xe^y + C$

6. $\int \frac{1-\sqrt{x}}{2\sqrt{x}}\, dx = ?$

A) $\frac{(1-\sqrt{x})^2}{2} + C$

B) $\sqrt{x} - \frac{1}{2}x + C$

C) $(\sqrt{x} + 1)^2 + C$

D) $\frac{x + \sqrt{x}}{2} + C$

E) $\frac{1 + \sqrt{2x}}{2} + C$

7. $\int \tan^2 x \, dx = ?$

A) sinx − x + C

B) cosx + x + C

C) tanx − x + C

D) cotx + x + C

E) secx − x + C

8. $\int \frac{d(x^3)}{x^2} = ?$

A) $\frac{x}{3} + C$

B) $\frac{x^2}{3} + C$

C) $x^3 + C$

D) $3x^2 + C$

E) $3x + C$

9. $\int (x-1)^2 \cdot x^2 \, dx = ?$

A) $\frac{x^5}{10} - \frac{x^4}{4} + \frac{x^3}{2} + C$

B) $\frac{x^5}{5} + \frac{x^4}{4} - \frac{x^3}{3} + C$

C) $\frac{x^4}{4} - \frac{x^3}{2} + \frac{x^2}{4} + C$

D) $\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} + C$

E) $\frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{6} + C$

10. $f''(x) = 20x^3 - 36x^2 + 12x - 4$,
$f'(0) = -1$,
$f(0) = 2 \Rightarrow$ **f(1) = ?**

A) 0

B) −1

C) −2

D) −3

E) −4

11. $\int \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx = ?$

A) $-\frac{1}{x^3} - \frac{2}{x^4} + C$

B) $\frac{3}{x^3} - \frac{8}{x^4} + C$

C) $\frac{1}{3x^3} - \frac{1}{2x^4} + C$

D) $-\frac{2}{x^3} - \frac{6}{x^4} + C$

E) $-\frac{1}{x} + \frac{1}{x^2} + C$

12. $\int \left(\sin x + \frac{4}{\sqrt{1-x^2}} \right) dx = ?$

A) −cosx + 4arcsinx + C

B) cosx + 4arcsinx + C

C) sinx + 4arccosx + C

D) −sinx + 4arccosx + C

E) 4arccosx + C

YANITLAR / ANSWERS

1. C	2. D	3. A	4. E	5. C	6. B	7. C	8. E
9. D	10. B	11. E	12. A				

1. $f'(x) = 12x^3 - 6x + 1$

$$\int f'(x)\,dx = \int (12x^3 - 6x + 1)\,dx$$

$$f(x) = 3x^4 - 3x^2 + x + C$$

$$f(-1) = 3\cdot(-1)^4 - 3\cdot(-1)^2 + (-1) + C$$

$$5 = -1 + C \Rightarrow C = 6$$

$$f(1) = 3\cdot 1^4 - 3\cdot 1^2 + 1 + 6 = 7$$

Yanıt / The Answer is C

---

2. $\left(\int (3x-2)\cdot f(x)\,dx\right)' = \left(2x^3 + \frac{5}{2}x^2 - 6x + C\right)'$

$$(3x-2)\cdot f(x) = 6x^2 + 5x - 6$$

$$(3x-2)\cdot f(x) = (3x-2)\cdot(2x+3)$$

$$f(x) = 2x + 3$$

$$f(-1) = 2\cdot(-1) + 3 = 1$$

Yanıt / The Answer is D

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3. $\int \frac{e^x - e^{-x}}{2}\,dx = \frac{1}{2}\int (e^x - e^{-x})\,dx$

$$= \frac{1}{2}\left(\int e^x dx - \int e^{-x} dx\right)$$

$$= \frac{1}{2}(e^x + e^{-x}) + C$$

Yanıt / The Answer is A

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4. $\int \frac{dx}{\sqrt{x\sqrt{x}}} = \int \frac{dx}{\sqrt[4]{x^3}}$

$$= \int x^{-\frac{3}{4}}\,dx$$

$$= 4\cdot x^{\frac{1}{4}} + C$$

$$= 4\sqrt[4]{x} + C$$

Yanıt / The Answer is E

---

5. $\int (x^2y + e^xy + e^y)\,dy = \int x^2y\,dy + \int e^xy\,dy + \int e^y\,dy$

$$= \frac{x^2y^2}{2} + \frac{e^xy^2}{2} + e^y + C$$

$$= \frac{y^2(e^y + e^x)}{2} + e^y + C$$

Yanıt / The Answer is C

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6. $\int \frac{1-\sqrt{x}}{2\sqrt{x}}\,dx = \int \frac{1}{2\sqrt{x}}\,dx - \int \frac{\sqrt{x}}{2\sqrt{x}}\,dx$

$$= \frac{1}{2}\int x^{-\frac{1}{2}}\,dx - \int \frac{1}{2}\,dx$$

$$= \frac{1}{2}\cdot 2x^{\frac{1}{2}} - \frac{1}{2}x + C$$

$$= \sqrt{x} - \frac{x}{2} + C$$

Yanıt / The Answer is B

7. $\int \tan^2 x\,dx = \int (1+\tan^2 x - 1)\,dx$

$= \int (1+\tan^2 x)\,dx - \int dx$

$= \tan x - x + C$

Yanıt / The Answer is C

8. $\int \frac{d(x^3)}{x^2} = \int \frac{3x^2}{x^2}\,dx$

$= \int 3\,dx$

$= 3x + C$

Yanıt / The Answer is E

9. $\int (x-1)^2 \cdot x^2\,dx = \int (x^2-2x+1)\cdot x^2\,dx$

$= \int (x^4-2x^3+x^2)\,dx$

$= \frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} + C$

$= \frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} + C$

Yanıt / The Answer is D

10. $\int f''(x)\,dx = \int (20x^3-36x^2+12x-4)\,dx$

$f'(x) = 5x^4-12x^3+6x^2-4x+C$

$f'(0) = 5\cdot 0^4 - 12\cdot 0^3 + 6\cdot 0^2 - 4\cdot 0 + C \Rightarrow C = -1$

$\int f'(x) = \int (5x^4-12x^3+6x^2-4x-1)\,dx$

$f(x) = x^5-3x^4+2x^3-2x^2-x+C$

$f(0) = 0^5 - 3\cdot 0^4 + 2\cdot 0^3 - 2\cdot 0^2 - 0 + C = 2$

$\Rightarrow C = 2$

$f(1) = 1^5 - 3\cdot 1^4 + 2\cdot 1^3 - 2\cdot 1^2 - 1 + 2 = -1$

Yanıt / The Answer is B

11. $\int \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx = \int x^{-2}\,dx - 2\int x^{-3}\,dx$

$= -\frac{1}{x} + \frac{1}{x^2} + C$

Yanıt / The Answer is E

12. $\int \left(\sin x + \frac{4}{\sqrt{1-x^2}}\right) dx = \int \sin x\,dx + 4\int \frac{dx}{\sqrt{1-x^2}}$

$= -\cos x + 4\arcsin x + C$

Yanıt / The Answer is A

1. $\int \frac{dx}{x-3} = ?$

A) $\frac{1}{3} \cdot \ln|x-3| + C$

B) $\frac{1}{3} \cdot \ln|x+3| + C$

C) $3 \cdot \ln|x+3| + C$

D) $\ln|x+3| + C$

E) $\ln|x-3| + C$

2. $\int \frac{x\,dx}{(x^2+4)^3} = ?$

A) $-\frac{1}{(x^2+4)^2} + C$

B) $\ln(x^2+4) + C$

C) $\arctan(2x) + C$

D) $-\frac{1}{4(x^2+4)^2} + C$

E) $(x^2+4)^{-3} + C$

3. $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = ?$

A) $\frac{1}{2} \arcsin e^x + C$

B) $\arcsin e^x + C$

C) $-\arcsin e^x + C$

D) $-\frac{1}{2} \arcsin e^x + C$

E) $\ln\left|\sqrt{1-e^{2x}}\right| + C$

4. $\int \frac{dx}{\sqrt{1-9x^2}} = ?$

A) $\arcsin x + C$

B) $\frac{1}{3} \arcsin x + C$

C) $\frac{1}{3} \arcsin 3x + C$

D) $\arcsin 3x + C$

E) $\frac{1}{9} \arcsin x + C$

5. $\int \frac{\sin\sqrt{x}}{2\sqrt{x}}\,dx = ?$

A) $\cos\sqrt{x} + C$

B) $-\cos\sqrt{x} + C$

C) $\frac{1}{2} \cos\sqrt{x} + C$

D) $-\frac{1}{2} \cos\sqrt{x} + C$

E) $-\frac{1}{2} \cos x + C$

6. $f(x) = \int \frac{1}{x \ln x}\,dx$ ve (and) $f(e) = 6 \Rightarrow$ **$f(x) = ?$**

A) $\ln x + 6$

B) $\ln^2 x + 6$

C) $2\ln x + 3$

D) $\ln|\ln x| + 6$

E) $x^2 \ln x + 9$

7. $\int \frac{dx}{x \cdot \cos^2(\ln x)} = ?$

A) $\frac{1}{2}\ln|\sin x| + C$

B) $\frac{1}{2}\tan(\ln x) + C$

C) $\tan(\ln x) + C$

D) $\frac{1}{\sin(\ln x)} + C$

E) $\cos^3(\ln x) + C$

8. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = ?$

A) $e^{\sqrt{x}} + C$

B) $\frac{1}{2}e^{\sqrt{x}} + C$

C) $2e^{\sqrt{x}} + C$

D) $e^{\sqrt{x}} \cdot x + C$

E) $e^{\sqrt{x}} + \sqrt{x} + C$

9. $\int \frac{\sin 2x}{8 - \cos^2 x}\,dx = ?$

A) $\ln|\cos^2 x| + C$

B) $\ln|8 - \cos^2 x| + C$

C) $\frac{(8 - \cos^2 x)^2}{2} + C$

D) $\frac{1}{2(8 - \cos^2 x)} + C$

E) $8x - \sin^2 x + C$

10. $\int \frac{x^4 - 2}{(x^5 - 10x)^5}\,dx = ?$

A) $\frac{\ln|x^5 - 10x|}{5} + C$

B) $\frac{\ln|x^5 - 10x|}{25} + C$

C) $\frac{-1}{20(x^5 - 10x)^4} + C$

D) $\frac{-1}{25(x^5 - 10x)^5} + C$

E) $\frac{-1}{30(x^5 - 10x)^6} + C$

11. $\int \frac{e^{2x}}{\cos^2(e^{2x})}\,dx = ?$

A) $\ln|\cos^2 e^{2x}| + C$

B) $2\ln|\cos e^{2x}| + C$

C) $\frac{\tan(e^{2x})}{2} + C$

D) $\frac{\tan^2(e^{2x})}{2} + C$

E) $\frac{-1}{2\cos(e^{2x})} + C$

12. $\int \frac{dx}{1 + \cos x} = ?$

A) $-\cot x - \frac{1}{\sin x} + C$

B) $-\cot x + \frac{1}{\sin x} + C$

C) $\cot x - \frac{1}{\sin x} + C$

D) $\cot x + \frac{1}{\cos x} + C$

E) $\cot x - \frac{1}{\cos x} + C$

YANITLAR / ANSWERS

1. E	2. D	3. B	4. C	5. B	6. D	7. C	8. C
9. B	10. C	11. C	12. B				

1.

$$\left.\begin{array}{l} x-3=u \\ dx=du \end{array}\right\} \Rightarrow \int \frac{dx}{x-3} = \int \frac{du}{u}$$

$$= \ln|u|+C$$

$$= \ln|x-3|+C$$

Yanıt / The Answer is E

2.

$$x^2+4=u \Rightarrow 2xdx=du$$

$$xdx=\frac{du}{2}$$

$$\int \frac{x\cdot dx}{(x^2+4)^3} = \int \frac{1}{u^3}\cdot\frac{du}{2}$$

$$=\frac{1}{2}\cdot\int u^{-3}du$$

$$=\frac{1}{2}\cdot\frac{u^{-2}}{-2}+C$$

$$=-\frac{1}{4\cdot(x^2+4)^2}+C$$

Yanıt / The Answer is D

3.

$$e^x=u \Rightarrow e^x dx=du$$

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}dx = \int \frac{du}{\sqrt{1-u^2}}$$

$$=\arcsin u+C$$

$$=\arcsin e^x+C$$

Yanıt / The Answer is B

4.

$$3x=u \Rightarrow 3dx=du$$

$$dx=\frac{du}{3}$$

$$\int \frac{dx}{\sqrt{1-9x^2}} = \frac{1}{3}\int \frac{du}{\sqrt{1-u^2}}$$

$$=\frac{1}{3}\arcsin u+C$$

$$=\frac{1}{3}\arcsin 3x+C$$

Yanıt / The Answer is C

5.

$$\sqrt{x}=u \Rightarrow \frac{1}{2\sqrt{x}}dx=du$$

$$\int \frac{\sin\sqrt{x}}{2\sqrt{x}}dx = \int \sin u\, du$$

$$=-\cos u+C$$

$$=-\cos\sqrt{x}+C$$

Yanıt / The Answer is B

6.

$$\left.\begin{array}{l} \ln x=u \\ \frac{1}{x}dx=du \end{array}\right\} \Rightarrow \int \frac{1}{x\cdot\ln x}dx = \int \frac{du}{u} = \ln|u|+C = \ln|\ln x|+C$$

$$f(x)=\ln|\ln x|+C$$

$$f(e)=\ln|\ln e|+C=6$$

$$\Rightarrow C=6$$

$$\Rightarrow f(x)=\ln|\ln x|+6$$

Yanıt / The Answer is D

7. $\ln x = u \Rightarrow \frac{1}{x}dx = du$

$$\int \frac{dx}{x\cdot\cos^2(\ln x)} = \int \frac{du}{\cos^2 u}$$

$$= \tan u + C$$

$$= \tan(\ln x) + C$$

Yanıt / The Answer is C

8.

$$\left.\begin{array}{l} \sqrt{x} = u \\ \frac{1}{2\sqrt{x}}dx = du \\ \frac{1}{\sqrt{x}}dx = 2\cdot du \end{array}\right\} \Rightarrow \int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int e^u\ 2du$$

$$= 2e^u + C$$

$$= 2e^{\sqrt{x}} + C$$

Yanıt / The Answer is C

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9. $u = 8 - \cos^2 x \Rightarrow \sin 2x\,dx = du$

$$\int \frac{\sin 2x}{8-\cos^2 x}dx = \int \frac{du}{u}$$

$$= \ln u + C$$

$$= \ln|8 - \cos^2 x| + C$$

Yanıt / The Answer is B

10.

$$\left.\begin{array}{l} u = x^5 - 10x \\ du = (5x^4 - 10)dx \\ \frac{1}{5}du = (x^4 - 2)dx \end{array}\right\} \Rightarrow \int \frac{x^4 - 2}{(x^5 - 10x)^5}dx = \int \frac{\frac{1}{5}du}{u^5}$$

$$= \frac{1}{5}\int \frac{du}{u^5}$$

$$= \frac{1}{5}\cdot\frac{u^{-4}}{-4} + C$$

$$= -\frac{1}{20\cdot u^4} + C$$

$$= -\frac{1}{20\cdot(x^5 - 10^4)^4} + C$$

Yanıt / The Answer is C

11.

$$\left.\begin{array}{l} u = e^{2x} \\ du = 2e^{2x}dx \\ e^{2x}dx = \frac{du}{2} \end{array}\right\} \Rightarrow \int \frac{e^{2x}}{\cos^2(e^{2x})}dx = \int \frac{\frac{1}{2}du}{\cos^2 u}$$

$$= \frac{1}{2}\int \frac{du}{\cos^2 u}$$

$$= \frac{1}{2}\cdot\tan u + C$$

$$= \frac{1}{2}\cdot\tan(e^{2x}) + C$$

Yanıt / The Answer is C

12.

$$\int \frac{dx}{1+\cos x} = \int \frac{1-\cos x}{1-\cos^2 x}dx$$

$$= \int \frac{1-\cos x}{\sin^2 x}dx = \int \frac{1}{\sin^2 x}dx - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$\left.\begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array}\right\} \Rightarrow -\cot x - \int \frac{du}{u^2}$$

$$= -\cot x - \frac{u^{-1}}{-1} + C$$

$$= -\cot x + \frac{1}{\sin x} + C$$

Yanıt / The Answer is B

1. $\displaystyle\int \frac{2x^3-6x-4}{x^2-4}\,dx = ?$

A) $x^2+\ln(x+2)^2+C$

B) $\frac{x^2}{2}+\ln(x+2)^2+C$

C) $\frac{x^2}{2}+\ln|x+2|+C$

D) $2x^2+\frac{1}{2}\ln|x+2|^2+C$

E) $x^2+\frac{1}{2}\ln|x+2|+C$

2. $\displaystyle\int \frac{3x+1}{x^2-1}\,dx = ?$

A) $\ln\big((x-1)^2\cdot(x+1)\big)+C$

B) $\ln\big((x-1)^2\cdot(x-2)\big)+C$

C) $\frac{3}{2}\ln(x^2-1)+C$

D) $\frac{2}{3}\ln(x^2-1)+C$

E) $6\ln(x^2-1)+C$

3. $\displaystyle\int \frac{dx}{x^2-10x+29} = ?$

A) $\frac{1}{5}\cdot\arctan\left(\frac{x-5}{5}\right)+C$

B) $\frac{1}{4}\cdot\arctan\left(\frac{x-4}{4}\right)+C$

C) $\frac{1}{3}\cdot\arctan\left(\frac{x-8}{3}\right)+C$

D) $\frac{1}{2}\cdot\arctan\left(\frac{x-5}{2}\right)+C$

E) $\frac{1}{3}\cdot\arctan\left(\frac{x-2}{3}\right)+C$

4. $\displaystyle\int \frac{dx}{x^2-4} = ?$

A) $\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

B) $-\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

C) $-\frac{1}{2}\ln\left|\frac{x+2}{x-2}\right|+C$

D) $\frac{1}{2}\ln\left|\frac{x+2}{x-2}\right|+C$

E) $2\ln\left|\frac{x+2}{x-2}\right|+C$

5. $\displaystyle\int \frac{x+2}{x^2+x}\,dx = ?$

A) $\ln\left|\frac{x+1}{x^2}\right|+C$

B) $\ln\left|\frac{2x^2}{x-1}\right|+C$

C) $\ln\left|\frac{2x^2}{x+1}\right|+C$

D) $\ln\left|\frac{x^2}{x-1}\right|+C$

E) $\ln\left|\frac{x^2}{x+1}\right|+C$

6. $\displaystyle\int x\cdot\ln x\,dx = ?$

A) $x\cdot\ln x-\frac{x^2}{2}+C$

B) $x^2\cdot\ln x-\frac{x^3}{3}+C$

C) $\frac{x^2}{2}\cdot\ln x-\frac{x^2}{4}+C$

D) $x\cdot\ln x-\frac{x^2}{2}+C$

E) $\ln^2 x-\frac{x^2}{2}+C$

7. $\int (x^2 - x + 1)e^x \, dx = ?$

A) $e^x \cdot (x^2 - 3x + 4) + C$

B) $e^x \cdot (x^2 + x - 5) + C$

C) $e^x \cdot (x^2 - 2x - 3) + C$

D) $e^x \cdot (x^2 + x + 6) + C$

E) $e^x \cdot (x^2 + 5x - 1) + C$

8. $\int x^2 \sin x \, dx = ?$

A) $\sin x(2 - x^2) + 2x \cdot \cos x + C$

B) $\cos x(2 - x^2) + 2x \cdot \sin x + C$

C) $\cos x(2 - x^2) + x \cdot \sin x + C$

D) $\sin x(2 - x^2) + x \cdot \cos x + C$

E) $\tan x(2 - x^2) + x \cdot \sin x + C$

9. $\int \sin^3 x \, dx = ?$

A) $-\cos x + \dfrac{\cos^2 x}{3} + C$

B) $\cos x + \dfrac{\cos^2 x}{3} + C$

C) $-\cos x + \dfrac{\cos^3 x}{3} + C$

D) $-\cos x - \dfrac{\cos^3 x}{3} + C$

E) $-\cos x - \dfrac{\cos^2 x}{3} + C$

10. $\int \cos x \cdot \cos 3x \, dx = ?$

A) $\dfrac{\sin 2x}{4} + \dfrac{\sin 4x}{8} + C$

B) $\dfrac{\sin 3x}{6} + \dfrac{\sin 4x}{8} + C$

C) $\dfrac{\sin 2x}{2} + \dfrac{\sin 4x}{4} + C$

D) $\dfrac{\sin^2 6x}{6} + \cos x + C$

E) $\dfrac{\cos^2 3x}{3} - \dfrac{\sin 2x}{2} + C$

11. $\int \cos(\cos^2 x) \cdot \sin 2x \, dx = ?$

A) $-\sin(\cos^2 x) + C$

B) $-\cos(\cos^2 x) + C$

C) $\sin(\cos^2 x) + C$

D) $-2\sin(\cos x) + C$

E) $-\sin(\cos x) + C$

12. $\int e^x \sin x \, dx = ?$

A) $e^x(\sin x - \cos x) + C$

B) $e^x(\sin x + \cos x) + C$

C) $e^x\left(\dfrac{\sin x + \cos x}{2}\right) + C$

D) $e^x\left(\dfrac{\sin x - \cos x}{2}\right) + C$

E) $e^x \cdot 2(\sin x - \cos x) + C$

YANITLAR / ANSWERS

1. A	2. A	3. D	4. A	5. E	6. C	7. A	8. B
9. D	10. A	11. A	12. D				

1. $\dfrac{2x^3-6x-4}{x^2-4} = 2x + \dfrac{2x-4}{x^2-4}$

$$\int \frac{2x^3-6x-4}{x^2-4}dx = \int 2x\,dx + \int \frac{2(x-2)}{(x-2)\cdot(x+2)}\,dx$$

$$= x^2 + 2\ln|x+2| + C$$

$$= x^2 + \ln(x+2)^2 + C$$

Yanıt / The Answer is A

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2. $\dfrac{3x+1}{x^2-1} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$

$$\left.\begin{array}{l} A+B=3 \\ A-B=1 \end{array}\right\} \Rightarrow A=2\ ,\ B=1$$

$$\int \frac{3x+1}{x^2-1}\,dx = \int \frac{2}{x-1}\,dx + \int \frac{dx}{x+1}$$

$$= 2\cdot\ln|x-1| + \ln|x+1| + C$$

$$= \ln((x-1)^2\cdot(x+1)) + C$$

Yanıt / The Answer is A

---

3. $\dfrac{1}{x^2-10x+29} = \dfrac{1}{x^2-10x+25+4} = \dfrac{1}{(x-5)^2+4}$

$$\int \frac{dx}{x^2-10x+29} = \int \frac{dx}{4+(x-5)^2} = \frac{1}{4}\int \frac{dx}{1+\left(\dfrac{x-5}{2}\right)^2}$$

$$\left.\begin{array}{l} u = \dfrac{x-5}{2} \\ du = \dfrac{1}{2}\,dx \end{array}\right\} \Rightarrow \frac{1}{4}\cdot 2\int \frac{dx}{1+u^2} = \frac{1}{2}\cdot \arctan u + C$$

$$= \frac{1}{2}\cdot\arctan\frac{x-5}{2} + C$$

Yanıt / The Answer is D

---

4. $\dfrac{1}{x^2-4} = \dfrac{A}{x-2} + \dfrac{B}{x+2}$

$$\frac{1}{x^2-4} = \frac{(A+B)x+(2A-2B)}{x^2-4}$$

$$\left.\begin{array}{l} A+B=0 \\ 2A-2B=1 \end{array}\right\} \Rightarrow A=\frac{1}{4}\ ,\ B=-\frac{1}{4}$$

$$\int \frac{dx}{x^2-4} = \frac{1}{4}\left(\int\frac{dx}{x-2} - \int\frac{dx}{x+2}\right)$$

$$= \frac{1}{4}\left(\ln|x-2| - \ln|x+2|\right) = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$$

Yanıt / The Answer is A

---

5. $\dfrac{x+2}{x^2+x} = \dfrac{A}{x} + \dfrac{B}{x+1}$

$$\frac{x+2}{x^2+x} = \frac{Ax+A+Bx}{x^2+x} \Rightarrow A=2\ ,\ B=-1$$

$$\int \frac{x+2}{x^2+x}\,dx = \int\left(\frac{2}{x} - \frac{1}{x+1}\right)dx$$

$$= 2\int\frac{dx}{x} - \int\frac{dx}{x+1}\,dx$$

$$= 2\ln|x| - \ln|x+1|$$

$$= \ln\left|\frac{x^2}{x+1}\right| + C$$

Yanıt / The Answer is E

---

6. $\ln x = u \Rightarrow \dfrac{1}{x}dx = du$

$$x\,dx = dv \Rightarrow \frac{x^2}{2} = v$$

$$\int x\cdot\ln x\,dx = \ln x\cdot\frac{x^2}{2} - \frac{1}{4}\int x^2\cdot\frac{1}{x}\,dx$$

$$= \ln x\cdot\frac{x^2}{2} - \frac{x^2}{4} + C$$

$$= \frac{x^2}{2}\cdot\ln x - \frac{x^2}{4} + C$$

Yanıt / The Answer is C

7. $\int (x^2-x+1)e^x\,dx$

Türev / *Derivative*		İntegral / *Integral*
$+\ x^2-x+1$	$\searrow$	e^x
$-\ 2x-1$	$\searrow$	e^x
$+\ 2$	$\searrow$	e^x
$-\ 0$		e^x

$$\int x^2-x+1)e^x\,dx=(x^2-x+1)\cdot e^x-(2x-1)e^x+2e^x+C$$

$$=e^x(x^2-x+1-2x+1+2)+C$$

$$=e^x(x^2-3x+4)+C$$

Yanıt / The Answer is A

8. $\int x^2\sin x\,dx$

Türev / *Derivative*		İntegral / *Integral*
$+\ x^2$	$\searrow$	$\sin x$
$-\ 2x$	$\searrow$	$-\cos x$
$+\ 2$	$\searrow$	$-\sin x$
$-\ 0$		$\cos x$

$$\int x^2\sin x\,dx=-\cos x\cdot x^2+2\cdot x\sin x+2\cos x$$

$$=\cos x(2-x^2)+2x\cdot\sin x$$

Yanıt / The Answer is B

9. $\int\sin^3x\,dx=\int\sin^2x\cdot\sin x\,dx$

$$=\int(1-\cos^2x)\cdot\sin x\,dx$$

$$\left.\begin{array}{l}u=\cos x\\ du=-\sin x\,dx\end{array}\right\}\Rightarrow\ =-\int(1-u^2)\,du$$

$$=-\left(u-\frac{u^3}{3}\right)+C$$

$$=-\cos x+\frac{\cos^3x}{3}+C$$

Yanıt / The Answer is D

10. $\cos x\cdot\cos 3x=\frac{1}{2}\cdot[\cos(x+3x)+\cos(3x-x)]$

$$=\frac{1}{2}\cdot[\cos 4x+\cos 2x]$$

$$\int(\cos x\cdot\cos 3x)\,dx=\int\frac{1}{2}(\cos 4x+\cos 2x)\,dx$$

$$=\frac{1}{2}\left(\frac{1}{4}\sin 4x+\frac{1}{2}\sin 2x\right)+C$$

$$=\frac{1}{8}\sin 4x+\frac{1}{4}\sin 2x+C$$

$$=\frac{\sin 2x}{4}+\frac{\sin 4x}{8}+C$$

Yanıt / The Answer is A

11. $\left.\begin{array}{l}u=\cos^2x\\ du=-\sin 2x\,dx\end{array}\right\}\Rightarrow\ \int\cos(\cos^2x)\sin 2x\,dx=-\int\cos u\,du$

$$=-\sin u+C$$

$$=-\sin(\cos^2x)+C$$

Yanıt / The Answer is A

12. $\sin x=u \qquad e^x dx=du$

$\cos x dx=du \qquad e^x=v$

$$\int e^x\sin x\,dx=\sin x\cdot e^x-\int e^x\cdot\cos x\,dx$$

$\cos x=u \qquad e^x dx=du$

$-\sin x dx=du \qquad e^x=v$

$$\int e^x\cdot\sin x=\mathrm{I}\ \Rightarrow \mathrm{I}=\sin x\cdot e^x-(\cos x\cdot e^x)+\int\sin x\cdot e^x\,dx$$

$$\mathrm{I}=\sin x\cdot e^x-\cos x e^x-\mathrm{I}$$

$$2\mathrm{I}=(\sin x-\cos x)e^x$$

$$\mathrm{I}=\frac{(\sin x-\cos x)e^x}{2}+C$$

Yanıt / The Answer is D

1. $\int (2x+1)f(x)\ dx = 2x^3+5x^2+10x+C \Rightarrow \mathbf{f(-1) = ?}$

Yanıt / The Answer is: **−6**

---

2. $f'(x) = 4x^3 + 6x^2 + 2x + 3$ ve(and) $f(2) = 56 \Rightarrow \mathbf{f(x) = ?}$

Yanıt / The Answer is: $\mathbf{x^4 + 2x^3 + x^2 + 3x + 14}$

---

3. $\int \frac{x^2-1}{x^2+1} dx = ?$

Yanıt / The Answer is: $\mathbf{x - 2 \cdot \arctan x + C}$

---

4. $\int \frac{3x}{3x+4} dx = ?$

Yanıt / The Answer is: $\mathbf{x - \frac{4}{3} \ln|3x+4| + C}$

---

5. $\int \frac{2x+3}{x^2+3x+4} dx = ?$

Yanıt / The Answer is: $\mathbf{\ln|x^2+3x+4| + C}$

---

6. $\int \frac{x}{(x^2+3)^3} dx = ?$

Yanıt / The Answer is: $\mathbf{\frac{-1}{4(x^2+3)^2} + C}$

7. $\int \frac{\operatorname{cosec}^2 x}{1+\cot x}\,dx = ?$

Yanıt / The Answer is: $-\ln|1+\cot x|+C$

8. $\int \frac{x^2-4}{x^2+4}\,dx = ?$

Yanıt / The Answer is: $x-4\cdot\arctan\frac{x}{2}+C$

9. $\int x\cdot e^{-x^2}\,dx = ?$

Yanıt / The Answer is: $-\frac{1}{2}e^{-x^2}+C$

10. $\int \frac{2x+1}{x^2+x-2}\,dx = ?$

Yanıt / The Answer is: $\ln|x^2+x-2|+C$

11. $\int \frac{e^x\,dx}{1+e^{2x}} = ?$

Yanıt / The Answer is: $\arctan e^x + C$

12. $\int \frac{x-3}{x^2-4x-5}\,dx = ?$

Yanıt / The Answer is: $\ln\sqrt[3]{(x+1)^2\,|x-5|}+C$

13. $\displaystyle\int \frac{2x^2 + 4x + 1}{2x^2 + 1}\,dx = ?$

Yanıt / The Answer is: $\mathbf{x + \ln|2x^2 + 1| + c}$

14. $f'(x) = 4x^3 - 3x^2 - 1$ ve(and) $f(1) = 2 \Rightarrow \mathbf{f(3) = ?}$

Yanıt / The Answer is: **54**

15. $\int (x + f(x))\,dx = x^2 + ax + b,\ f(3) = 5 \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **2**

16. $\displaystyle\int \frac{1}{x \cdot \ln x}\,dx = ?$

Yanıt / The Answer is: $\mathbf{\ln|\ln x| + c}$

17. $\displaystyle\int \frac{\sin x}{\cos^2 x}\,dx = ?$

Yanıt / The Answer is: $\displaystyle\frac{1}{\cos x} + c$

18. $\displaystyle\int \frac{dx}{\sqrt{4 - (x-3)^2}} = ?$

Yanıt / The Answer is: $\displaystyle\mathbf{\arcsin\frac{x-3}{2}} + c$

19. $\int (3x^2 + 2\sqrt{x} + 4)\,dx = ?$

Yanıt / The Answer is: $x^3 + \frac{4}{3}\sqrt{x^3} + 4x + C$

20. $\int \sec x\,dx = ?$

Yanıt / The Answer is: $\ln|\sec x + \tan x| + C$

21. $\int \frac{\cos 2x}{\sin^2 x}\,dx = ?$

Yanıt / The Answer is: $-2x - \cot x + C$

22. $\int \frac{1}{x^2 + 10x + 26}\,dx = ?$

Yanıt / The Answer is: $\arctan(x + 5) + C$

23. $\int \frac{dx}{1 + \cos 2x} = ?$

Yanıt / The Answer is: $\frac{1}{2}\tan x + C$

24. $\int \sin 2x \cdot \cos x\,dx = ?$

Yanıt / The Answer is: $-\frac{2\cos^3 x}{3} + C$

25. $\int \left(\frac{3}{\sin^2 x} + \frac{1}{1+x^2} \right) dx = ?$

Yanıt / The Answer is: **$-3\cot x + \arctan x + C$**

---

26. $\int (x^2 + x) \cdot e^x \, dx = ?$

Yanıt / The Answer is: **$e^x (x^2 - x + 1) + C$**

---

27. $\int x^3 \cdot e^{2x} \, dx = ?$

Yanıt / The Answer is: $e^{2x} \left(\frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{4} - \frac{3}{8} \right) + C$

---

28. $\int x^2 \cdot \ln x \, dx = ?$

Yanıt / The Answer is: $\frac{x^3}{3} \left(\ln x - \frac{1}{3} \right) + C$

---

29. $\int x \cdot (x-1)^8 \, dx = ?$

Yanıt / The Answer is: $\frac{(x-1)^{10}}{10} - \frac{(x-1)^9}{9} + C$

---

30. $\int \sin^2 x \cdot \cos^2 x \, dx = ?$

Yanıt / The Answer is: $\frac{x}{8} - \frac{1}{32} \sin 4x + C$

31. $\int \cos 4x \cdot \cos 2x \, dx = ?$

Yanıt / The Answer is: $\frac{1}{12}\sin 6x + \frac{1}{4}\sin 2x + C$

32. $\int \sin 5x \cdot \sin 2x \, dx = ?$

Yanıt / The Answer is: $\frac{1}{6}\sin 3x - \frac{1}{14}\sin 7x + C$

33. $\int \left(x^2 + 2x\right) \cdot \cos x \, dx = ?$

Yanıt / The Answer is: $\sin x(x^2 + 2x - 2) + (2x + 2) \cdot \cos x + C$

34. $\int \frac{dx}{4x - x^2} = ?$

Yanıt / The Answer is: $\ln\left|\frac{x}{4-x}\right| + C$

35. $\int \tan 2x \, dx = ?$

Yanıt / The Answer is: $-\frac{1}{2}\ln|\cos 2x| + C$

36. $\int \cos^5 x \, dx = ?$

Yanıt / The Answer is: $\sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$

1. $f''(x) = 24x^2 - 36x + 12$

$f'(0) = 1,$

$f(0) = -2 \Rightarrow \mathbf{f(-1) = ?}$

A) 8

B) 9

C) 10

D) 11

E) 12

---

2. $\int \frac{x^3 - 3x^2 + x}{x} dx = ?$

A) $\frac{x^3}{2} - \frac{x^2}{3} + x + C$

B) $\frac{x^4}{4} - x^3 + x^2 + C$

C) $x^3 - \frac{x^2}{3} + \frac{x}{2} + C$

D) $\frac{x^3}{3} - \frac{3x^2}{2} + x + C$

E) $\frac{x^3}{3} + \frac{x^2}{2} + C$

---

3. $\int (ax^3 - bx^2 + x + 4)\, dx = ?$

A) $\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{x^2}{2} + 4x + C$

B) $\frac{ax^3}{2} + \frac{bx^2}{2} + \frac{x}{2} + 4x + C$

C) $\frac{ax^4}{4} - \frac{bx^3}{3} + \frac{x^2}{2} + 4x + C$

D) $\frac{ax^2}{4} - \frac{bx^3}{3} + \frac{x^2}{2} + C$

E) $\frac{ax^4}{4} - \frac{bx^3}{3} + 4x + C$

---

4. $\int \left(x^2 - \frac{3}{x^4} + 5\right) dx = ?$

A) $\frac{x^3}{3} + \frac{1}{x^3} + 5x + C$

B) $\frac{x^3}{3} - \frac{15}{x^5} + 5x + C$

C) $x^3 - \frac{1}{5x^5} + 5x + C$

D) $\frac{x^3}{3} - \frac{3}{5x^5} + 5x + C$

E) $\frac{x^3}{3} - \frac{9}{x^3} + 5x + C$

---

5. $\int x \cdot (x-2)^2\, dx = ?$

A) $\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} + C$

B) $\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2 + C$

C) $\frac{x^4}{12} - \frac{x^3}{9} + x^2 + C$

D) $\frac{x^4}{3} - \frac{x^3}{2} + x^2 + C$

E) $\frac{x^4}{6} - \frac{x^3}{12} + \frac{x^2}{3} + C$

---

6. $\int \left(e^{2x} + 3^x\right) dx = ?$

A) $\frac{e^{3x}}{3} + \frac{3^x}{\ln 3} + C$

B) $\frac{e^{2x}}{2} + 3^x + C$

C) $\frac{e^{3x}}{3} + \frac{3 \cdot 3^x}{\ln 3} + C$

D) $\frac{e^{2x}}{2} + \frac{3^x}{3\ln 3} + C$

E) $\frac{e^{2x}}{2} + \frac{3^x}{\ln 3} + C$

7. $\int \frac{d(\cos x)}{\sin x} = ?$

A) $\ln|\sin x| + C$

B) $\frac{\sin^2 x}{2} + C$

C) $\tan x + C$

D) $x^2 + C$

E) $-x + C$

8. $\frac{d^2f(x)}{dx^2} = -x^2 - 2x,\ \frac{df(1)}{dx} = 0,\ f(1) = 1 \Rightarrow \mathbf{f(x) = ?}$

A) $f(x) = -\frac{1}{6}x^3 - \frac{1}{3}x^2 + \frac{4}{3}x + 2$

B) $f(x) = -\frac{1}{12}x^3 - \frac{1}{6}x^2 + \frac{4}{3}x + 1$

C) $f(x) = -\frac{1}{12}x^3 - \frac{1}{3}x^2 + \frac{4}{3}x - \frac{1}{24}$

D) $f(x) = -\frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{4}{3}x + \frac{1}{12}$

E) $f(x) = -\frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{4}{3}x + \frac{1}{12}$

9. $\frac{d^3f(x)}{dx^3} = e^x + 1,\ \frac{d^2f(0)}{dx^2} = 1,\ \frac{df(0)}{dx} = 2,\ f(0) = 3 \Rightarrow \mathbf{f(x) = ?}$

A) $f(x) = e^x + \frac{x^2}{3} + x + 9$

B) $f(x) = e^x + \frac{x^2}{3} + 3x + 6$

C) $f(x) = 2e^x + \frac{x \cdot e^x}{3} + x^2 \cdot e^x + 12$

D) $f(x) = e^x + \frac{e^{2x}}{3} + x \cdot e^x + 6$

E) $f(x) = e^x + \frac{1}{6}x^3 + x + 2$

10. $\int (2\sin x + 5^x)dx = ?$

A) $2\sin x + \ln 5 + C$

B) $-2\sin x + \frac{5^x}{5} + C$

C) $2\cos x + \frac{5^x}{\ln 5} + C$

D) $-2\cos x + \frac{5^x}{\ln 5} + C$

E) $-2\sin x + 5^x \cdot \ln 5 + C$

11. $\int \left(\frac{-2}{\sin^2 x} + \frac{5}{\cos^2 x} \right) dx = ?$

A) $5 \cdot \tan x - 2 \cdot \cot x + C$

B) $2 \cdot \tan x - 2 \cdot \cot x + C$

C) $2 \cdot \cot x - 5 \cdot \tan x + C$

D) $2 \cdot \tan x + 5 \cdot \cot x + C$

E) $5 \cdot \tan x + 2 \cdot \cot x + C$

12. $\int \frac{2^{3x} + 1}{2^{2x} - 2^x + 1} dx = ?$

A) $2^x \cdot \ln 2 + C$

B) $\frac{2^x}{\ln 2} + x + C$

C) $2^x \cdot \ln 2 + x + C$

D) $\frac{2^x}{\ln 2} + C$

E) $\frac{2^x}{\ln 2} - x + C$

YANITLAR / ANSWERS

1. D	2. D	3. C	4. A	5. B	6. E	7. E	8. D
9. E	10. D	11. E	12. B				

1. $\int (x^2-3x+5)^3 \cdot (2x-3)\, dx = ?$

A) $\frac{(x^2-3x+5)^2}{2}+C$

B) $\frac{(x^2-3x)^4}{4}+x+C$

C) $\frac{(x^2-3x+5)\cdot x^2}{4}+C$

D) $\frac{(x^2-3x+5)^4}{4}+C$

E) $\frac{(x^2-3x+5)\cdot(x^2-3x)}{8}+C$

2. $\int \sqrt{x^2+3x+1} \cdot (2x+3)\, dx = ?$

A) $-2(x^2+3x+1)^{-\frac{1}{2}}+C$

B) $2(x^2+3x+1)^{\frac{1}{2}}+C$

C) $\frac{2}{3}\cdot(x^2+3x+1)^{\frac{3}{2}}+C$

D) $\frac{2}{5}\cdot(x^2+3x+1)^{\frac{5}{2}}+C$

E) $(x^2+3x+1)^{\frac{3}{2}}\cdot(x^2+3x)+C$

3. $\int \cos(2x-1)\, dx = ?$

A) $\frac{\sin(2x-1)}{2}+C$

B) $\frac{-\sin(2x-1)}{2}+C$

C) $\frac{\cos(2x-1)}{2}+C$

D) $\frac{-\cos(2x-1)}{2}+C$

E) $\frac{\tan(2x-1)}{2}+C$

4. $\int \frac{dx}{x-2} = ?$

A) $\frac{(x-2)^2}{2}+C$

B) $\frac{1}{2(x-2)^2}+C$

C) $\ln|x-2|+C$

D) $\frac{1}{2x^2}-\frac{1}{2x}+C$

E) $\frac{1}{3(x-2)^3}+C$

5. $\int \cot x\, dx = ?$

A) $\ln|\sin x| + C$

B) $\ln|\cos x| + C$

C) $\ln|\tan x| + C$

D) $\ln|\cot x| + C$

E) $\frac{\cot^2 x}{2}+C$

6. $\int \left(\frac{1}{x}-\frac{2x}{x^2+1}\right) dx = ?$

A) $\ln\left|\frac{x}{x+1}\right|+C$

B) $\ln\left|\frac{x^2}{x-1}\right|+C$

C) $\ln\left|\frac{x^2}{x+1}\right|+C$

D) $\ln\left|\frac{x}{x^2+1}\right|+C$

E) $\ln\left|\frac{x^2}{x-2}\right|+C$

7. $\int (12x^2-5x+3)\cdot(8x^3-5x^2+6x-3)^4\,dx=?$

A) $\frac{(8x^3-5x^2+6x-3)^5}{10}+C$

B) $\frac{(8x^3-5x^2+6x-3)^5}{5}+C$

C) $10(8x^3-5x^2+6x-3)^5+C$

D) $5(8x^3-5x^2+6x-3)+C$

E) $\ln|8x^3-5x^2+6x-3|+C$

---

8. $\int \frac{3x^5-2x^3-3x}{(x^6-x^4-3x^2)^3}\,dx=?$

A) $\frac{\ln|x^6-x^4-3x^2|}{3}+C$

B) $\frac{\ln|x^6-x^4-3x^2|}{9}+C$

C) $\frac{-1}{3(x^6-x^4-3x^2)^4}+C$

D) $\frac{-1}{6(x^6-x^4-3x^2)^4}+C$

E) $\frac{-1}{4(x^6-x^4-3x^2)^2}+C$

---

9. $\int \sin^4 x\cdot\cos x\,dx=?$

A) $\frac{\sin^5 x}{5}+C$

B) $\frac{\cos^5 x}{5}+C$

C) $\frac{-\sin^5 x}{5}+C$

D) $\frac{-\cos^5 x}{5}+C$

E) $\frac{\tan^5 x}{5}+C$

---

10. $\int \frac{\ln^3 x}{x}\,dx=?$

A) $\frac{\ln^2 x}{2}+C$

B) $\frac{\ln^3 x}{3}+C$

C) $\frac{\ln^4 x}{4}+C$

D) $\frac{2}{\ln^2 x}+C$

E) $\ln|\ln x|+C$

---

11. $\int \frac{e^{5x}}{e^{5x}+5}\,dx=?$

A) $\frac{(e^{5x}+5)^2}{10}+C$

B) $\frac{-1}{10(e^{5x}+5)^2}+C$

C) $\frac{\ln|e^{5x}+5|}{5}+C$

D) $\ln|e^{5x}+5|+C$

E) $\frac{\ln(e^{5x}+5)^2}{10}+C$

---

12. $\int \frac{3x^2+4}{\left(x^3+4x\right)^2}\,dx = ?$

A) $\ln|x^3+4x|+C$

B) $\ln|3x^2+4|+C$

C) x^3+4x+C

D) $-x^3-4x+C$

E) $-\frac{1}{x^3+4x}+C$

YANITLAR / ANSWERS

1. D	**2. C**	**3. A**	**4. C**	**5. A**	**6. D**	**7. A**	**8. E**
9. A	**10. C**	**11. C**	**12. E**				

1. $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx = ?$

A) $-\cos(\ln x) + C$
B) $\cos(\ln x) + C$
C) $-\sin(\ln x) + C$
D) $\sin(\ln x) + C$
E) $\sin x + \ln x + C$

2. $\displaystyle\int \frac{\sin 5x}{1+\cos 5x}\,dx = ?$

A) $\ln|1+\cos 5x|+C$
B) $\frac{1}{5}\ln|1+\cos 5x|+C$
C) $-\frac{1}{5}\ln|1+\cos 5x|+C$
D) $-\frac{1}{5}\ln|1+\cos x|+C$
E) $\frac{1}{5}\ln|1+\cos x|+C$

3. $\displaystyle\int \frac{dx}{\sqrt{x}\cdot(1+x)} = ?$

A) $\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}+C$
B) $-\frac{2x^{\frac{3}{2}}}{3}-\frac{2x^{\frac{5}{2}}}{5}+C$
C) $2\cdot\arctan x+C$
D) $-2\cdot\arctan\sqrt{x}+C$
E) $2\cdot\arctan\sqrt{x}+C$

4. $\displaystyle\int \frac{\sin^3 x}{\cos^5 x}\,dx = ?$

A) $\tan x + C$
B) $\frac{\tan^2 x}{2}+C$
C) $\frac{\tan^3 x}{3}+C$
D) $\frac{\tan^4 x}{4}+C$
E) $\frac{\tan^5 x}{5}+C$

5. $\displaystyle\int \frac{\sin 4x}{e^{\cos^2 2x}}\,dx = ?$

A) $\frac{1}{2}e^{\cos^2 2x}+C$
B) $\frac{1}{2}e^{\sin^2 2x}+C$
C) $-\frac{1}{2}e^{\sin^2 2x}+C$
D) $-\frac{1}{2}e^{\cos^2 2x}+C$
E) $e^{\cos^2 2x}+C$

6. $\displaystyle\int e^{\cos x+4}\cdot\sin x\,dx = ?$

A) $e^{\cos x+4}+C$
B) $-e^{\cos x+4}+C$
C) $-e^{\cos x}+C$
D) $2e^{\cos x+4}+C$
E) $e^{\cos x+4}+\sin x+C$

7. $\int \frac{\arctan x}{1+x^2}\,dx = ?$

A) $\arctan^2 x + C$

B) $\arctan x + C$

C) $\frac{1}{2}\arctan x + C$

D) $-\frac{1}{2}\arctan x + C$

E) $\frac{1}{2}\arctan^2 x + C$

8. $\int \frac{dx}{\cos^2 4x} = ?$

A) $\tan 4x + C$

B) $\frac{1}{4}\tan x + C$

C) $\frac{1}{4}\tan 4x + C$

D) $\frac{1}{12}\cos^3 x + C$

E) $-\frac{1}{4}\cos^3 x + C$

9. $\int \frac{dx}{1+\sin x} = ?$

A) $\tan x + \frac{1}{\cos x} + C$

B) $\tan x - \frac{1}{\cos x} + C$

C) $\ln|1+\sin x| + C$

D) $\tan x - \ln|\cos x| + C$

E) $\ln|\cos^2 x| + C$

10. $\int \frac{\sin 2x}{\sqrt{1-\sin^4 x}}\,dx = ?$

A) $-\arcsin(\sin^2 x) + C$

B) $\arcsin(\sin^2 x) + C$

C) $\arcsin(\cos^2 x) + C$

D) $-\arcsin(\cos^2 x) + C$

E) $\arccos(\sin^2 x) + C$

11. $\int \frac{x^7}{1-x^4}\,dx = ?$

A) $-\frac{1}{4}(\ln(1-x^4)+x^4)+C$

B) $\frac{1}{4}(\ln(1-x^4)+x^4)+C$

C) $\frac{1}{4}(\ln(1-x^4)-1+x^4)+C$

D) $-\frac{1}{4}(\ln(1-x^4)-1+x^4)+C$

E) $\frac{1}{4}(\ln(1-x^4)-1)+C$

12. $\int \frac{dx}{\sqrt{x(1-x)}} = ?$

A) $\arcsin(1-2x) + C$

B) $-\arcsin(1-2x) + C$

C) $\arcsin(1-x) + C$

D) $-\arcsin(1-x) + C$

E) $\arcsin(1-x) + \sqrt{x} + C$

YANITLAR / ANSWERS

1. A	2. C	3. E	4. D	5. A	6. B	7. E	8. C
9. B	10. B	11. A	12. B				

1. $\int \frac{dx}{\sqrt{4-x^2}} = ?$

A) $\frac{\arcsin\frac{x}{2}}{2} + C$

B) $2\arcsin\frac{x}{2} + C$

C) $\arcsin\frac{x}{2} + C$

D) $2\arctan\frac{x}{2} + C$

E) $\arctan\frac{x}{2} + C$

2. $\int \frac{dx}{\sqrt{7+6x-x^2}} = ?$

A) $\arcsin(x-7)(x-6) + C$

B) $\arcsin\frac{x-3}{2} + C$

C) $\arcsin\frac{x-3}{3} + C$

D) $\arcsin\frac{x-3}{4} + C$

E) $\arcsin\frac{x-3}{6} + C$

3. $\int \frac{x}{\sqrt{1-x^4}}\,dx = ?$

A) $\frac{\arcsin x^2}{2} + C$

B) $\frac{\arcsin x^3}{3} + C$

C) $\frac{\arcsin x^4}{4} + C$

D) $\arcsin(1-x^2) + C$

E) $\arctan x^4 + C$

4. $\int \frac{dx}{25+x^2} = ?$

A) $\arctan 5x + C$

B) $\arctan(5x)^2 + C$

C) $\frac{\arctan^5 x}{5} + C$

D) $\arctan\frac{x}{5} + C$

E) $\frac{\arctan\frac{x}{5}}{5} + C$

5. $\int \frac{x-4}{x^2+4}\,dx = ?$

A) $\frac{\ln|x^2-4|}{2} - 4\arctan\frac{x}{2} + C$

B) $\frac{\ln|x^2+4|}{2} - 2\arctan\frac{x}{2} + C$

C) $\ln|x-4| - 4\arctan\frac{x}{2} + C$

D) $\ln|x+4| - 4\arctan(x-4) + C$

E) $\ln|x^2+4| - 8\arctan(x^2+4) + C$

6. $\int \frac{dx}{x^2+6x+9} = ?$

A) $\ln|x+3| + C$

B) $\frac{-1}{x+3} + C$

C) $\frac{-2}{(x+3)^2} + C$

D) $\arctan(x+3) + C$

E) $\frac{\arctan(x+3)}{3} + C$

7. $\int \frac{dx}{x^2-5x+6} = ?$

A) $\ln\left|\frac{x-3}{x-2}\right| + C$

B) $\ln\left|\frac{x-2}{x-3}\right| + C$

C) $\ln\left|\frac{x-6}{x+1}\right| + C$

D) $\ln\left|\frac{x+1}{x-6}\right| + C$

E) $\ln\left|\frac{x-2}{x+3}\right| + C$

8. $\int \frac{x^3-2x^2+1}{x-1}\, dx = ?$

A) $\frac{x^3}{3} - x^2 + \ln(x-1) + C$

B) $\frac{x^3}{3} - x + \ln(x-1) + C$

C) $\frac{x^3}{3} + \frac{x^2}{2} + 2x + C$

D) $\frac{x^3}{3} - \frac{x^2}{2} - x + C$

E) $\frac{x^3}{3} - x^2 + 3x + C$

9. $\int \frac{dx}{x(x-2)} = ?$

A) $\ln\left|\frac{x}{x-2}\right| + C$

B) $\ln\left|\frac{x-2}{x}\right| + C$

C) $-\ln\left|\frac{x}{x-2}\right| + C$

D) $-\ln\left|\frac{x-2}{x}\right| + C$

E) $\frac{1}{2}\ln\left|\frac{x-2}{x}\right| + C$

10. $\int \frac{dx}{\sqrt{4-x^2}} = ?$

A) $\arcsin\frac{x}{2} + C$

B) $2\arcsin\frac{3x}{2} + C$

C) $\arctan\sqrt{x} + C$

D) $\arcsin\sqrt{x} + C$

E) $2\arcsin\sqrt{x} + C$

11. $0 < x < \frac{\pi}{2} \Rightarrow \int \sqrt{36-4x^2}\, dx = ?$

A) $x \cdot \sqrt{9-x^2} + \arcsin\frac{x}{3} + C$

B) $9x \cdot \sqrt{9-x^2} + \arcsin\frac{x}{3} + C$

C) $x \cdot \sqrt{9-x^2} + 9\arcsin\frac{x}{3} + C$

D) $x \cdot \sqrt{9-x^2} + \frac{x}{3} + C$

E) $x \cdot \sqrt{9-x^2} + \arcsin x + C$

12. $0 < x < \frac{\pi}{2} \Rightarrow \int 3\sin^2(\pi-x)\cdot\cos(\pi+x)\, dx = ?$

A) $\frac{\sin^3 x}{3} + C$ B) $\frac{\cos^3 x}{3} + C$ C) $\frac{\tan^2 x}{2} + C$

D) $-\frac{\sin^3 x}{3} + C$ E) $-\sin^3 x + C$

YANITLAR / ANSWERS

1. C 2. D 3. A 4. E 5. B 6. B 7. A 8. D
9. E 10. A 11. C 12. E

BELİRLİ İNTEGRAL
DEFINITE INTEGRAL

BÖLÜM / CHAPTER 2

f: [a, b] → R, y = f(x) [a, b] aralığında integrallenebilir bir fonksiyon olmak üzere,

$\int f(x)\,dx = F(x) + C \Rightarrow \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ ifadesine f fonksiyonunun a dan b ye kadar belirli integrali denir.

Let f: [a, b] → R be an integrable function on the interval [a, b].

The expression $\int f(x)\,dx = F(x) + C \Rightarrow \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ *is called definite integral of the function f from a to b.*

ÖRNEK / EXAMPLE 1

$\int_0^2 3x^2\,dx = ?$

A) 4 B) 8 C) 16 D) 20 E) 24

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\int_0^2 3x^2\,dx = \frac{3x^3}{3}\Big|_0^2 = 2^3 - 0^3 = 8$$

ÖRNEK / EXAMPLE 2

$\int_0^{\frac{\pi}{4}} \sin 2x\,dx = ?$

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int_0^{\frac{\pi}{4}} \sin 2x\,dx = -\frac{1}{2}\cos 2x\Big|_0^{\frac{\pi}{4}} = -\frac{1}{2}(\cos\frac{\pi}{2} - \cos 0) = -\frac{1}{2}(0-1) = \frac{1}{2}$$

NOTLAR / NOTES

- ✓ f(x) tek fonksiyon ise,
  If f(x) is odd function, then
  f(−x) = −f(x).
- ✓ f(x) çift fonksiyon ise
  If f(x) is even function, then
  f(−x) = f(x).

PEKİŞTİRME / REINFORCE

$$\int_{1}^{2} (3x^2 + 4x)\, dx = ?$$

YANIT / ANSWER : 13

BELİRLİ İNTEGRALİN ÖZELLİKLERİ
PROPERTIES of DEFINITE INTEGRAL

a, b, c ∈ R,

1. $$\int_a^b k \cdot f(x)\, dx = k \cdot \int_a^b f(x)\, dx \qquad (k \in R)$$

2. $$\int_a^b (f(x) \pm g(x))\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$$

3. f(x) tek fonksiyon / *f(x) is odd function.*

$$\int_{-a}^{a} f(x)\, dx = 0$$

f(x) çift fonksiyon / *f(x) is even function.*

$$\int_{-a}^{a} f(x)\, dx = 2 \cdot \int_{0}^{a} f(x)\, dx$$

4. $$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$$

5. $a < c < b$

$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$$

ÖRNEK / EXAMPLE 3

$$\int_{-1}^{2} (x^2 - 2x + 3)\, dx = ?$$

A) 9 B) 7 C) 5 D) 3 E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\int_{-1}^{2} (x^2 - 2x + 3)\, dx = \left.\frac{x^3}{3} - \frac{2x^2}{2} + 3x\right|_{-1}^{2}$$

$$= \left(\frac{8}{3} - 2^2 + 3 \cdot 2\right) - \left(-\frac{1}{3} - 1 - 3\right)$$

$$= \frac{14}{3} + \frac{13}{3} = 9$$

ÖRNEK / EXAMPLE 4

$$\int_{\frac{1}{4}}^{\frac{e^2}{4}} \frac{\ln 4x}{x}\,dx = ?$$

A) $-\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$\ln 4x = u \Rightarrow \frac{1}{x}dx = du$

$x = \frac{1}{4} \Rightarrow \ln 4x = 0 \Rightarrow u = 0$

$x = \frac{e^2}{4} \Rightarrow \ln 4x = 2 \Rightarrow u = 2$

$$\int_{\frac{1}{4}}^{\frac{e^2}{4}} \frac{\ln 4x}{x}\,dx = \int_0^2 u\,du = \left.\frac{u^2}{2}\right|_0^2 = \left(\frac{2^2}{2} - \frac{0^2}{2}\right) = 2$$

PEKİŞTİRME / REINFORCE

$$\int_e^{e^2} \frac{\ln x}{x}\,dx = ?$$

YANIT / ANSWER : 3 / 2

ÖRNEK / EXAMPLE 5

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x}\,dx = ?$$

A) $\ln 3$ B) $\ln\frac{3}{4}$ C) $\ln\frac{1}{4}$ D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$\sin^2 x = u \Rightarrow \sin 2x\,dx = du$

$x = \frac{\pi}{6} \Rightarrow \sin^2\frac{\pi}{6} = \frac{1}{4} \Rightarrow u = \frac{1}{4}$

$x = \frac{\pi}{3} \Rightarrow \sin^2\frac{\pi}{3} = \frac{3}{4} \Rightarrow u = \frac{3}{4}$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x}\,dx = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{du}{u} = \ln u\left.\vphantom{\int}\right|_{\frac{1}{4}}^{\frac{3}{4}} = \ln\frac{3}{4} - \ln\frac{1}{4} = \ln 3$$

ÖRNEK / EXAMPLE 6

$$\int_0^2 (x^2+3x)\cdot(2x+3)\,dx = ?$$

A) 55 B) 50 C) 44 D) 36 E) 28

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$x^2+3x=u \Rightarrow (2x+3)\,dx = du$

$x=0 \Rightarrow x^2+3x=0 \Rightarrow u=0$

$x=2 \Rightarrow x^2+3x=0 \Rightarrow u=10$

$$\int_0^2 (x^2+3x)\cdot(2x+3)\,dx = \int_0^{10} u\,du$$

$$= \left.\frac{u^2}{2}\right|_0^{10} = \frac{100}{2}-\frac{0}{2}=50$$

ÖRNEK / EXAMPLE 7

$$\int_0^{\frac{\pi}{2}} \cos^3 x\,dx = ?$$

A) $-\frac{2}{3}$ B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) $\frac{2}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\int_0^{\frac{\pi}{2}} \cos^3 x\,dx = \int_0^{\frac{\pi}{2}} \cos^2 x\cdot\cos x\,dx = \int_0^{\frac{\pi}{2}} (1-\sin^2 x)\cdot\cos x\,dx$$

$\sin x = u \Rightarrow \cos x\,dx = du$

$x=0 \Rightarrow \sin 0 = 0 \Rightarrow u=0$

$x=\frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2}=1 \Rightarrow u=1$

$$\int_0^{\frac{\pi}{2}} (1-\sin^2 x)\cdot\cos x\,dx = \int_0^{1} (1-u^2)\,du$$

$$= u-\left.\frac{u^3}{3}\right|_0^1 = \left(1-\frac{1}{3}\right)-\left(0-\frac{0^3}{3}\right)=\frac{2}{3}$$

ÖRNEK / EXAMPLE 8

$$\int_{-2}^{2} (5x^4+3x^3+3x^2+4x+5)\,dx = ?$$

A) 10 B) 50 C) 100 D) 115 E) 120

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$3x^3$, $4x$ tek fonksiyondur. / *$3x^3$, $4x$ are odd function.*

$$\int_{-2}^{2} (5x^4+3x^3+3x^2+4x+5)\,dx = \int_{-2}^{2} (5x^4+3x^2+5)\,dx$$

$$= 2\int_{0}^{2} (5x^4+3x^2+5)\,dx$$

$$= 2\cdot\left[5\cdot\frac{x^5}{5}+\frac{3\cdot x^3}{3}+5x\right]\Bigg|_0^2$$

$$= 2\cdot\left[2^5+2^3+5\cdot 2\right]\Big|_0^2 = 100$$

ÖRNEK / EXAMPLE 9

$$\int_{0}^{2} (x-1)\cdot(x+2)\,dx = ?$$

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\int_{0}^{2} (x-1)\cdot(x+2)\,dx = \int_{0}^{2} (x^2+x-2)\,dx$$

$$= \frac{x^3}{3}+\frac{x^2}{2}-2x\Big|_0^2$$

$$= \left(\frac{2^3}{3}+\frac{2^2}{2}-2\cdot 2\right)-\left(\frac{0}{3}+\frac{0}{2}-2\cdot 0\right)$$

$$= \frac{2}{3}$$

İNTEGRALİN TÜREVİ / DERIVATIVE of INTEGRAL

f : [a, b] → R fonksiyonu integrallenebilir olmak üzere,

Let f : [a, b] → R be an integrable function.

1. $F(x)=\int_a^x f(t)\,dt \Rightarrow F'(x)=f(x)$

2. $F(x)=\int_a^{u(x)} f(t)\,dt \Rightarrow F'(x)=u'(x)\cdot f(u(x))$

3. $F(x)=\int_{v(x)}^{u(x)} f(t)\,dt \Rightarrow F'(x)=u'(x)\cdot f(u(x))-v'(x)\cdot f(v(x))$

ÖRNEK / EXAMPLE 10

$F(x)=\int_0^{\sin x}(t^2-t)\,dt \Rightarrow F'\left(\frac{\pi}{4}\right)=?$

A) $\frac{2-\sqrt{2}}{2}$

B) $\frac{\sqrt{2}-2}{2}$

C) $\frac{\sqrt{2}-2}{4}$

D) $\frac{2-\sqrt{2}}{4}$

E) $2-\sqrt{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int_0^{\sin x}(t^2-t)\,dt \Rightarrow F'(x)=(\sin x)'\cdot(\sin^2 x-\sin x)$$

$$=\cos x\cdot(\sin^2 x-\sin x)$$

$$F'\left(\frac{\pi}{4}\right)=\cos\frac{\pi}{4}\cdot\left(\sin^2\frac{\pi}{4}-\sin\frac{\pi}{4}\right)$$

$$=\frac{\sqrt{2}}{2}\cdot\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)$$

$$=\frac{\sqrt{2}-2}{4}$$

ÖRNEK / EXAMPLE 11

$F(x)=\int\limits_{0}^{\sqrt{x}}(t+2)^{\frac{1}{2}}\,dt \Rightarrow F'(4)=?$

A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int\limits_{0}^{\sqrt{x}}(t+2)^{\frac{1}{2}}\,dt \Rightarrow F'(x)=(\sqrt{x})'\cdot(\sqrt{x}+2)^{\frac{1}{2}}$$

$$=\frac{1}{2\sqrt{x}}\cdot\left(\sqrt{\sqrt{x}+2}\right)$$

$$F'(4)=\frac{1}{2\sqrt{4}}\cdot\left(\sqrt{\sqrt{4}+2}\right)$$

$$=\frac{1}{4}\cdot 2=\frac{1}{2}$$

ÖRNEK / EXAMPLE 12

$F(x)=\int\limits_{x}^{x^2}\frac{\ln t}{2+\ln^2 t}\,dt \Rightarrow F'(e)=?$

A) $\frac{2e}{3}$ B) $\frac{2}{3}$ C) $\frac{2e-1}{4}$

D) $\frac{2e-1}{3}$ E) $\frac{2e+1}{3}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\int\limits_{x}^{x^2}\frac{\ln t}{2+\ln^2 t}\,dt \Rightarrow F'(x)=(x^2)'\cdot\left(\frac{\ln x^2}{2+\ln^2 x^2}\right)-(x)'\cdot\left(\frac{\ln x}{2+\ln^2 x}\right)$$

$$=2x\cdot\frac{\ln x^2}{2+\ln^2 x^2}-\frac{\ln x}{2+\ln^2 x}$$

$$F'(e)=2e\cdot\frac{\ln e^2}{2+\ln^2 e^2}-\frac{\ln e}{2+\ln^2 e}$$

$$=2e\cdot\frac{2}{6}-\frac{1}{3}$$

$$=\frac{2e-1}{3}$$

MUTLAK DEĞER FONKSİYONUNUN İNTEGRALİ
INTEGRATION of the ABSOLUTE VALUE FUNCTION

$\int_a^b |f(x)|\,dx$ integrali hesaplanırken $f(x) = 0$ denkleminin kökleri bulunarak kritik noktalar belirlenir. Bu noktalara göre integral parçalanarak hesaplanır.

While the integral $\int_a^b |f(x)|\,dx$ *is found, firstly the roots of the equation* $f(x) = 0$ *and critical points are found. Integral is found by part up to the these points.*

ÖRNEK / EXAMPLE 13

$$\int_{-4}^{1} |x+2|\,dx = ?$$

A) $\frac{5}{2}$ B) $\frac{7}{2}$ C) $\frac{9}{2}$ D) $\frac{11}{2}$ E) $\frac{13}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x + 2 = 0 \Rightarrow x = -2$

x		−2	
x + 2	−	○	+

$$\int_{-4}^{1} |x+2|\,dx = \int_{-4}^{-2} (-x-2)\,dx + \int_{-2}^{1} (x+2)\,dx$$

$$= -\left(\frac{x^2}{2} + 2x\right)\Bigg|_{-4}^{-2} + \left(\frac{x^2}{2} + 2x\right)\Bigg|_{-2}^{1}$$

$$= \frac{13}{2}$$

ÖRNEK / EXAMPLE 14

$\int_{0}^{2} | x^2 - 3x + 2 | \, dx = ?$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{5}{6}$ D) 1 E) $\frac{6}{5}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$x^2 - 3x + 2 = 0 \Rightarrow (x-2)\cdot(x-1) = 0$

$x = 2\,,\; x = 1$

x		1		2	
$x^2 - 3x + 2$	+	○	−	○	+

$$\int_{0}^{2} | x^2 - 3x + 2 | \, dx = \int_{0}^{1} (x^2 - 3x + 2)\, dx + \int_{1}^{2} -(x^2 - 3x + 2)\, dx$$

$$= \frac{x^3}{3} - \frac{3x^2}{2} + 2x \Bigg|_{0}^{1} - \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right) \Bigg|_{1}^{2} = 1$$

ÖRNEK / EXAMPLE 15

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} | \sin x | \, dx = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$x \in \left(-\frac{\pi}{2}, 0 \right) \Rightarrow \sin x < 0$

$x \in \left(0, \frac{\pi}{2} \right) \Rightarrow \sin x > 0$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} | \sin x | \, dx = \int_{-\frac{\pi}{2}}^{0} -\sin x \, dx + \int_{0}^{\frac{\pi}{2}} \sin x \, dx$$

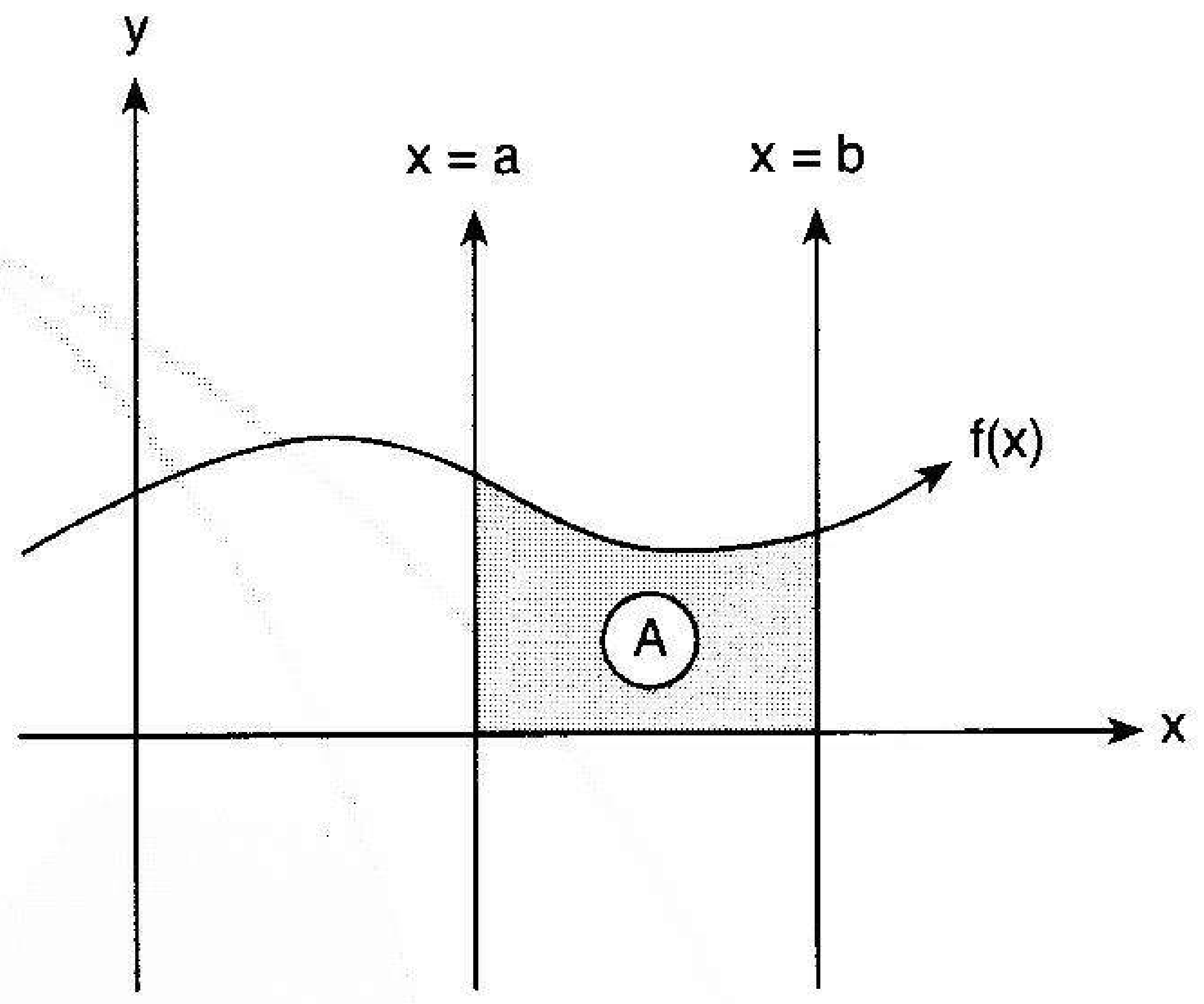
$$= \cos x \Bigg|_{-\frac{\pi}{2}}^{0} - \cos x \Bigg|_{0}^{\frac{\pi}{2}} = 2$$

BELİRLİ İNTEGRALİN UYGULAMALARI
APPLICATIONS of the DEFINITE INTEGRAL

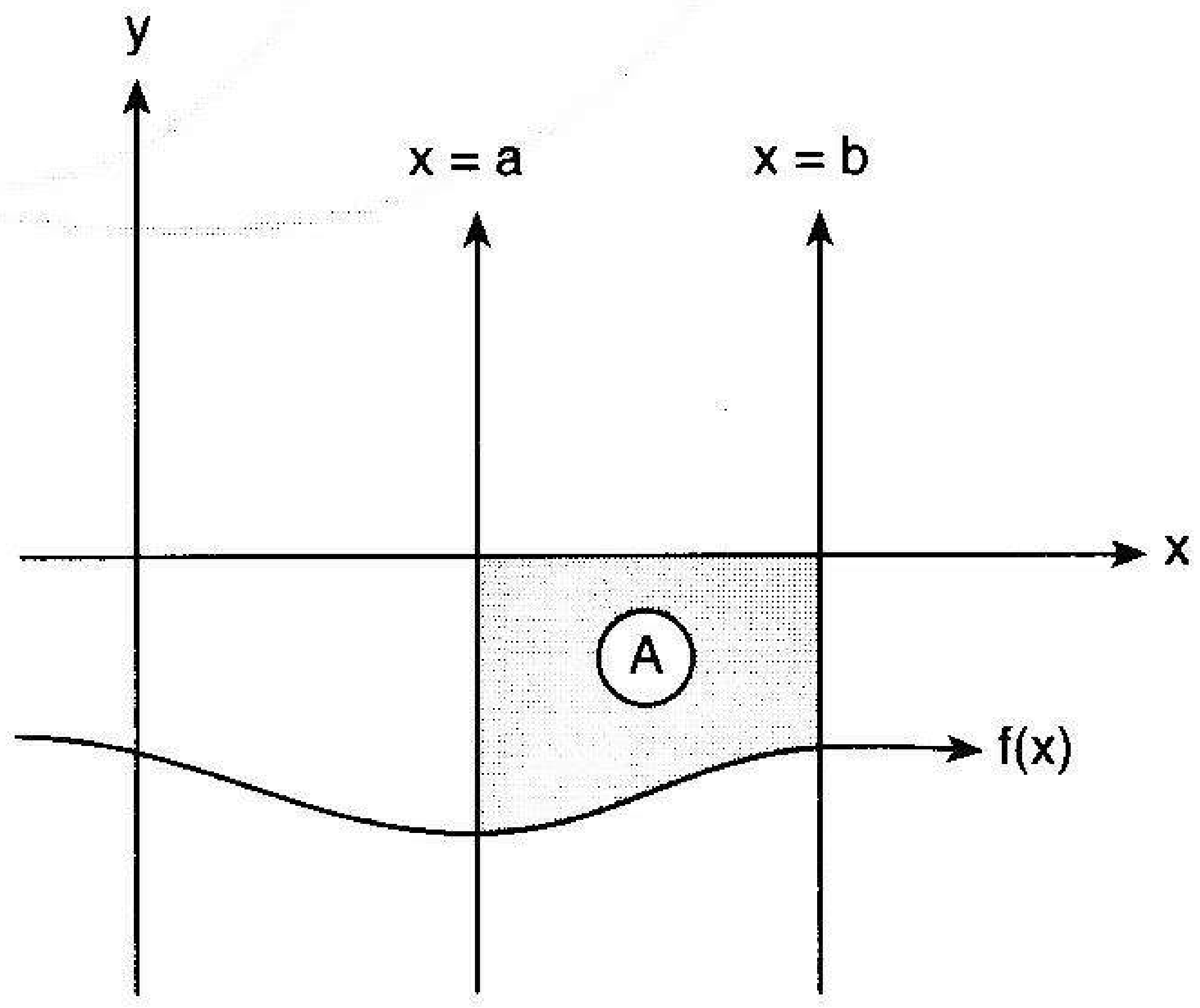
Alan Hesabı / Area Calculus

1. $y = f(x)$ fonksiyonu, x – ekseni, $x = a$, $x = b$ doğruları tarafından sınırlandırılan bölgenin alanı, aşağıdaki gibi hesaplanır.

   The area of the region bounded by the function $y = f(x)$, the lines $x = a$, $x = b$ and the x – axis is founded as follows.



$$A = \int_a^b f(x)\,dx$$



$$A = -\int_a^b f(x)\,dx$$

ÖRNEK / EXAMPLE 16

Taralı alan / *The shaded area* = ?

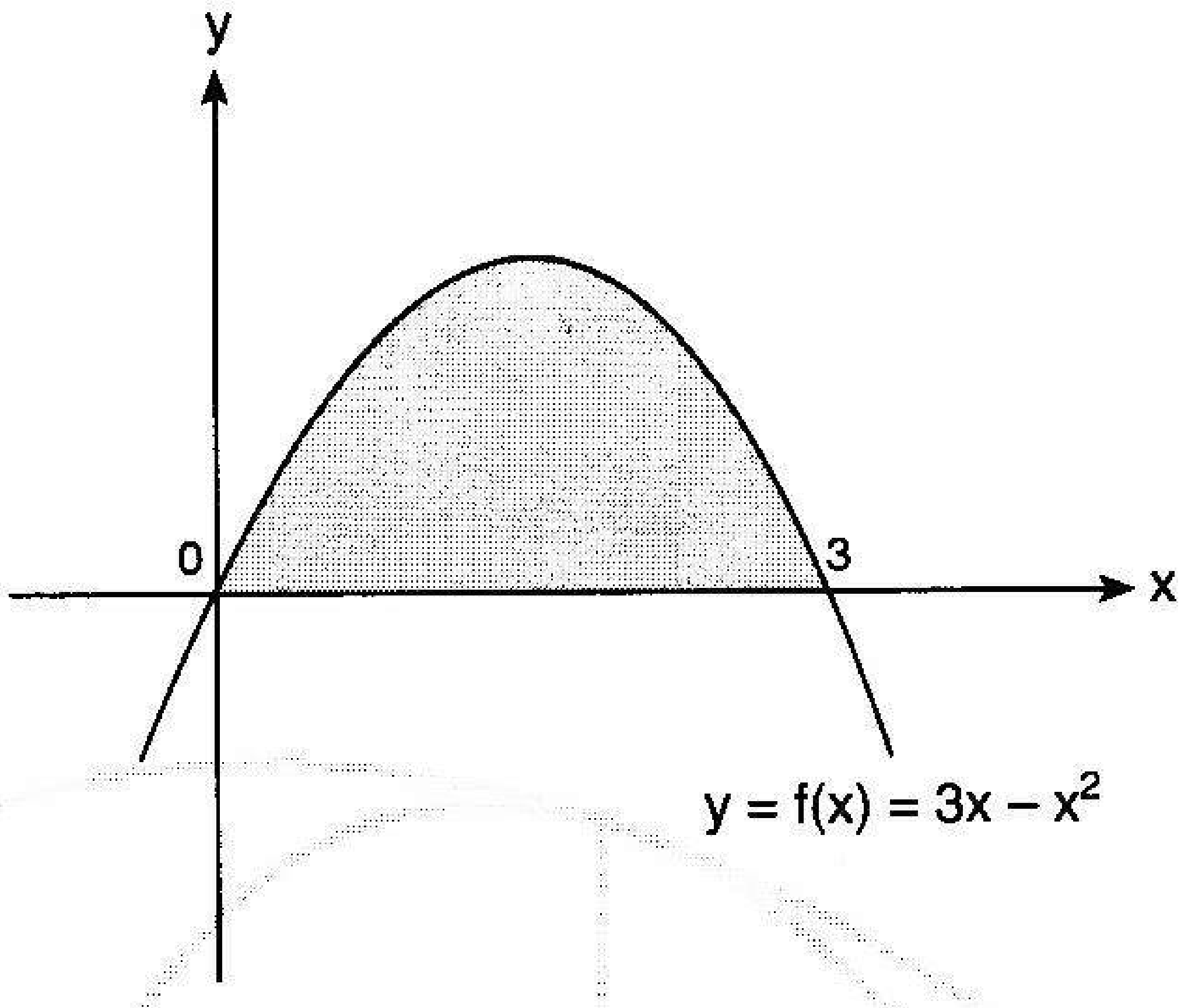
A) $\frac{27}{2}$

B) 9

C) $\frac{27}{4}$

D) $\frac{27}{5}$

E) $\frac{9}{2}$



ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$TA/SA=\int_0^3 (3x-x^2)\,dx=\left(\frac{3x^2}{2}-\frac{x^3}{3}\right)\Bigg|_0^3=\left(\frac{27}{2}-\frac{27}{3}\right)-0=\frac{27}{6}=\frac{9}{2}$$

ÖRNEK / EXAMPLE 17

$A_1 = 5$ br², $A_2 = 12$ br²

$$\Rightarrow \int_{-5}^{2} f(x)\,dx = ?$$

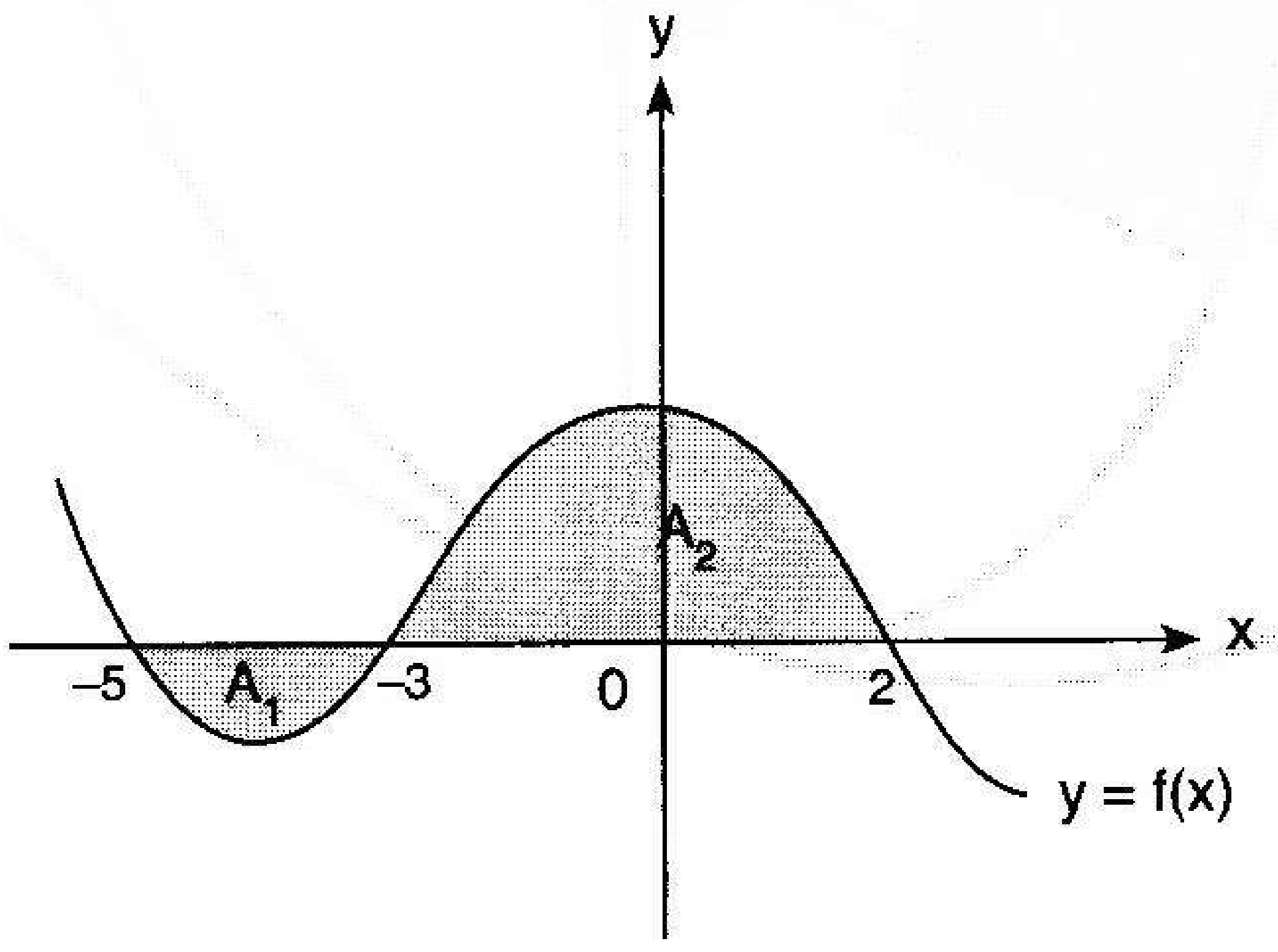
A) –17

B) –7

C) 0

D) 7

E) 17



ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

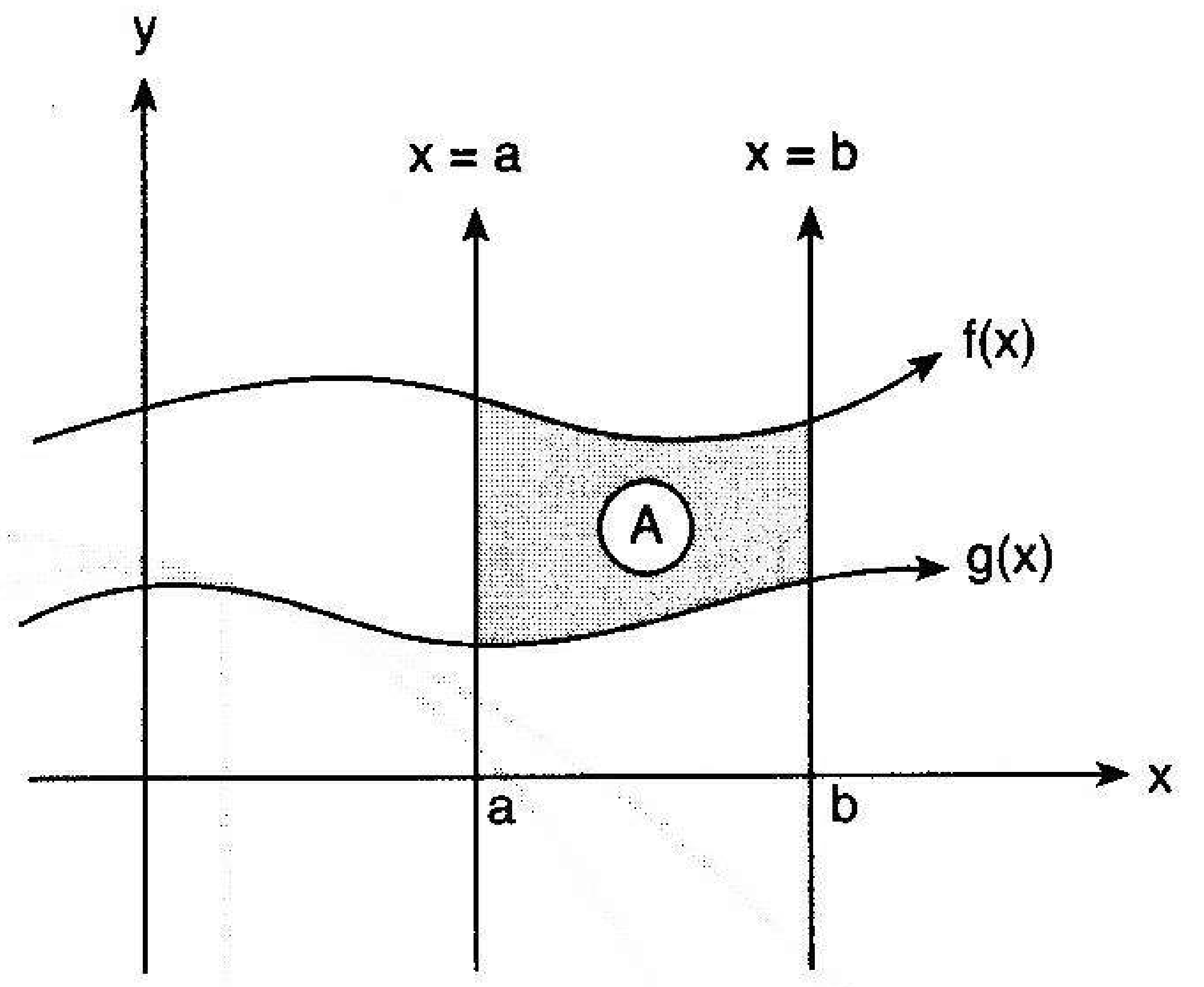
$$A_1=-\int_{-5}^{-3} f(x)\,dx=5 \;\Rightarrow\; \int_{-5}^{-3} f(x)\,dx=-5$$

$$A_2=\int_{-3}^{2} f(x)\,dx=12$$

$$\Rightarrow \int_{-5}^{2} f(x)\,dx=\int_{-5}^{-3} f(x)\,dx+\int_{-3}^{2} f(x)\,dx=-5+12=7$$

2. $y = f(x)$, $y = g(x)$ fonksiyonları ile $x = a$, $x = b$ doğruları tarafından sınırlanan bölgenin alanı aşağıdaki gibi bulunur.

The area of the region bounded by the functions $y = f(x)$, $y = g(x)$, the lines $x = a$ and $x = b$ are founded as follow.



$$A = \int_a^b |f(x) - g(x)|\,dx$$

ÖRNEK / EXAMPLE 18

Taralı alan / *The shaded area* = ?

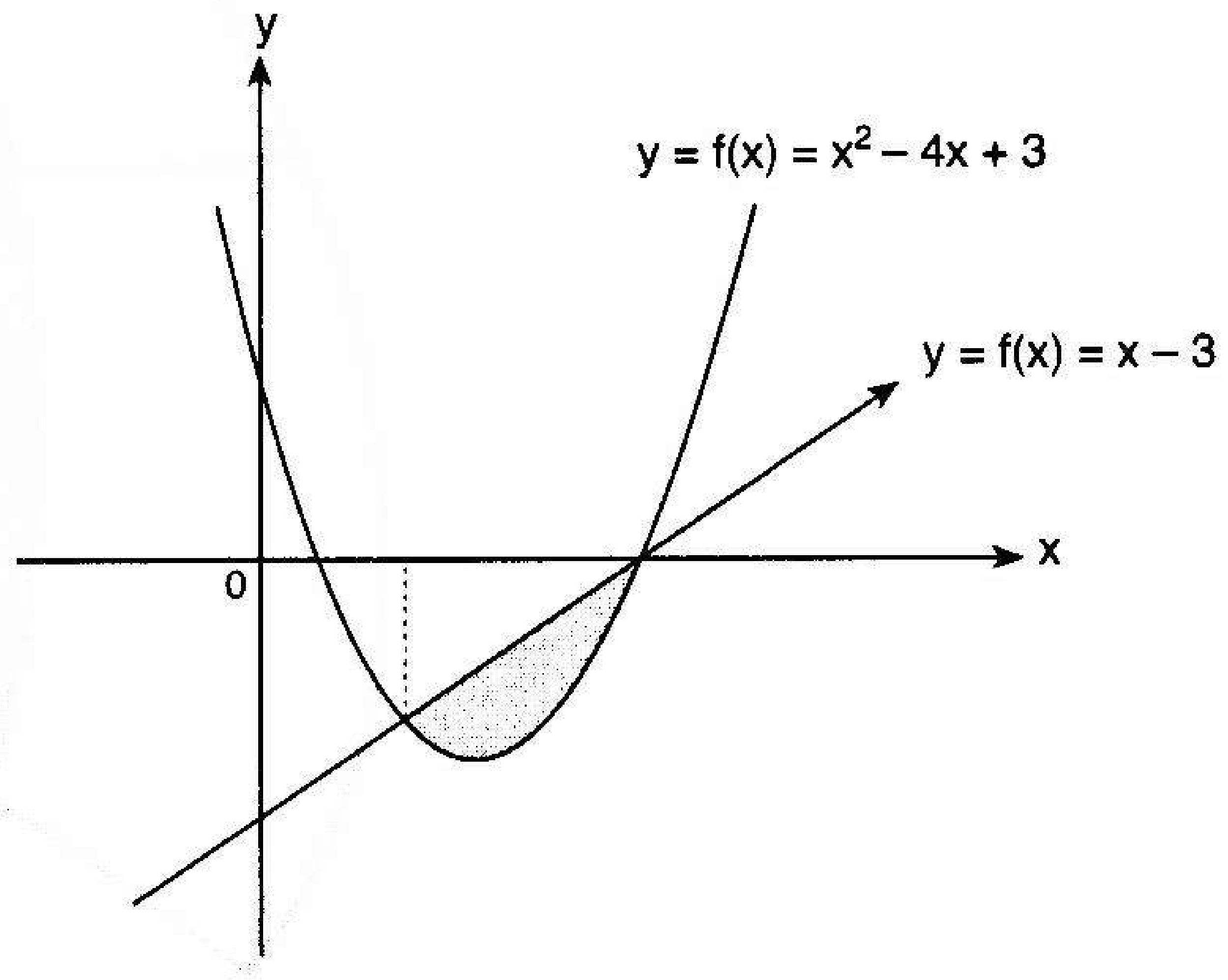
A) $\frac{1}{6}$

B) $\frac{2}{3}$

C) 1

D) $\frac{4}{3}$

E) $\frac{5}{3}$



ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$x^2 - 4x + 3 = x - 3 \Rightarrow x^2 - 5x + 6 = 0$$

$$(x-2)\cdot(x-3) = 0$$

$$x = 2,\ x = 3$$

$$\text{TA / SA} = \int_2^3 \left|x^2 - 4x + 3 - (x - 3)\right| dx = \int_2^3 |x^2 - 5x + 6|\,dx$$

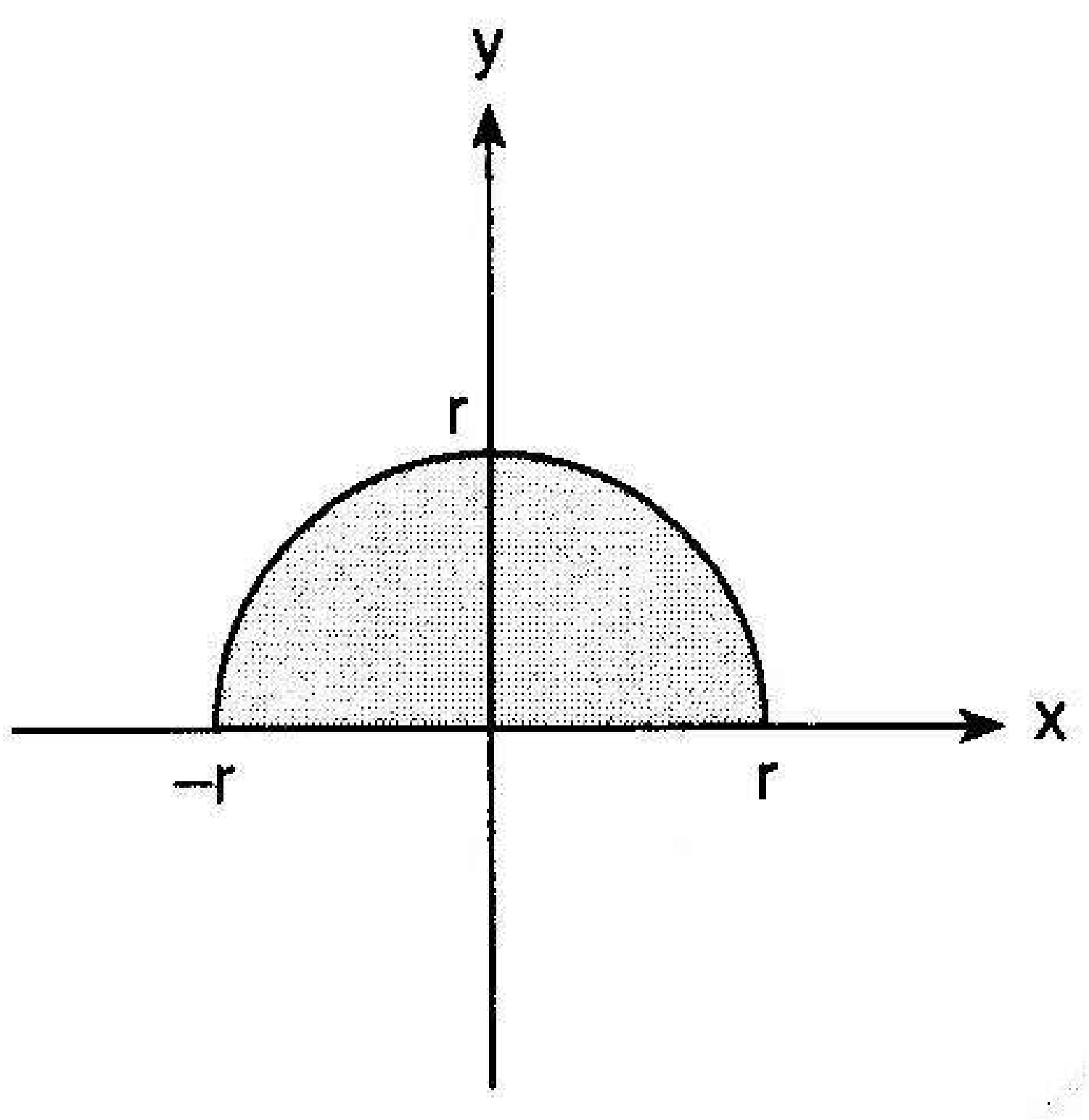
$$= \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right)\Bigg|_2^3$$

$$= \left|\left(9 - \frac{45}{2} + 18\right) - \left(\frac{8}{3} - 10 + 12\right)\right|$$

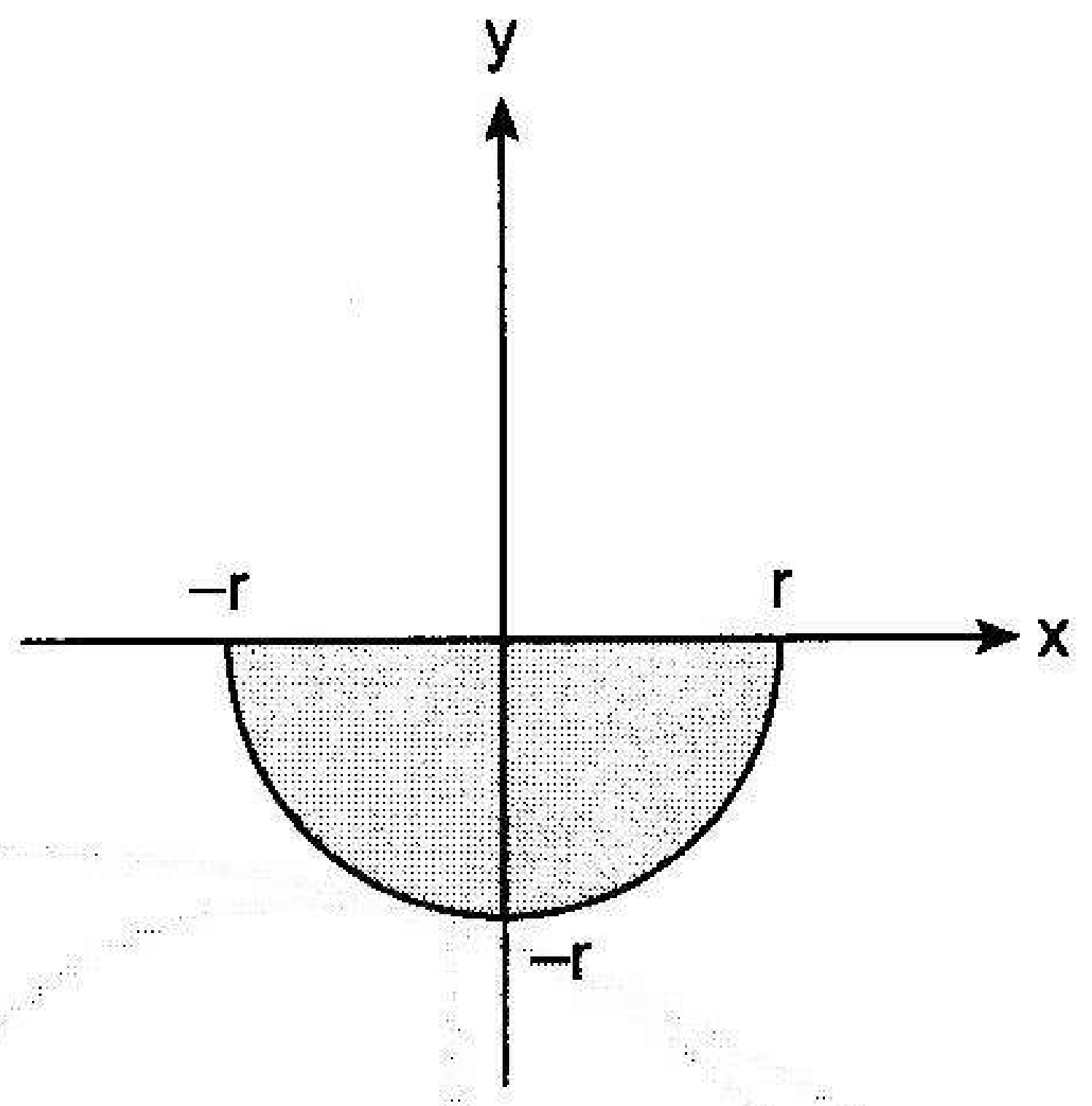
$$= \left|\frac{9}{2} - \frac{14}{3}\right| = \frac{1}{6}$$

Alan Yardımıyla İntegral Bulma / Finding Integral with Area

$x^2 + y^2 = r^2 \Rightarrow y = \pm\sqrt{r^2 - x^2}$



$y = \sqrt{r^2 - x^2}$



$y = -\sqrt{r^2 - x^2}$

$$\int_{-r}^{r} f(x) = \frac{\pi r^2}{2}$$

NOTLAR / NOTES

İfade / *Expression*	Yöntem / *Method*
✓ $\sqrt{a^2 - x^2}$	$x = a\sin t$
✓ $\sqrt{x^2 - a^2}$	$x = a\sec t$
✓ $\sqrt{x^2 + a^2}$	$x = a\tan t$

ÖRNEK / EXAMPLE 19

$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx = ?$$

A) $\frac{9\pi}{2}$ B) $\frac{7\pi}{2}$ C) $\frac{5\pi}{2}$ D) $\frac{3\pi}{2}$ E) $\frac{\pi}{2}$

ÇÖZÜM / SOLUTION

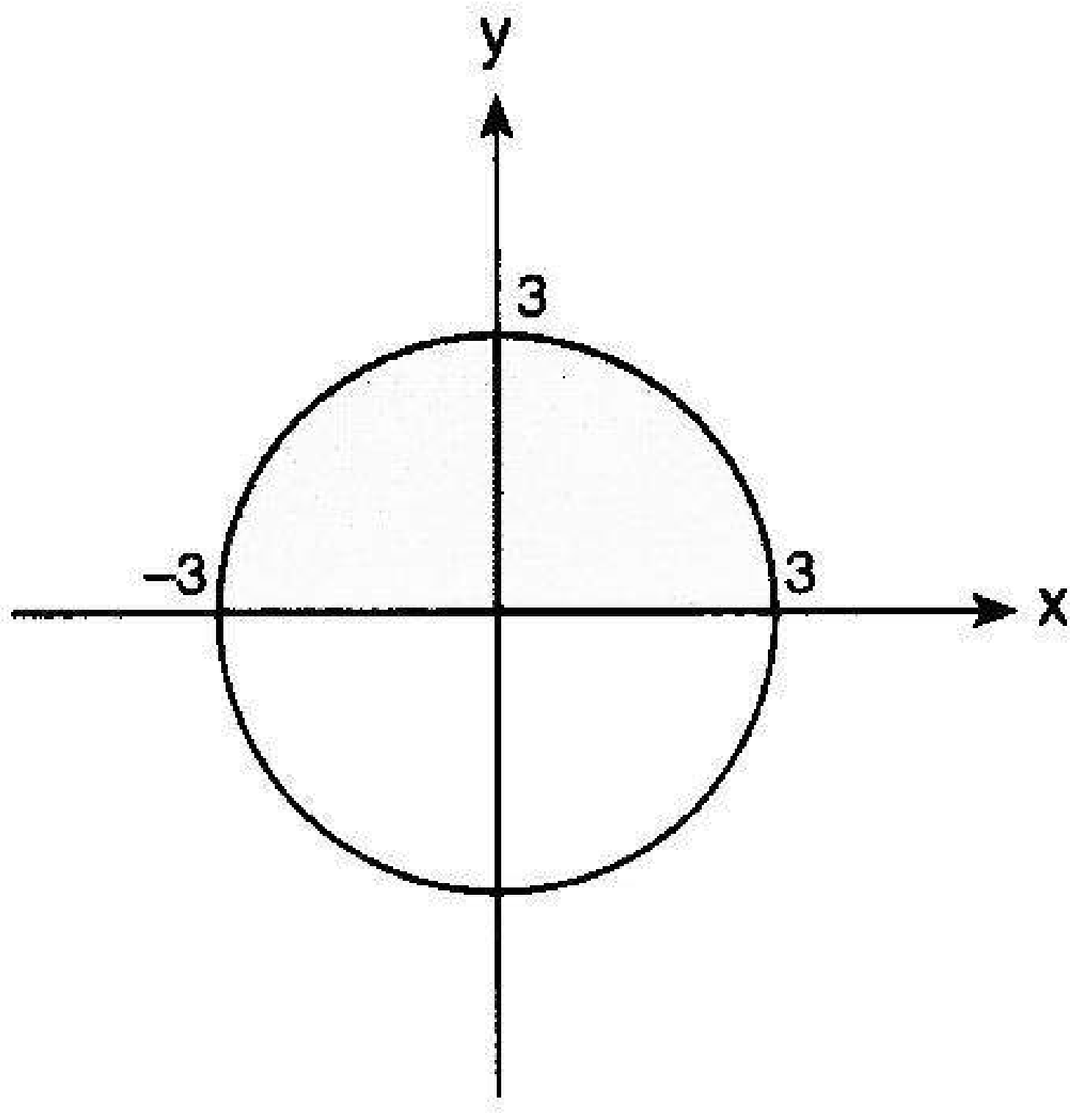
Yanıt / Answer **A**

$\sqrt{9 - x^2} = y$

$9 - x^2 = y^2$ [Merkezi (0,0) yarıçapı 3 br olan çemberin denklemi *Equation of circle that centre is (0,0) and radius is 3*]

$x^2 + y^2 = 9$

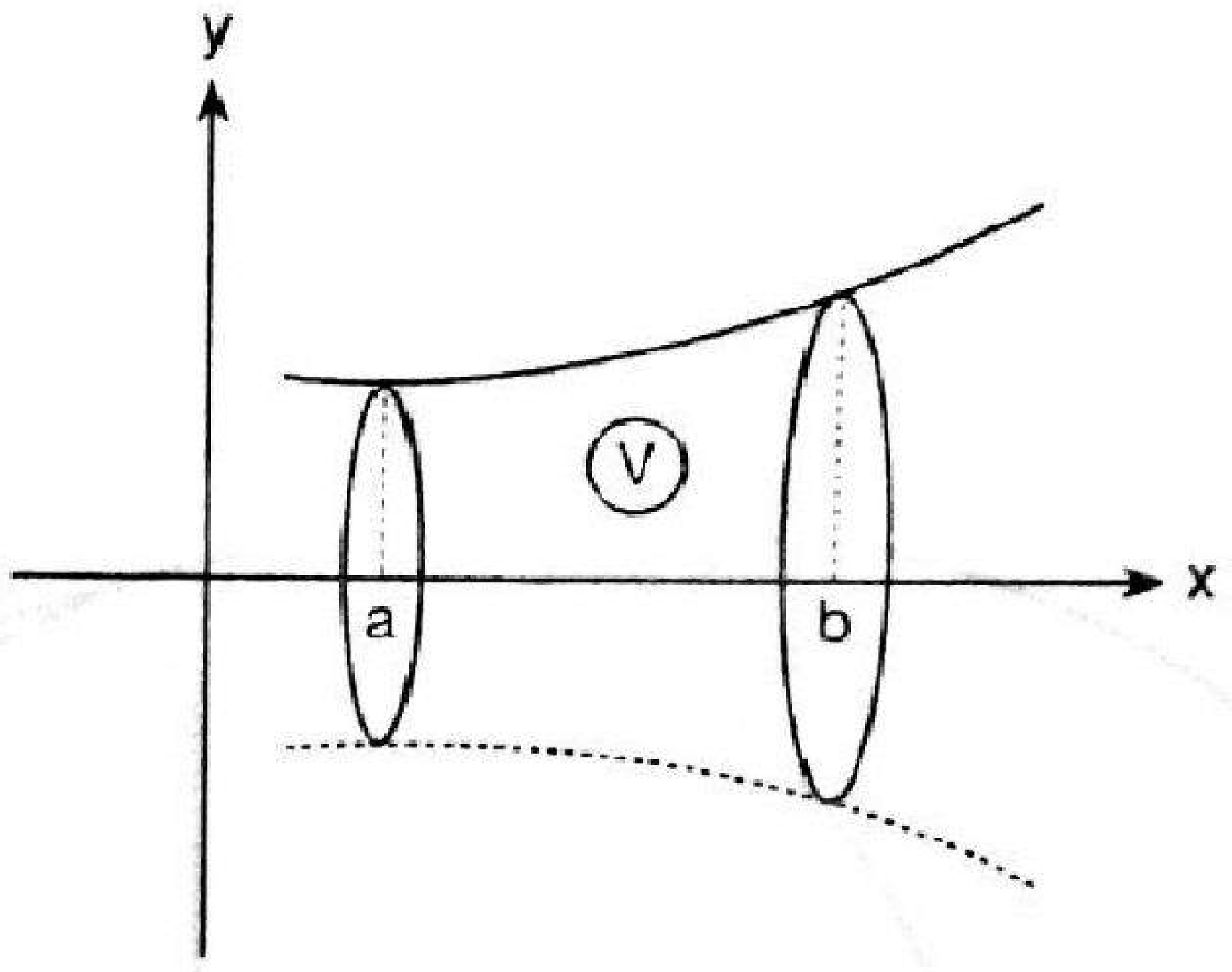
$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx = \frac{\pi \cdot r^2}{2} = \frac{\pi \cdot 3^2}{2} = \frac{9\pi}{2}$$



Hacim Hesabı / Volume Calculus

y = f(x) fonksiyonu, x ekseni ve x = a, x = b doğruları arasında kalan bölgenin x – ekseni etrafında döndürülmesiyle oluşan cismin hacmi aşağıdaki gibi bulunur.

The volume of the solid rotating around the x – axis is founded as follow that A region bounded by the function y = f(x), the lines x = a, x = b and x – axis.



$$\Rightarrow V = \pi \int_a^b y^2\,dx$$

ÖRNEK / EXAMPLE 20

$V = ?\pi$

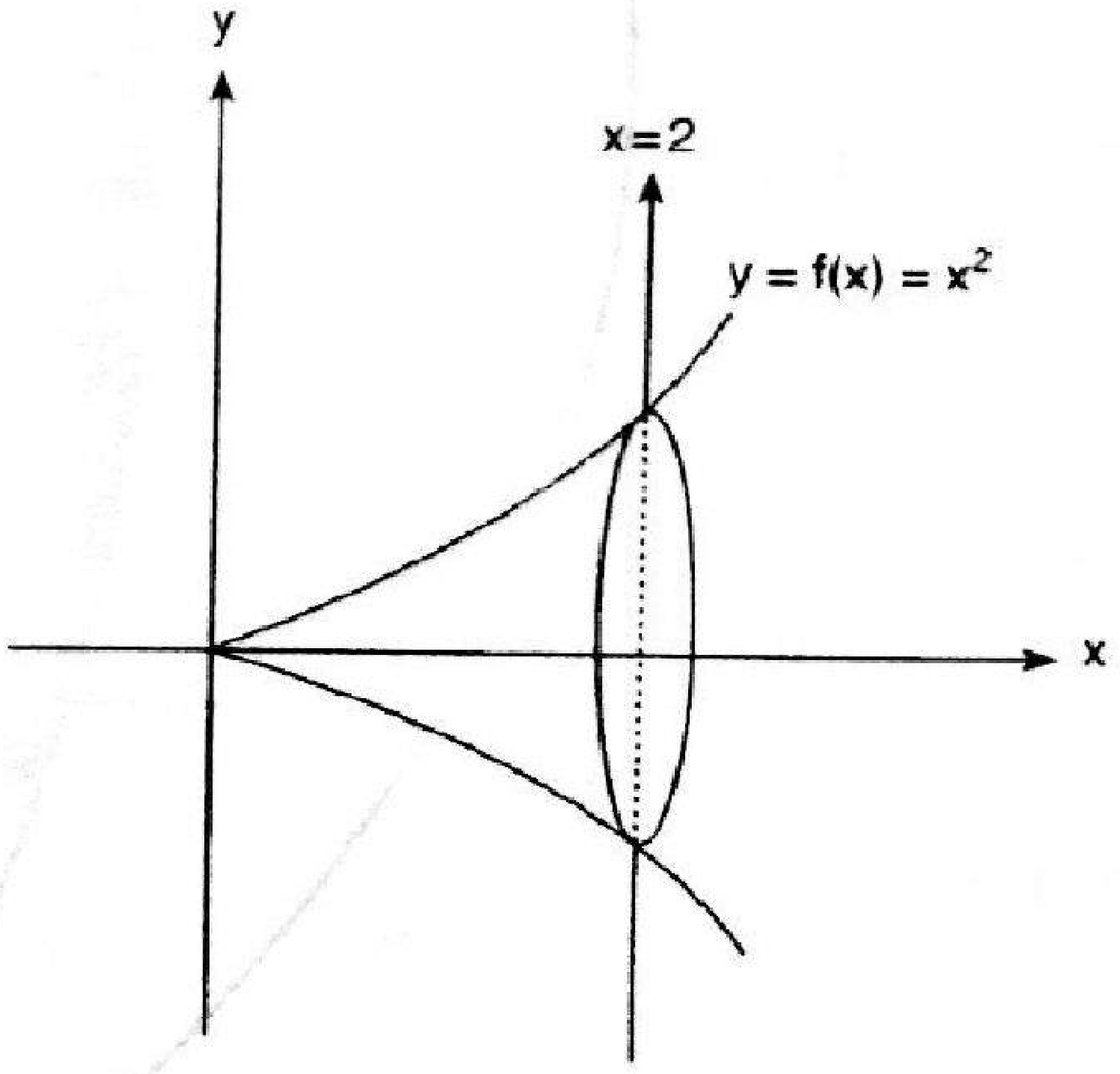
A) 16

B) $\frac{32}{3}$

C) 8

D) $\frac{32}{5}$

E) 4



ÇÖZÜM / SOLUTION

Yanıt / Answer D

$$V = \pi \int_0^2 [f(x)]^2\,dx$$

$$= \pi \int_0^2 (x^2)^2\,dx$$

$$= \pi \int_0^2 x^4\,dx$$

$$= \pi \cdot \frac{x^5}{5}\Big|_0^2 = \frac{32}{5}\pi$$

ÖRNEK / EXAMPLE 21

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$f(x)=\sqrt{\frac{3x^2}{(x^3+1)^5}}\ ,\ a<c<b$$

$$\int_a^a f(x)\,dx+\int_a^c f(x)\,dx+\int_c^b f(x)\,dx+\int_b^a f(x)\,dx=?$$

A) –2 B) –1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\int_a^a f(x)\,dx=0$$

$$\int_a^c f(x)\,dx+\int_c^b f(x)\,dx=\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx+\int_b^a f(x)\,dx=0$$

ÖRNEK / EXAMPLE 22

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$\int_0^\pi 3x\sin t\,dt=?$$

A) $3\pi x$ B) $6x$ C) 3π D) 0 E) 1

ÇÖZÜM / SOLUTION

Yanıt / Answer **B**

$$\begin{aligned}\int_0^\pi 3x\sin t\,dt&=3x\int_0^\pi \sin t\cdot dt\\&=3x\left[(-\cos t)\Big|_0^\pi\right]\\&=3x(-\cos\pi-(-\cos 0))\\&=3x(1+1)\\&=6x\end{aligned}$$

ÖRNEK / EXAMPLE 23

YILDIZ TEKNİK ÜNİ. YÖS 2013

$$\int_{-\pi}^{\pi} \cos(2x)\, d(2x) = ?$$

A) −2 B) −1 C) 0 D) 1 E) 2

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$\begin{aligned}\int_{-\pi}^{\pi} \cos(2x)\, d(2x) &= \int_{-\pi}^{\pi} \cos(2x)\, 2\, dx \\ &= \sin 2x \Big|_{-\pi}^{\pi} \\ &= \sin 2\pi - \sin(-2\pi) \\ &= 2\sin 2\pi \\ &= 0\end{aligned}$$

ÖRNEK / EXAMPLE 24

YILDIZ TEKNİK ÜNİVERSİTESİ YÖS 2013

$$\int_{x=0}^{4} \left(\int_{y=0}^{3} x\, dy \right) dx = ?$$

A) 4 B) 12 C) 18 D) 24 E) 26

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\int_{y=0}^{3} x\, dy = (xy)\Big|_{0}^{3} = 3x - 0 = 3x$$

$$\int_{x=0}^{4} 3x\, dx = \left(\frac{3x^2}{2} \right)\Big|_{0}^{4} = 24 - 0 = 24$$

ÖRNEK / EXAMPLE 25

MARMARA ÜNİVERSİTESİ YÖS 2013

$\int_a^b (4x-3)dx = 68, b-a=4 \Rightarrow \mathbf{b+a=?}$

A) 10 B) 15 C) 18 D) 20 E) 24

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\int_a^b (4x-3)dx = 68$$

$$\left(\frac{4x^2}{2}-3x\right)\Bigg|_a^b = 68$$

$$(2x^2-3x)\Big|_a^b = 68$$

$$2b^2-3b-2a^2+3a=68$$

$$2(b^2-a^2)-3(b-a)=68$$

$$(b-a)(2b+2a-3)=68$$

$$4\cdot(2b+2a-3)=68$$

$$2\cdot(b+a)-3=17$$

$$2\cdot(b+a)=20$$

$$b+a=10$$

ÖRNEK / EXAMPLE 26

İSTANBUL ÜNİVERSİTESİ YÖS 2013

$\int_1^3 \frac{1}{x\ln 2}dx = ?$

A) $\frac{\ln 3}{\ln 2}$ B) $2\ln 2$ C) $\frac{1}{\ln 2}$ D) $\ln 4$ E) e^{2x}

ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

$$\int_1^3 \frac{1}{x\ln 2}dx = \frac{1}{\ln 2}\int_1^3 \frac{1}{x}dx = \frac{1}{\ln 2}\ln x\Big|_1^3$$

$$= \frac{1}{\ln 2}[\ln 3 - \ln 1]$$

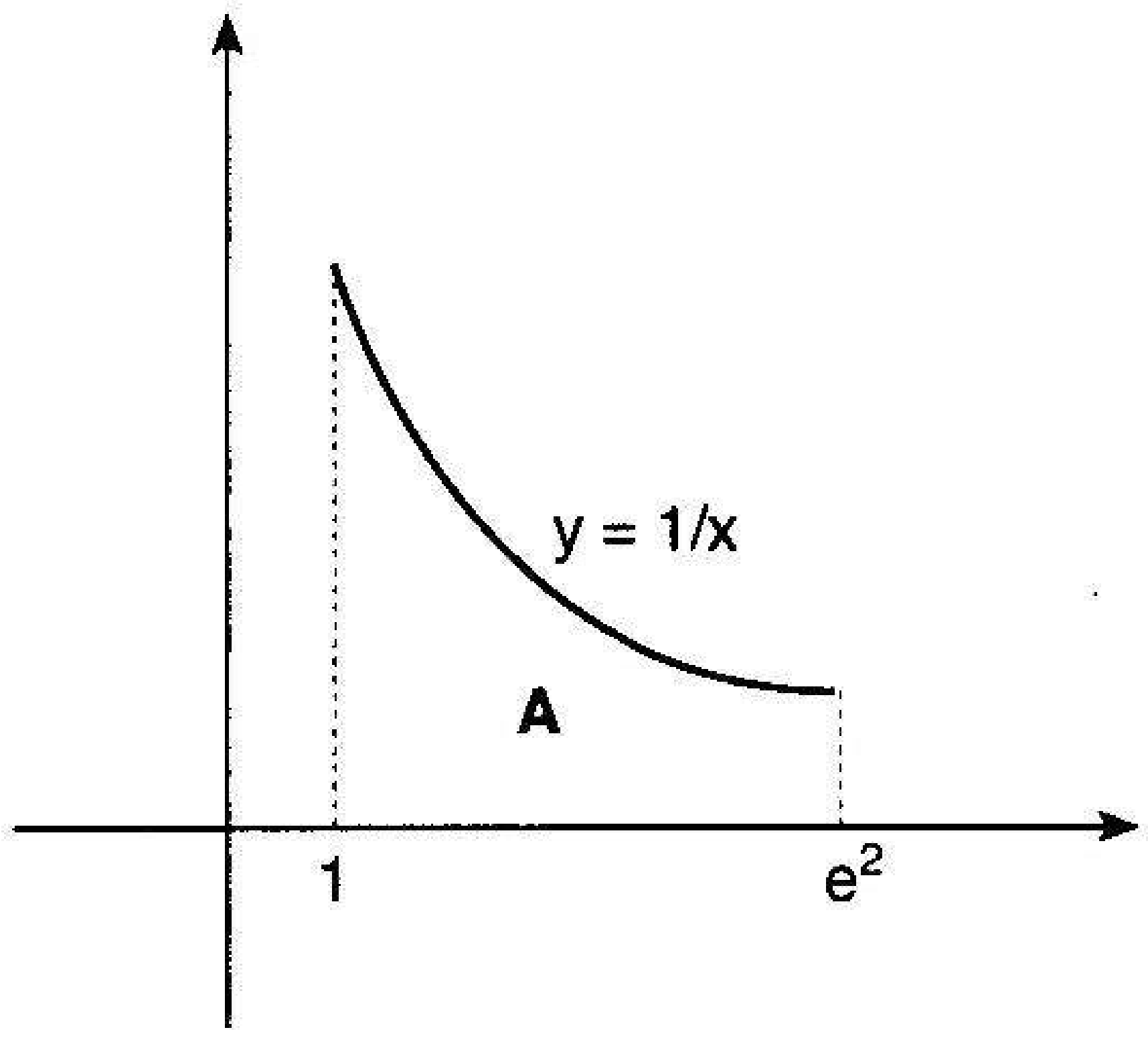
$$= \frac{\ln 3}{\ln 2}$$

ÖRNEK / EXAMPLE 27

İSTANBUL ÜNİVERSİTESİ YÖS 2013

$y=\frac{1}{x}$, $x=1$, $x=e^2$ ⇒ **A = ?**

A) 2
B) e
C) 4
D) e^2
E) 6



ÇÖZÜM / SOLUTION

Yanıt / Answer **A**

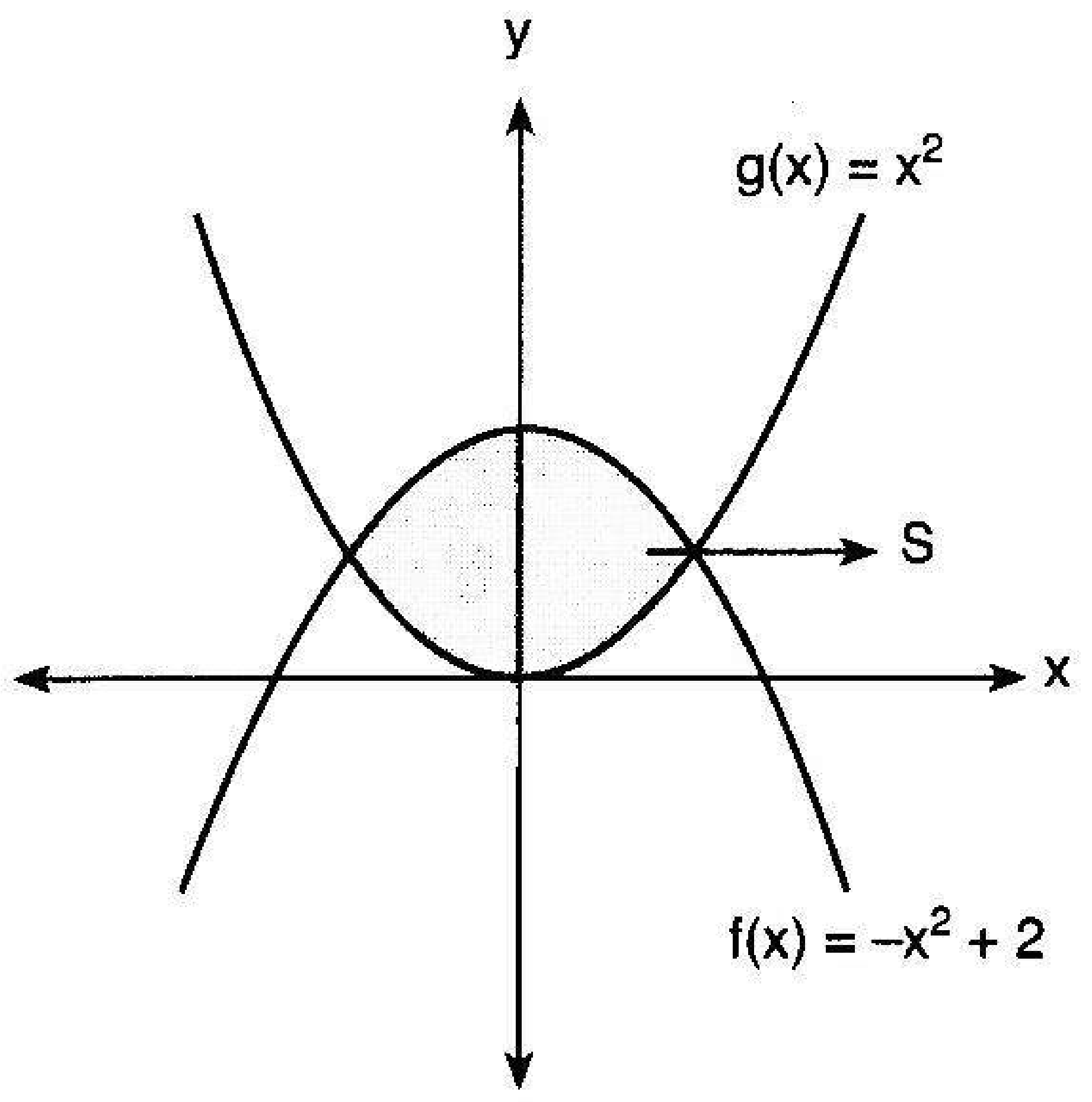
$$A=\int_1^{e^2}\frac{1}{x}\,dx$$

$$A=\ln x\Big|_1^{e^2}=\ln e^2-\ln 1$$

$$=2-0=2$$

ÖRNEK / EXAMPLE 28

AKDENİZ ÜNİVERSİTESİ YÖS 2014

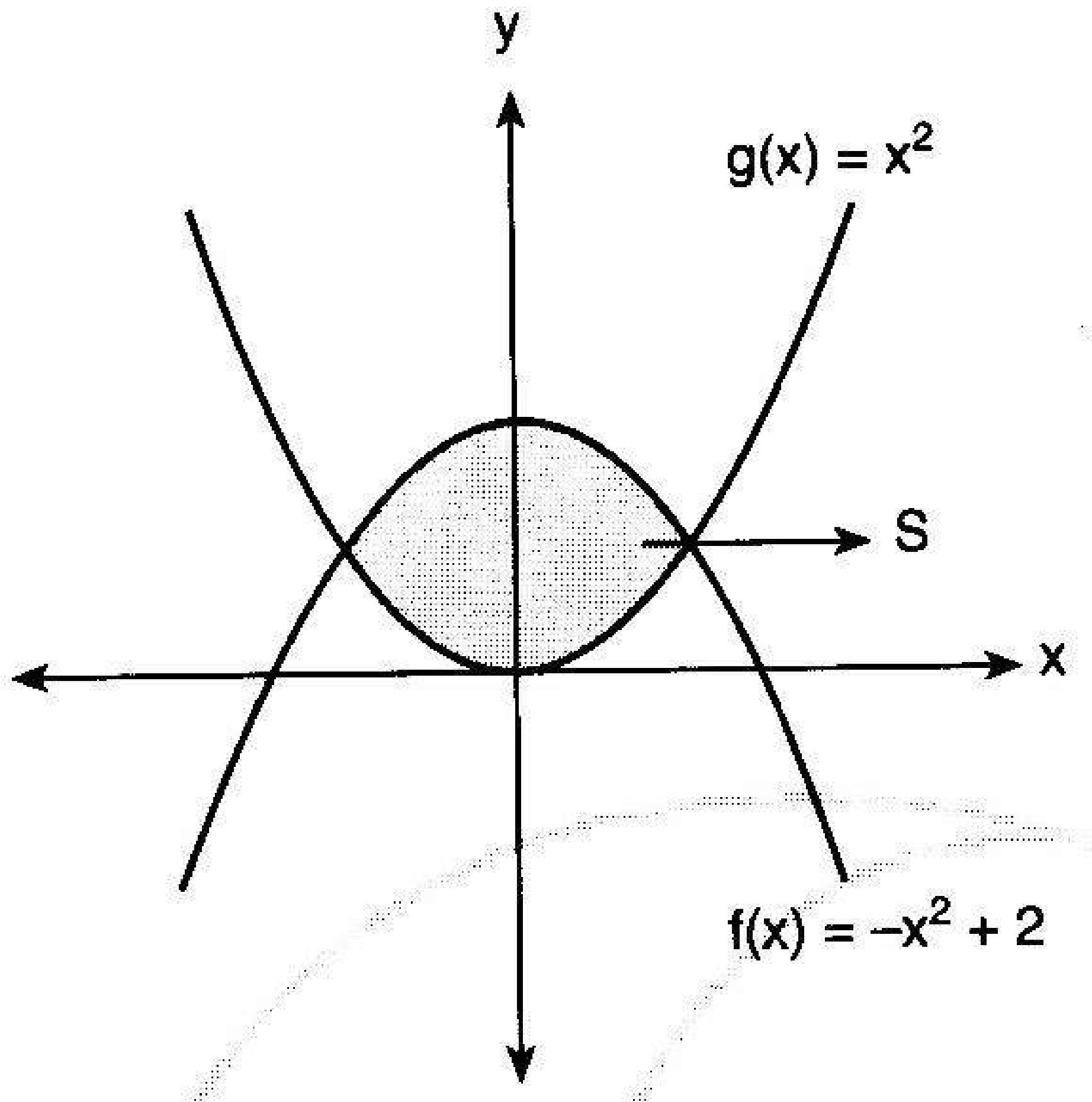


S = ?

A) $\frac{9}{2}$ B) 3 C) $\frac{7}{3}$ D) $\frac{8}{3}$ E) $\frac{5}{2}$

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**



$$x^2 = -x^2 + 2$$

$$x^2 = 1 \Rightarrow x = \pm 1$$

$$\int_{-1}^{1} (-2x^2 + 2)\,dx = \left(-\frac{2x^3}{3} + 2x\right)\Bigg|_{-1}^{1}$$

$$= \left(-\frac{2}{3} + 2\right) - \left(\frac{2}{3} - 2\right)$$

$$= \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$$

ÖRNEK / EXAMPLE 29

AKDENİZ ÜNİVERSİTESİ YÖS 2014

$$\int_{0}^{1} x \cdot e^x \,dx = ?$$

A) e

B) e + 1

C) –1

D) +1

E) 1 – e

ÇÖZÜM / SOLUTION

Yanıt / Answer **D**

$$\begin{array}{ccc} + & x & e^x \\ - & 1 & e^x \\ + & 0 & e^x \end{array} \Rightarrow (x \cdot e^x - e^x)\Big|_{0}^{1} = (e^1 - e^1) - (0 - e^0)$$

$$= 0 - (-e^0) = 1$$

ÖRNEK / EXAMPLE 30

İSTANBUL ÜNİVERSİTESİ YÖS 2014

$$\int_{2}^{10} |2x-8|\,dx = ?$$

A) 0 B) 20 C) 40 D) 45 E) 60

ÇÖZÜM / SOLUTION

Yanıt / Answer **C**

$$2x-8=0$$

$$2x=8$$

$$x=4$$

$$\int_{2}^{10} |2x-8|\,dx = \int_{2}^{4} |12x-8| + \int_{4}^{10} |12x-8|\,dx$$

$$= \int_{2}^{4} (8-2x)\,dx + \int_{4}^{10} (2x-8)\,dx$$

$$= 8x-x^2\Big|_{2}^{4} + x^2-8x\Big|_{4}^{10} = 16-12+20+16=40$$

ÖRNEK / EXAMPLE 31

YILDIZ TEKNİK ÜNİ. YÖS 2016

$$\int_{23}^{44} f(x)\cdot dx = 36 \Rightarrow \int_{8}^{15} f(3x-1)\cdot dx = ?$$

A) 108 B) 72 C) 36 D) 18 E) 12

ÇÖZÜM / SOLUTION

Yanıt / Answer **E**

$$\int f(x)\,dx = F(x)$$

$$\int_{23}^{44} f(x)\cdot dx = F(x)\Big|_{23}^{43} = F(44)-F(23)=36$$

$$\int_{8}^{15} f(3x-1)\cdot dx = \frac{F(3x-1)}{3}\Bigg|_{8}^{15}$$

$$= \frac{F(44)-F(23)}{3} = \frac{36}{3} = 12$$

1. $\int_{-2}^{3} x^2\, dx = ?$

A) $\frac{32}{3}$

B) $\frac{34}{3}$

C) $\frac{35}{3}$

D) $\frac{37}{3}$

E) $\frac{38}{3}$

2. $\int_{-1}^{1} (4x^3 - 2x + 7)\, dx = ?$

A) –14

B) –7

C) 0

D) 7

E) 14

3. $\int_{\ln 2}^{\ln 8} e^{2x}\, dx = ?$

A) 24

B) 26

C) 28

D) 30

E) 32

4. $\int_{-1}^{1} \frac{1}{1+x^2}\, dx = ?$

A) $\frac{\pi}{2}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{5}$

E) $\frac{\pi}{6}$

5. $\int_{1}^{e^3} \frac{dx}{x} = ?$

A) 4

B) 3

C) 2

D) 1

E) 0

6. $\int_{\frac{\pi}{2}}^{\pi} \sin 3x\, dx = ?$

A) $\frac{1}{5}$

B) $\frac{1}{4}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

E) 1

7. $\int_{-2}^{4} |x-2|\,dx = ?$

A) 8

B) 10

C) 12

D) 14

E) 16

8. $\int_{-3}^{1} |x+2|\,dx = ?$

A) –2

B) 0

C) 2

D) 5

E) 6

9. $\int_{0}^{\pi} \cos x\,dx = ?$

A) -1

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) 1

10. $\int_{a}^{b} (4x-1)\,dx = 12,\ a-b=-4 \Rightarrow \mathbf{a \cdot b = ?}$

A) 4

B) 3

C) 0

D) –3

E) –4

11. $\int_{0}^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\,dx = ?$

A) $\frac{1}{4}$

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) $\frac{1}{5}$

E) 1

12. $\frac{d}{dx}\left(\int_{1}^{8} (x^3-5x)\,dx\right) = ?$

A) –2 B) –1 C) 0

D) 1 E) 2

YANITLAR / ANSWERS							
1. C	2. E	3. D	4. A	5. B	6. C	7. B	8. D
9. C	10. D	11. B	12. C				

1. $$\int_{-2}^{3} x^2\,dx = \left.\frac{x^3}{3}\right|_{-2}^{3} = \left(\frac{3^3}{3} - \frac{(-2)^3}{3}\right)$$

$$= 9 + \frac{8}{3} = \frac{35}{3}$$

Yanıt / The Answer is C

---

2. $$\int_{-1}^{1} (4x^3 - 2x + 7)\,dx = \left.\frac{4x^4}{4} - \frac{2x^2}{2} + 7x\right|_{-1}^{1}$$

$$= \left. x^4 - x^2 + 7x \right|_{-1}^{1}$$

$$= (1^4 - 1^2 + 7\cdot 1) - ((-1)^4 - (-1)^2 + 7\cdot(-1))$$

$$= 7 + 7 = 14$$

Yanıt / The Answer is E

---

3. $$\int_{\ln 2}^{\ln 8} e^{2x}\,dx = \left.\frac{1}{2}e^{2x}\right|_{\ln 2}^{\ln 8}$$

$$= \frac{1}{2}\left(e^{2\ln 8} - e^{2\ln 2}\right)$$

$$= \frac{1}{2}(64 - 4) = 30$$

Yanıt / The Answer is D

---

4. $$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \left.\arctan x\right|_{-1}^{1}$$

$$= \arctan 1 - \arctan(-1)$$

$$= \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{\pi}{2}$$

Yanıt / The Answer is A

---

5. $$\int_{1}^{e^3} \frac{dx}{x} = \left.\ln x\right|_{1}^{e^3} = (\ln e^3 - \ln 1)$$

$$= 3 - 0 = 3$$

Yanıt / The Answer is B

---

6. $$\int_{\frac{\pi}{2}}^{\pi} \sin 3x\,dx = \left.-\frac{1}{3}\cos 3x\right|_{\frac{\pi}{2}}^{\pi}$$

$$= -\frac{1}{3}\left[\cos 3\pi - \cos\frac{3\pi}{2}\right]$$

$$= -\frac{1}{3}[-1 - 0] = \frac{1}{3}$$

Yanıt / The Answer is C

7. $x-2=0 \Rightarrow x=2$

x		2	
x − 2	−	○	+

$$\int_{-2}^{4} |x-2|\,dx = \int_{-2}^{2} (2-x)\,dx + \int_{2}^{4} (x-2)\,dx$$

$$=\left(2x-\frac{x^2}{2}\right)\Bigg|_{-2}^{2} + \left(\frac{x^2}{2}-2x\right)\Bigg|_{2}^{4}$$

$$=(2-(-6))+(0-(-2))$$

$$=10$$

Yanıt / The Answer is B

8. $x+2=0 \Rightarrow x=-2$

x		−2	
x + 2	−	○	+

$$\int_{-3}^{1} |x+2|\,dx = \int_{-3}^{-2} (-x-2)\,dx + \int_{-2}^{1} (x+2)\,dx$$

$$=-\frac{x^2}{2}-2x\Bigg|_{-3}^{-2} + \frac{x^2}{2}+2x\Bigg|_{-2}^{1}$$

$$=\frac{1}{2}+\frac{9}{2}=\frac{10}{2}=5$$

Yanıt / The Answer is D

9. $\int_{0}^{\pi} \cos x\,dx = \sin x\Bigg|_{0}^{\pi} = \sin\pi - \sin 0 = 0$

Yanıt / The Answer is C

10. $\int_{a}^{b} (4x-1)\,dx = 12 \Rightarrow \left(\frac{4x^2}{2}-x\right)\Bigg|_{a}^{b} = 12$

$$(2b^2-b)-(2a^2-a)=12$$

$$2b^2-b-2a^2+a=12$$

$$2\cdot(b^2-a^2)+a-b=12$$

$$2\cdot(b-a)\cdot(b+a)+a-b=12$$

$$2\cdot 4\cdot(b+a)-4=12$$

$$8\cdot(b+a)=16$$

$$\begin{array}{rl} & b+a=2 \\ + & a-b=-4 \\ \hline & 2a=-2 \Rightarrow a=-1\ ,\ b=3 \end{array}$$

$a\cdot b=-1\cdot 3==-3$

Yanıt / The Answer is D

11. $\int_{0}^{\frac{\pi}{4}} (\cos^2 x-\sin^2 x)\,dx = \int_{0}^{\frac{\pi}{4}} \cos 2x\,dx$

$$=\frac{1}{2}(\sin 2x)\Bigg|_{0}^{\frac{\pi}{4}}$$

$$=\frac{1}{2}\left(\sin\frac{\pi}{2}-\sin 0\right)$$

$$=\frac{1}{2}(1-0)=\frac{1}{2}$$

Yanıt / The Answer is B

12. $\int_{1}^{8} (x^3-5x^2)\,dx = a\ ,\ a\in R \Rightarrow \frac{d}{dx}a=0$

Yanıt / The Answer is C

1. $\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot x \, dx = ?$

A) $-\ln\frac{1}{2}$ B) $-\ln\frac{\sqrt{3}}{2}$ C) $\ln\frac{1}{2}$

D) $\ln\frac{\sqrt{3}}{2}$ E) 0

---

2. $\int\limits_{1}^{e^2} \frac{\ln^2 x}{x} \, dx = ?$

A) $\frac{8}{3}$ B) $\frac{10}{3}$ C) $\frac{11}{3}$

D) $\frac{14}{3}$ E) $\frac{16}{3}$

---

3. $\int\limits_{0}^{1} \frac{x^2+1}{x^3+3x+5} \, dx = ?$

A) $\frac{1}{2}\ln\frac{9}{2}$ B) $\frac{1}{3}\ln\frac{8}{3}$ C) $\frac{1}{3}\ln\frac{9}{5}$

D) $\frac{1}{4}\ln\frac{9}{7}$ E) $\frac{1}{4}\ln\frac{12}{5}$

---

4. $\int\limits_{0}^{\frac{\pi}{2}} \frac{\cos x \, dx}{(1+\sin x)^3} = ?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$

D) $\frac{1}{2}$ E) $\frac{3}{4}$

---

5. $\int\limits_{0}^{2} \frac{dx}{4+x^2} = ?$

A) $\frac{\pi}{4}$ B) $\frac{3\pi}{4}$ C) $\frac{\pi}{8}$

D) 2 E) $\frac{1}{8}$

---

6. $\int\limits_{0}^{\pi} \frac{dx}{x^2-4} = ?$

A) $2\cdot\ln\left|\frac{\pi+2}{\pi-2}\right|$ B) $\frac{1}{4}\cdot\ln\left|\frac{\pi-2}{\pi+2}\right|$

C) $\frac{1}{4}\cdot\ln\left|\frac{\pi+2}{\pi-2}\right|$ D) $\frac{1}{2}\cdot\ln\left|\frac{\pi+2}{\pi-2}\right|$

E) $\frac{1}{2}\cdot\ln\left|\frac{\pi-2}{\pi+4}\right|$

7. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = ?$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$ E) π

8. $\displaystyle\int_1^{e^2} x \cdot \ln x \cdot dx = ?$

A) $\frac{e^2-1}{4}$ B) $\frac{4e^4-e^2+1}{4}$ C) $\frac{4e^4-e^2}{4}$ D) $\frac{3e^4+1}{4}$ E) $\frac{e^4-4e^2+1}{4}$

9. $\displaystyle\int_0^{\frac{\pi}{4}} \cos 2x \cdot \sin 4x\ dx = ?$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1 E) $\frac{5}{4}$

10. $F(x) = \displaystyle\int_0^{\sqrt{x}} e^{t^2} dt \Rightarrow F'(1) = ?$

A) $\frac{e}{4}$ B) $\frac{e}{3}$ C) $\frac{e}{2}$ D) e E) 2e

11. $F(x) = \displaystyle\int_0^{\cos x} \arccos t\ dt \Rightarrow F'\left(\frac{\pi}{2}\right) = ?$

A) $-\frac{\pi}{2}$ B) 0 C) $\frac{\pi}{6}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{2}$

12. $\displaystyle\int_{-5}^{5} \sqrt{25-x^2}\ dx = ?$

A) $\frac{5\pi}{2}$ B) 5π C) $\frac{15\pi}{2}$ D) 10π E) $\frac{25\pi}{2}$

YANITLAR / ANSWERS

1. B 2. A 3. C 4. C 5. C 6. B 7. B 8. D
9. B 10. C 11. A 12. E

1. $$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot x \, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} dx$$

$\sin x = u \Rightarrow \cos x \, dx = du$

$x = \frac{\pi}{3} \Rightarrow \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow u = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1 \Rightarrow u = 1$

$$\int_{\frac{\sqrt{3}}{2}}^{1} \frac{du}{u} = \ln u \Big|_{\frac{\sqrt{3}}{2}}^{1} = \ln 1 - \ln\frac{\sqrt{3}}{2}$$

$$= 0 - \ln\frac{\sqrt{3}}{2} = -\ln\frac{\sqrt{3}}{2}$$

Yanıt / The Answer is B

---

2. $\ln x = u \Rightarrow \frac{1}{x} dx = du$

$x = 1 \Rightarrow \ln 1 = 0 \Rightarrow u = 0$

$x = e^2 \Rightarrow \ln e^2 = 2 \Rightarrow u = 2$

$$\int_{1}^{e^2} \frac{\ln^2 x}{x} dx = \int_{0}^{2} u^2 \, du$$

$$= \frac{u^3}{3}\Big|_0^2 = \left(\frac{8}{3} - 0\right) = \frac{8}{3}$$

Yanıt / The Answer is A

---

3. $x^3 + 3x + 5 = u \Rightarrow (3x^2 + 3)\, dx = du \;,\; (x^2 + 1)dx = \frac{du}{3}$

$x = 0 \Rightarrow x^2 + 3x + 5 = 5 \Rightarrow u = 5$

$x = 1 \Rightarrow x^2 + 3x + 5 = 9 \Rightarrow u = 9$

$$\int_0^1 \frac{x^2 + 1}{x^3 + 3x + 5} dx = \frac{1}{3}\int_5^9 \frac{du}{u}$$

$$= \frac{1}{3}\left[\ln u \Big|_5^9\right]$$

$$= \frac{1}{3}\left[\ln 9 - \ln 5\right] = \frac{1}{3}\ln\frac{9}{5}$$

Yanıt / The Answer is C

---

4. $1 + \sin x = u \Rightarrow \cos x \, dx = du$

$x = 0 \Rightarrow \quad 1 + \sin 0 = 1 \quad \Rightarrow u = 1$

$x = \frac{\pi}{2} \Rightarrow 1 + \sin\frac{\pi}{2} = 2 \Rightarrow u = 2$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x \, dx}{(1 + \sin x)^3} dx = \int_1^2 \frac{du}{u^3}$$

$$= \frac{u^{-2}}{-2}\Big|_1^2$$

$$= \frac{-1}{2u^2}\Big|_1^2 = \left(-\frac{1}{8} - \left(-\frac{1}{2}\right)\right) = \frac{3}{8}$$

Yanıt / The Answer is C

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5. $$\int_0^2 \frac{dx}{4 + x^2} = \frac{1}{4}\int_0^2 \frac{dx}{1 + (\frac{x}{2})^2} \quad , \quad \frac{x}{2} = u \Rightarrow \frac{1}{2} dx = du$$

$x = 0 \Rightarrow \frac{x}{2} = 0 \Rightarrow u = 0$

$x = 2 \Rightarrow \frac{x}{2} = 1 \Rightarrow u = 1$

$$\frac{1}{4}\int_0^2 \frac{dx}{1 + (\frac{x}{2})^2} = \frac{1}{4}\int_0^1 \frac{2du}{1 + u^2}$$

$$= \frac{1}{4} \cdot 2(\arctan u)\Big|_0^1$$

$$= \frac{1}{2}(\arctan 1 - \arctan 0) = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$$

Yanıt / The Answer is C

---

6. $$\frac{1}{x^2 - 4} = \frac{A}{x - 2} + \frac{B}{x + 2}$$

$$\left.\begin{aligned} A + B &= 0 \\ 2A - 2B &= 1 \end{aligned}\right\} \Rightarrow A = \frac{1}{4}, \quad B = -\frac{1}{4}$$

$$\int_0^{\pi} \frac{dx}{x^2 - 4} = \int_0^{\pi}\left(\frac{\frac{1}{4}}{x - 2} - \frac{\frac{1}{4}}{x + 2}\right)dx$$

$$= \frac{1}{4} \cdot \int_0^{\pi}\left(\frac{1}{x - 2} - \frac{1}{x + 2}\right)dx$$

$$= \frac{1}{4} \cdot (\ln|x - 2| - \ln|x + 2|)\Big|_0^{\pi} = \frac{1}{4}\ln\left|\frac{\pi - 2}{\pi + 2}\right|$$

Yanıt / The Answer is B

7.
$$\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = \int_0^{\frac{\pi}{2}} \frac{\cos 2x+1}{2}\,dx$$
$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \cos 2x\,dx + \int_0^{\frac{\pi}{2}} dx\right]$$
$$= \frac{1}{2}\left[\frac{1}{2}\sin 2x\Big|_0^{\frac{\pi}{2}} + x\Big|_0^{\frac{\pi}{2}}\right]$$
$$= \frac{1}{2}\left[\frac{1}{2}(\sin\pi - \sin 0) + \left(\frac{\pi}{2} - 0\right)\right]$$
$$= \frac{1}{2}\cdot\left(0 + \frac{\pi}{2}\right) = \frac{\pi}{4}$$

Yanıt / The Answer is B

8.

$$\ln x = u \quad , \quad x\,dx = dv \quad , \quad \frac{1}{x}dx = du \quad , \quad \frac{x^2}{2} = v$$
$$\int_1^{e^2} x\ln x\,dx = \ln x\cdot\frac{x^2}{2} - \frac{1}{2}\int_1^{e^2} x^2\cdot\frac{1}{x}\,dx$$
$$= \ln x\cdot\frac{x^2}{2} - \frac{x^2}{4}\Big|_1^{e^2}$$
$$= \left(\ln e^2\cdot\frac{e^4}{2} - \frac{e^4}{4}\right) - \left(\ln 1\cdot\frac{1}{2} - \frac{1}{4}\right)$$
$$= \left(2\cdot\frac{e^4}{2} - \frac{e^4}{4}\right) + \frac{1}{4}$$
$$= \frac{3e^4}{4} + \frac{1}{4} = \frac{3e^4+1}{4}$$

Yanıt / The Answer is D

9.
$$\int_0^{\frac{\pi}{4}} \cos 2x\cdot\sin 4x\,dx = \int_0^{\frac{\pi}{4}} \cos 2x\cdot 2\cos 2x\cdot\sin 2x\,dx$$
$$= \int_0^{\frac{\pi}{4}} (\cos 2x)^2\cdot 2\sin 2x\,dx$$

$\cos 2x = u \Rightarrow -2\sin 2x\,dx = du$

$x = 0 \Rightarrow \cos 2x = 1 \Rightarrow u = 1$

$x = \frac{\pi}{4} \Rightarrow \cos 2x = 0 \Rightarrow u = 0$

$$\int_0^{\frac{\pi}{4}} (\cos 2x)^2\cdot 2\sin 2x\,dx = -\int_1^0 u^2 du$$
$$= -\left[\frac{u^3}{3}\Big|_1^0\right] = -\left(0 - \frac{1}{3}\right) = \frac{1}{3}$$

Yanıt / The Answer is B

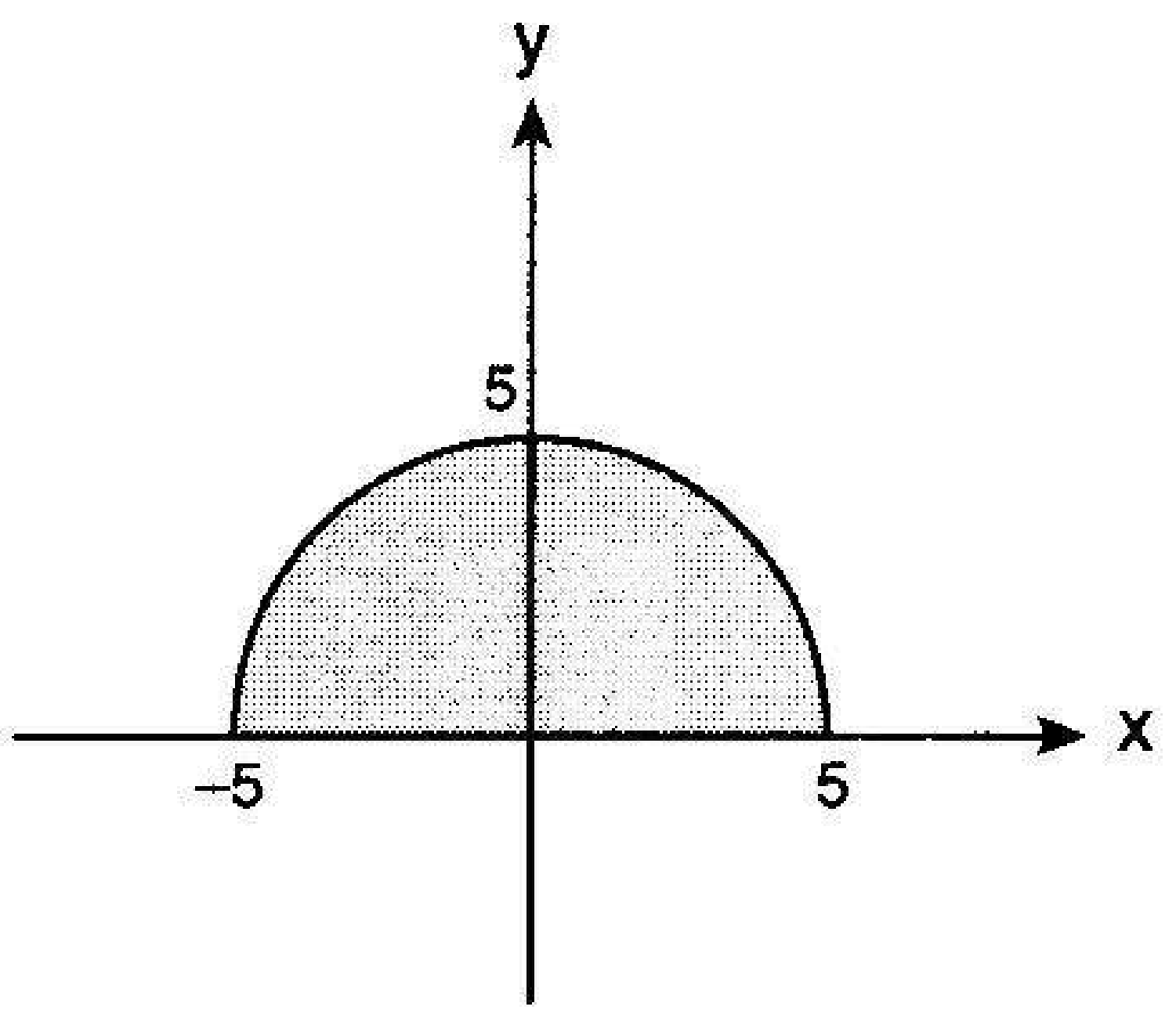
10.
$$F(x) = \int_0^{\sqrt{x}} e^{t^2}dt \Rightarrow F'(x) = (\sqrt{x})'\cdot e^{(\sqrt{x})^2} = \frac{1}{2\sqrt{x}}e^x$$
$$\Rightarrow F'(1) = \frac{1}{2\sqrt{1}}e^1 = \frac{e}{2}$$

Yanıt / The Answer is C

11.
$$F(x) = \int_0^{\cos x} \arccos t\,dt$$
$$\Rightarrow F'(x) = (\cos x)'\cdot(\arccos(\cos x)) = -\sin x\cdot x$$
$$F'\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2}\cdot\frac{\pi}{2} = \frac{-\pi}{2}$$

Yanıt / The Answer is A

12.
$$\left.\begin{aligned} y &= \sqrt{25-x^2} \\ y^2 &= 25-x^2 \\ x^2+y^2 &= 25 \end{aligned}\right\} \Rightarrow$$



$$\Rightarrow \int_{-5}^{5} \sqrt{25-x^2}\,dx = \frac{\pi r^2}{2} = \frac{\pi 5^2}{2} = \frac{25\pi}{2}$$

Yanıt / The Answer is E

1. **Taralı alan /**

The shaded area **= ? br^2 / u^2**

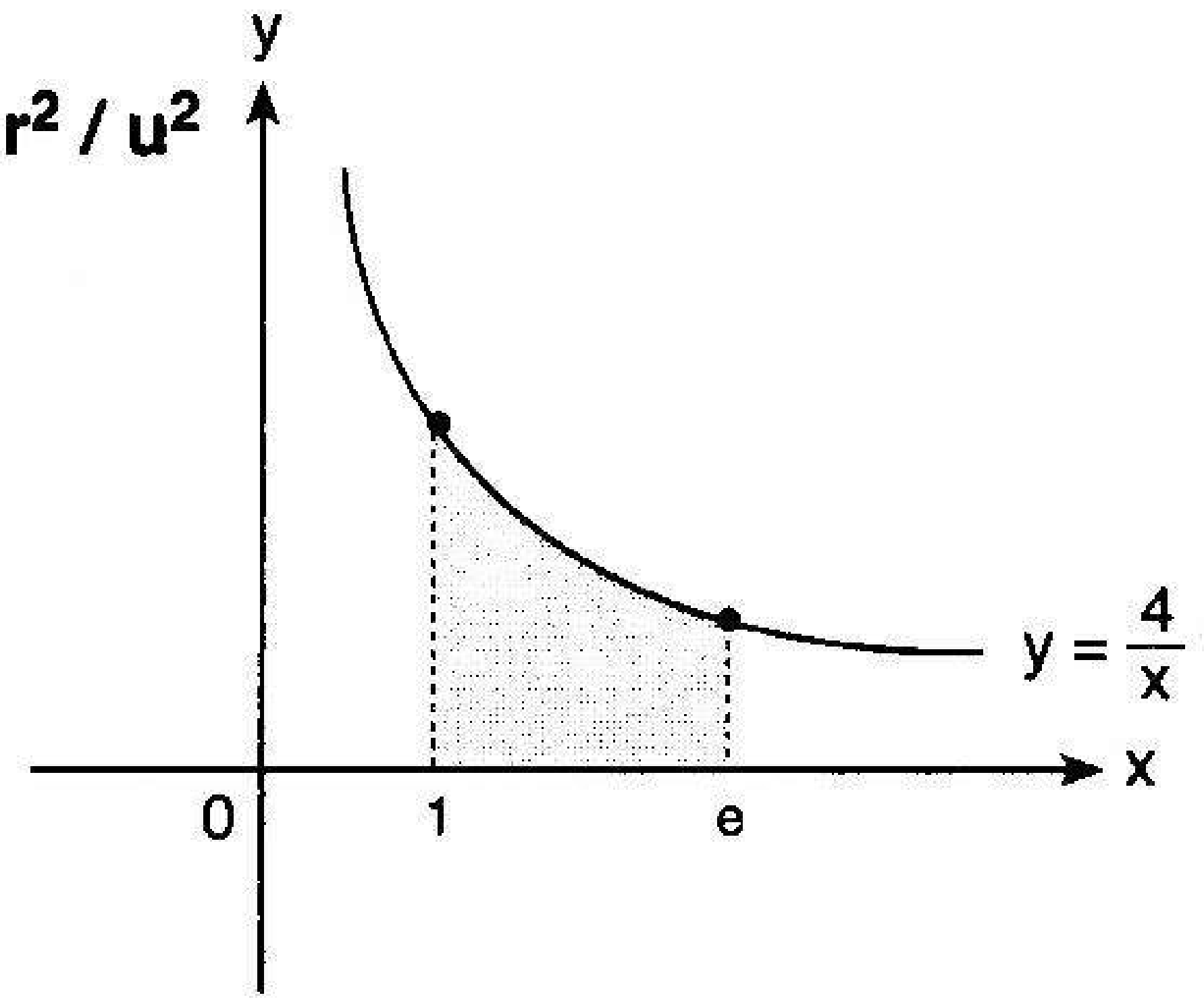
A) 2

B) 3

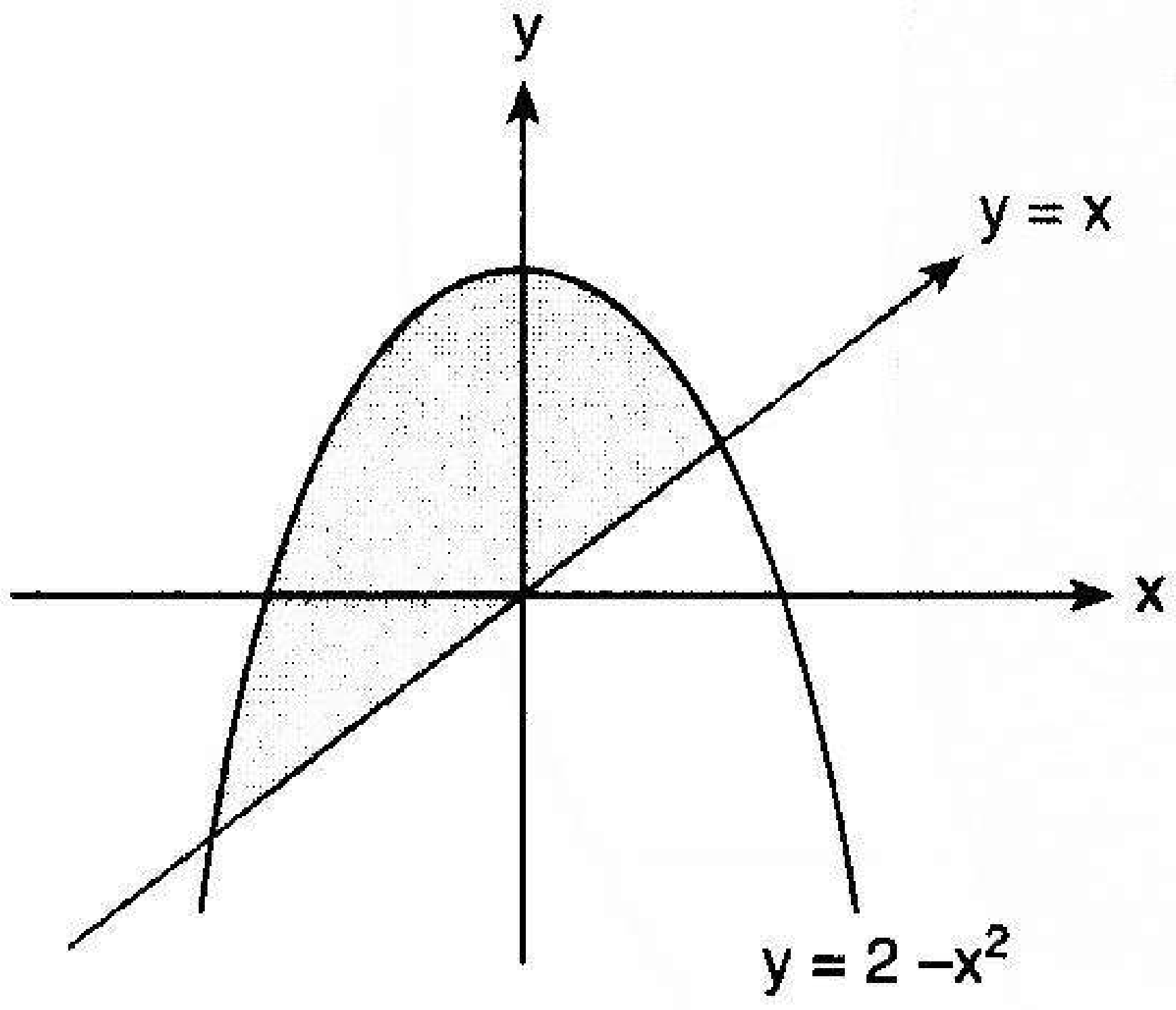
C) 4

D) 6

E) 7


y
x
0
1
e
$y = \frac{4}{x}$

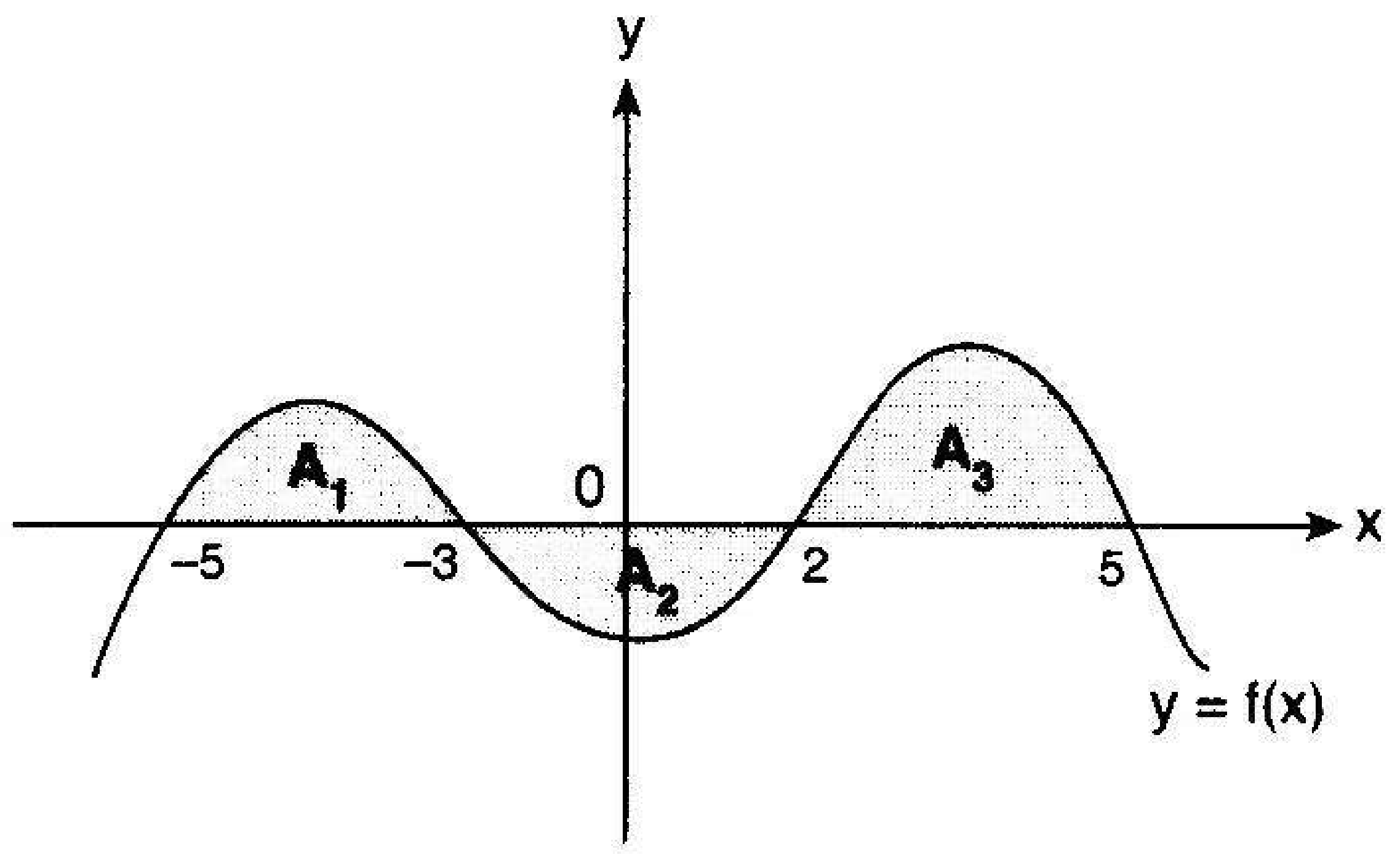
2.



Taralı alan / ***The shaded area*** **= ? br^2 / u^2**

A) $\frac{5}{2}$ B) 3 C) $\frac{7}{2}$ D) 4 E) $\frac{9}{2}$

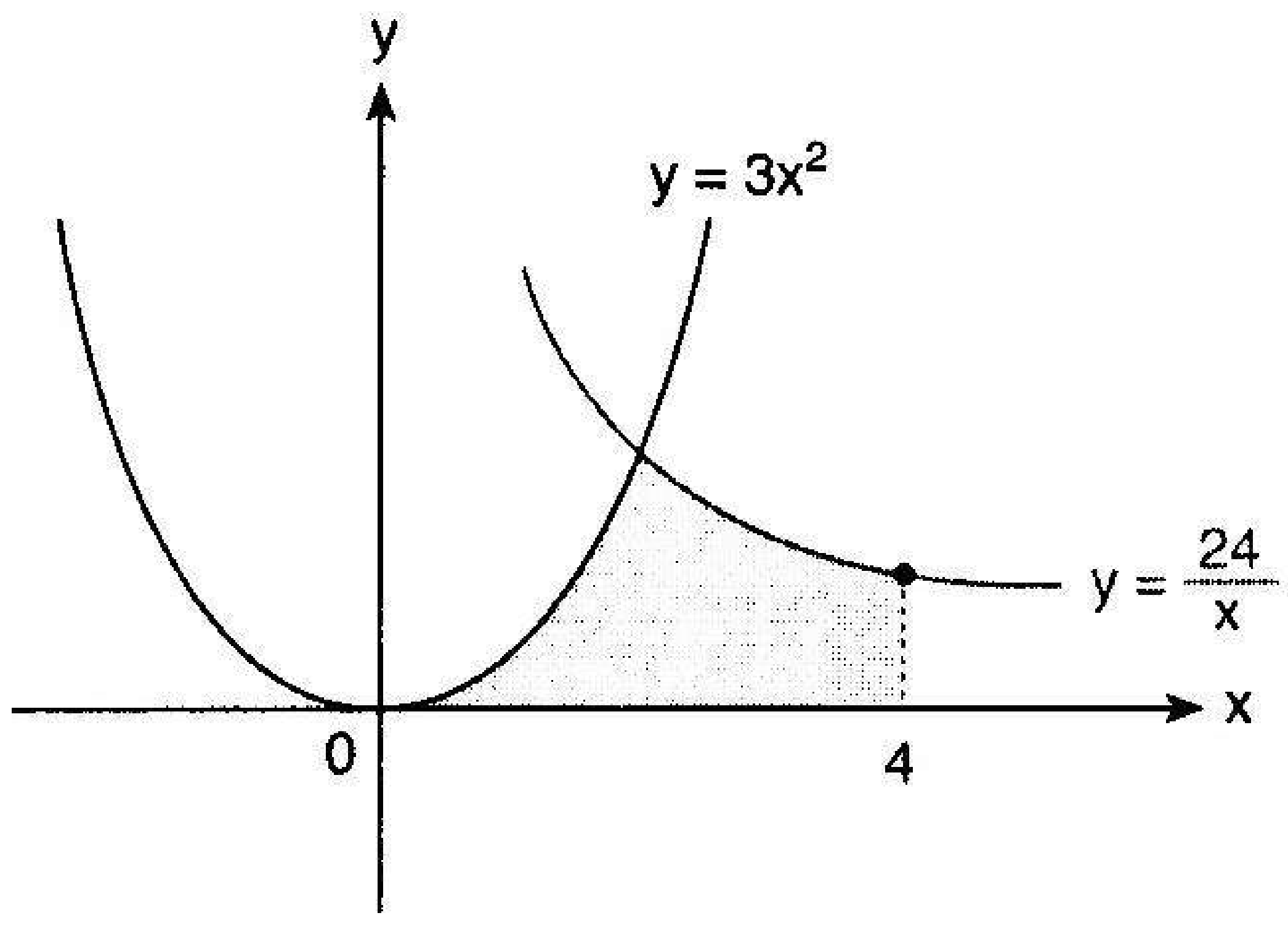
3.


y
x
0
−5
−3
2
5
A_1
A_2
A_3
y = f(x)

$\int_{-5}^{5} f(x)\,dx = 18$, $A_1 = 8$ br^2 / u^2, $A_2 = 5$ br^2 / u^2 $\Rightarrow$ $\mathbf{A_3} = ?$

A) 3 B) 6 C) 9 D) 12 E) 15

4.

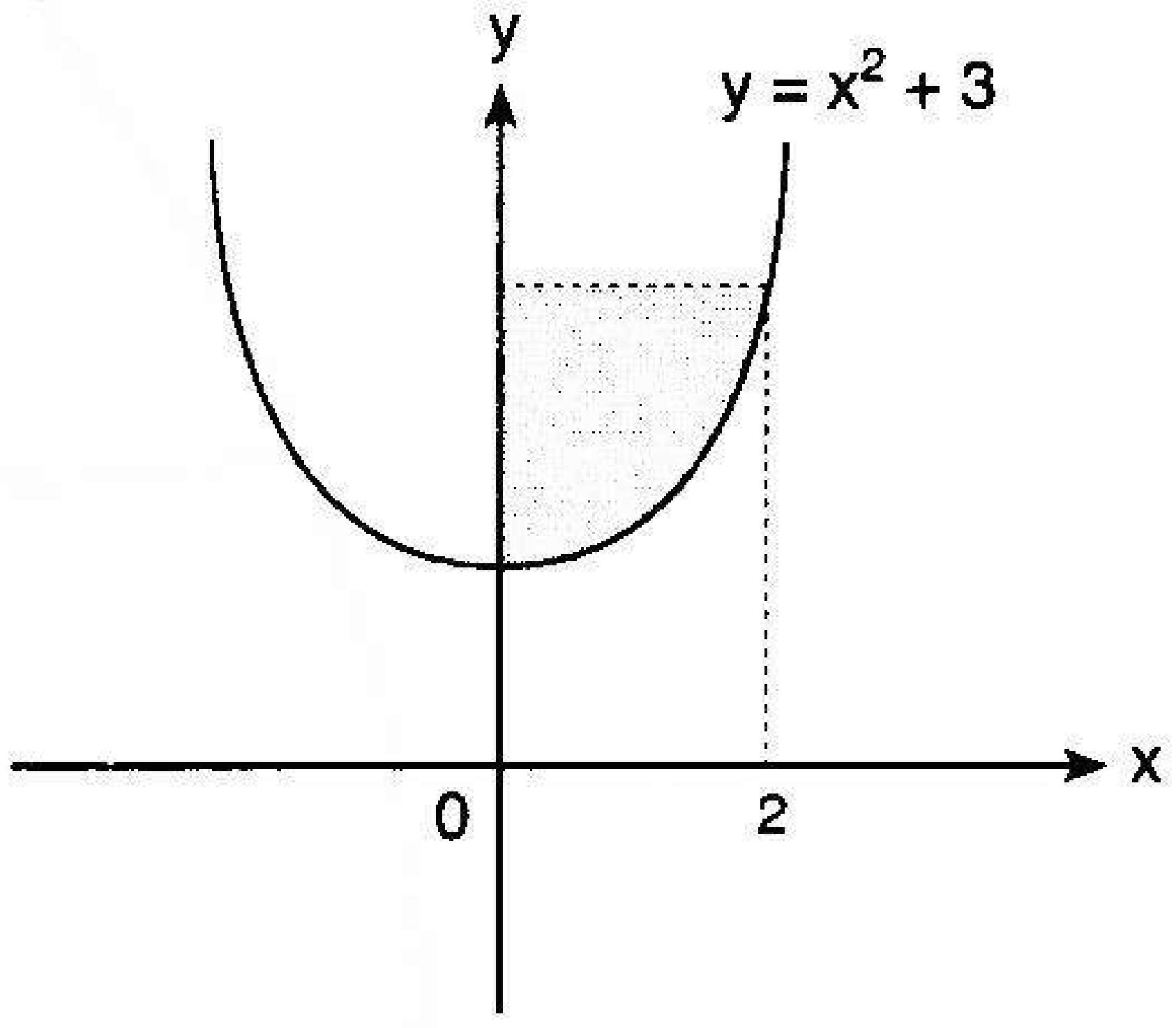

y
x
0
4
$y = 3x^2$
$y = \frac{24}{x}$

Taralı alan / ***The shaded area*** **= ? br^2 / u^2**

A) 8 + 18ln2 B) 16 + 36ln2 C) 8 + 16ln2

D) 12 + 24ln2 E) 8 + 24ln2

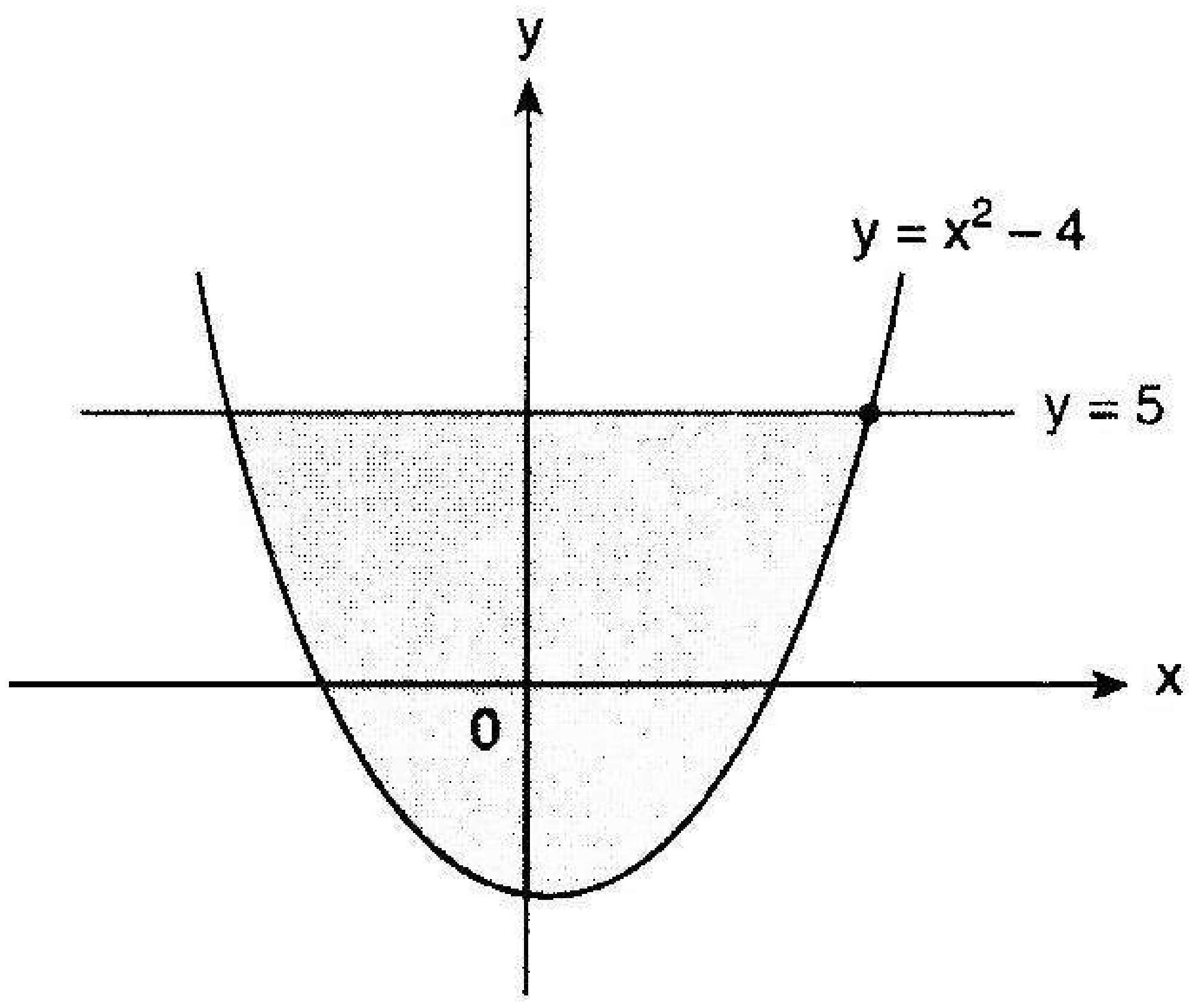
5.



Taralı alan / ***The shaded area*** **= ? br^2 / u^2**

A) $\frac{16}{3}$ B) $\frac{17}{3}$ C) $\frac{22}{3}$ D) $\frac{26}{3}$ E) $\frac{28}{3}$

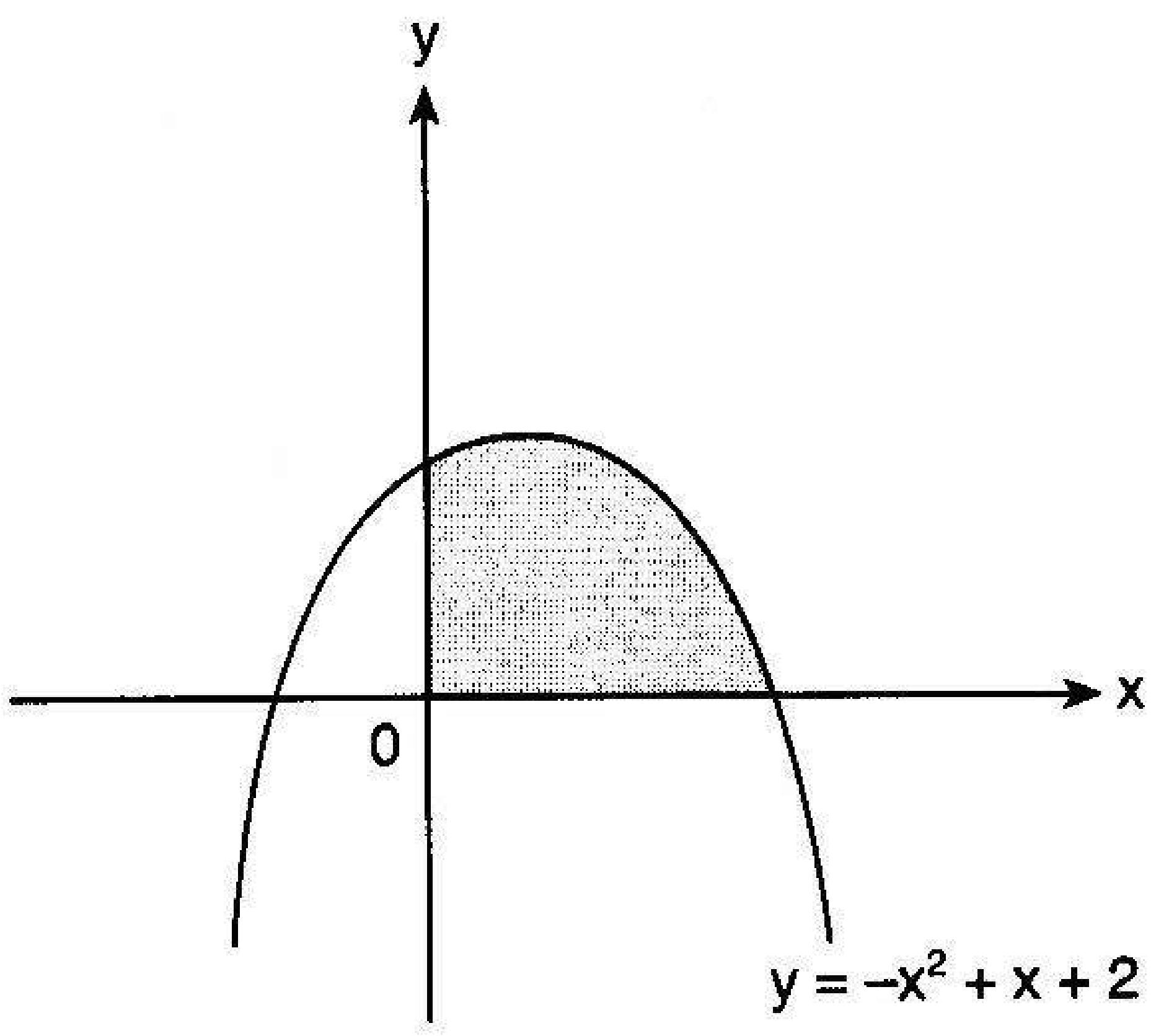
6.



Taralı alan / ***The shaded area*** **= ? br^2 / u^2**

A) 36 B) 40 C) 45 D) 48 E) 50

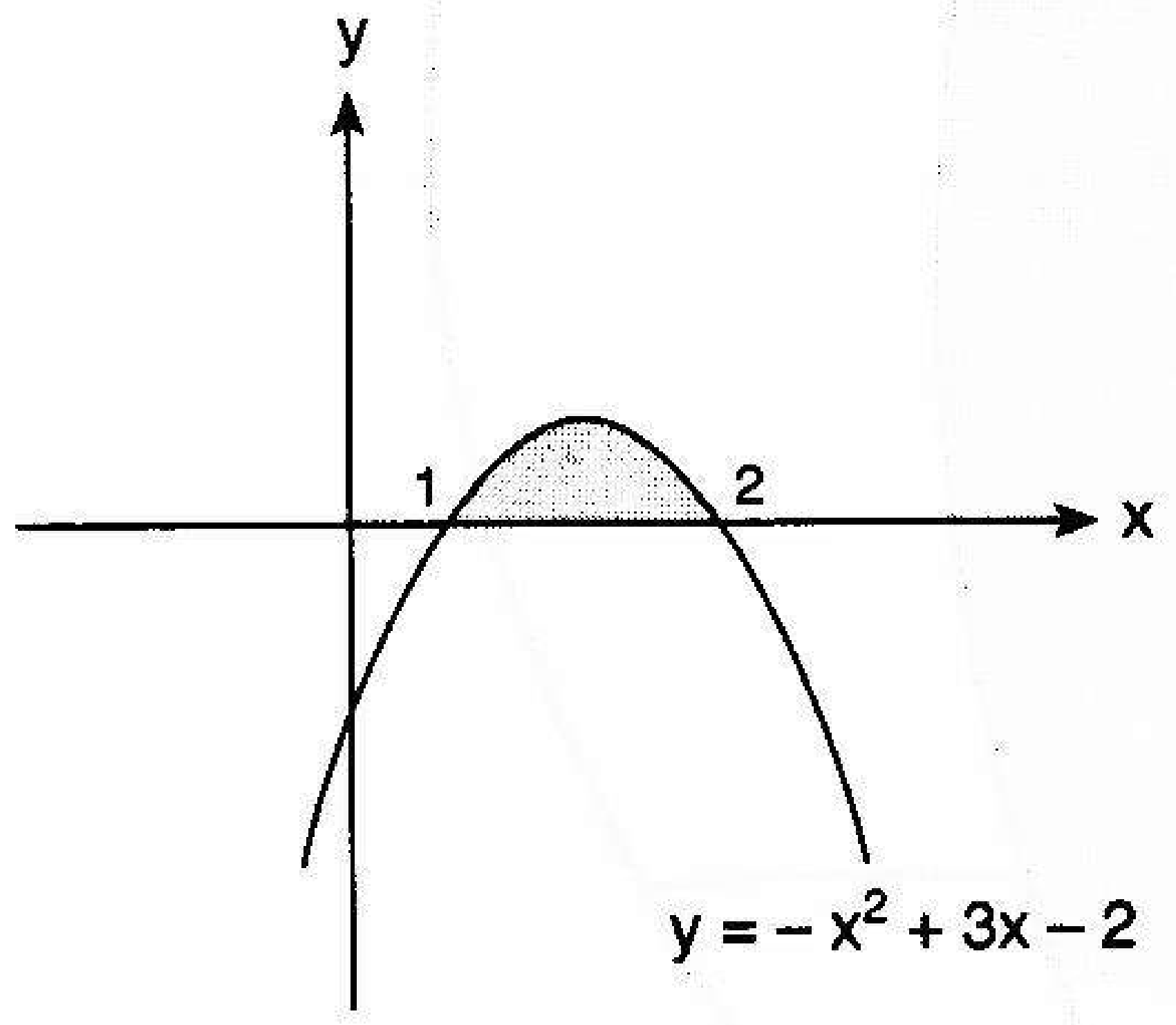
7.



Taralı alan / *The shaded area* = ? br^2 / u^2

A) $\frac{8}{3}$ B) $\frac{10}{3}$ C) $\frac{11}{3}$ D) $\frac{13}{3}$ E) $\frac{14}{3}$

8.



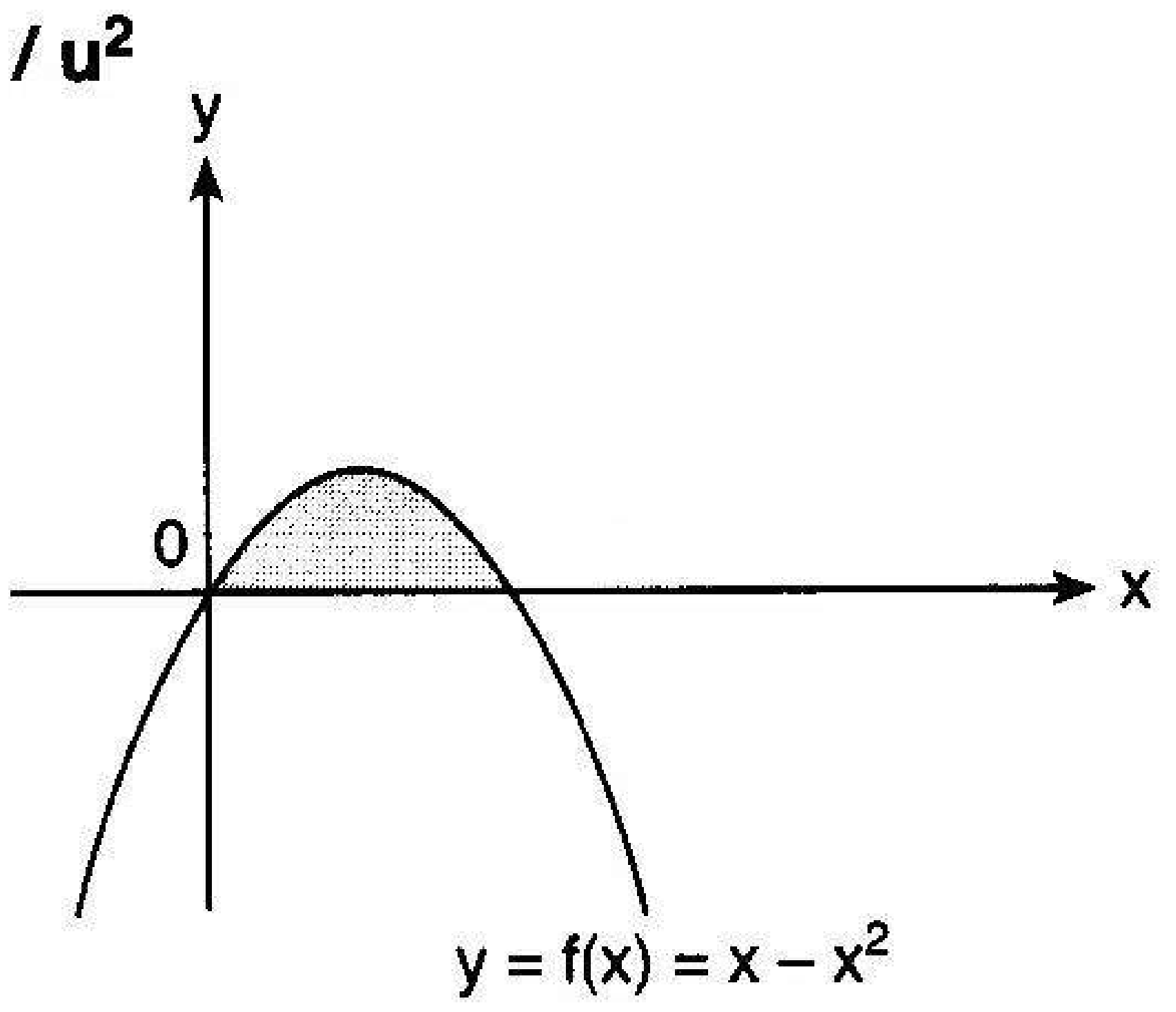
Taralı alan

/ *The shaded area* = ? br^2 / u^2

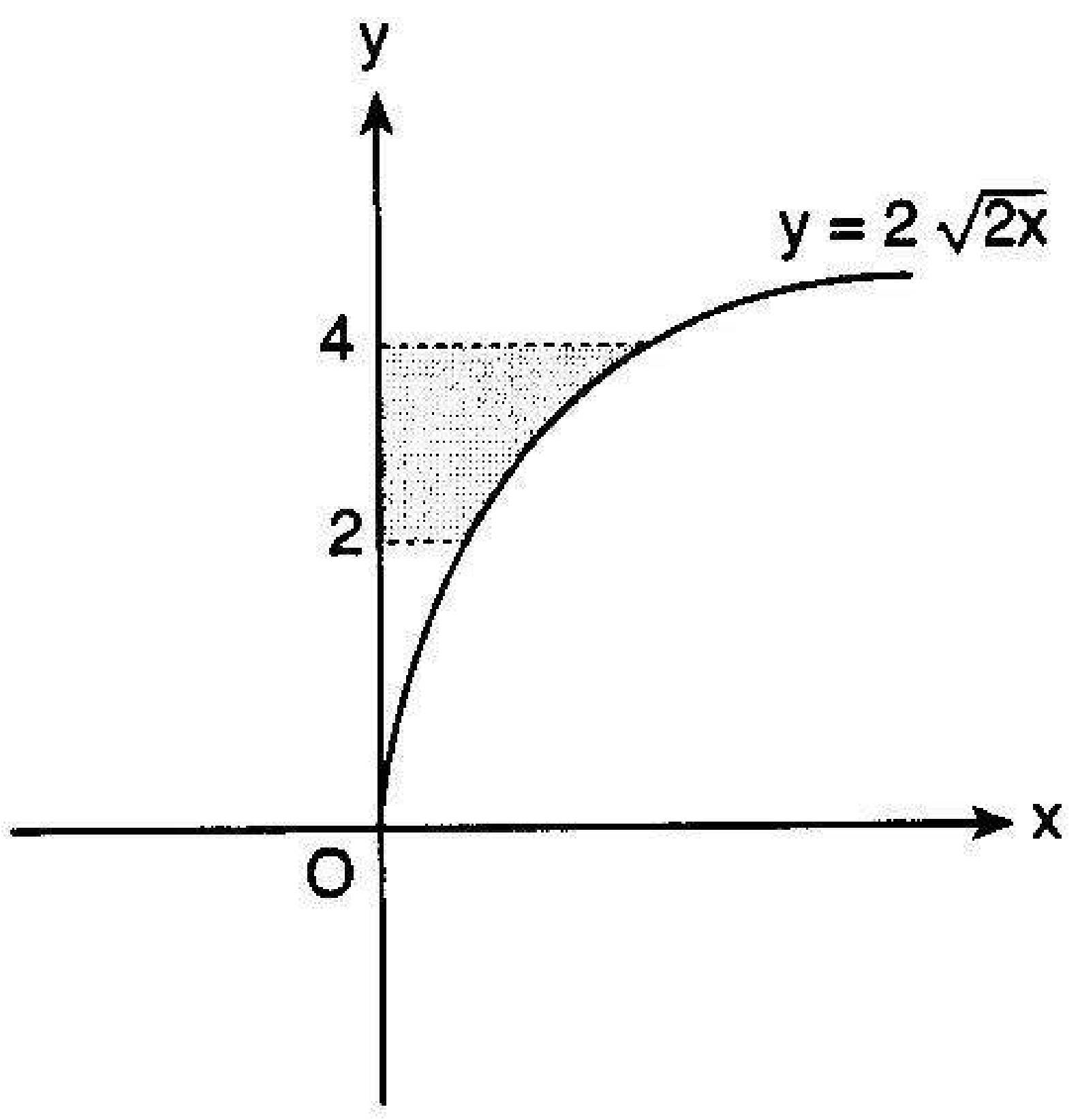
A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{6}$ E) $\frac{1}{5}$

9. **Taralı alan /**
***The shaded area* = ? br^2 / u^2**

A) 4
B) 2
C) $\frac{1}{2}$
D) $\frac{1}{4}$
E) $\frac{1}{6}$



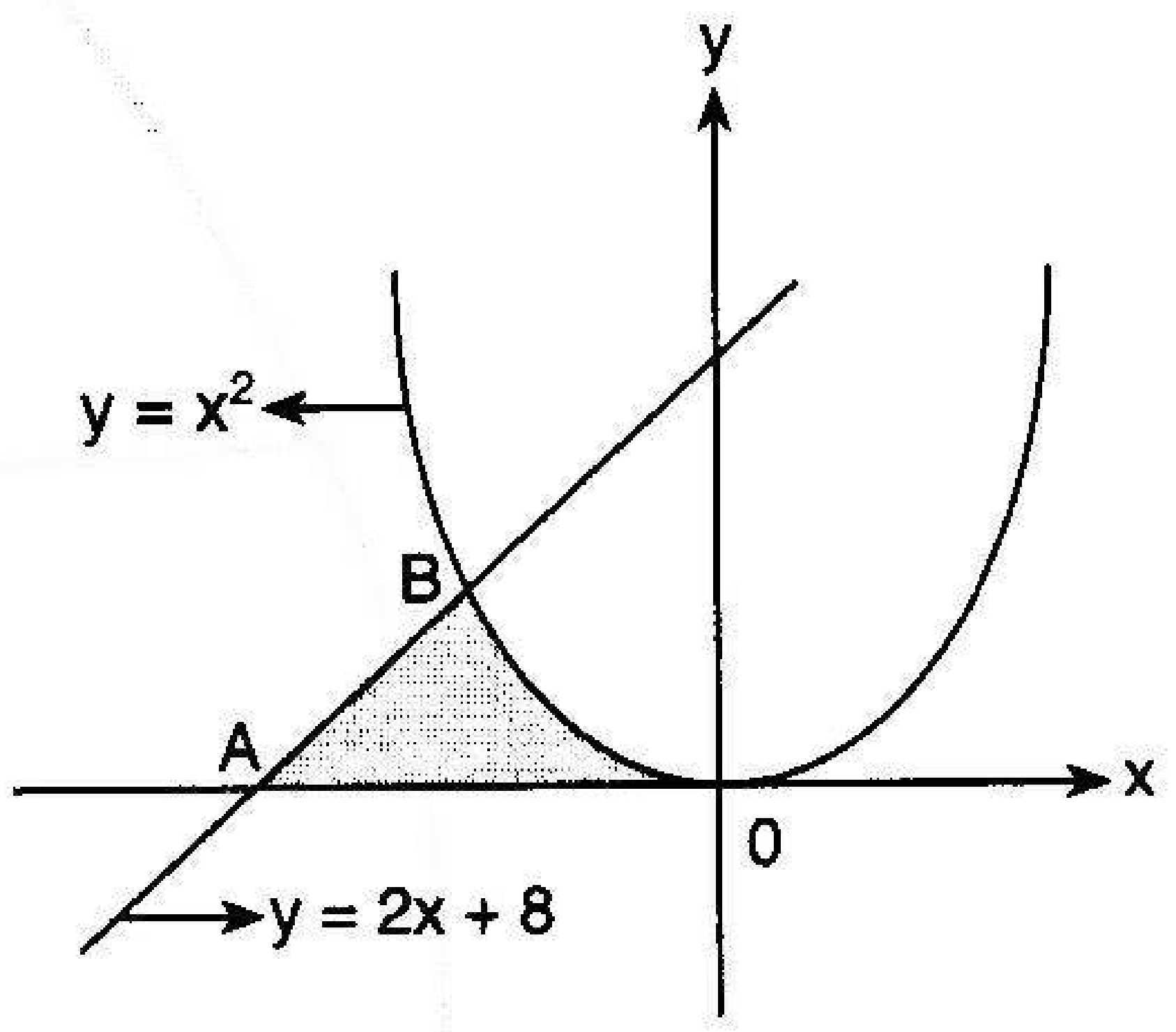
10.



Taralı alan / *The shaded area* = ? br^2 / u^2

A) $\frac{7}{3}$ B) $\frac{10}{3}$ C) $\frac{14}{3}$ D) $\frac{17}{3}$ E) $\frac{22}{3}$

11.

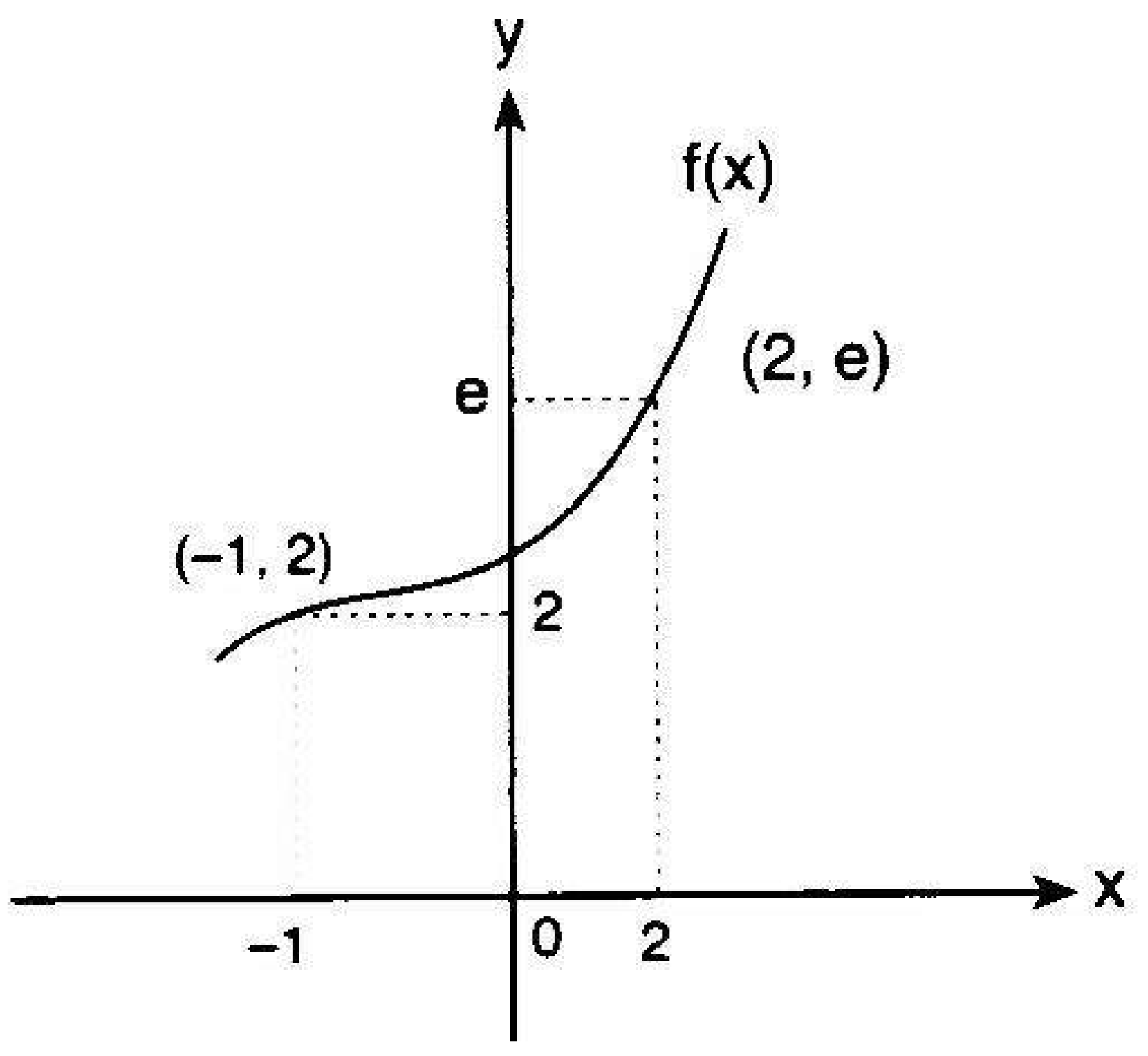


Taralı alan / *The shaded area* = ? br^2 / u^2

A) $\frac{10}{3}$ B) $\frac{16}{3}$ C) $\frac{20}{3}$ D) $\frac{31}{3}$ E) $\frac{47}{3}$

12. $\displaystyle\int_{-1}^{2} \frac{f'(x)}{f(x)}\,dx = ?$

A) ln2 – 1
B) 1 – ln2
C) 1
D) 0
E) ln2



YANITLAR / ANSWERS

1. C	2. E	3. E	4. E	5. A	6. A	7. B	8. D
9. E	10. A	11. C	12. B				

1. $f(x)=\frac{4}{x} \Rightarrow TA/SA=\int_1^e \frac{4}{x}\,dx$

$$TA/SA=4\cdot \ln x\Big|_1^e=4\cdot(\ln e-\ln 1)=4\ \text{br}^2/\text{u}^2$$

Yanıt / The Answer is C

2.

$$-x^2+2x+3=x+1$$

$$x^2-x-2=0$$

$$x_1=2,\quad x_2=-1$$

$$\Rightarrow TA/SA=\int_{-1}^{2}(-x^2+2x+3)-(x+1)\,dx$$

$$=\int_{-1}^{2}(-x^2+x+2)\,dx$$

$$=\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\Big|_{-1}^{2}\right)=\frac{9}{2}\ \text{br}^2/\text{u}^2$$

Yanıt / The Answer is E

3. $A_1=8\ \text{br}^2/\text{u}^2 \Rightarrow \int_{-5}^{-3} f(x)=8$

$$A_2=5\ \text{br}^2/\text{u}^2 \Rightarrow \int_{-3}^{2} f(x)=-5$$

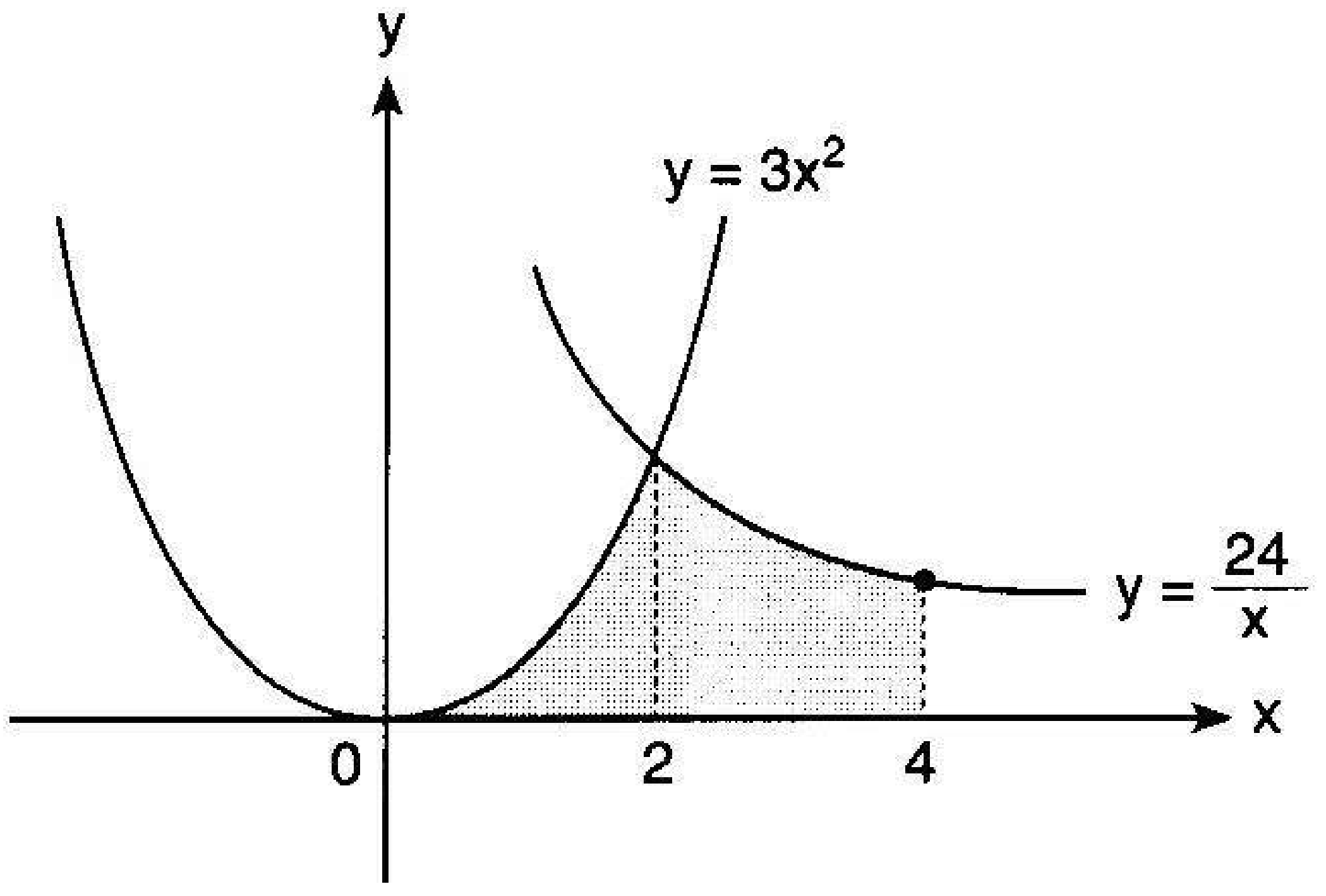
$$\int_{-5}^{5} f(x)\,dx=\int_{-5}^{-3} f(x)dx+\int_{-3}^{2} f(x)dx+\int_{2}^{5} f(x)dx=18$$

$$18=8-5+A_3$$

$$A_3=15\ \text{br}^2/\text{u}^2$$

Yanıt / The Answer is E

4.



$$3x^2=\frac{24}{x} \Rightarrow x=2$$

$$TA/SA=\int_0^2 3x^2\,dx+\int_2^4 \frac{24}{x}\,dx=x^3\Big|_0^2+24\ln x\Big|_2^4=8+24\ln 2$$

Yanıt / The Answer is E

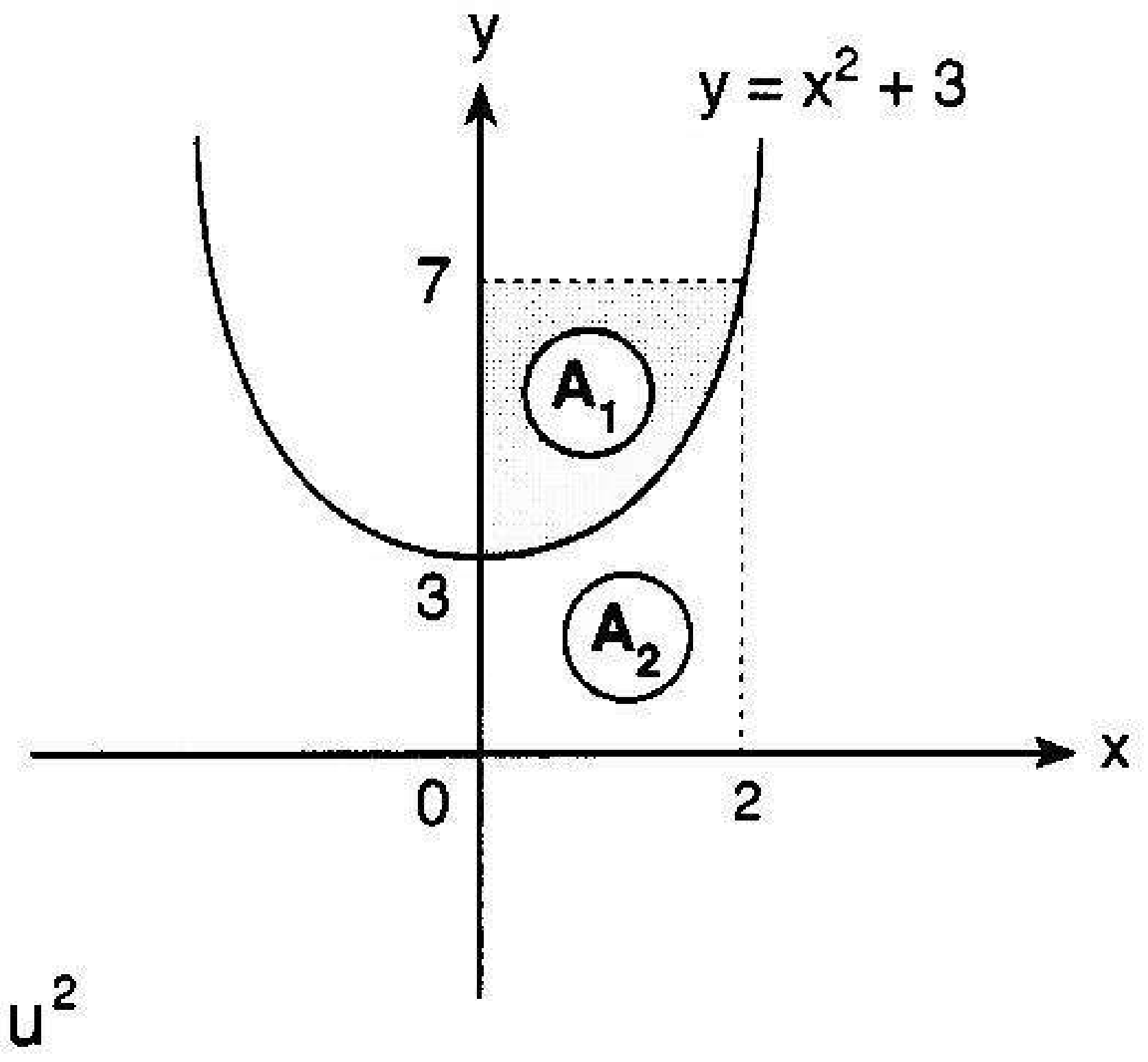
5. $A_1=(A_1+A_2)-A_2$

$$=2\cdot 7-\int_0^2 (x^2+3)\,dx$$

$$=14-\left(\frac{x^3}{3}+3x\Big|_0^2\right)$$

$$=14-\left(\frac{8}{3}+6\right)$$

$$=14-\frac{26}{3}=\frac{16}{3}\ \text{br}^2/\text{u}^2$$



Yanıt / The Answer is A

6. $TA/SA=\int_{-3}^{3}\left(5-(x^2-4)\right)dx$

$$=\int_{-3}^{3}\left(-x^2+9\right)dx$$

$$=\frac{-x^3}{3}+9x\Big|_{-3}^{3}$$

$$=(-9+27)-(9-27)$$

$$=18+18=36\ \text{br}^2/\text{u}^2$$

Yanıt / The Answer is A

7 $-x^2+x+2=0 \Rightarrow x_1=2 \ , x_2=-1$

$$TA/SA=\int_0^2(-x^2+x+2)\,dx$$

$$=-\frac{x^3}{3}+\frac{x^2}{2}+2x\Big|_0^2$$

$$=\left(-\frac{8}{3}+\frac{4}{2}+4\right)-0$$

$$=\frac{10}{3}\ br^2/u^2$$

Yanıt / The Answer is B

---

8. $S=\int_1^2(-x^2+3x-2)\,dx$

$$=\left(-\frac{x^3}{3}+3\cdot\frac{x^2}{2}-2x\right)\Bigg|_1^2$$

$$=\left(-\frac{2^3}{3}+3\cdot\frac{2^2}{2}-2\cdot2\right)-\left(-\frac{1^3}{3}+3\cdot\frac{1}{2}-2\cdot1\right)$$

$$=\left(-\frac{8}{3}+6-4\right)-\left(-\frac{1}{3}+\frac{3}{2}-2\right)$$

$$=-\frac{8}{3}+2+\frac{1}{3}+\frac{1}{2}$$

$$=-\frac{7}{3}+\frac{5}{2}=\frac{1}{6}\ br^2/u^2$$

Yanıt / The Answer is D

---

9. $x^2-x=0$

$x\cdot(x-1)=0$

$x=0$ veya (or) $x=1$

$$TA/SA=\int_0^1(x-x^2)\,dx$$

$$=\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Bigg|_0^1$$

$$=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}\ br^2/u^2$$

Yanıt / The Answer is E

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10. $y=2\sqrt{2x} \Rightarrow y^2=8x$

$$x=\frac{y^2}{8}$$

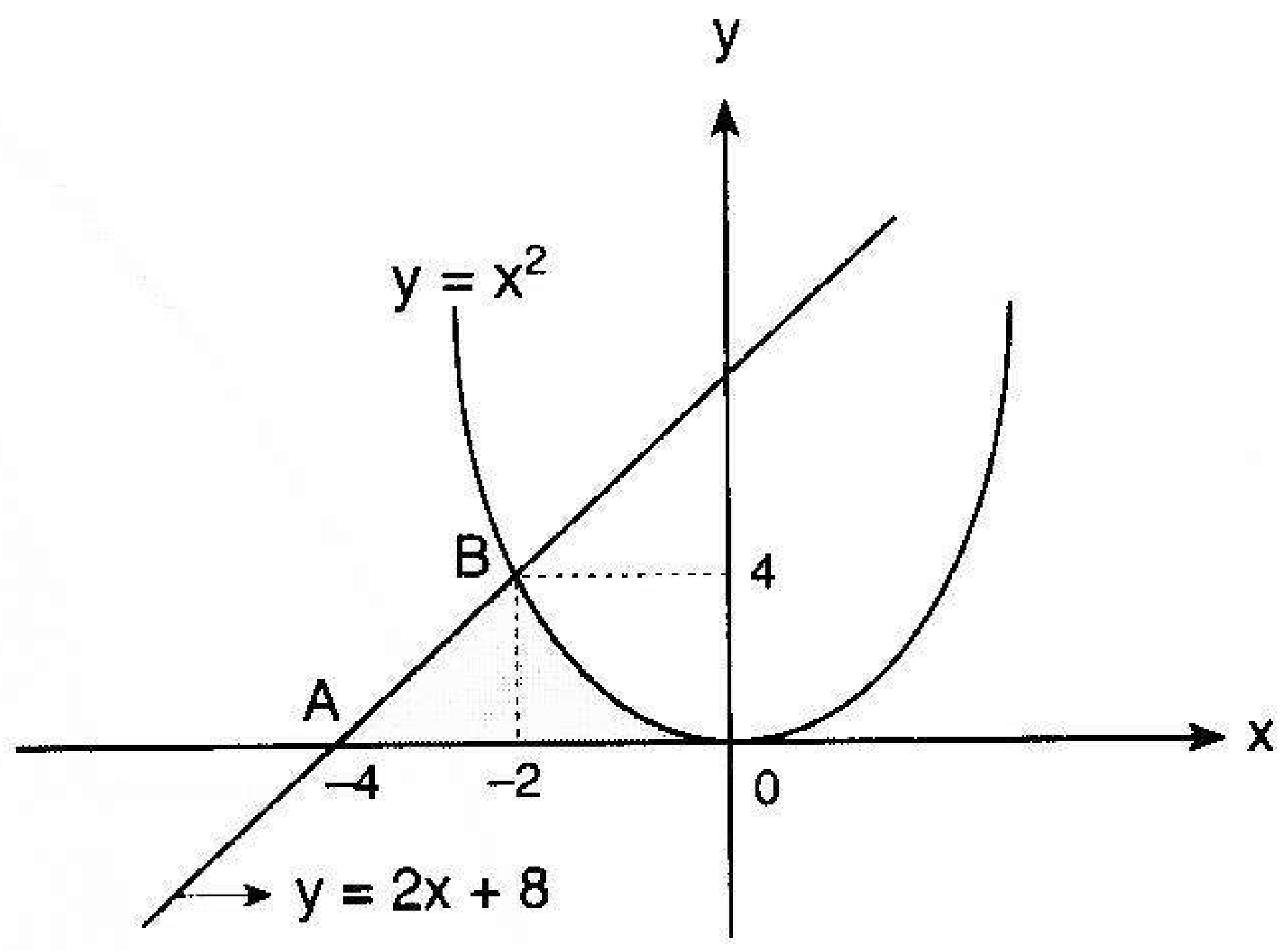
$$A=\int_2^4|f(y)|\,dy$$

$$A=\int_2^4\left|\frac{y^2}{8}\right|dy=\frac{y^3}{24}\Bigg|_2^4=\frac{64}{24}-\frac{8}{24}$$

$$=\frac{56}{24}=\frac{7}{3}\ br^2/u^2$$

Yanıt / The Answer is A

11.



$$TA/SA=\int_{-4}^{-2}(2x+8)\,dx+\int_{-2}^{0}x^2\,dx$$

$$=x^2+8x\Big|_{-4}^{-2}+\frac{x^3}{3}\Big|_{-2}^{0}$$

$$=(-12-(-16))+(0-(-\frac{8}{3}))=4+\frac{8}{3}=\frac{20}{3}\ br^2/u^2$$

Yanıt / The Answer is C

---

12. $f(x)=u \Rightarrow f'(x)\,dx=du$

$x=-1 \Rightarrow f(-1)=2$

$x=2 \Rightarrow f(2)=e$

$$\int_{-1}^{2}\frac{f'(x)}{f(x)}\,dx=\int_2^e\frac{du}{u}$$

$$=\ln u\Big|_2^e$$

$$=\ln e-\ln 2$$

$$=1-\ln 2$$

Yanıt / The Answer is B

1. $\int_{0}^{1} \frac{2x^2+3x}{x}\,dx = ?$

Yanıt / The Answer is: **4**

2. $\int_{1}^{3} \left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx = ?$

Yanıt / The Answer is: $\frac{2}{9}$

3. $\int_{0}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx = ?$

Yanıt / The Answer is: **0**

4. $\int_{1}^{e^2} \frac{3}{x}\,dx = ?$

Yanıt / The Answer is: **6**

5. $\int_{\ln 2}^{\ln 8} (e^{2x} - 2e^{x})\,dx = ?$

Yanıt / The Answer is: **18**

6. $\int_{0}^{\sqrt{3}} \frac{1}{1+x^2}\,dx = ?$

Yanıt / The Answer is: $\frac{\pi}{3}$

7. $\int_{-a}^{a} (x^3+4)\,dx = 24 \Rightarrow \mathbf{a = ?}$

Yanıt / The Answer is: **3**

8. $\int_{2}^{5} \frac{\mathbf{3x^2+2}}{\mathbf{x^3+2x}}\,\mathbf{dx} = \mathbf{?}$

Yanıt / The Answer is: $\mathbf{\ln\frac{45}{4}}$

9. $\int_{3}^{5} \frac{e^{\ln x}}{x}\,\mathbf{dx} = \mathbf{?}$

Yanıt / The Answer is: **2**

10. $\int_{\frac{\pi}{6}}^{0} \mathbf{\tan x\,dx} = \mathbf{?}$

Yanıt / The Answer is: $\mathbf{\ln\frac{\sqrt{3}}{2}}$

11. $\int_{4}^{1} f(x)\,dx = -6 \Rightarrow \int_{2}^{5} \mathbf{f(6-x)\,dx} = \mathbf{?}$

Yanıt / The Answer is: **6**

12. $\int_{0}^{1} \mathbf{2xe^{x^2}\,dx} = \mathbf{?}$

Yanıt / The Answer is: **e – 1**

13. $\int_{-2}^{2} |x+2|\, dx = ?$

Yanıt / The Answer is: **8**

14. $\int_{-3}^{2} \left|x^2-2x-3\right| dx = ?$

Yanıt / The Answer is: $\frac{59}{3}$

15. $\int_{0}^{\pi} (|\cos x| - |\sin x|)\, dx = ?$

Yanıt / The Answer is: **0**

16. $\int_{-1}^{1} \frac{|x|}{x}\, dx = ?$

Yanıt / The Answer is: **0**

17. $\int_{0}^{\frac{\pi}{4}} (\cos^4 x - \sin^4 x)\, dx = ?$

Yanıt / The Answer is: $\frac{1}{2}$

18. $\int_{0}^{e-1} \frac{x}{x+1}\, dx = ?$

Yanıt / The Answer is: **e – 2**

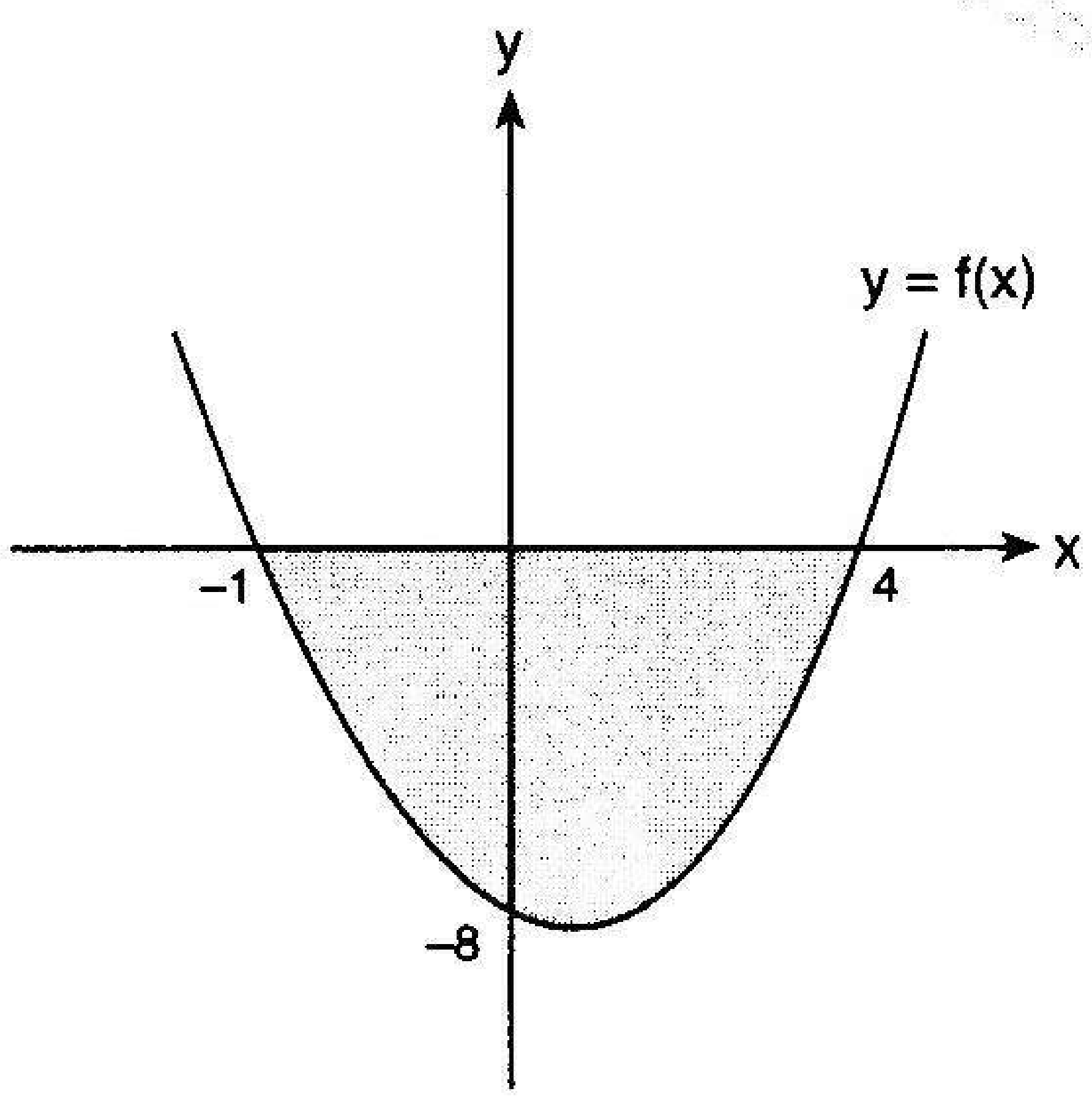
19. $\int_0^4 \sqrt{16-x^2}\ dx = ?$

Yanıt / The Answer is: 4π

20. $\frac{d}{dx}\left[\int_1^4 (x^2+x)dx\right] = ?$

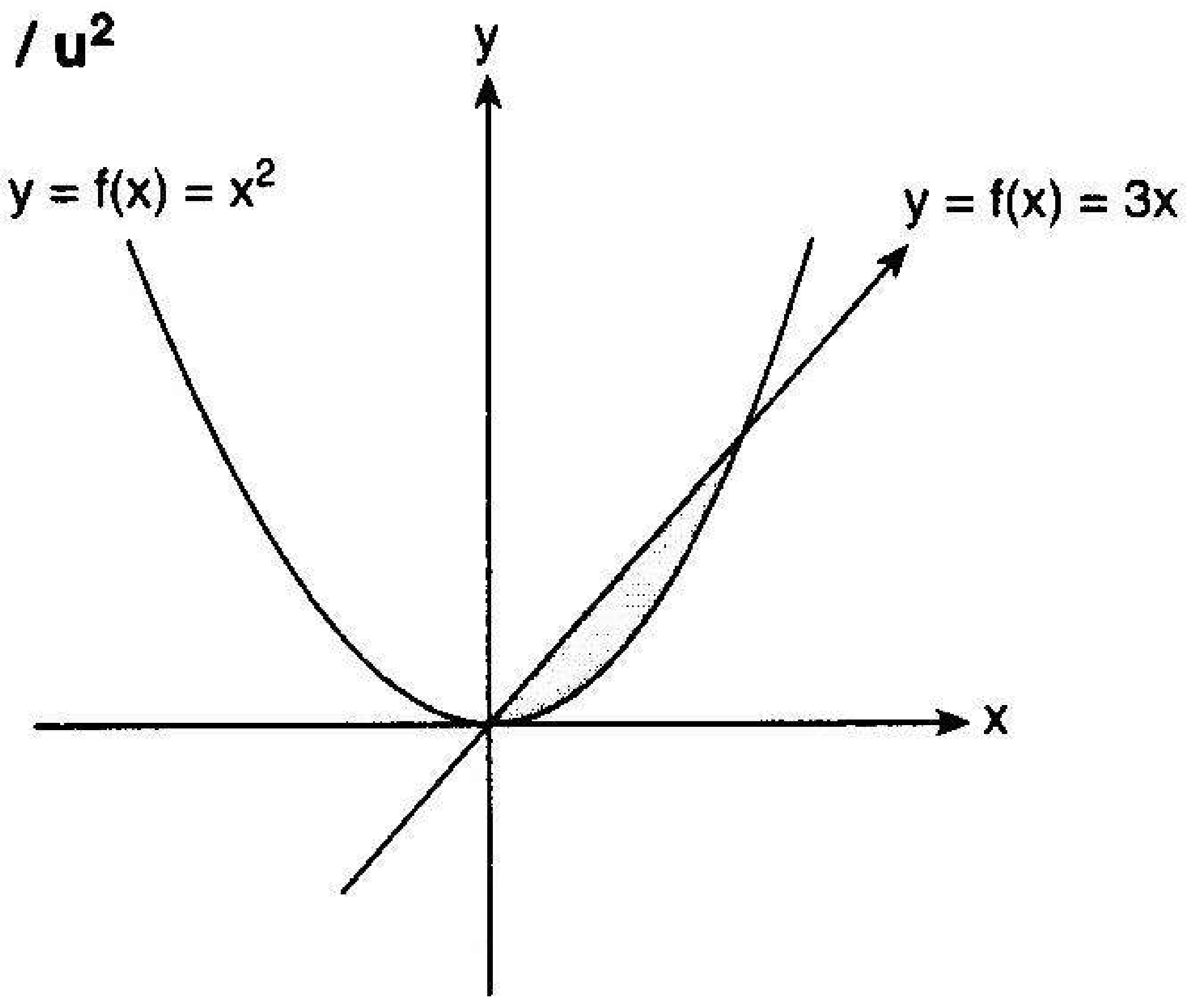
Yanıt / The Answer is: 0

21. TA / SA = ? br^2 / u^2



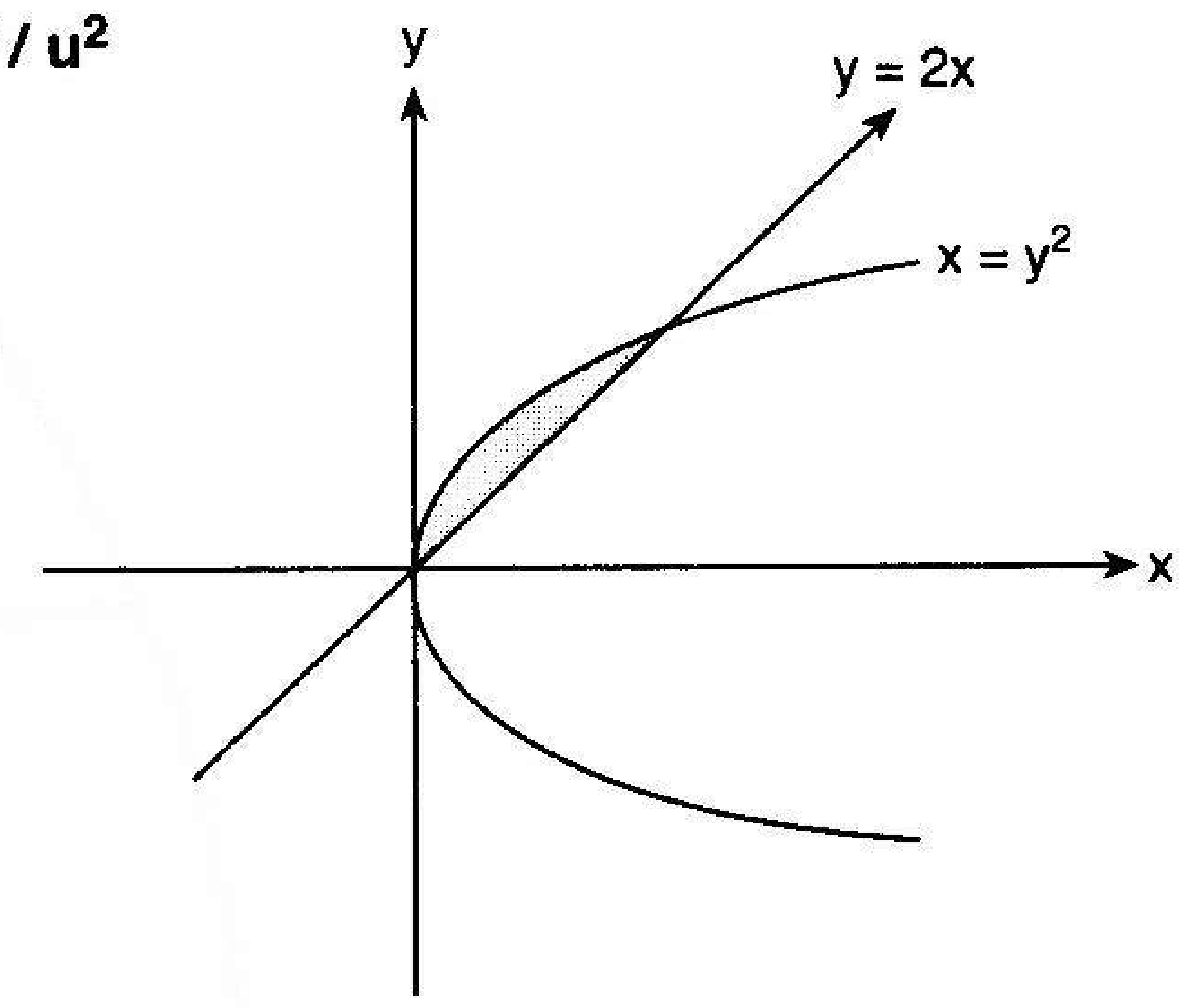
Yanıt / The Answer is: $\frac{125}{3}$

22. TA / SA = ? br^2 / u^2


y
y = f(x) = x^2
y = f(x) = 3x
x

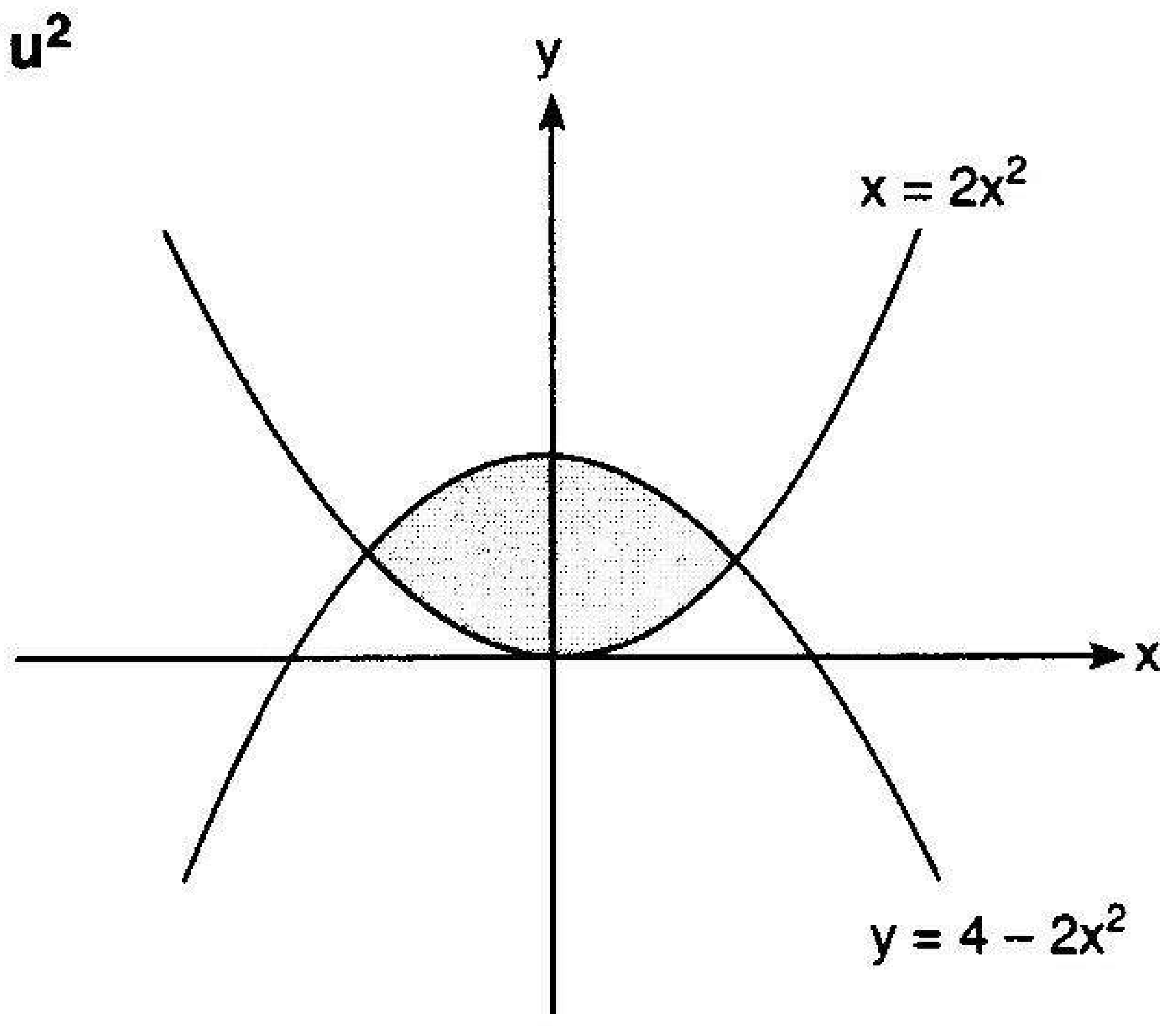
Yanıt / The Answer is: $\frac{9}{2}$

23. TA / SA = ? br^2 / u^2



Yanıt / The Answer is: $\frac{1}{48}$

24. TA / SA = ? br^2 / u^2


y
x = 2x^2
x
y = 4 – 2x^2

Yanıt / The Answer is: $\frac{16}{3}$

1. $\int_1^2 \frac{2x^3+1}{x^2}\,dx = ?$

A) 3

B) $\frac{7}{2}$

C) 4

D) $\frac{9}{2}$

E) 5

2. $\int_1^2 \frac{dx}{3x+1} = ?$

A) $\frac{1}{2}\ln 14$

B) $\frac{1}{4}\ln\frac{7}{3}$

C) $\frac{1}{4}\ln\frac{3}{7}$

D) $\frac{1}{3}\ln\frac{4}{7}$

E) $\frac{1}{3}\ln\frac{7}{4}$

3. $\int_0^1 \left(x^2+e^x\right)dx = ?$

A) $e+\frac{4}{3}$

B) $e-\frac{4}{3}$

C) $e-\frac{2}{3}$

D) $e+\frac{2}{3}$

E) $e-\frac{3}{4}$

4. $b-a=5,\ \int_a^b (2x+1)\,dx = 25 \Rightarrow b = ?$

A) $\frac{5}{2}$

B) 3

C) $\frac{7}{2}$

D) 4

E) $\frac{9}{2}$

5. $\int_0^{\frac{\pi}{4}} (\cos x+\sin x)\,dx = ?$

A) 1

B) $\frac{\sqrt{2}}{2}$

C) $\sqrt{2}$

D) $2\sqrt{2}$

E) 4

6. $\int_0^1 \left(\frac{1-\sqrt{x}}{\sqrt{x}}\right)dx = ?$

A) 1

B) 2

C) 3

D) 4

E) 5

7. $\int_{-1}^{2} (6x^2 - 4x)\, dx = ?$

A) 4

B) 6

C) 8

D) 10

E) 12

8. $\int_{0}^{3} |x-1|\, dx = ?$

A) 1

B) $\frac{3}{2}$

C) 2

D) $\frac{5}{2}$

E) 3

9. $\frac{d}{dx}\left(\int_{-2}^{3} (x^2 + 5x - 2)\, dx\right) = ?$

A) $-\frac{93}{6}$

B) $-\frac{85}{6}$

C) 0

D) $\frac{45}{6}$

E) $\frac{85}{6}$

10. $\int_{-2}^{4} |x-3|\, dx = ?$

A) 12

B) 13

C) 14

D) 15

E) 16

11. $\int_{\frac{\pi}{4}}^{\pi} \sin 2x\, dx = ?$

A) $-\frac{1}{2}$

B) $-\frac{1}{3}$

C) $-\frac{1}{4}$

D) $-\frac{1}{5}$

E) $-\frac{1}{6}$

12. $\int_{-2}^{2} \frac{x^3}{1+x^4}\, dx = ?$

A) –8

B) –6

C) –4

D) –2

E) 0

YANITLAR / ANSWERS

1. B	2. E	3. C	4. E	5. A	6. A	7. E	8. D
9. C	10. B	11. A	12. E				

1. $\int_0^{10} 2^x\,dx = ?$

A) $\frac{2^{10}}{\ln 2}$

B) $\frac{2^{10}+1}{\ln 2}$

C) $\frac{2^{10}-1}{\ln 2}$

D) 2^{10}

E) $2^{10}-1$

2. $\int_0^{8} \sqrt[3]{x^2}\,dx = ?$

A) $\frac{91}{5}$

B) $\frac{92}{5}$

C) $\frac{93}{5}$

D) $\frac{94}{5}$

E) $\frac{96}{5}$

3. $\int_0^{1} \frac{3x\,dx}{x+1} = ?$

A) $\ln\frac{e^3}{2}$

B) $\ln\frac{e^3}{8}$

C) $\ln\frac{e}{2}$

D) $\ln\frac{e}{8}$

E) $\ln 8e$

4. $\int_0^{\frac{\pi}{2}} \sin^2 x \cdot \cos x\,dx = ?$

A) $\frac{1}{3}$

B) $\frac{1}{6}$

C) 0

D) $-\frac{1}{3}$

E) $-\frac{1}{6}$

5. $\int_{-2}^{3} \left|x^2-2x\right|dx = ?$

A) 8

B) $\frac{25}{3}$

C) $\frac{26}{3}$

D) 9

E) $\frac{28}{3}$

6. $\int_0^{\frac{3}{2}} \frac{dx}{9+4x^2} = ?$

A) $\frac{\pi}{6}$

B) $\frac{\pi}{9}$

C) $\frac{\pi}{12}$

D) $\frac{\pi}{24}$

E) $\frac{\pi}{30}$

7. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{1-\cos 2x}\, dx = ?$

A) $\dfrac{\sqrt{6}+2}{2}$

B) $\dfrac{\sqrt{6}-1}{4}$

C) $\dfrac{\sqrt{6}+1}{2}$

D) $\dfrac{\sqrt{6}-1}{2}$

E) $\dfrac{\sqrt{6}-2}{2}$

---

8. $\displaystyle\int_{0}^{\frac{\pi}{2}} x\cdot\cos x\, dx = ?$

A) $\pi-1$

B) $\dfrac{\pi}{2}-1$

C) $\dfrac{\pi}{2}-2$

D) $\pi-2$

E) $\dfrac{\pi}{3}-1$

---

9. $\displaystyle\int_{0}^{\frac{\pi}{4}} \sqrt{1-\cos 2x}\, dx = ?$

A) $\sqrt{2}+1$

B) $\dfrac{\sqrt{2}}{2}+1$

C) $\sqrt{2}-1$

D) $\dfrac{\sqrt{2}}{2}-1$

E) $2\sqrt{2}$

---

10. $\displaystyle F(x)=\int_{0}^{2x} \sin\left(\frac{\pi}{4}t\right) dt \Rightarrow F'(1) = ?$

A) 1

B) $\dfrac{3}{2}$

C) 2

D) 3

E) $\dfrac{7}{2}$

---

11. $\displaystyle\int_{0}^{2} \left(\sqrt{8-x^2}-x\right) dx = ?$

A) $\pi-1$

B) π

C) $\pi+1$

D) $\pi+\dfrac{3}{2}$

E) $\pi+\dfrac{5}{2}$

---

12. $\displaystyle\int_{0}^{\frac{\pi}{3}} \frac{\sin^3 x}{\cos^2 x}\, dx = ?$

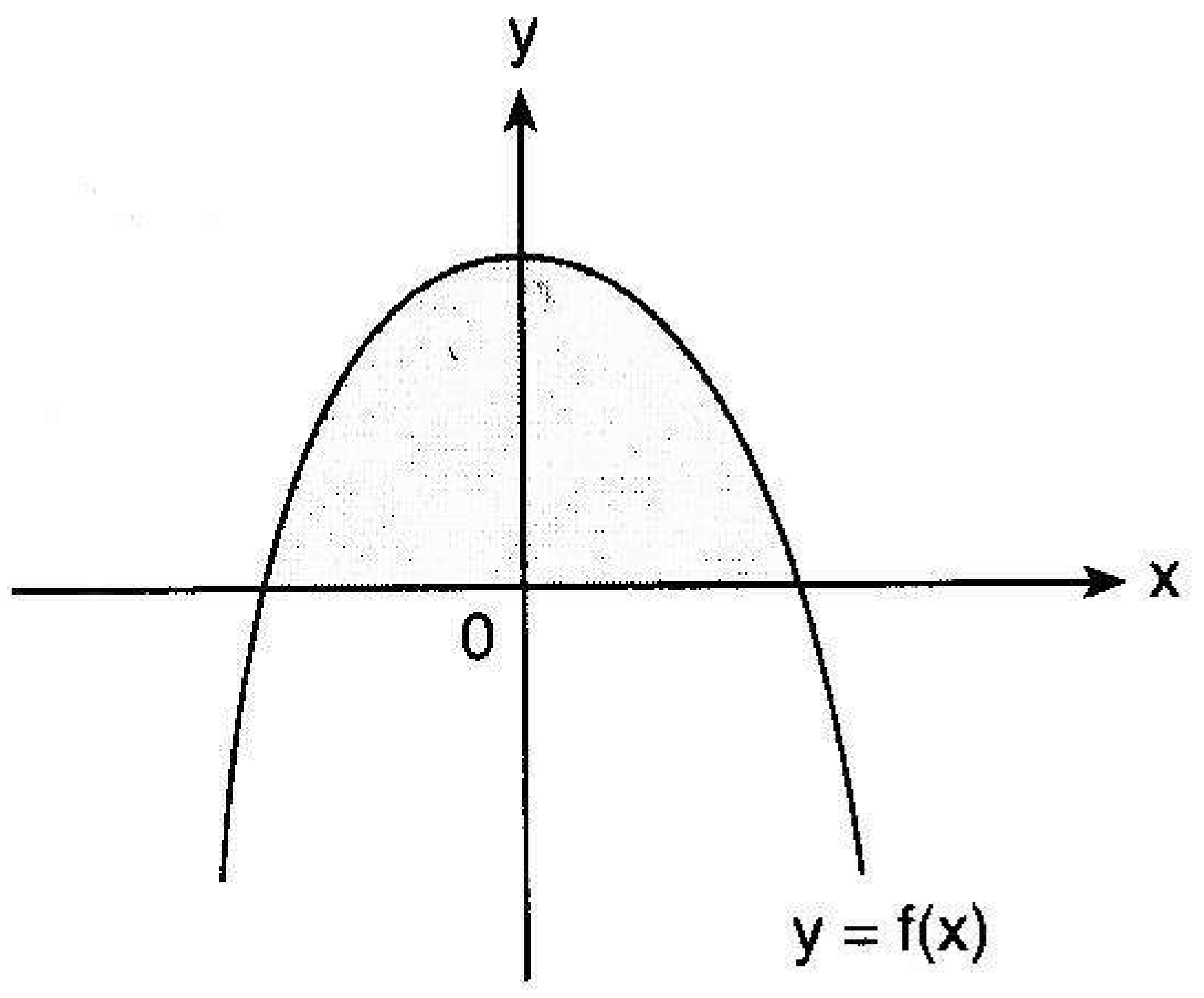
A) $\dfrac{3}{2}$ B) 1 C) $\dfrac{3}{4}$

D) $\dfrac{1}{2}$ E) $\dfrac{1}{4}$

YANITLAR / ANSWERS

1. C	2. E	3. B	4. A	5. E	6. D	7. E	8. B
9. C	10. C	11. B	12. D				

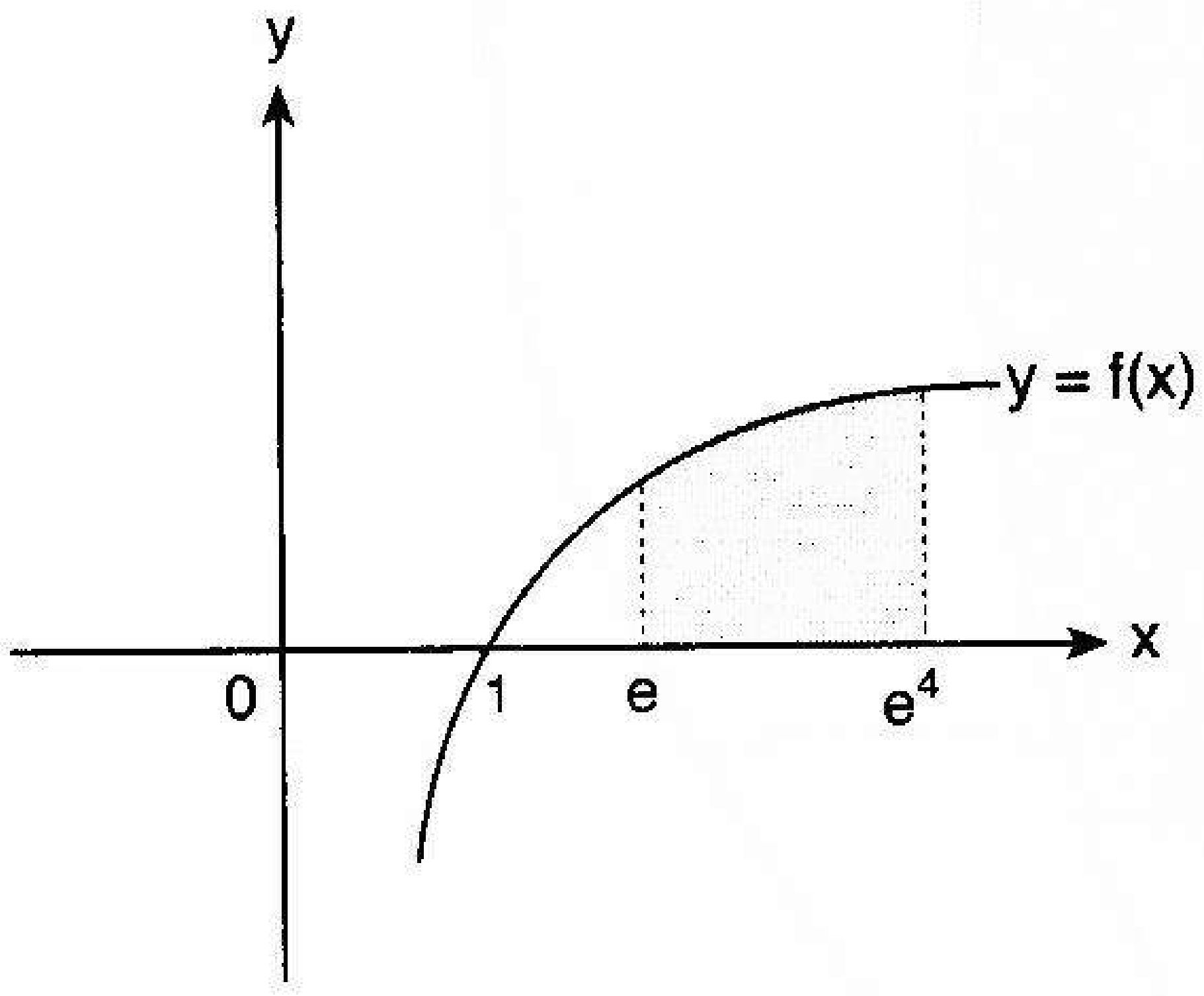
1.



$f(x) = -x^2 + 9 \Rightarrow$ **Taralı alan /** ***The shaded area*** **= ? br^2 / u^2**

A) 24
B) 28
C) 32
D) 36
E) 40

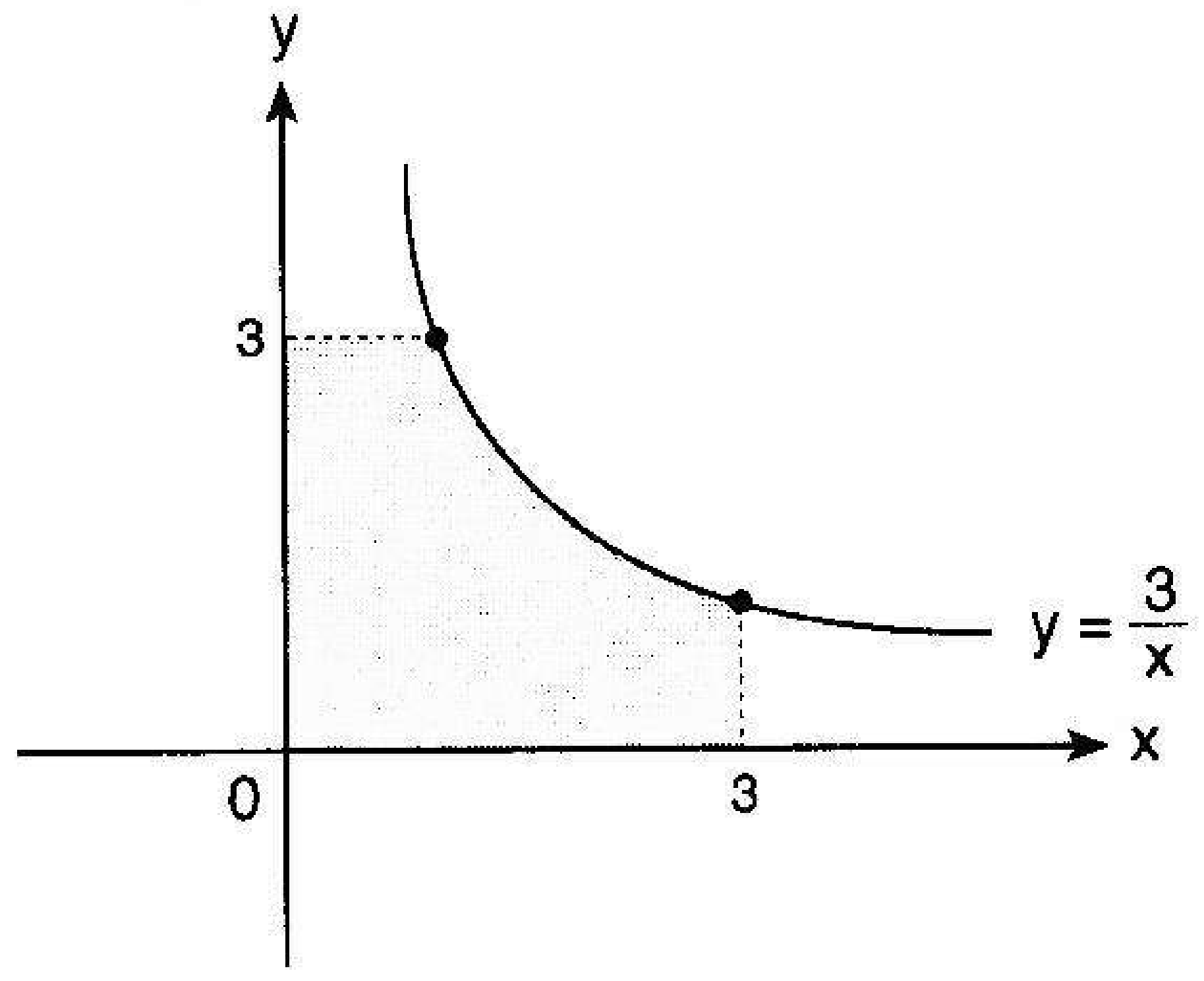
2.


y
y = f(x)
x
0
1
e
e^4

$f(x) = \ln x \Rightarrow$ **Taralı alan /** ***The shaded area*** **= ? br^2 / u^2**

A) $e^4 - 1$
B) $e^4 + 2$
C) $4e^4 - e$
D) $3 \cdot e^4$
E) $e^4 + 5$

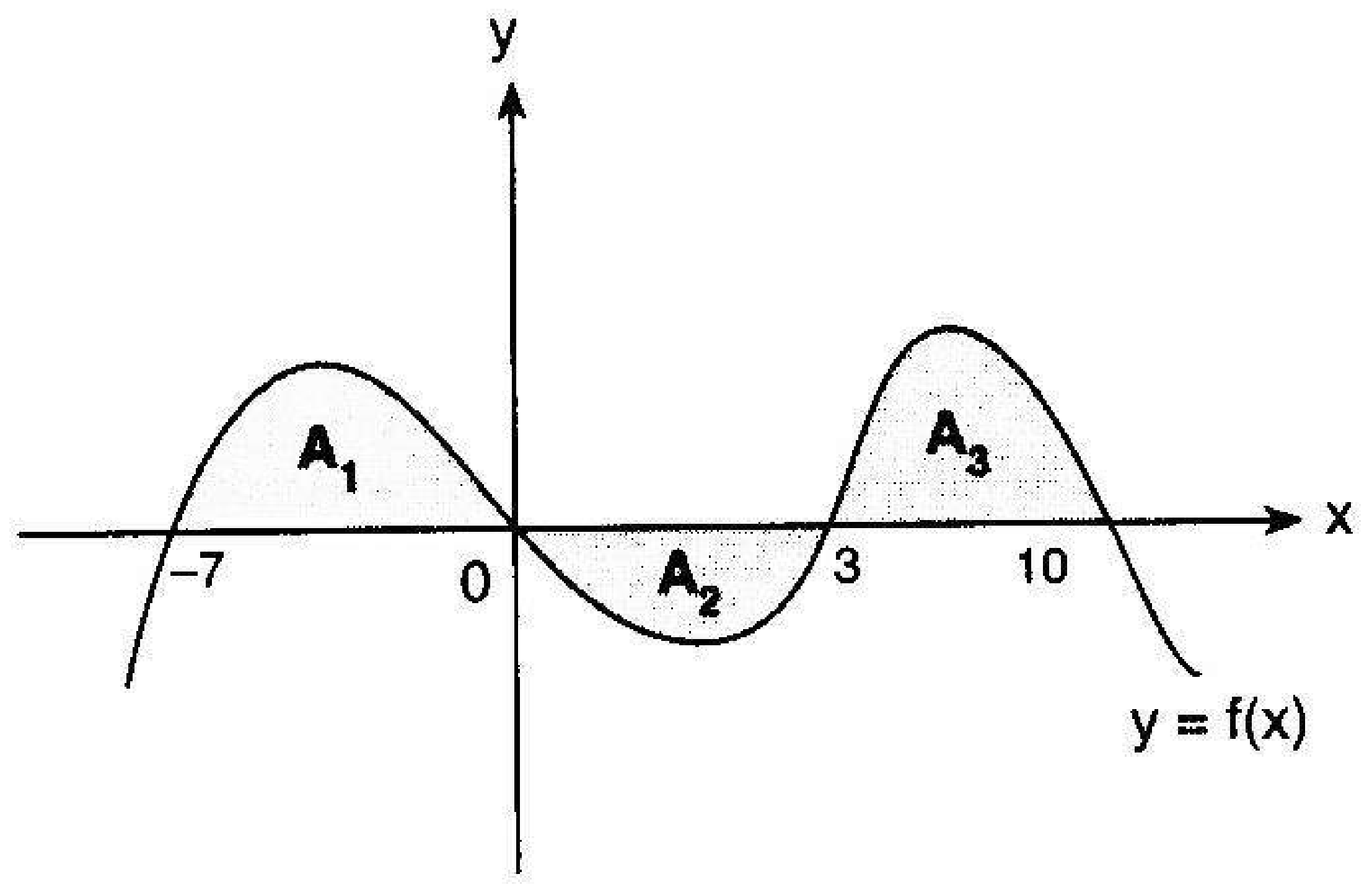
3.


y
3
$y = \frac{3}{x}$
x
0
3

Taralı alan / ***The shaded area*** **= ? br^2 / u^2**

A) ln18 + 1
B) ln18 + 3
C) ln27
D) ln27 + 1
E) ln27 + 3

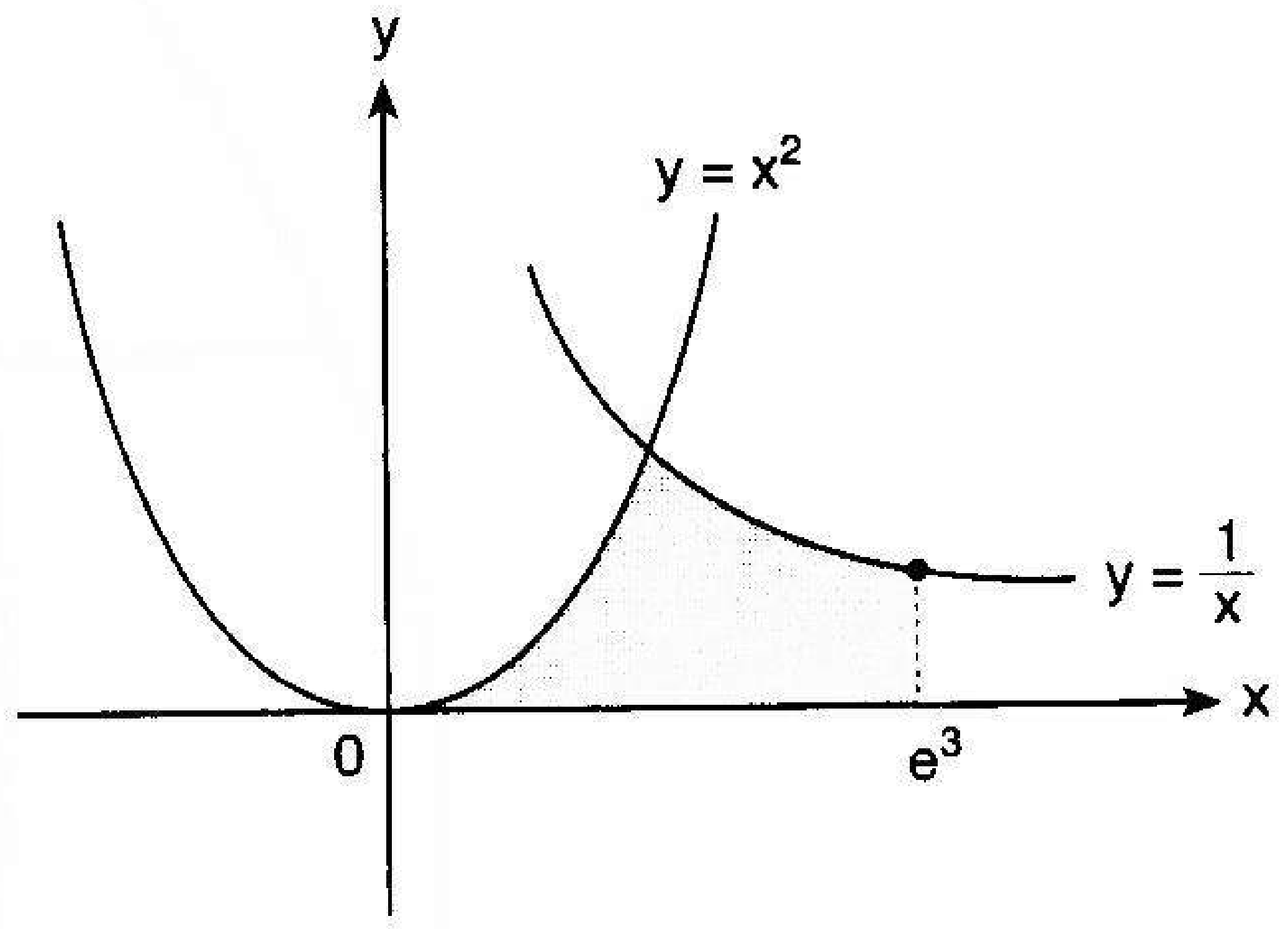
4.


y
A_1
A_2
A_3
x
–7
0
3
10
y = f(x)

$A_1 = 10\text{br}^2/\text{u}^2,\ A_2 = 2\text{br}^2/\text{u}^2,\ A_3 = 15\text{br}^2/\text{u}^2 \Rightarrow \int_{-7}^{10} f(x)\,dx = ?$

A) 27
B) 23
C) 15
D) 7
E) –3

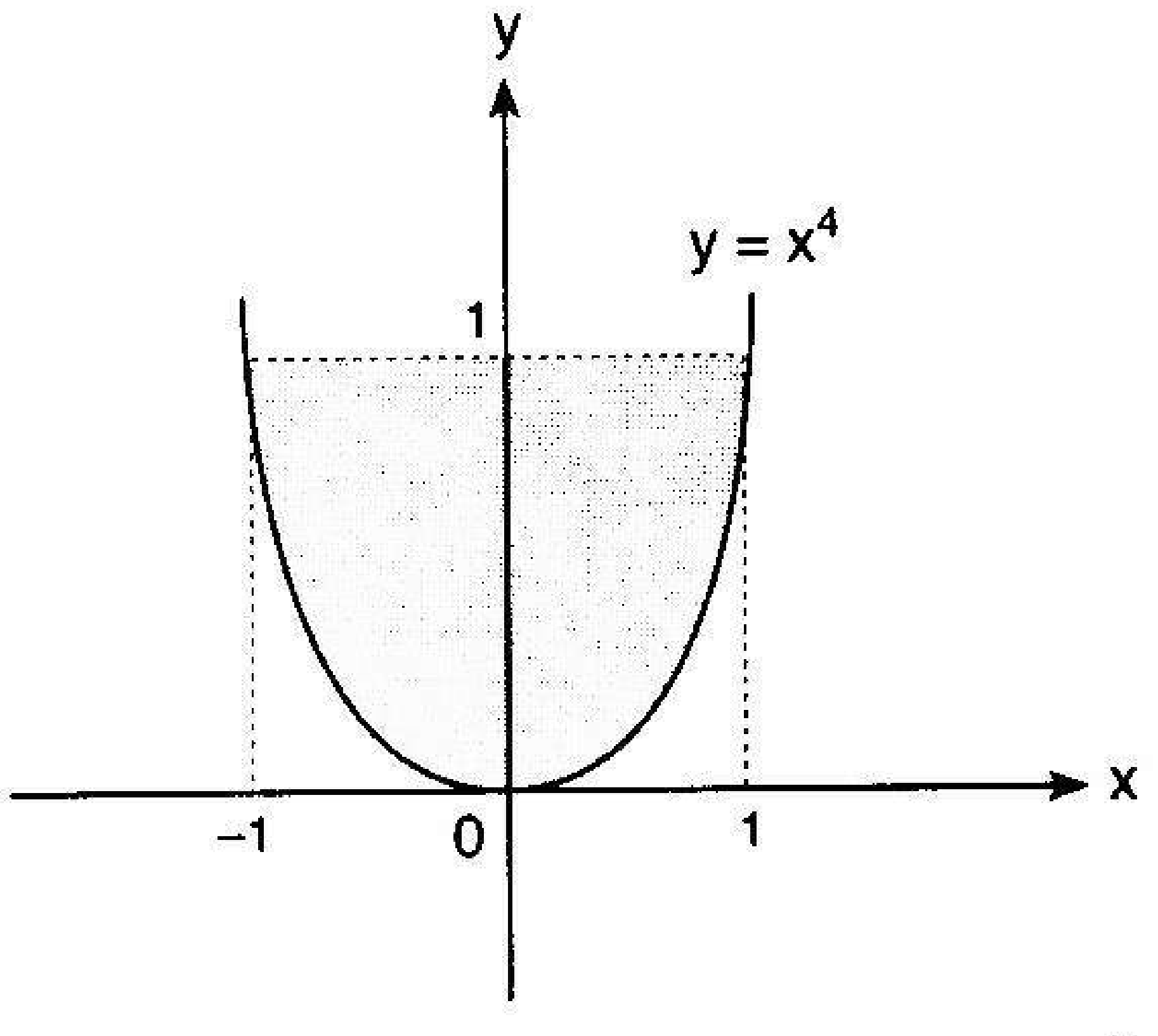
5.


y
$y = x^2$
$y = \frac{1}{x}$
x
0
e^3

Taralı alan / ***The shaded area*** **= ? br^2 / u^2**

A) $\frac{4}{3}$
B) $\frac{5}{3}$
C) $\frac{8}{3}$
D) $\frac{10}{3}$
E) $\frac{14}{3}$

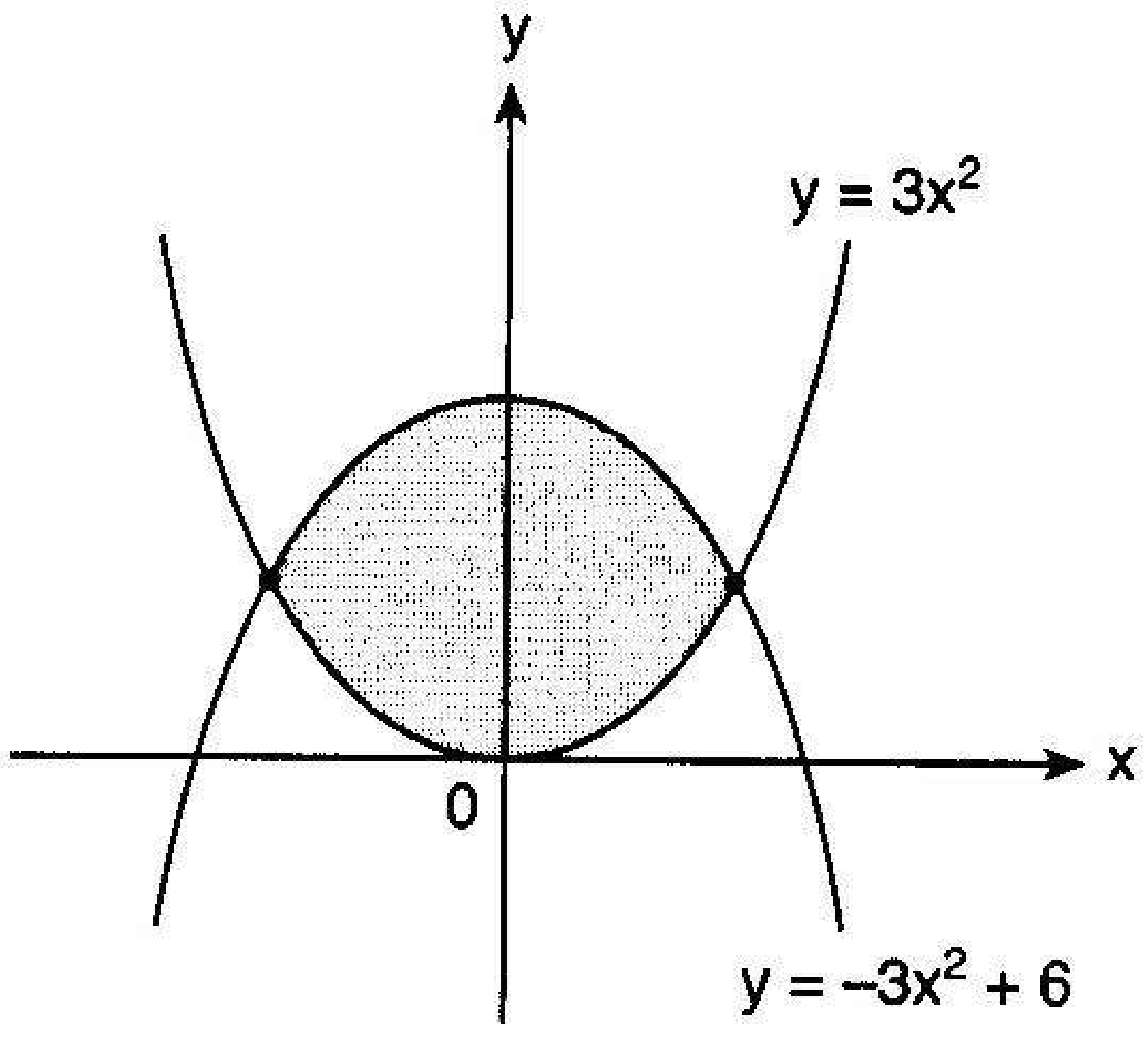
6.



Taralı alan / ***The shaded area*** **= ? br^2 / u^2**

A) $\frac{8}{5}$
B) $\frac{7}{5}$
C) $\frac{6}{5}$
D) 1
E) $\frac{4}{5}$

7.

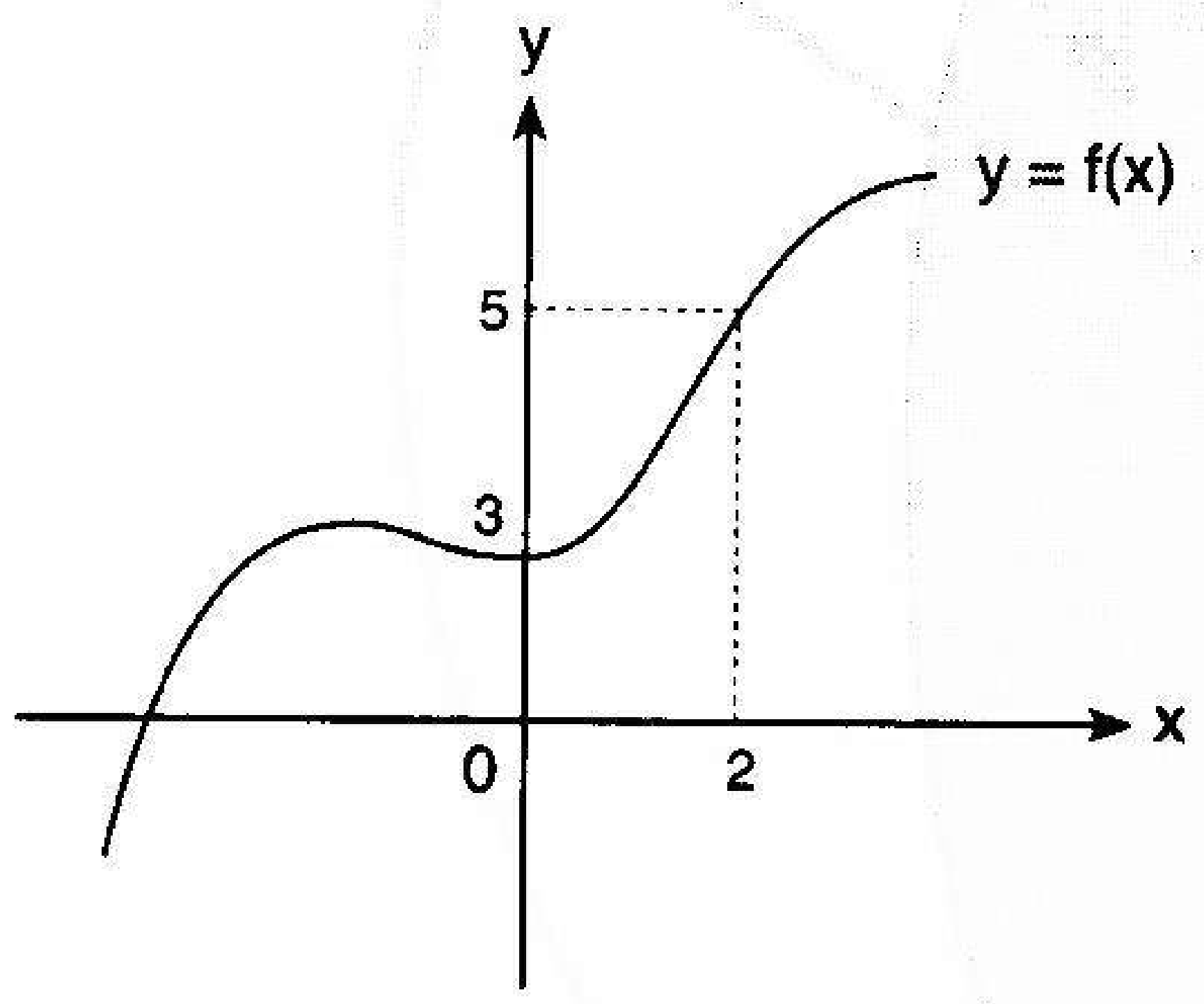

y
y = 3x^2
x
0
y = −3x^2 + 6

Taralı alan / *The shaded area* = ? br^2 / u^2

A) 2 B) 4 C) 6

D) 8 E) 10

8.

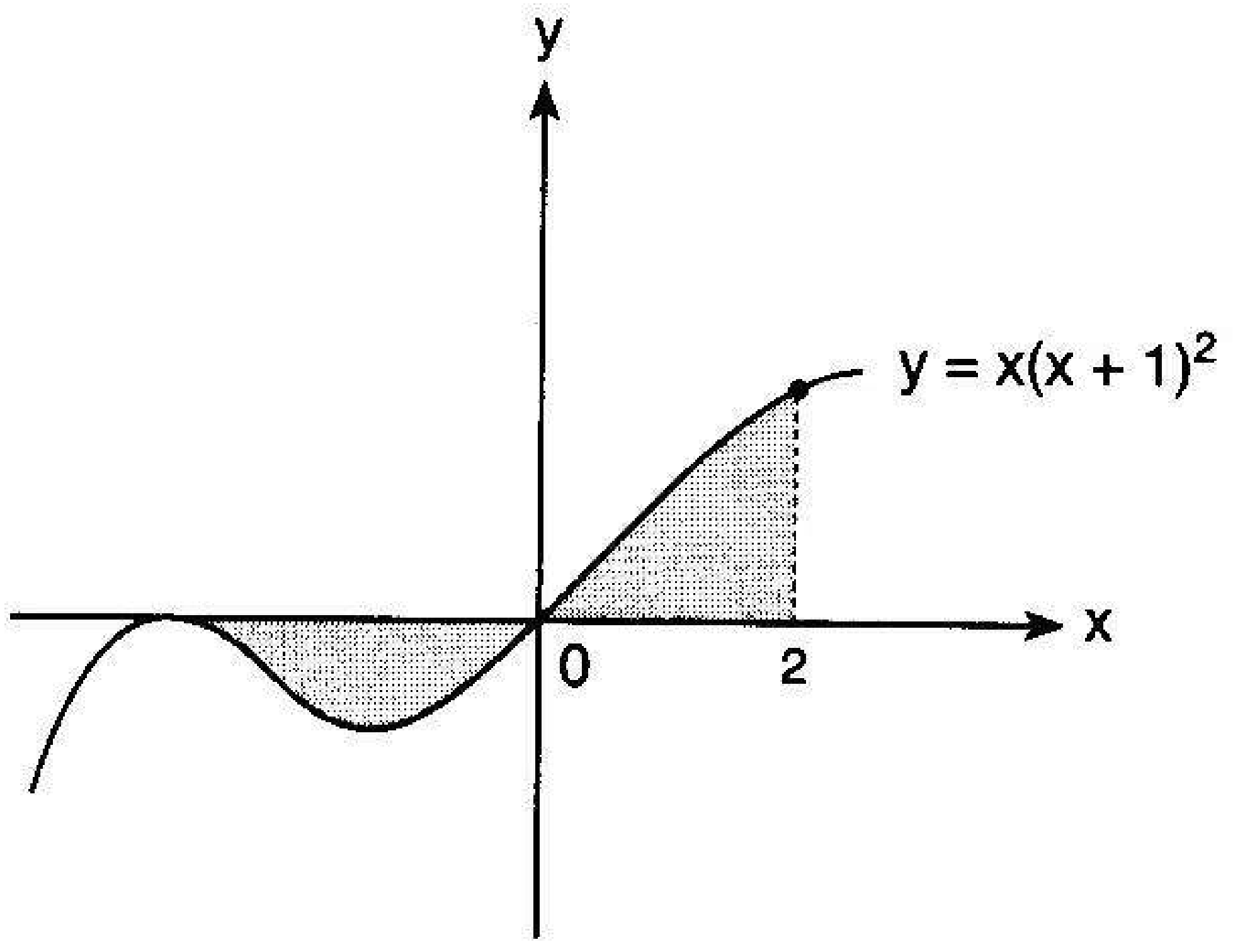


$$\Rightarrow \int_0^2 f'(x)\,dx = ?$$

A) 2 B) 4 C) 6

D) 8 E) 10

9.

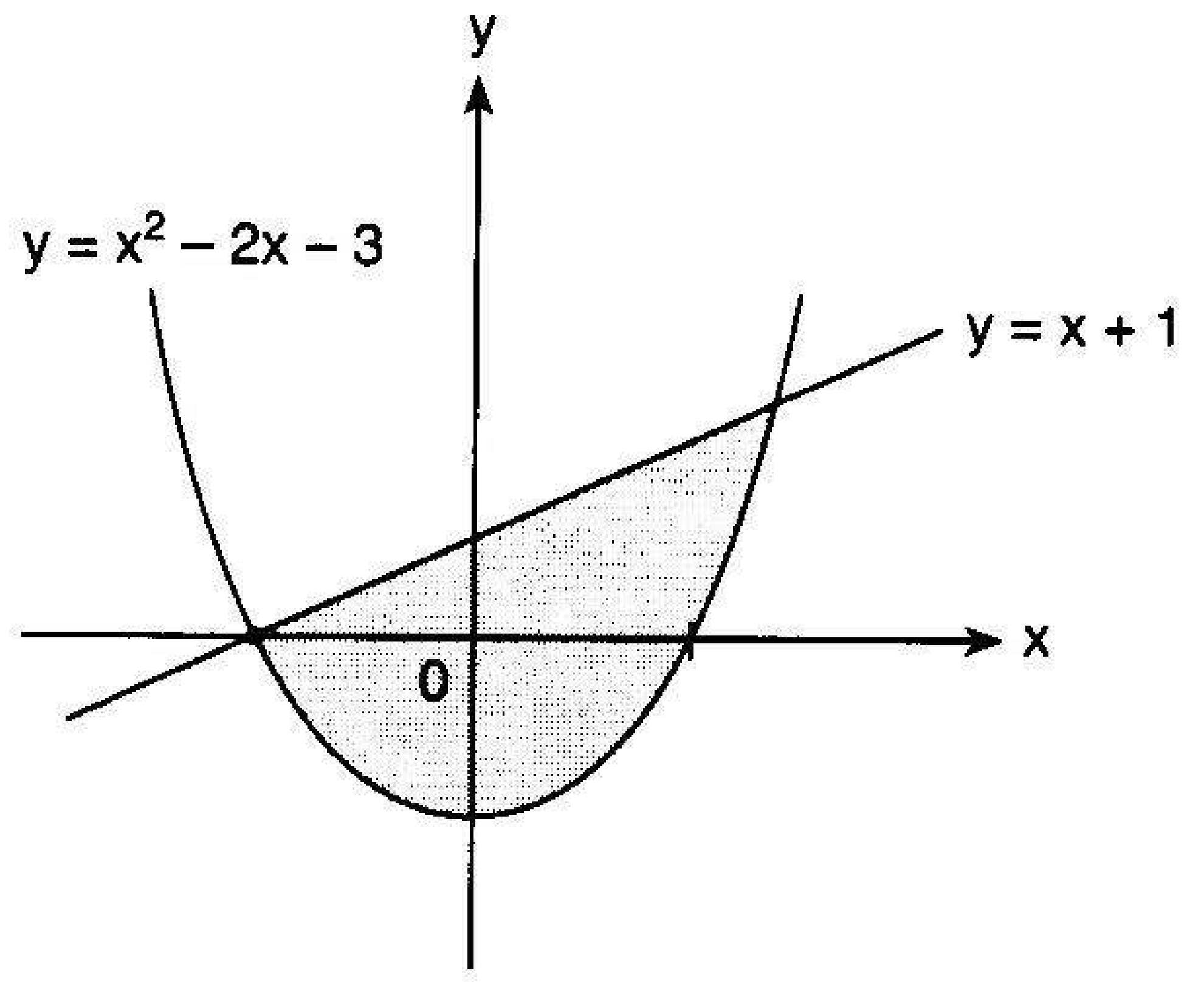


Taralı alan / *The shaded area* = ? br^2 / u^2

A) $\frac{127}{12}$ B) $\frac{135}{12}$ C) $\frac{137}{12}$

D) $\frac{141}{12}$ E) $\frac{143}{12}$

10.

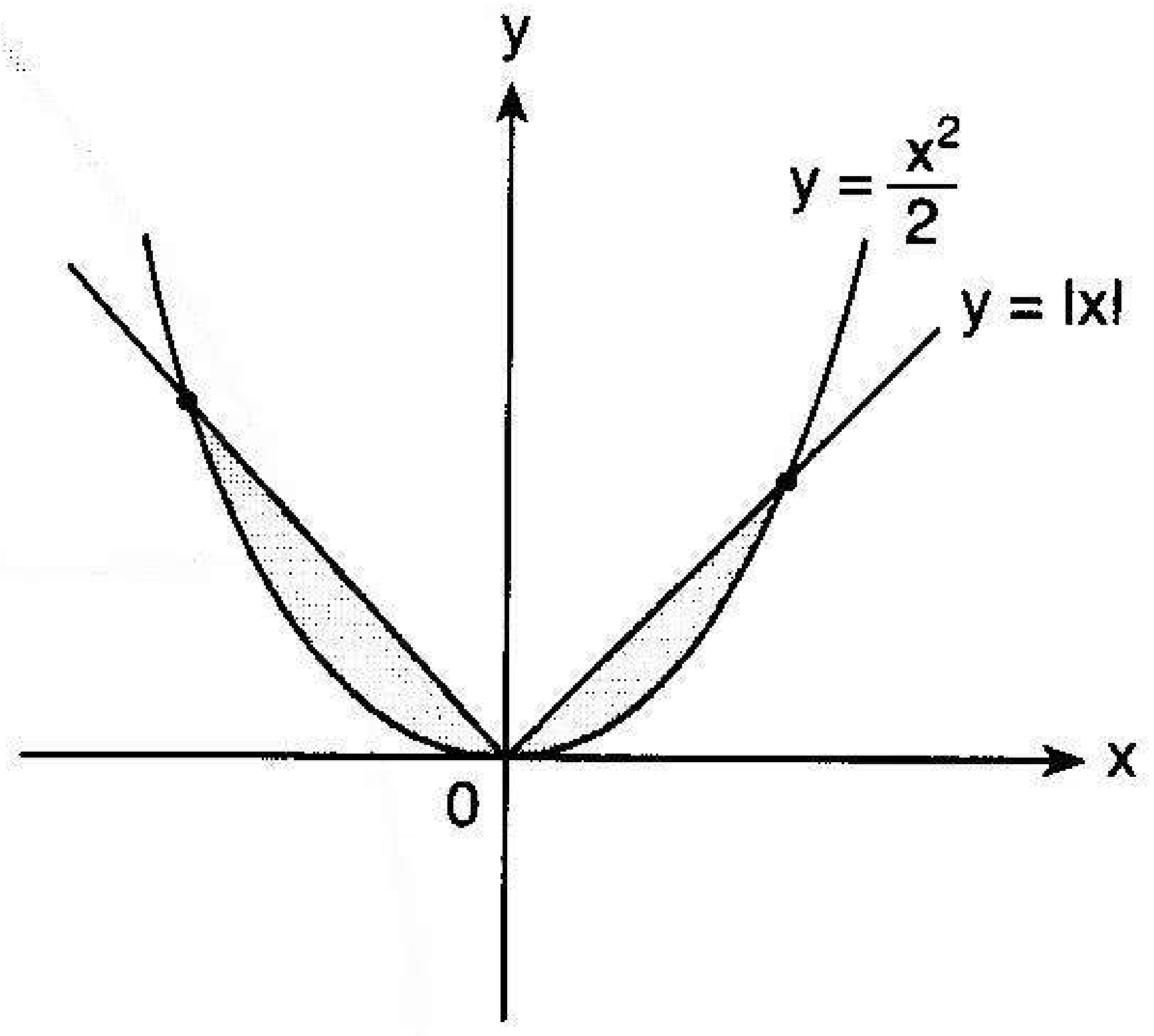

y
y = x^2 − 2x − 3
y = x + 1
x
0

Taralı alan / *The shaded area* = ? br^2 / u^2

A) $\frac{125}{6}$ B) $\frac{135}{6}$ C) $\frac{145}{6}$

D) $\frac{155}{6}$ E) $\frac{165}{6}$

11.


y
y = $\frac{x^2}{2}$
y = |x|
x
0

Taralı alan / *The shaded area* = ? br^2 / u^2

A) $\frac{4}{3}$ B) 2 C) $\frac{7}{3}$

D) $\frac{8}{3}$ E) 3

12. S = ln 27 ⇒ **a = ?**

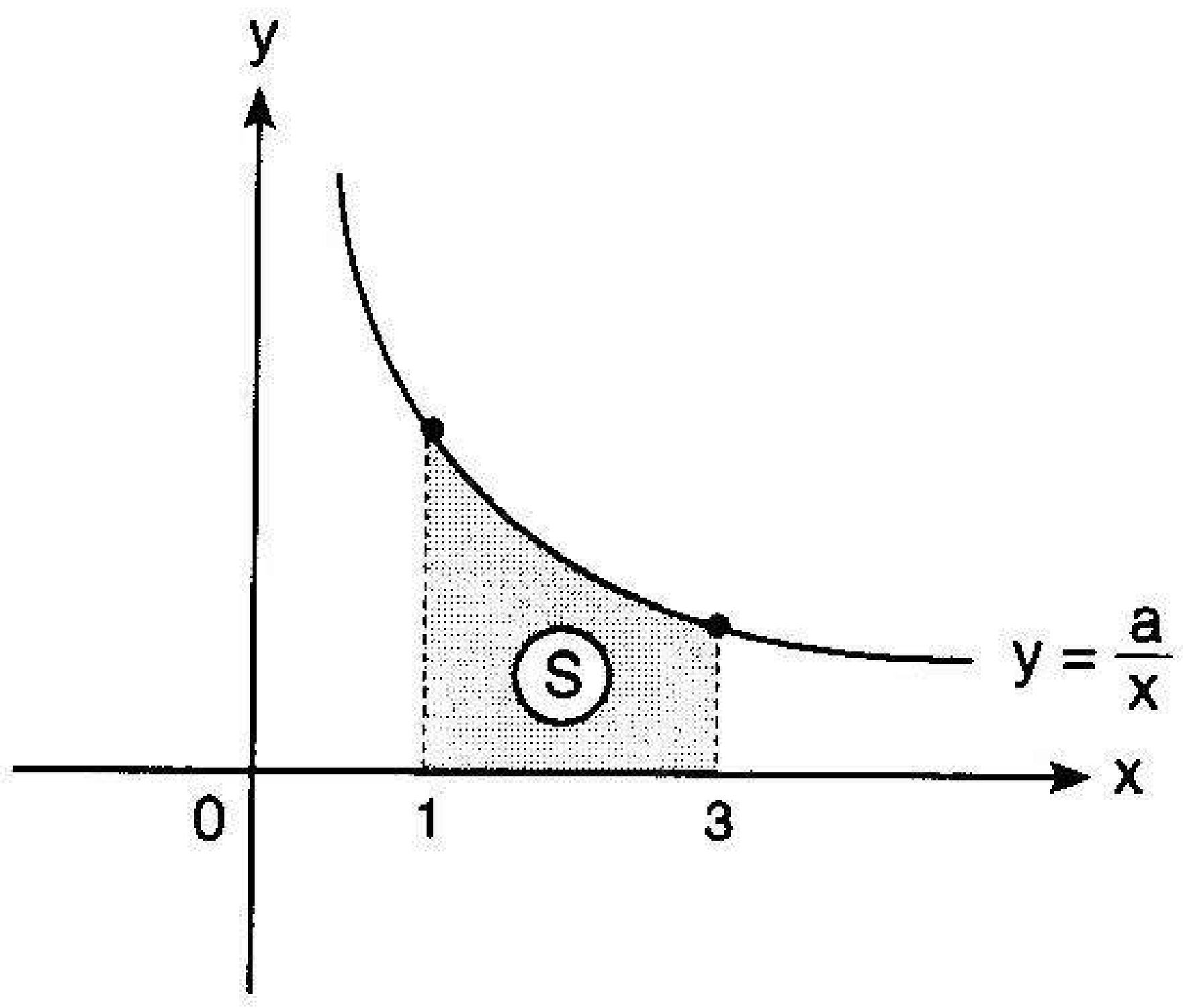
A) 1

B) 2

C) 3

D) 4

E) 5


y
S
y = $\frac{a}{x}$
x
0
1
3

YANITLAR / ANSWERS

1. D	**2. D**	**3. E**	**4. B**	**5. D**	**6. A**	**7. D**	**8. A**
9. C	**10. A**	**11. A**	**12. C**				

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